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Decay constants of charm and beauty pseudoscalar heavy-light mesons on fine lattices

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Abstract

We compute decay constants of heavy-light mesons in quenched lattice QCD with a lattice spacing of $a \simeq 0.04$ fm using non-perturbatively $O(a)$ improved Wilson fermions and $O(a)$ improved currents. We obtain $f_{D_s} = 220(6)(5)(11)$ MeV, $f_D = 206(6)(3)(22)$ MeV, $f_{B_s} = 205(7)(26)(17)$ MeV and $f_B = 190(8)(23)(25)$ MeV, using the Sommer parameter $r_0 = 0.5$ fm to set the scale. The first error is statistical, the second systematic and the third from assuming a $\pm 10\%$ uncertainty in the experimental value of r_0. A detailed discussion is given in the text. We also present results for the meson decay constants f_K and f_π and the ρ meson mass.

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1. Weak decays of heavy-light mesons with c and b quarks are interesting for studies of CP violation and determination of the CKM mixing angles. New experimental data on such decays are emerging (e.g. [1–4]) and their interpretation requires knowledge of hadronic matrix elements governed by the strong interaction. Lattice QCD allows one to calculate the strong matrix elements from first principles. However, if the heavy quark mass m_Q is of the order of the inverse lattice spacing a, considerable discretization effects proportional to powers of am_Q occur.

One possibility for coping with this problem is to use an effective theory such as Heavy Quark Effective Theory (HQET) [5] or Nonrelativistic QCD (NRQCD) [6]. These formalisms start from an infinitely heavy quark and consider corrections to this limit in the form of an expansion in the inverse of m_Q. However, to study the charm quark in HQET or NRQCD requires a considerable number of correction terms, and one still has to worry about the uncertainty from the truncation of the $1/m_Q$ expansion. A formulation for relativistic quarks where masses can be of $O(1)$ in lattice units is the Fermilab approach [7] and modifications thereof, developed in [8, 9] and [10]. One expects that the dominating discretization effects are then proportional to powers of momenta of $O(\Lambda_{QCD})$. While within HQET non-perturbative renormalization is possible [11], in many of the calculations using effective theories the renormalization constants are calculated only in perturbation theory (e.g. in Ref. [12]), leading to further uncertainties.

Another possibility is to simulate on very fine lattices, and this is the approach we have adopted in the present paper. We have performed a quenched lattice study of heavy mesons with a lattice spacing a of about 0.04 fm. On such a fine lattice a relativistic treatment of the charm quark should be justified and we expect that discretization errors are small compared to previous calculations on coarser lattices. We also make an attempt to study B mesons in our relativistic framework. Even on our fine lattice we cannot simulate B mesons directly, but the required extrapolation becomes relatively short-range. We expect that the resulting uncertainty is not much larger than the systematic error caused by the use of an effective theory. For example, recent unquenched calculations of f_B and f_{B_s} [12,13] employ NRQCD for the b quark and quote a $\sim 10\%$ error based on perturbation theory and other systematic effects.

In this article we present results for the leptonic decay constants of the D_s, B_s, D and B mesons. We also evaluate light meson masses and decay constants to compare with previous quenched calculations of the light spectrum on coarser lattices and in order to be able to disentangle discretization and quenching effects.

2. Our results are based on the analysis of 114 quenched Wilson gauge configurations simulated at the coupling parameter $\beta = 6.6$ with a mixed heatbath and microcanonical overrelaxation algorithm using the publicly available MILC code [14]. The lattice volume is $40^3 \times 80$, i.e. our lattice extends over 40 points (~ 1.59 fm) in space and 80 points in time. The lattice spacing is determined using the Sommer parameter $r_0 = 0.5$ fm. This choice is motivated by a previous calculation [15] which used r_0 to determine the lattice spacings and found that the results for f_{D_s} from a quenched lattice and a lattice with $N_f = 2$ agreed ($a \approx 0.1$ fm in these calculations). From the interpolating formula given in [16], one finds for our lattice $a^{-1} = 4.97$ GeV.

For the quarks we use the $O(a)$ improved clover formulation [17], with the nonperturbative value of the clover coefficient $c_{SW} = 1.467$ determined in Ref. [18]. We work with seven quark masses corresponding to three "light" hopping parameters $\kappa = 0.13519, 0.13498, 0.13472$ and four "heavy" hopping parameters, $\kappa = 0.13000, 0.12900, 0.12100, 0.11500$. Statistical errors are estimated by means of a bootstrap procedure using 500 bootstrap samples. For the central values we take the median. The error bars are calculated including 34% of the sample values below and above the median, respectively. Since the upper and lower error bars are found to be quite symmetric for most of our data, we just quote the larger of the two. The autocorrelation times for the pseudoscalar meson propagator appear to be small. In the worst case we studied, the autocorrelations decay after a distance of one configuration.

To extract the decay constants we follow the procedure described in Ref. [19]. For light and for heavy-light mesons we calculate the correlation functions

$$\begin{aligned} C^{SL}_{PA4}(t) &= V \sum_{\vec{x}} \langle A_4(\vec{x},t) P^{S\dagger}(0)\rangle , \\ C^{Si}_{PP}(t) &= V \sum_{\vec{x}} \langle P^i(\vec{x},t) P^{S\dagger}(0)\rangle , \end{aligned} \tag{1}$$

where A_4 is the local axial vector current operator, P the pseudoscalar density which can be local ($i = L$) or Jacobi smeared ($i = S$), and V is the spatial lattice volume.

κ_1	κ_2	am_{PS}	am_V	$af^{(0)}$	$af^{(1)}$	af
0.13519	0.13519	0.1059(13)	0.1928(62)	0.0376(11)	0.0248(13)	0.0312(09)
0.13498	0.13519	0.1231(12)	0.2005(48)	0.0388(11)	0.0251(12)	0.0324(09)
0.13498	0.13498	0.1388(10)	0.2107(43)	0.0403(10)	0.0258(10)	0.0337(08)
0.13472	0.13519	0.1422(11)	0.2065(39)	0.0405(11)	0.0261(11)	0.0339(09)
0.13472	0.13498	0.1560(10)	0.2186(33)	0.0418(10)	0.0270(10)	0.0351(08)
0.13472	0.13472	0.1722(09)	0.2292(27)	0.0434(10)	0.0283(09)	0.0366(08)

Table 1: Light meson masses and decay constants in lattice units.

Masses and amplitudes are determined from fits of the correlation functions with

$$C^{Si}_{PP}(t) = A^{Si}_{PP} \left(e^{-Et} + e^{-E(T-t)}\right) , \tag{2}$$

$$C^{SL}_{PA4}(t) = A^{SL}_{PA4} \left(e^{-Et} - e^{-E(T-t)}\right) , \tag{3}$$

where E is the ground state energy. In Table 1 we give the raw data for the light pseudoscalar meson masses, determined from C^{SL}_{PP}, and for light vector meson masses from smeared-local correlation functions of the spatial components of the vector currents.

To determine the bare quark masses, we calculate κ_{crit}, the κ value corresponding to massless quarks, from a fit of the squared mass of a pseudoscalar meson ("pion") consisting of quarks with mass parameters κ_1 and κ_2 as a function of the averaged $O(a)$ improved quark mass

$$(am_{PS})^2 = a_1\, a\tilde{m}_q , \tag{4}$$

with

$$\tilde{m}_q = (1 + b_m\, am_q)\, m_q \,, \; m_q = \frac{1}{2}(m_{q1} + m_{q2})$$

and $am_{qi} = \frac{1}{2}(\frac{1}{\kappa_i} - \frac{1}{\kappa_{crit}})$, $i = 1, 2$. We use the non-perturbative value of -0.6636 for the improvement parameter b_m using an interpolating formula from Ref. [20]. The fit includes all data with $\kappa_{1,2} \geq 0.13472$, where we find the improved quark masses to just lie on a straight line. We find $\kappa_{crit} = 0.135472(11)$. The hopping parameter corresponding to the average u and d quark mass, κ_ℓ, is determined by setting m_{PS} on the left hand side of Eq. (4) equal to the physical pion mass, $m_{PS} = 138$ MeV. We find $\kappa_\ell = 0.135456(10)$.

We parameterize the quark mass dependence of light meson decay matrix elements with hopping parameters κ_1 and κ_2 by fitting them to a function of the form

$$c_0 + c_1\, a\tilde{m}_q. \tag{5}$$

The light quark mass dependence of masses and decay matrix elements of heavy-light mesons is parameterized using a linear fit as in Eq. (5), with $\tilde{m}_q$ being the light quark mass instead of the average quark mass.

We also calculate the vector ("ρ meson") mass. The fit and the chiral extrapolation assuming a quark mass dependence as in Eq. (5) are shown in Fig. 1. At κ_ℓ we find 846(37) MeV (the error is statistical), which is roughly a 10% (2σ) discrepancy with experiment. We compare our result to other recent quenched calculations in Table 2. Within errors our result agrees with Ref. [21], where a continuum extrapolation from coarser lattices with $O(a)$ improved clover fermions is performed. We also list studies employing chiral lattice fermions where smaller quark masses can be reached while coarser lattices are used [22–24]. Ref. [23] quotes results from two lattice spacings. In Table 2 we present the results from their finer lattice. To determine the strange quark mass parameter κ_s, we interpolate the vector meson mass to the physical ϕ meson mass, $M_\phi = 1.01946(19)$ GeV. We find $\kappa_s = 0.13502(6)$. This is our "method I" for determining the κ value corresponding to the strange quark mass. Using Eq. (4) and setting m_K^2 to the experimental value for $(m_{K_+}^2 + m_{K_0}^2)/2$ gives a value in very close agreement: $\kappa_s = 0.134981(9)$.

The raw data for the heavy-light meson masses are given in Table 3. To find the physical values of the heavy-light meson masses, we extrapolate for each heavy quark mass linearly in m_q, see Eq. (5). The fits are shown in Fig. 1. The quark mass dependence is linear to very good accuracy. This is in contrast to the findings of, e.g., Ref. [25]. In the final step, the calculation of the decay constants, the physical values of the c and b quark masses will be reached by interpolating or extrapolating the heavy-light meson mass to the D or B mass and the heavy-strange meson mass to the D_s or B_s mass.

In a quenched calculation, different methods to choose the input for determining physical parameters may give different answers. In order to investigate the influence of this arbitrariness we also use the heavy-light spectrum to determine κ_s and call this procedure "method II". We consider the splitting between mesons with a heavy quark and a strange quark (generically denoted by M_s) and a meson with a heavy quark and a quark with the u, d quark mass (denoted by M_ℓ). In our data, as well as in experiment, the $M_s - M_\ell$ mass difference is fairly independent of the heavy quark mass. To fix κ_s in method II, we choose a heavy quark close to the charm mass from our simulation points, namely $\kappa = 0.129$, and set the splitting between the M_s and the M_ℓ masses equal to the experimental value for the D meson, $m_{D_s} - m_D = 98.85(30)$ MeV. The corresponding value for the strange hopping parameter is $\kappa_s = 0.134929(15)$.

We calculate the pseudoscalar decay constants from the improved axial vector current A_μ^I

$$A_4^I = Z_A(1 + ab_A m_q)\,(A_4 + c_A a\partial_4 P)\,, \tag{6}$$

where $A_\mu(x) = \overline{q}_{1x}\gamma_\mu\gamma_5 q_{2x}$ and $P(x) = \overline{q}_{1x}\gamma_5 q_{2x}$. We take the nonperturbatively determined values for Z_A from [26] and for c_A from [18]. For our calculation, this gives $Z_A = 0.8338$ and

Ref.	a^{-1}[GeV]	quark action	gauge action	m_ρ[GeV]
this work	4.97	clover	Wilson	0.849(38)
[21]	cont	clover	Wilson	0.797(13)
[22]	1.33	chirally improved	Lüscher-Weisz	0.791(42)
[22]	1.29	fixed point	fixed point	0.828(25)
[23]	2.09	overlap	Lüscher-Weisz	0.79(2)
[24]	1.60	overlap hypercube	Wilson	1.017(40)

Table 2: ρ meson masses from quenched lattice calculations. The lattice scale has been determined using $r_0 = 0.5$ fm in all calculations except in [23] where $r_0 = 0.56$ fm is used. The quoted errors are only statistical.

$c_A = -0.01967$. The coefficient b_A is calculated from 1-loop perturbation theory [27]. Using a boosted coupling $g_0^2 \to g_0^2/u_0^4$ with $u_0 = \langle \frac{1}{3} Tr U_P \rangle^{1/4}$, we find $b_A = 1.2143$ which is close to the result one finds using the tadpole-improved scheme of [28]. A non-perturbative determination of b_A on coarser lattices ($\beta \leq 6.4$) [29] also gives values in agreement with boosted perturbation theory within errors.

The meson matrix elements of the currents

$$\begin{aligned} f^{(0)} &= \frac{1}{M}\langle 0|A_4|M\rangle, \\ f^{(1)} &= \frac{1}{M}\langle 0|a\partial_4 P|M\rangle = -\frac{1}{M}\sinh(aM)\langle 0|P|M\rangle, \\ f &= \frac{1}{M}\langle 0|A_4^I|M\rangle, \end{aligned} \tag{7}$$

are related to the amplitudes by

$$f^{(0)} = -2\sqrt{\kappa_1\kappa_2}\frac{\sqrt{2}A_{PA4}^{SL}}{\sqrt{MVA_{PP}^{SS}}}, \tag{8}$$

$$f^{(1)} = 2\sqrt{\kappa_1\kappa_2}\sinh(aM)\frac{\sqrt{2}A_{PP}^{SL}}{\sqrt{MVA_{PP}^{SS}}}, \tag{9}$$

where M denotes the meson mass. The convention for the factors of $\sqrt{2}$ corresponds to the normalization where $f_\pi \simeq 130$ MeV.

3. The fit of the light meson decay constants according to Eq. (5) is shown in Fig. 2. Mesons with degenerate and nondegenerate quark masses fall on the same straight line. For f_π, the value at the physical u, d quark mass, and f_K, the value extrapolated to the averaged strange and u, d quark mass, we find

$$f_\pi = 140(4) \text{ MeV}, \quad f_K = 153(4) \text{ MeV}. \tag{10}$$

This result for f_π agrees well with the value of 137(2) MeV determined by [21] using an extrapolation to the continuum from coarser lattices. Both values are slightly larger than the experimental value of $f_{\pi^+} = 130.7(4)$ MeV. Our value for f_K is 6% or 2σ lower than the result from [21] of 163(1) MeV. The experimental value is $f_{K^+} = 159.8(15)$ MeV.

κ_1	κ_2	am_{PS}	af^0	af^1	af
0.11500	0.13519	0.8363(15)	0.0371(11)	0.0532(17)	0.0423(13)
0.12100	0.13519	0.6676(13)	0.0417(14)	0.0496(17)	0.0432(14)
0.12900	0.13519	0.4065(11)	0.0475(13)	0.0417(13)	0.0435(12)
0.13000	0.13519	0.3685(12)	0.0478(13)	0.0399(13)	0.0431(12)
0.11500	0.13498	0.8431(12)	0.0383(12)	0.0551(19)	0.0437(13)
0.12100	0.13498	0.6747(11)	0.0429(12)	0.0517(16)	0.0446(12)
0.12900	0.13498	0.4145(10)	0.0488(15)	0.0431(14)	0.0448(14)
0.13000	0.13498	0.3765(09)	0.0490(13)	0.0412(12)	0.0443(12)
0.11500	0.13472	0.8517(11)	0.0402(12)	0.0584(19)	0.0460(13)
0.12100	0.13472	0.6836(10)	0.0446(13)	0.0541(16)	0.0466(14)
0.12900	0.13472	0.4242(08)	0.0508(13)	0.0453(13)	0.0469(12)
0.13000	0.13472	0.3866(08)	0.0507(13)	0.0430(12)	0.0460(12)

Table 3: Pseudoscalar heavy-light meson masses and decay constants at the simulation points.

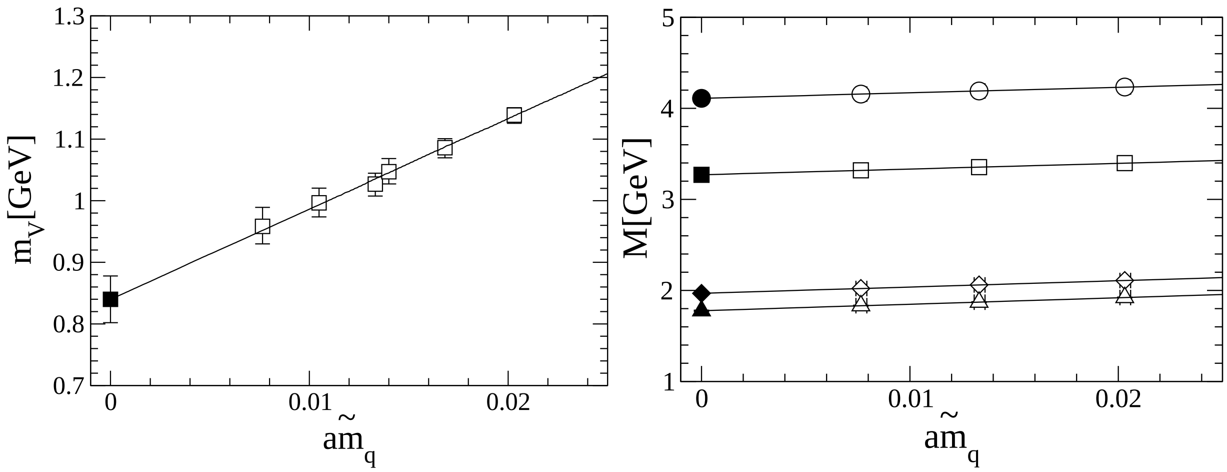

Figure 1: Chiral extrapolation of meson masses. On the left, light vector meson masses, on the right, heavy-light pseudoscalar meson masses for the heavy hopping parameters $\kappa = 0.115$ (circles), $\kappa = 0.121$ (squares), $\kappa = 0.129$ (diamonds) and $\kappa = 0.130$ (triangles). Open symbols denote the simulation points, closed symbols the chiral extrapolation.

The SU(3) flavor breaking ratio of the light decay constants in our calculation turns out to be relatively small. We find

$$f_K/f_\pi - 1 = 0.088(12)\,. \tag{11}$$

Our number is substantially lower than the experimental value of 0.222, but is consistent with a recent quenched calculation using overlap fermions [30], which finds $f_K/f_\pi - 1 = 0.09(4)$ using the same scale setting with $r_0 = 0.5$ fm. It is also consistent with other quenched determinations (see [31]).

4. Next we consider the heavy-light decay constants. To determine values at the physical quark masses, we extrapolate or interpolate the decay constants separately in the light and the

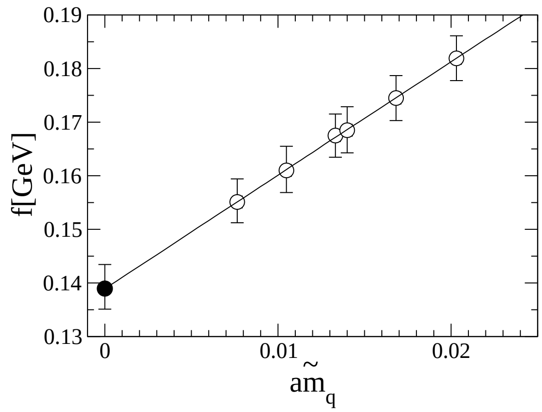

Figure 2: Chiral fit of light meson decay constants. The chirally extrapolated value is denoted by the filled circle.

heavy quark mass. For the fits in the light quark mass we use a function of the form (5) with m_q being the mass of the quark with the light κ parameter of the heavy-light meson instead of the average quark mass. For the extrapolation to the b quark mass and also for an interpolation

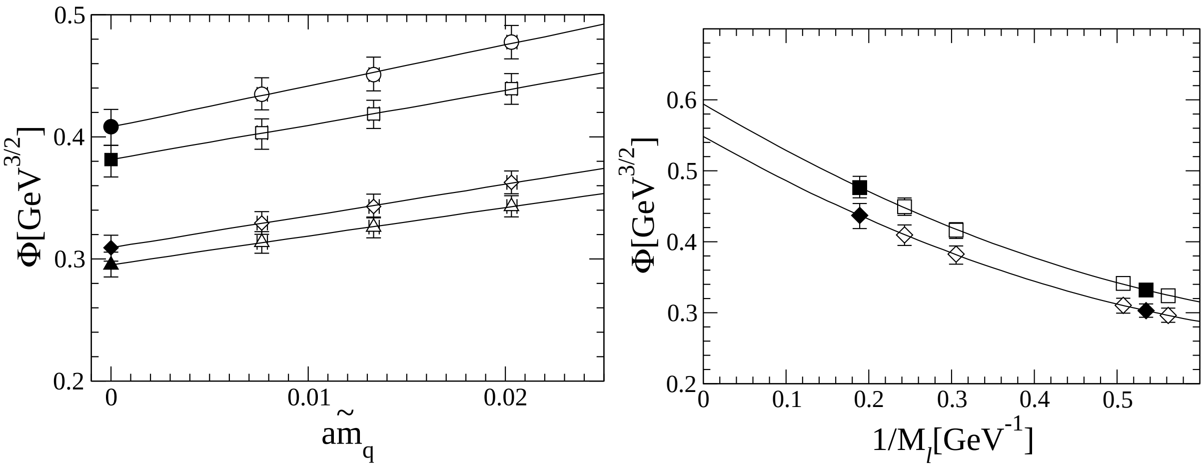

Figure 3: On the left, chiral extrapolation of heavy-light decay matrix elements. Symbols have the same meaning as in the right part of Fig. 1. On the right, heavy quark mass dependence of heavy-light decay matrix elements. Squares denote strange, and diamonds denote physical light quarks. Closed symbols denote heavy quark masses extrapolated to the b or interpolated to the c quark mass.

to the c quark mass we use a formula motivated by HQET (see e.g. [32]). In the heavy quark limit, matching of the decay matrix element in the effective theory to the matrix element in full QCD introduces logarithmic corrections in the heavy quark mass which have to be resummed. In addition, power corrections in $1/m_Q$ have to be added. Since the extrapolation to the b mass in our case is rather short, the precise form of the extrapolation formula is not important. We use only the lowest order running for α_s, and take the heavy-light meson mass M_ℓ (and not the quark mass m_Q) as an expansion (scale) parameter:

$$\Phi \equiv \left(\frac{\alpha_s(M_B)}{\alpha_s(M_\ell)}\right)^{\gamma_0/(2b_0)} \times f\sqrt{M_\ell} = c_0\left(1+\frac{c_1}{M_\ell}+\frac{c_2}{M_\ell^2}\right). \tag{12}$$

Here $\gamma_0 = -4$ is the leading order anomalous dimension of the axial vector current, and $b_0 = 11$ is the leading coefficient of the QCD β function for zero dynamical flavors. The fits and the interpolated values are shown in Fig. 3. The values of the fit parameters are $c_0 = 0.55(4)$ GeV$^{3/2}$, $c_1 = -0.66(19)$ GeV and $c_2 = 0.38(21)$ GeV^2 if the light quark mass is the u, d quark mass, and $c_0 = 0.59(4)$ GeV$^{3/2}$, $c_1 = -0.70(14)$ GeV and $c_2 = 0.39(15)$ GeV^2 for the s quark.

Decay constant ratios			
f_{D_s}/f_D	f_{B_s}/f_B	f_{D_s}/f_{B_s}	f_D/f_B
1.068(18)(20)	1.080(28)(31)	1.069(28)(160)	1.082(42)(168)

Table 4: Ratios of heavy-light decay constants. The first error is statistical, and the second systematic. The systematic errors are discussed in the text.

Our final results for the ratios of heavy-light decay constants are presented in Table 4, and the heavy-light decay constants are given along with a comparison in Table 5.

Estimation of systematic errors is notoriously difficult. One source of uncertainty concerns setting the quark masses to their physical values. For the strange quark this can be estimated by comparing the results from our methods I and II and suggests an error of 4 MeV for f_{D_s} and f_{B_s}. For the u, d quarks a chiral extrapolation is required. The corresponding error is difficult to estimate. Our data are consistent with the simplest linear chiral extrapolation. Quenched chiral perturbation theory provides a more sophisticated formula. However, it is not clear if it is applicable to our data. The uncertainty in fixing the heavy quark mass can be estimated by comparing the difference between the mass fixed from quarkonium and from the heavy-light meson system. Since the η_c meson mass using the charm quark hopping parameter determined from the D_s meson agrees with the physical value, we assume that this uncertainty is rather small in our calculation. In addition, for the B system there is an uncertainty from the extrapolation in the heavy quark mass. The difference between a quadratic fit to the matrix elements $f\sqrt{M}$ and a quadratic fit to Φ is very small and changes the values for the decay constants by less than 1 MeV. If only the three lighter heavy quark masses are included in the extrapolation to the b mass, f_{B_s} (f_B) changes by -3 ($+1$) MeV.

Since we have results only from one lattice spacing, we cannot perform a continuum extrapolation from our data alone and have to estimate the discretization effects as a systematic error. Leading discretization effects are $O(a^2)$. A rough estimate of them can be obtained by squaring the $O(a)$ corrections appearing in the Symanzik improvement program. For the charm quark, the correction proportional to c_A is small, around 2%, while the term proportional to the quark mass and b_A is around 10% of the size of the matrix element itself. The square of the sum of these variations is around 1%, which we take as our estimate for the discretization error of f_D and f_{D_s}. A similar consideration for the B and B_s systems results in an estimate of a discretization error of roughly 12%. For the error in the renormalization constants we use the estimate given for Z_A in Ref. [26] of 1%. Since the heavy-light meson masses in lattice units

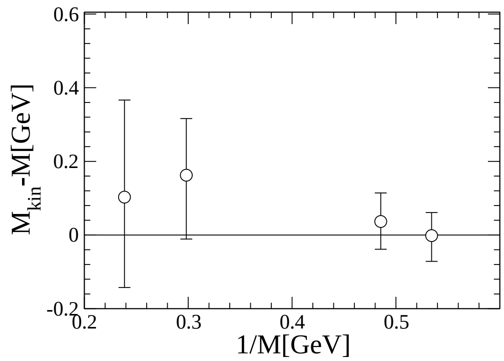

Figure 4: Difference of the kinetic mass and the rest mass for heavy-light mesons with the light hopping parameter $\kappa = 0.13498$. The line at zero is plotted to guide the eye.

in our simulation increase up to values of ~ 0.8 one might be concerned about cutoff effects in the dispersion relation for the heavy-light meson. We therefore compare the kinetic mass [7] $M_{\rm kin}$ calculated from

$$E^2 = M^2 + \frac{M}{M_{\rm kin}}\vec{p}^{\,2} + O(p^4), \qquad (13)$$

where M is the rest energy of the meson. Results for the light quark mass close to the strange quark mass are shown in Fig. 4. We find that discretization errors in the dispersion relation are smaller than the statistical errors.

The finite volume effects of the ratio f_{B_s}/f_B have been investigated in the framework of heavy meson chiral perturbation theory [33]. For quenched lattices of spatial extent 1.6 fm and pseudoscalar meson masses around 500 MeV (which corresponds to the smallest quark mass used in our simulation) they are quoted to be around 1% or smaller. It is plausible that the finite volume effects for D and D_s mesons are of similar size.

We estimate the total systematic error due to discretization effects, errors in Z_A, finite volume effects and ambiguities in fixing the physical quark masses by collecting all contributions and adding them in quadrature. It is given as the second error in Table 5.

The ratios are less sensitive to some of the systematic effects. The dominating ones for f_{D_s}/f_{B_s} and f_D/f_B are the discretization effects. For f_{D_s}/f_D and f_{B_s}/f_B we find a variation depending on how the strange quark mass is set, while the estimated discretization effects are smaller than the statistical errors. The uncertainties from fixing the physical quark masses and the discretization errors (added in quadrature) are given as the second error in Table 4.

We have used the value of the Sommer parameter $r_0 = 0.5$ fm to set the scale in physical units. This choice allows for a direct comparison with previous lattice determinations (see below) but is not universally accepted. With a different value of the Sommer parameter our results have to be modified accordingly. The variation of the decay constants if r_0 is changed by $\pm 10\%$ is given as third error bar for our results in Table 5 [1].

5. Finally, we compare our results to other lattice calculations of decay constants. There exist recent quenched results for f_{D_s} from nonperturbatively $O(a)$ improved clover fermions [34–36]

[1] We note, however, that a value of $r_0 = 0.45$ fm leads to seemingly unphysical results. In particular the $SU(3)$ breaking in the meson masses and decay constants becomes very small. Also, different methods to set the strange quark mass produce more noticeable differences in the results.

Ref.	N_f, HQ action, scale	f_{D_s}[MeV]	f_D[MeV]
		Lattice	
this work	0, clover, $r_0 = 0.5$ fm	220(6)(5)(11)	206(6)(3)(22)
[34]	0, clover, $r_0 = 0.5$ fm	$243(2)(^{03}_{24})$	$222(3)(^{04}_{33})$
[35]	0, clover, $r_0 = 0.5$ fm	252(9)	
[36]	0, clover, $r_0 = 0.5$ fm	225(6)	
[9]	0, mod. Fermilab, $r_0 = 0.5$ fm	237(5)	
[25]	0, overlap, f_π	266(10)(18)	235(8)(14)
[42]	$2+1$, Fermilab, Υ spectrum	249(3)(16)	201(3)(17)
		Experiment	
[3]		280(12)(6)	
[4]		283(17)(16)	
[2]			223(17)(3)

Ref.	N_f, HQ action, scale	f_{B_s}[MeV]	f_B[MeV]
		Lattice	
this work	0, clover, $r_0 = 0.5$ fm	205(7)(26)(17)	190(8)(23)(25)
[34]	0, clover, $r_0 = 0.5$ fm	$240(4)(^{12}_{42})$	$217(5)(^{13}_{40})$
[38]	0, clover+static, $r_0 = 0.5$ fm	205(12)	
[39]	0, clover+static, $r_0 = 0.5$ fm	191(6)	
[12]	$2+1$, NRQCD, Υ spectrum	260(7)(28)	
[13]	$2+1$, NRQCD, Υ spectrum		216(9)(20)
		Experiment	
[1]	experiment		$229(^{36}_{31})(^{34}_{37})$
[41]	UTfit	227(9)	

Table 5: Heavy-light decay constants from lattice calculations and experiment for the D (upper table) and for the B system (lower table). For the lattice calculations, the number of flavors in the simulation (N_f), the heavy quark (HQ) action, and the quantity used to set the scale are also indicated. The first error bar is the statistical, the second (where given) the systematic error except for the uncertainty in r_0. For our work we quote a third error assuming a $\pm 10\%$ uncertainty in the physical value of r_0. For the result from [36] we quote the value from the finest lattice instead of the continuum extrapolated result.

for a range of lattice spacings ($0.03 \leq a \leq 0.1$ fm) as well as for overlap quarks [25]. The comparison with the clover results is particularly interesting because it sheds some light on the discretization effects and might indicate the possibility of a joint continuum extrapolation. We plot the clover data in Fig. 5 as a function of the squared lattice spacing together with the overlap data. First we notice that on coarser lattices there is a discrepancy between the clover data of Refs. [34] and [35]. The discrepancy corresponds roughly to the difference one obtains when c_A values from different nonperturbative calculations for a meson mass > 2.4 GeV are used, as discussed in [34]. Furthermore, the work [35] uses a nonperturbatively determined value for b_A [37]. On the finer lattice of Ref. [34] ($\beta = 6.2$) the value used in Ref. [35] is about 6% larger than the perturbative number, which according to our estimates would affect the decay constants by at most 2%. At $\beta = 6.0$ the difference is even smaller. On the finest lattice used by [35] ($\beta = 6.45$) the difference between the perturbative and nonperturbative values of

b_A is $\sim 7\%$, which translates on a fine lattice into only a very small difference in the decay constants. In a more recent calculation [29] the nonperturbative value at that β value has come into agreement with perturbation theory, as mentioned in Section 2.

Our data is in good agreement with the value obtained by Jüttner on his finest lattice [36]. The overlap value from [25] is on the other hand substantially larger. Being determined on a relatively coarse lattice it might be affected by discretization errors. It is important that all data shown in Fig. 5 come from lattices with similar spatial extent between 1.5 and 1.6 fm. So, finite size effects can be expected to be roughly the same in all calculations.

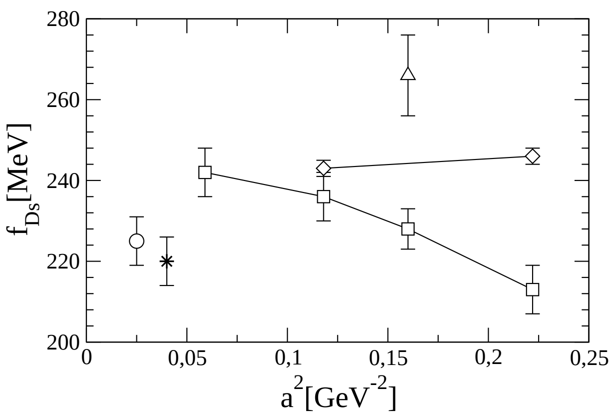

Figure 5: Lattice spacing dependence of quenched f_{D_s} from $O(a)$ improved clover quarks (this work, star), (UKQCD [34], diamonds), (ALPHA [35], squares), (Jüttner [36], circle), and overlap quarks (Ref. [25], triangle). The error bars show statistical and fitting uncertainties only. The scale is set using $r_0 = 0.5$ fm with the exception of [25] where f_π is employed for the conversion of the decay constant to physical units. If $r_0 = 0.5$ fm is used instead, their lattice spacing decreases by 12%, which would increase their result for the decay constant even further.

Our result for f_{B_s} is consistent with the quenched calculations of [38, 39], but considerably lower than the nonrelativistic (but unquenched) calculation of [12]. The fit to the standard model gives a value with a relatively small error in between these two numbers.

For f_D and f_B, the values obtained from lattice calculations are consistent with the experimental results. Since the experimental errors are still large, this comparison is not conclusive, however.

6. Let us summarize our main findings. We have calculated decay constants of heavy-light pseudoscalar mesons on a very fine quenched lattice using clover fermions. Our extrapolations to the b quark mass appear reasonable. Nevertheless, from a comparison of the results at the charm mass to data obtained on coarser lattices we obtain the impression that discretization errors with the relativistic formalism adopted here are still significant for the b sector, unless the inverse lattice spacing becomes larger than ~ 10 GeV.

Our results and those of Ref. [36] for f_{D_s} are $10-15\%$ smaller than the central values quoted for other recent lattice calculations, and roughly 20% smaller than recent experimental values. Eventually one would like to determine the decay constants to an accuracy of a few percent. Our work and the result of Ref. [36] indicate that discretization errors for the clover results on lattices with $a^{-1} \leq 2-3$ GeV are too large to reach this precision, and that even a continuum extrapolation from a set of coarser lattices has a large uncertainty for heavy quarks. On the other hand, we do not find any source of large systematic errors, other than quenching, that could affect our calculation. It seems, therefore, that the new lattice results on fine lattices (this work and [36]) indicate a relatively small value for f_{D_s} from lattice QCD. Quenching effects are notoriously difficult to estimate. However, since in previous calculations with $a^{-1} \approx 2$ GeV [15] it was found that the quenching error is insignificant with our choice of lattice parameters, we expect that they will not be too large.

The systematic uncertainties on our results are larger for the B system than for the D system and more difficult to estimate reliably. Our results are in agreement with several other recent lattice calculations, but smaller than the values from recent unquenched calculations using nonrelativistic methods.

We find a rather small $SU(3)$ symmetry breaking ratio of the heavy-light and light decay constants compared to experiment and also to several recent unquenched lattice calculations. The difference between our numbers and the unquenched results may be partially due to the use (see e.g. [42]) of a chiral extrapolation formula for the unquenched data which is inspired by chiral perturbation theory and predicts a particularly strong decrease of the decay constant at lighter quark mass values than are accessible in the simulation. This is in contrast to the use of a simple linear extrapolation in our calculation.

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