A quantitative model for structured microfinance

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Abstract

We develop a quantitative model for structured microfinance instruments, which are still regarded as an important means for refinancing microfinance institution after the financial crisis of 2008/2009. The quantitative credit risk model presented takes into account the peculiarities of microfinance institutions and can be used for pricing purposes and analyzing the risk inherence in different tranches of a structured microfinance investment vehicle. Additionally, we introduce an innovative pricing methodology that abstains from using the martingale probability measure. This approach seems more appropriate for illiquid securitized debt of microfinance institutions. In a realistic application we check the robustness and demonstrate the advantages of the model presented.

Keywords: microfinance, structured products, quantitative model, MC based pricing, risk analysis, robustness check
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1. Introduction

While a few years ago microcredit was mainly seen as a means to reduce poverty and boost the economies of developing countries, it has now become an increasingly attractive investment opportunity. As Lensink et al. (2010) points out, adding microfinance funds to a portfolio consisting of international bonds and stocks yields diversification gains. Furthermore investors might profit from an additional social return.[1] The investment instruments at hand are stocks, bonds, microfinance investment funds (MFIFs) and microfinance collateralized debt obligations (MiCDOs).[2]

Taxonomies of structured microfinance vary: According to Byström (2007) structured microfinance is defined as direct and indirect securitization. Direct refers to the securitization of a microcredit portfolio by the MFI itself. An example is the "BRAC Micro Credit Securitization Series I Trust" by the Bangladeshian MFI BRAC[3]. As opposed to this, indirect securitization means the securitization of a portfolio of debt instruments issued by MFIs. The indirect transactions are more frequent than the direct ones. One example for the indirect type is the "BlueOrchard Microfinance Securities I" which was the first MiCDO ever issued. However, using the example of the European Fund for Southeast Europe (EFSE), Glaubitt et al. (2008) show how structured elements can also be applied to MFIFs. This supports the broader definition by Jobst (2010), who identifies three forms of structured microfinance, namely MFIFs, direct securitization or local issuance, and indirect securitization or external issuance, all of which we comprise by the term "structured microfinance investment vehicle" (SMIV). In any case the instrument has a CDO-type cash flow structure implying that the SMIV holds a pool of debt instruments on the asset side of the balance sheet and several tranches on the liability side. The accruing cash flows of the pool are distributed to the investors, i.e. the holders of the tranches, according to an a-priori fixed scheme with regard to the different seniorities of the tranches, ranging from the highest ranked senior tranche to equity at the bottom. The first losses in the asset pool affect the equity tranche. After this tranche is eliminated the other tranches are hit.

Structured microfinance entails several advantages often cited in literature in the context of MiCDOs, which we shortly want to summarize in the following.

According to Glaubitt et al. (2008) structured microfinance can reduce regulatory capital, as MFIs securitize parts of their loan portfolio. However this holds true only for direct securitizations.

MFIs can profit from improved access to capital markets. Compared to commercial bonds the volume of MFI bond issues is considerably smaller, which makes it more difficult for MFIs to refinance themselves by utilizing the international capital markets. Pooling the smaller liabilities of MFIs is a means to reach

[1] Cf. Dorfleitner et al. (2010b).

[2] See Dorfleitner et al. (2010a) for an overview.

[3] Cf. Hüttenrauch and Schneider (2008).

a customary size. Another aspect is that MFIs are often rated lower than institutions of equal credit risk in developed countries due to higher country specific risks[4], which makes MFI bonds less interesting for institutional investors. By participating in a SMIV the MFI can separate its credit risk from the country risk and thereby also attain improved access to capital markets. This improvement allows MFIs to diversify their founding resources.[5] According to Watson (2009) MFIs were financed by middle- to long-term debt with an average share of 36,2% in 2009. If this part of the debt is refinanced only by some few lenders the MFIs can be affected by serious financial distress if one of these is not willing to endure their engagement.

Glaubitt et al. (2008) suggest that MFIs can acquire reputation on international capital markets by joining MiCDOs issued by recognized financial institutions. Due to the tranching process different risk-return profiles can be created from a relatively homogeneous asset pool. This helps to attract new investors, whose individual risk preferences can be met more easily.[6] According to Byström (2008) structured microfinance also helps to overcome the asymmetric information problem introduced by Akerlof (1970) as "lemon phenomenon", which is particularly evident in microfinance. Since publicly available information is very limited for individual investors they in turn demand additional premia for compensation. This problem can be mitigated if the much better informed issuer invests in the equity tranche and hence signals that he is willing to take the most risky part himself.

All of these advantages contributed to the development that has made structured products, in particular MiCDOs, the preferred investment vehicle in microfinance in recent years.[7] However, due to the global financial crisis[8], the issuing of structured microfinance products did become impossible in 2009 (Littlefield and Kneiding (2009)) and it seemed unclear whether there was any future to this kind of refinancing instrument. Nevertheless Dorfleitner et al. (2010a) reveal in a survey addressed to relevant world-wide experts on investing in microfinance that structured microfinance is still regarded as a very important means for refinancing microcredit lending. In Section 2 we introduce a quantitative credit risk model for MFI debt instruments, which can be used for pricing and the risk analyzing of the tranches of a SMIV by performing MC simulation. We develop a modular concept in the sense of an easy-to-use toolbox for SMIV analysis. Section 3 provides an application of this model to a realistic SMIV consisting of a MFI bond portfolio. Section 4 concludes.

[4] Cf. Glaubitt et al. (2008).
[5] Cf. Glaubitt et al. (2008).
[6] Cf. Byström (2008).
[7] Cf. Jobst (2008).
[8] See Dwyer and Tkac (2009) for an general overview of the financial crisis 2008. Dooley and Hutchison (2009) shows how emerging markets were affected by the financial crisis.

2. A quantitative model for structured microfinance

Default risk modeling of a microcredit portfolio in general should consider the peculiarities of microcredits, namely the small nominal amounts for each loan, the aim of stimulating productive activities and the often missing collateral. Further properties that can often be observed empirically are high interest rates, low default rates, a greater number of loans assigned to women than men and lower default rates for female borrowers.

However, since most of the SMIVs like MFIFs and MiCDOs are indirect investment instruments, we restrict our quantitative model to a portfolio of debt instruments of MFIs. Therefore the above-mentioned properties are only of indirect importance and we rather have to capture peculiarities of MFIs as obligors, namely the low correlation with worldwide stock markets[9], a large variety in profitability, and a possible geographical dependence of MFIs active in the same region induced for example by climatic risks[10]. In our model we try to capture these properties.

The model that we develop in this section serves the purpose of being the basis for Monte Carlo simulations, i.e. applying the model for risk analysis or the valuation of tranches is done by simulating cash flows of the structured instrument according to the model.

First we will develop a basic version of the structured instruments' cash flow model. In a second subsection we will discuss variations and extensions of this model.

2.1. The basic model

We start with a set of n obligors (MFIs) whose debt instruments (loans or bonds), which we refer to as the asset pool, are held by the structured instrument. All cash flows originated by the asset pool are distributed to the tranche owners via an a-priori fixed distribution scheme following the so-called waterfall principle. As the cash flows of the asset pool in turn depend only on the possible defaults of the obligors, the default times of the asset pool and their dependency structure are crucial for an exact description of cash flows received by the tranche owners.

Modeling dependent default times. Our considerations build on the modern approach presented by Bluhm and Overbeck (2007), which is based on factor models[11] and uses copulae for modelling the dependency structure between individual obligors, as frequently demanded by various authors.[12]

The model can be summarized as follows. The PD term structure captures the probability of default over time and is obligor specific. We denote the PD

[9] Cf. Krauss and Walter (2010).

[10] Cf. Churchill and Frankiewicz (2006).

[11] Cf. Crouhy et al. (2000), Gordy (2000), Gordy (2003),Vasicek (1987).

[12] Cf. Bluhm and Overbeck (2007), Choros et al. (2009), Lando (2004), Lucas et al. (2006), Schönbucher (2003), Mounfield (2008).

term structure of obligor i by $t \mapsto F_i(t) := PD_i^{(t)}$ with $i = 1, \ldots, n$. This function captures the probability that obligor i defaults within the given time interval $[0, t]$. The question of how to specify the PD term structure is treated below. The stochastic default time τ_i of obligor i is now defined by

$$\tau_i = F_i^{-1}[N(CWI_i)] \tag{1}$$

where F_i^{-1} is the inverse of F_i and CWI_i is a time-independent latent random variable called the credit worthiness index corresponding to obligor i. The symbol N denotes the cumulative distribution function of the standard normal distribution.

To capture the default dependence between the obligors the variable CWI_i is modeled linearly dependent on a systematic and an idiosyncratic risk component. The systematic factor is denoted by Ψ_c, which can be interpreted as a macroeconomic measure of the country c. This means that

$$CWI_i = \underbrace{\alpha_i \Psi_{c(i)}}_{\text{system. risk}} + \underbrace{\varepsilon_i}_{\text{idiosync. risk}} \tag{2}$$

with $c(i)$ denoting the country to which obligor i belongs and

$$\Psi_{c(i)} \in \{\Psi_1, \ldots, \Psi_C\}.$$

The parameter α_i is determined as the square root of the R-squared obtained by a linear regression of the obligor's i asset returns against the returns corresponding to the same time interval of the macroeconomic variable.

It is assumed that the residuals ε_i are independent and identically distributed and also independent of the systematic factor $\Psi_{c(i)}$. For the calculation of the default time τ_i according to equation (1) the variable CWI_i must be standard normally distributed. To this end we make specific assumptions concerning the distributions of systematic factor and residuals, namely

$$\Psi_{c(i)} \sim N(0,1) \quad \text{and} \quad \varepsilon_i \sim N(0, 1-\alpha_i^2). \tag{3}$$

Note that by the construction given the CWI_i follows a standard normal distribution and $N(CWI_i)$ follows a uniform distribution on $[0, 1]$.

It is common practice in credit risk modeling to use a country specific stock index as macroeconomic variable $\Psi_{c(i)}$.[13] However, this approach is not suitable for the context of microfinance for the following reason. According to Krauss and Walter (2010), who used MFI-data from 1998 to 2006, one can expect a very low dependence between the returns of MFIs and country specific or even global stock indices. Krauss and Walter (2010) also indicate that stock indices of developing countries are often dominated by only a few larger companies and therefore poorly reflect the country's economical development and are often not published by financial data providers like e.g. Thomson Financial Datastream.

[13] Cf. Bluhm and Overbeck (2007).

Therefore, in our context using stock indices (if available) as systematic factors would result in α_i estimates close to zero, which in turn leads to an approximate independence between the different obligors. This would have a distorting impact on the model, because diversification effects would be overestimated.

As a solution to this problem we suggest a self-constructed MFI index for each country. In particular, a capital-weighted return-on-equity index $ROEX^{(t)}$ seems to be a good alternative. The calculation is as follows:

$$ROEX^{(t)} = \sum_{i=1}^{I} \frac{E_i^{(t)}}{\sum_{j=1}^{I} E_j^{(t)}} ROE_i^{(t)} \tag{4}$$

with $E_i^{(t)}$ representing the book value[14] of obligor's i equity and analogously $ROE_i^{(t)}$ as the rate of return on equity. The alpha factors of the MFIs with respect to each country's ROEX can be derived via linear regression, where α_i equals the square root of the regression's R-squared[15].

Alternatively, one can replace the equity with asset values, which are widely acknowledged in credit risk modeling due to the pioneering work of Merton (1974). An appropriate return-on-assets index $ROAX^{(t)}$ is constructed analogously to equation (4) with substituting the returns on equity by the returns on assets and the equity values by the asset values.

Furthermore we take into account the dependency structure of the systematic factors by assuming a multivariate normal distribution N_C with covariance matrix Γ_C.

Cash flow distribution. As already mentioned, the owner's cash flows are determined by the dependent default times τ_i of the asset pool's debt instruments and the SMIV's distribution scheme. Every debt instrument in the portfolio has a particular cash profile depending on whether and when it defaults. A typical bond's profile may for instance be the following.

- If no default occurs during time to maturity then the coupon payment takes place in every period t (including T) as well as the repayment of the nominal at maturity T.
- If a default occurs during time to maturity then the coupon payments in the periods before default time τ_i take place but no more payments[16] afterwards.

The distribution scheme works as follows. The pool's incoming cash flows are distributed top-down according to every tranche's seniority. This means that

[14] As the market value of MFI equity is often not observable due to the lack of public trading, we use the book value as a proxy.

[15] Cf. Bluhm and Overbeck (2007, p. 125).

[16] We do without a recovery payment in order to keep things simple. But for more precise results we recommend implementing a microfinance specific recovery rate.

after deducting a non-obligatory management fee the remaining cash flows are used to pay each tranche an a-priori committed payment following the cascade-principle. The cash remaining is retained by the equity tranche. Consequently, the equity tranche suffers the first losses caused by defaulting obligors before the more senior tranches are affected. Due to the focus on cash-based instruments in microfinance[17], realizing losses is equivalent to a reduction of a tranche's nominal value.

The remaining nominal of tranche κ at time t is defined as $N_\kappa^{(t)}$ and can be described with regard to the upper attachment points UAP_κ and lower LAP_κ attachment point and the (cumulated) portfolio loss $L_P^{(t)}$ at time t:

$$\begin{aligned} N_\kappa^{(t)} &= N_\kappa^{(0)} - L_\kappa^{(t)} \\ \text{with} \quad N_\kappa^{(0)} &= (UAP_\kappa - LAP_\kappa) \cdot N_P^{(0)} \\ \text{and} \quad L_\kappa^{(t)} &= \min[(UAP_\kappa - LAP_\kappa) \cdot N_P^{(0)}, \max(L_P^{(t)} - LAP_\kappa \cdot N_P^{(0)}, 0)] \end{aligned} \tag{5}$$

The variable $N_P^{(0)}$ is the nominal value of the portfolio at time of initiation and $L_\kappa^{(t)}$ the realized tranche loss at time t.

According to the seniority and therefore to the inherent risk a fixed rate is arranged initially between the SPV and the tranche owners. Then the pool's cash flow is distributed each period t, here quarters, in the following way.

First of all management fees are conducted, then the senior tranche receives a payment which equals the fixed tranche rate r_1 times the remaining nominal $N_1^{(t)}$. Then the remaining cash is used to pay the following tranches analogously according to the waterfall principle. The equity tranche instead receives the complete cash that remains after the distribution. Additionally, at maturity every tranche's remaining nominal is repaid.

One advantage of the model is the modular concept. Thus one can easily implement different specifications for the PD term structure and the dependence modeling. In the next subsection we present some model extensions.

2.2. Further model elements and extensions

Copulae as an alternative for modeling dependency structures. In credit risk, particularly in CDO-modeling, copula based approaches have become popular in recent years.[18] One reason for this development is that according to the Sklar (1959) any multivariate distribution can be described by its marginals and a suitable copula function which allows modelers to use distributions closer to the real one as standard approaches.

Here we present two important copula functions and show how these can be integrated in the model.[19]

[17] Because the raising of funds and not regulatory benefits is in the center of interest.

[18] Cf. Bluhm and Overbeck (2007), Choros et al. (2009), Lando (2004), Mounfield (2008), Lucas et al. (2006), Schönbucher (2003).

[19] The notation used is based on Bluhm and Overbeck (2007).

Gaussian copula

The Gaussian copula is equivalent to the multivariate normal distribution N_m used in the core-model with correlation matrix Γ in which the inverses of the univariate normal distribution N^{-1} are inserted.

$$C_{n,\Gamma}(u_1, \ldots, u_n) = N_n[N^{-1}(u_1), \ldots, N^{-1}(u_n); \Gamma] \tag{6}$$

Gaussian copulas are often used[20] as they are easy to implement, because the correlation matrix Γ is the only parameter to be estimated. Because of the lack of tail dependence the use of Gaussian copulas could lead to errors in cash flow mapping, as the probability of extreme events is underestimated.

Student-t copula

As the Student-t copula equals the multivariate t-distribution $\Theta_{\Gamma,d}$ with correlation matrix Γ and d degrees of freedom in which the inverses of the univariate t-distribution Θ_d^{-1} have been inserted, it entails fatter tails. This copula is defined as:

$$C_{n,\Gamma,d}(u_1, \ldots, u_n) = \Theta_{n,\Gamma,d}[\Theta_d^{-1}(u_1), \ldots, \Theta_d^{-1}(u_n)] \,. \tag{7}$$

With increasing d the Student-t copula converges to the multivariate normal distribution. The integration into our model is done using the link function[21]

$$CWI_i^{t-copula} = \sqrt{d} \cdot \underbrace{(\alpha_i \Psi_i + \varepsilon_i)}_{CWI_i} / \sqrt{X} \qquad \text{with} \quad X \sim \chi^2(d) \,. \tag{8}$$

and CWI_i defined as the original latent variable of the basic model (2) are mixed with additional $\chi^2(d)$-distributed random variables.[22] As $CWI_i^{t-copula}$ is now Student-t distributed equation (1) is modified to

$$\tau_i = F_i^{-1}[\Theta_d(CWI_i^{t-copula})] \,. \tag{9}$$

Specification of the PD term structure. As mentioned above the PD term structure $t \mapsto PD_i^{(t)}$ describes the distribution of the default times of obligor i. Markov chains are often used in this context because they allow us to model the future dependent solely on the last observed state.[23] We present two different approaches using Markov chains.

The first and most straight-forward approach is the **exponential distribution** in the form of

$$PD_i^{(t)} = 1 - e^{-t \cdot PD_i} \tag{10}$$

[20] Cf. Choros et al. (2009), Finger (2000), Bluhm and Overbeck (2007).
[21] Cf. Bluhm and Overbeck (2007, p. 127).
[22] The calculation of CWI_i is the same as in the basic model. So the ROE factors are still multivariate normally distributed and the assumptions on the parameter distributions (Equation 3) are valid as well.
[23] Cf. Kreinin and Sidelnikova (2001), Bluhm and Overbeck (2007).

where PD_i is an default intensity parameter capturing the continuous propensity to default. The idea behind this formula is that the probability of default of obligor i for a given time horizon is constant no matter how long the preceding history without a default is.

A second approach is the time-homogeneous, continuous Markov chain approach **HCTMC approach** presented by Bluhm and Overbeck (2007). It is a generalization of the exponential distribution term structure where we only have two states (default, non-default). HCTMC captures changes in the obligor's creditworthiness represented by rating changes. This feature is an important advantage because rating changes can be observed very often as opposed to real defaults. We start considering a rating migration matrix as published every year by all major rating agencies[24]. As an example Table 1 displays the rating migration matrix of Fitch Rating.

	AAA	**AA**	**A**	**BBB**	**BB**	**B**	**CCCtoC**	**Default**
AAA	94.90	5.10	0.00	0.00	0.00	0.00	0.00	0.00
AA	0.08	91.65	7.84	0.35	0.02	0.02	0.00	0.04
A	0.02	2.34	92.48	4.73	0.21	0.07	0.06	0.09
BBB	0.00	0.24	4.29	90.70	3.71	0.53	0.24	0.28
BB	0.03	0.06	0.16	8.53	80.63	7.20	1.83	1.55
B	0.00	0.00	0.26	0.72	10.62	82.07	4.34	1.99
CCCtoC	0.00	0.00	0.00	0.27	1.08	19.46	54.59	24.59

Table 1: Fitch's global rating migration matrix for the period 1990-2008 (data in %) according to FitchRating (2009)

Every line displays relative frequencies of one-year rating migrations based on corporate bond data from 1990 to 2008. We use these historical rating migrations as a proxy for the real transition probabilities from the rating in the first column to the other rating classes. We are still interested in the continuous PD term structure by taking into account the probabilities of different stages between the default.

Since we consider a continuous time setting we need to calculate a Markov generator Q that satisfies the following equation[25]:

$$MM^{(1)} \stackrel{!}{=} \exp(Q * 1)\,. \tag{11}$$

The PD term structure $PD_i^{(t)}$ is then derived by[26]:

$$PD_i^{(t)} = [\exp(tQ)]_{l,m} \tag{12}$$

where l is the row of the initial rating of obligor i and m is the last column, which represents the default stage. Figure 1 shows the PD term structures derived from Fitch's rating migration matrix for all of the initial rating classes.

[24] These are Fitch Rating, Standard & Poor's, Moody's Investor Service.

[25] For an overview of how to calculate Markov generators see Kreinin and Sidelnikova (2001).

[26] Cf. Bluhm and Overbeck (2007).

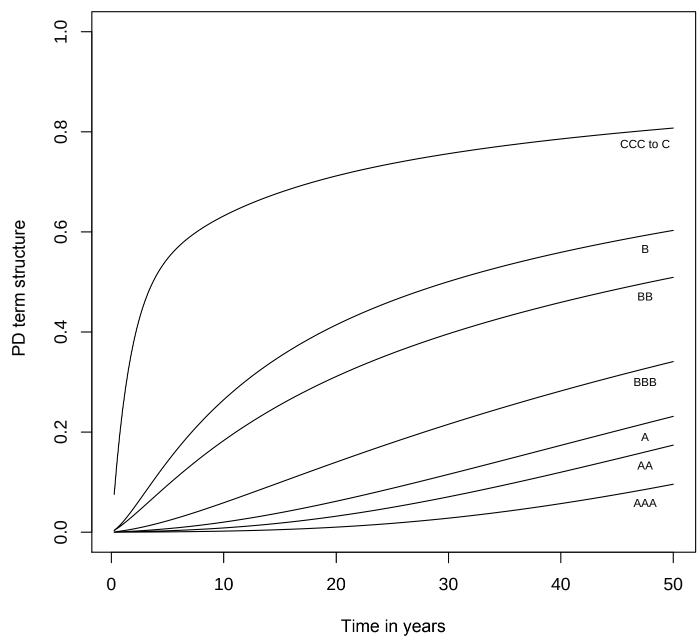

Figure 1: PD term structures based on the HCTMC approach using Fitch's rating migration matrix(Tabelle 1)

Risk-neutral tranche pricing. The basic model does not yet make any statements on the valuation or on determining the spreads of the different tranches. In this subsection we develop a possible extension for calculating credit spreads for senior and mezzanine tranches of a SMIV on the assumption that they are supposed to be priced to par.

It is well-known that investors demand risk premia for accepting credit risk. There are several ways to take these risk premia into account. The standard approach in derivative pricing is to use a risk neutral probability measure and then to take expectations. This methodology that is based on arbitrage-free pricing is also standard in CDO valuation in the form of risk neutral probabilities of default. According to Duffie and Singleton (2003, p. 101) the risk neutral probabilities of default are chosen in a way such that the market price equals the present value of the expected cash flows discounted with the risk free interest rate. It should be indicated that the PD term structures introduced in the previous subsection describe the physical, actually observable values, which are remarkably lower than the risk neutral ones because the demanded risk compensation has to be covered. When doing MC simulation and pricing a tranche we can discount the cash flow profile of each path derived by using a risk neutral

PD term structure simply with the risk free interest rate. In a second step, an averaging over all paths approximates taking the expectation and thus yields the price of a specific tranche.

Bluhm and Overbeck (2007, p. 268) show a way of deriving the risk neutral $PD_{i,rn}^{(t)}$ from any physical PD term structure $PD_i^{(t)}$ with regard to a proxy for the obligor's i one year risk neutral $PD_{i,rn}^{(1)}$ by

$$PD_{i,rn}^{(t)} = N\left[N^{-1}(PD_i^{(t)}) + [N^{-1}(PD_{i,rn}^{(1)}) - N^{-1}(PD_i^{(1)})]\sqrt{t}\right]. \qquad (13)$$

The proxy $PD_{i,rn}^{(1)}$ can be derived from suitable spreads of one-year credit default swaps. This suggestion is a comfortable way to get a hold of risk neutral PD term structures in line with the HCTMC-approach, but it causes some hurdles that are difficult to overcome in an empirical application, especially in microfinance, due to the lack of corresponding CDS data. As there are no CDS for MFIs, further assumptions are necessary.

Using the $PD_{i,rn}^{(t)}$ a tranche is priced, if the present value of the expected loss equals the expected cash flows caused by the spread.

$$\sum_{t=1}^{T} \delta^{(t)} E(L_\kappa^{(t)} - L_\kappa^{(t-1)}) = \sum_{t=1}^{T} \delta^{(t)} S_\kappa \eta E(N_\kappa^{(t)}) \qquad (14)$$

where S_κ is the spread of tranche κ, η is the payment period[27] and $\delta^{(t)}$ is the discount factor. The expected values $E(.)$ are calculated as mean of the simulation data.

Solving equation (14) for S_κ leads to:

$$S_\kappa = \frac{\sum_{t=1}^{T} \delta^{(t)} E(L_\kappa^{(t)} - L_\kappa^{(t-1)})}{\sum_{t=1}^{T} \delta^{(t)} \eta E(N_\kappa^{(t)})} \qquad (15)$$

A pricing approach based on risk analysis. Given a specific SMIV with fixed spreads of the tranches and given an interest rate term structure one can still price tranches of the instrument by using the above risk-neutral pricing approach. However, we rather suggest using the real-world PD term structure and considering the risk explicitly for several reasons: CDS spreads for deriving risk neutral PDs are not available for MFIs. One can use a proxy for the same rating class but then the MFI is forced into resembling companies from a completely different industry. The risk neutral approach assumes no-arbitrage pricing which is not possible with the illiquid debt titles of MFIs. Furthermore microfinance investors need not necessarily behave in a neoclassical way. They might want to have a look at the complete risk profile of a tranche and then set

[27] In the following we set the payment period to 0.25, which equals 4 interest payments per year.

their price and the risk premium individually. They also might include a social return in their considerations[28].

For that purpose we use a MC based risk analysis in the spirit of Hertz (1964), where we use different risk measures such as the value-at-risk (VaR) and the shortfall probability with regard to the internal rate of return (IRR).

With an suitable iterative algorithm we can derive tranche specific spreads using the the IRR distribution, so that the tranches are priced at par. Our approach is based on the view of an investor in a particular SMIV tranche. For optimizing her return she uses as much debt as possible to refinance the investment. The amount of equity needed depends on the risk of the tranche's specific cash flows. Taking into account the IRR distribution, the minimum equity ratio w needed to allow a maximum debt financing is driven only by the yearly debt interest rate r_d and the probability that the investor defaults on debt services PD_{inv} at the end of the investment period T in a way that

$$w_\kappa = \frac{r_{d,T} - \min\left[Q_{\kappa,T}(PD_{inv}), r_{d,T}\right]}{1 + r_{d,T}} \tag{16}$$

with $r_{d,T}$ symbolizing the time-to-maturity interest rate on debt and with $Q_{\kappa,T}(PD_{inv})$ representing the quantile function of the T-years IRR distribution of tranche κ and with PD_{inv} as the investor-specific T-year default probability for the investment into the tranche. The investment in the SMIV tranche is therefore only profitable for the investor, if

$$\mu_\kappa \geq w_\kappa \cdot r_e + (1 - w_\kappa) \cdot r_d = wacc_\kappa \tag{17}$$

with μ_κ as the expected IRR of tranche κ and r_e representing the return on equity the investor is able to receive from alternative risky investments. Note that at this spot alternative views on r_e as compared to the standard CAPM-based approach, which might be due to investors' social responsibility, could easily be implemented and made transparent.

With the micro foundation of an individual investor's calculus in equation (17) we can formulate the whole SMIV pricing problem as follows: Choose the spreads S_κ of superior tranches in such a way, that equation (18) holds, i.e.

$$\max_{S_1, \cdots, S_{K-1}} \mu_{equ} \tag{18}$$

$$\text{s.t.} \quad \mu_\kappa \geq w_\kappa \cdot r_e + (1 - w_\kappa) \cdot r_d \quad \text{for } \kappa < K - 1 \tag{19}$$

It is obvious that the expected 1-year IRR of the equity tranche μ_K reaches the maximum when each μ_κ of the superior tranches equals $wacc_\kappa$.

In contrast to the risk neutral pricing approach presented above this approach is also economically quite intuitive. Whereas the other approach above uses (often under critically assumptions) market-implied risk neutral PDs as

[28] Cf. Dorfleitner et al. (2010b).

main drivers for spreads, we recommend a sensible individual calculus based on r_e and r_d with a corresponding PD_{inv}. We see our approach in line with Wilmott (2009), who suggests making models not too complicated and putting them on a sound empirical basis. Furthermore the approach allows, in contrast to risk neutral pricing, easily made adoptions in the cash flow distribution scheme.

In addition to pricing purposes this analysis is a well-suited tool for making risk inherence apparent and therefore it can be used in risk management.

Check for robustness. It is clear that the estimation of the input parameters, in particular of the correlation matrix Γ, may suffer due to relatively short time series for MFIs. Additionally, it can also be less reliable because it is taken from balance sheet data that may be biased. For this reason we suggest checking the robustness of the model against an erroneous Γ by adding a random error to the ROEX indices. This can be achieved in our simulation based approach by interfering one randomly chosen value in the dataset in the following way:

$$\tilde{ROEX}_c^{(t)} = ROEX_c^{(t)} \cdot e^{\nu} \tag{20}$$

where ν is a normal distributed random variable. With this extension we are in line with Wilmott (2009) who requires financial models to be made more robust as one lesson learned from the financial crisis of 2008/2009. Furthermore one can use the difference in the resulting IRR distributions to derive a risk premium for the model risk inherence. We regard this as a responsible way of dealing with the still evident problem of data availability in microfinance.

A multi-factor extension incorporating climatic risk. In credit risk modeling it is also common to extend equation (2) with industry factors. Since MFIs belong to only one industry the more important question would be whether MFIs lend their money mostly to one type of entrepreneurial activity. The most important type could be agricultural activity, where we have a strong influence of the weather. Therefore it is natural to assume the credit risk of MFIs as dependent on the credit exposure in the agricultural sector, which in turn is dependent on the weather. In this sense we suggest a multi-factor extension also comprising a local weather index variable, namely

$$CWI_i = \alpha_i \frac{y_i \cdot \Psi_{c(i)} + z_i \cdot \Xi_{c(i)}}{\sqrt{y_i^2 + z_i^2 + 2 y_i z_i CORR[\Psi_{c(i)}, \Xi_{c(i)}]}} + \varepsilon_i \tag{21}$$

with $\Psi_{c(i)} \sim N(0,1)$, $\Xi_{c(i)} \sim N(0,1)$, $\varepsilon_i \sim N(0, 1-\alpha_i^2)$ and $\Xi_{c(i)}$ representing the weather index for country $c(i)$. The weights y_i and z_i represent the strength of the influence of both factors. We let y_i equal the incremental contribution of factor one to the R-squared of a regression of the ROEX on both factors divided by the corresponding incremental contribution of factor two and v_i equal to 1. This is a generalization of the approach by Bluhm and Overbeck (2007), who set both weights equal to 1. As above, α_i is the root of the R-squared of the regression comprising both factors.

The model does not necessarily need to be restricted to a special weather index. However, according to World-Bank (2005) growing degree days are a common index in the agricultural sector and a biologically accurate measure for growing activity of grains and fruits. Therefore we use a growing-degree-day index regarding the following structure:

$$GDD = \sum_{d=1}^{D} \max(\vartheta_d - L; 0) \tag{22}$$

with ϑ_d denoting the daily average temperature, L the critical threshold temperature under which no growing activity is observable, and D the number of days in the year under consideration.

3. Empirical Application

In this section we show how the model developed in the previous section can be used in a realistic application. For that purpose we arrange a SMIV like a plain vanilla CDO or a structured MFIF. In this example we first derive the risk neutral tranche spreads for the upper tranches. As we see, this part cannot be carried out without extensive assumptions. Alternatively we then calculate the spreads by the risk analysis based approach that we introduced above. Nevertheless we take the risk neutral spreads as given by the market in the next steps, where we show the risk inherence in each tranche by analyzing the simulated internal rate of returns (IRR) on the basis of the one and two factor case and check for the robustness of the model.

SMIV structure. To keep the example simple, the SMIV considered consists only of senior, mezzanine and equity tranche. The exact structure and the attachment points are shown in Table 2.

Tranche	lower attachement point	upper attachement point
senior	0.350	1.000
mezz.	0.125	0.350
equity	0.000	0.125

Table 2: Structure of the SMIV

For the underlying asset pool we assume that it consists entirely of bullet bonds with identical time to maturity to the SMIV, namely five years with quarterly interest payments. The obligors are existing Latin American MFIs having been assigned a commercial rating of Fitch Ratings which is necessary for applying the HCTMC-approach.[29]

[29] For unrated MFIs we would recommend using the exponential PD term structure with estimated one year PDs.

MFI	Country	Rating	Exposure	LGD	R_E^2	R_A^2	PDrn (1 Year)	Coupon
AgroCapital	BO	BBB	2,000	1	0.68094	0.82289	0.00638	0.05912
BancoSol	BO	A	3,400	1	0.82168	0.65359	0.00374	0.05612
CRECER	BO	BBB	1,100	1	0.84264	0.83954	0.00638	0.05901
FIE	BO	A	4,500	1	0.52401	0.27703	0.00374	0.05613
Fundacin MICROS	GT	B	7,500	1	0.14992	0.10802	0.03151	0.10961
CompartamosBanco	MX	AA	10,000	1	0.09191	0.52474	0.00223	0.05132
BANEX (Ex FINDESA)	NI	BBB	4,600	1	0.01467	0.82220	0.00638	0.05907
FAMA	NI	BBB	4,900	1	0.52147	0.00003	0.00638	0.05895
EDPYME Crear Arequipa	PE	B	7,000	1	0.27352	0.62926	0.03151	0.10949
SUM			45,000					

Table 3: Portfolio structure of the SMIV (Exposure in 1000 USD)

The portfolio of MFI bonds is displayed in Table 3. The information on the MFIs stems from MixMarket, a web-based platform providing lots of data on MFIs. Each debt instrument's share of the pool is chosen with respect to the obligor's rating and the aim of balancing the overall investment per country. We take a USD investor's perspective and realistically assume that the bonds are issued in USD. Since MFIs are not expected to have lots of accessible collateral we assume recovery payment of zero, which equals a loss given default (LGD) of 1. This assumption can of course easily be relaxed. The value of R_E^2 (R_A^2) is derived by linear regression with the MFIs ROE (ROA) as dependent and the ROEX (ROAX) index (see Table 4) as an independent variable. The risk neutral 1-year PDs are estimated from Itraxx-Data of equivalent rated CDS provided by Giaccherini and Pepe (2008)[30]. As the Itraxx contains no B rated CDS the corresponding risk neutral PD is estimated by regression with log PDs and extrapolation. Finally we assume the single bonds to be priced at par and therefore derive the coupons on the basis of simulation.

The pool's cash flows after deducting a realistic management fee of 1,3119% p.a.[31] are distributed following the cascade principle according to seniority: The upper tranches receive the risk free interest rate (2,79% p.a.)[32] plus spread on the remaining nominal and the equity tranche acquires the residuals. At maturity each tranche receives up to the remaining nominal.

The ROEX (ROAX) indices are calculated based on all MFIs for which information about equity, ROE, assets and ROA has been available from MixMarket for at least three years.[33] Our results for Bolivia (BO), Guatemala (GT), Mexico (MX), Nicaragua (NI) und Peru (PE) are shown in Table 4 and the corresponding correlation matrix Γ_C estimated by the standard procedure

[30]In particular we used the mean of risk neutral PDs of equal rated CDSs. This step is quite critical as we "mix" the Risk neutral PDs of different industries, which shows once again the problems inherent in this method.

[31]This equals the EFSE's management fees in 2007 with regard to European Fund for Southeast Europe (2007).

[32]This is equivalent to the 5 year Treasury bond yield for 11.06.2009 (Source: Thomson Datastream).

[33]Note that a lot more (at least 18 per country) MFIs than the ones participating in the SMIV contribute to these indices.

in Table 5 which is required as a parameter of the multivariate normal distribution. The resulting ROAX time series are presented in Table A.11 and the correlation matrix in Table A.12 in the Appendix.

	ROEX(BO)	ROEX(GT)	ROEX(MX)	ROEX(NI)	ROEX(PE)
2007	0.159644	0.095693	0.302575	0.190256	0.259006
2006	0.130380	0.061891	0.309043	0.217373	0.271205
2005	0.131750	0.021587	0.289835	0.226171	0.316358
2004	0.145820	0.065640	0.365253	0.172658	0.266086
2003	0.130886	0.063580	0.527800	0.117495	0.274590
2002	0.056725	0.095500	0.495148	0.099309	0.339920
2001	0.007760	0.042900	0.363944	0.113040	0.214262
2000	0.007808	0.049900	0.444282	0.139155	0.144459
1999	0.051953	0.229200	0.163467	0.140115	0.077709

Table 4: Country specific ROEX

	ROEX(BO)	ROEX(GT)	ROEX(MX)	ROEX(NI)	ROEX(PE)
ROEX(BO)	1.0000	-0.1449	-0.1121	0.6502	0.5359
ROEX(GT)	-0.1449	1.0000	-0.5219	-0.2423	-0.6102
ROEX(MX)	-0.1121	-0.5219	1.0000	-0.5357	0.4663
ROEX(NI)	0.6502	-0.2423	-0.5357	1.0000	0.2159
ROEX(PE)	0.5359	-0.6102	0.4663	0.2159	1.0000

Table 5: Correlation matrix Γ_C of the ROEX

As we wish to model the PD term structure according to the HCTMC-approach we need to derive a Markov generator Q with regard to Fitch's rating migration matrix (see Table 1). We do this by following Kreinin and Sidelnikova (2001)[34].

Risk neutral pricing. Now that the structure of the SMIV and its input parameters have been clarified we can derive the spreads of the superior tranches so that the tranches are priced at par. We start by analyzing the generation of different cash flow scenarios with Monte Carlo simulation. Then we derive the tranche spreads based on risk neutral pricing. For this purpose, the following steps have to be taken:

1. Derive the **risk neutral** PD term structure of each MFI i from the physical PD term structure calculated by the HCTMC approach and the risk neutral one-year PD by following (13).
2. MC-simulate the pool's cash flows in the following way:

[34] In particular we used the "weighted adjustment" for its superior adaptability compared to "diagonal adjustment".

(a) Generate the random variables ϵ_i and $\Psi_1, ..., \Psi_C$ with regard to the multivariate normal distribution of systematic factors N_C with covariance matrix Γ_C.
(b) Calculate the CWI_i following equation (2).
(c) Calculate the dependent default times τ_i by inserting the CWI_i into the inverse of the **risk neutral** PD term structure.
(d) Derive the pool's cash flows determined by the default times τ_i.

3. Calculate each tranche's expected remaining nominal value $N_\kappa^{(t)}$ following equation (5) and occurring losses for each t. Repeat 2. and 3. many times.
4. Calculate the risk-neutral spreads for the senior and mezzanine tranche according to equation (15).

To achieve an appropriate approximation to the real distribution we used 10^6 simulation runs. As result of our simulation we receive these calculated spreads: 15.1 bp for senior and 397.2 bp for the mezzanine tranche. Note again, that this approach requires a mainstream rating and is therefore only applicable a limited number of MFIs.

Pricing based on risk analysis. More suitable for MFIs might be the pricing methodology based on risk analysis introduced above, that abstains form using proxies for risk neutral PDs as those are not observable for MFI debt. Therefore also a mainstream rating is no longer required, if a more simple term structure like the exponential distribution is used and the corresponding PDs are estimated individually on basis of suitable values like write-off ratio, gross portfolio yield and microfinance specific ratings.

However as the sample portfolio contains only MFIs with Fitch rating we keep using the HCTMC while demonstrating our pricing approach, which contains the following steps:

1. MC-simulate the pool's cash flows in the following way:
   (a) Generate the random variables ϵ_i and $\Psi_1, ..., \Psi_C$ with regard to the multivariate normal distribution of systematic factors N_C with covariance matrix Γ_C.
   (b) Calculate the CWI_i following equation (2).
   (c) Calculate the dependent default times τ_i by inserting CWI_i into the inverse of the **real-world** PD term structure.
   (d) Derive the pool's cash flows determined by the default times τ_i. Repeat a) to d) many times.
2. Derive spread for senior tranche S_1 using the following iterative algorithm[35]:
   (a) Calculate the tranche's cash flows depending on the distribution scheme and the IRR of each tranche.
   (b) Calculate the fictitious equity ratio w according to equation (16).

[35] We set the start-value of $S_1 = 0$.

(c) Check: If $wacc_\kappa \approx \mu_\kappa$ then terminate; otherwise adjust S_1 by adding $wacc_\kappa - \mu_\kappa$ and repeat.

3. Reduce simulated pool's cash flows by payments to senior tranche holders based on S_1.
4. Derive spread for mezzanine tranche S_2 by repeating 2. analogously using the from 3. resulting cash flow distribution.

To achieve an appropriate approximation to the real distribution we used 10^6 simulation runs. Furthermore we set r_e equal 8.0% and r_d to be 3.0% with a corresponding $PD_{inv} = 0.2815\%$.[36] As result of our simulation we receive these calculated spreads: 17.6 bp for senior and 554.5 bp for the mezzanine tranche.

Analyzing the tranche's risk. In the following we assume the risk neutral spreads to be existent and analyze the risk inherence in the different tranches of the SMIV. It should be mentioned once again that we use the real-world PD term structure for this purpose as opposed to the risk-neutral pricing approach.

In order to simulate the tranches' cash flows with regard to (pseudo-)random multivariate default times the following steps have to be conducted:

1. Generate the random variables ϵ_i and $\Psi_1, ..., \Psi_C$ with regard to the multivariate normal distribution of systematic factors N_C with covariance matrix Γ_C.
2. Calculate the CWI_i following equation (2).
3. Calculate the dependent default times τ_i by inserting CWI_i into the inverse of the **real-world** PD term structure.
4. Derive the pool's cash flows determined by the default times τ_i.
5. Calculate each tranche's cash flows depending on the distribution scheme and the IRR of each tranche. Repeat 1. to 5. many times.

Tranches	"VaR (90%)"	"VaR (99%)"	SFP (¡ 0)	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
senior	0.02980	0.02980	0.00042	-0.16490	0.02980	0.02980	0.02975	0.02980	0.02980
mezz.	0.02922	-0.14229	0.02273	-1.00000	0.06942	0.06942	0.05600	0.06942	0.06942
equity	-0.13925	-0.24698	0.23540	-0.93780	0.04819	0.22150	0.13700	0.22150	0.22150

Table 6: Simulation results basic model using ROEX: Risk measures and descriptive analysis of the IRRs of the tranches

Tranches	"VaR (90%)"	"VaR (99%)"	SFP (¡ 0)	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
senior	0.02980	0.02980	0.00021	-0.05811	0.02980	0.02980	0.02977	0.02980	0.02980
mezz.	0.02922	-0.24136	0.02759	-1.00000	0.06942	0.06942	0.05479	0.06942	0.06942
equity	-0.13925	-0.25573	0.23829	-0.80560	0.03164	0.22150	0.13330	0.22150	0.22150

Table 7: Simulation results basic model using ROAX: Risk measures and descriptive analysis of the IRRs of the tranches

[36] This equals the 5-years PD of an AA rated obligor calculated on basis of the HCTMC approach.

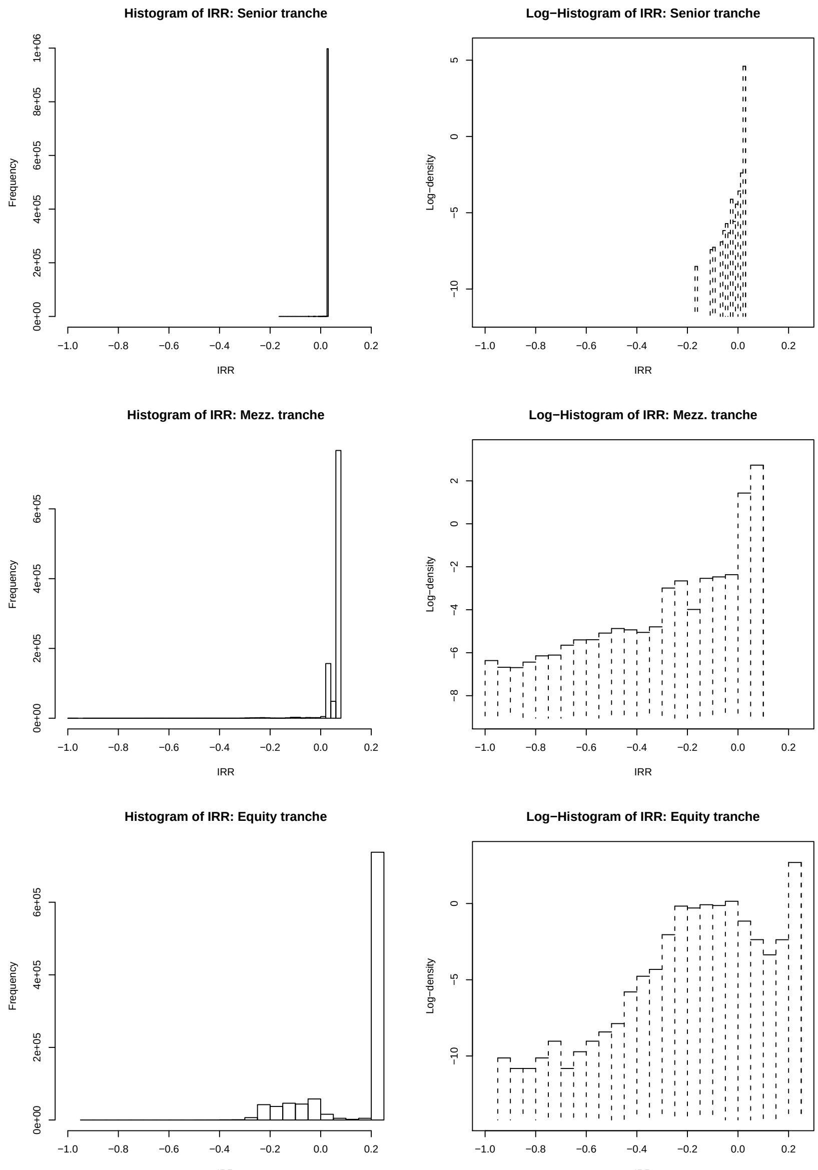

Figure 2: Histograms and Log-Histograms of the IRR_κ (Basic Model using ROEX)

In Table 6 we present the "VaR" of the IRRs at the 99% and the 90% level and also the probability of receiving an IRR smaller than 0% (also shortfall probability or SFP) as results of our simulations. In addition it shows the standard descriptive measures of the IRR distribution for a better overview. Figure

2 shows histograms of the entirely IRR distribution for each tranche and—as losses occur rather rarely—also in logarithmic form. The reason for the mininmal IRR of mezzanine tranche being smaller than that of the equity tranche is caused by the distribution scheme in addition to the small and very heterogeneous asset pool. Thus the equity tranche can, in contrast to the mezzanine tranche, in very rare cases, still receive cash flow payments even if it has already completely defaulted upon. The reason is that in these cases only higher rated obligors have defaulted and lower rated bonds are still paying high interest rates.
In addition we present also the results obtained by using ROAX instead of ROEX in Table 7 and the histograms in Figure B.3 in the appendix. Comparing the results it appears that both indices leads to quite similar IRR distributions. For that reason we confine to use only ROEX in the following.

Robustness check. In the next step the model presented is checked for robustness against Γ errors. Therefore we interfere each set of 500 MC simulation runs with a random entry of the REOX indices according to (20) with re-estimating Γ in each run. For this reason, we set mean and standard deviation of Θ equal to 0 and 0.1. For performance reasons we reduce the number of MC runs for each of the 500 cases to 10^5.
The resulting barplots representing the different realizations of risk and descriptive measures are shown in Figures B.5 and B.6 in the Appendix. For a better overview we present the span between minimum and maximum realization of each measure in Table 8. Note that only the realizations of the minimum differ by more than 1% for all tranches. Furthermore, remarkable spans are only observable for the equity tranche's first quartile and the mezzanine's tranche "VAR (99%)". This indicates that our model reacts robustly to measurement errors in ROEX values.

	senior	mezz.	equity
VAR (99%)	0.00000	**0.08875**	0.00000
VAR (90%)	0.00000	0.00068	0.00625
SFP	0.00043	0.00409	0.00955
Min.	**0.17172**	**0.04020**	**0.57310**
1st Qu.	0.00000	0.00000	**0.02654**
Median	0.00000	0.00000	0.00000
Mean	0.00003	0.00124	0.00320
3rd Qu.	0.00000	0.00000	0.00000
Max	0.00000	0.00000	0.00000

Table 8: Simulation results robustness check: Span of realizations of different risk and descriptive measures (values $\geq$ 1% in bold print)

Two factor extension. In the following we demonstrate the implementation of a second factor. We use the GDD index for each obligor's country calculated according to (22) with L equaling 10 Celsius based on the average daily temperature of weather stations in the following cities: Santa Cruz (BO), Guatemala (GT), Mexico-City (MX), Managua (NI), Lima (PE).[37] The resulting GDD indices are shown in Table A.13 and the correlation matrix Γ_{2F} in Table A.14 in the Appendix.

Table 9 now presents the R-squared values as results of linear regressions of each obligor's ROE on ROEX, on GDD and on both factors. Note that adding GDD as second factor leads to higher R-squared values for many obligors, which in turn implies an increase in the systematic risk component of the CWI.

MFI	Country	R^2_E	R^2_{GDD}	R^2_{2F}
AgroCapital	BO	0.37071	0.38717	0.67453
BancoSol	BO	0.68007	0.14838	0.69210
CRECER	BO	0.35469	0.22276	0.48498
FIE	BO	0.13932	0.04595	0.16805
Fundacin MICROS	GT	0.02679	0.00793	0.03894
CompartamosBanco	MX	0.00746	0.01001	0.01522
BANEX (ex FINDESA)	NI	0.04377	0.53169	0.56308
FAMA	NI	0.12278	0.15929	0.18924
EDPYME Crear Arequipa	PE	0.45761	0.51307	0.64789

Table 9: R-squareds of linear regressions of MFI ROE with of the ROEX, GDD and both factors

With all parameters at hand we MC simulate 10^6 runs according to the following scheme:

1. Generate the random variables ϵ_i and $\Psi_1, ..., \Psi_C, \Xi_1, ..., \Xi_C$ with regard to the multivariate normal distribution of systematic factors N_{2C} with correlation matrix Γ_{2F}
2. Calculate the CWI_i following (21).
3. to 5. are identical to the above procedure.

Tranches	"VaR (90%)"	"VaR (99%)"	SFP (¡ 0)	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
senior	0.02980	0.02980	0.00196	-0.30110	0.02980	0.02980	0.02960	0.02980	0.02980
mezz.	0.02574	-0.30737	0.04904	-1.00000	0.06942	0.06942	0.04697	0.06942	0.06942
equity	-0.15577	-0.27084	0.24736	-1.00000	0.00932	0.22150	0.12990	0.22150	0.22150

Table 10: Simulation results two factor extension: Risk measures and descriptive analysis of the IRRs of the tranches

Table 10 presents the "VaR" of the IRRs to the 99% and the 90% level as well as the shortfall probability. In addition it also shows the standard

[37]The data was drawn from www.wunderground.org. We solve the problem of lacking data between February 22th 2000 and May 02th 2000 by interpolation of the accumulated GDD for that period.

descriptive measures of the IRR distribution for a better overview. Figure B.4 in the Appendix shows histograms of the whole IRR distribution for each tranche and also in logarithmic form. When comparing the results with the results of the basic model (Table 6) one can see that the risk in the mezzanine and the equity tranche now has increased a little bit, a fact that is economically sensible since identifying the weather risk reveals a higher default dependency. However, overall the results a relatively stable which is in favour of the applicability of our approach.

4. Conclusion

In this paper we develop a quantitative model for structured microfinance instruments, which have – like all structured instruments – suffered from the financial crisis of 2008/2009 but are still regarded as a very important means for refinancing microcredit lending.

As one lesson learnt from the financial crisis being that it is desirable to make investment issues as transparent as possible, we restrict ourselves to considering structured microfinance as a real cash flow instrument without any opaque peculiarities, a case that is very realistic in the microfinance context. The microfinance vehicle we analyze has an asset pool consisting of debt instruments of several MFIs.

We model the joint default risk of several MFIs with a state-of-the-art factor approach and several extensions with a special focus on peculiarities of MFIs. Dependent on the multivariate vector of default times, that is basically the random variable we model and MC simulate, we can construct the cash flows of the structured instruments tranches. Even if we suggest a sophisticated model, we make things as transparent as possible and suggest to use MFI return-on-equity or return-on-asset indices for each country under consideration instead of the usual stock market index. Additionally, we introduce an innovative pricing methodology on basis of risk analysis utilizing the IRR distribution rather than a blind trust in risk-neutral pricing approaches. Furthermore we highlight the robustness of the model with respect to the possibility of incorrect MFI balance sheet data. Also, the results of the risk analysis approach are relatively robust to model variations. This implies that the suggested procedure can well be applied in practice.

As a next step the implementation of a model that uses MFI specific values for estimating PDs instead of mainstream ratings would allow the application of the model presented to a major number of MFIs. One possible solution might be seen in logit models.

In summary this paper is a supporting contribution to the future refinancing of microcredits in general and the development of structured microfinance in particular. We feel that this is still a promising topic for investors as well as for the entrepreneurs among the poor who still do not have enough access to microcredit markets.

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Appendix A. Tables

	ROAX(BO)	ROAX(GT)	ROAX(MX)	ROAX(NI)	ROAX(PE)
2007	0.022243	0.046229	0.065924	0.038628	0.035661
2006	0.019338	0.035348	0.056407	0.040687	0.039553
2005	0.019586	0.014522	0.042060	0.049511	0.050688
2004	0.019745	0.015857	0.045547	0.040164	0.035949
2003	0.022612	0.000000	0.189523	0.056177	0.047801
2002	0.015712	0.028800	0.153663	0.055239	0.051001
2001	0.006547	0.028300	0.130059	0.041417	0.026070
2000	0.006370	0.014000	0.300702	0.084657	0.033076
1999	0.012666	0.016500	0.156920	0.096670	0.020451

Table A.11: Country specific ROAX

	ROAX(BO)	ROAX(GT)	ROAX(MX)	ROAX(NI)	ROAX(PE)
ROAX(BO)	1.0000	0.0495	-0.6131	-0.4895	0.5761
ROAX(GT)	0.0495	1.0000	-0.4263	-0.4464	-0.1564
ROAX(MX)	-0.6131	-0.4263	1.0000	0.7072	-0.1831
ROAX(NI)	-0.4895	-0.4464	0.7072	1.0000	-0.4031
ROAX(PE)	0.5761	-0.1564	-0.1831	-0.4031	1.0000

Table A.12: Correlation matrix Γ_C of the ROAX

	GDD(BO)	GDD(GT)	GDD(MX)	GDD(NI)	GDD(PE)
2007	4838	3525	2281	6222	3011
2006	4954	3465	2275	6297	3426
2005	4825	3523	2483	6075	3206
2004	4330	3209	2133	5538	2904
2003	4785	3531	2393	6167	3083
2002	4965	3491	2400	6097	3168
2001	4700	3354	2068	6093	2925
2000	3438	2919	1900	5553	2887
1999	4807	3250	2055	5860	3329

Table A.13: Country specific GDD indices

	ROEX(BO)	ROEX(GT)	ROEX(MX)	ROEX(NI)	ROEX(PE)	GDD(BO)	GDD(GT)	GDD(MX)	GDD(NI)	GDD(PE)
ROEX(BO)	1.0000	-0.1449	-0.1121	0.6502	0.5359	0.4276	0.5792	0.6306	0.3197	0.2220
ROEX(GT)	-0.1449	1.0000	-0.5219	-0.2423	-0.6102	0.2253	-0.1040	-0.2521	-0.0868	0.4045
ROEX(MX)	-0.1121	-0.5219	1.0000	-0.5357	0.4663	-0.2447	0.0198	0.1881	-0.0158	-0.4424
ROEX(NI)	0.6502	-0.2423	-0.5357	1.0000	0.2159	0.1396	0.2143	0.2810	0.1679	0.3546
ROEX(PE)	0.5359	-0.6102	0.4663	0.2159	1.0000	0.4464	0.6863	0.8254	0.4631	0.0417
GDD(BO)	0.4276	0.2253	-0.2447	0.1396	0.4464	1.0000	0.9045	0.7183	0.8059	0.6294
GDD(GT)	0.5792	-0.1040	0.0198	0.2143	0.6863	0.9045	1.0000	0.8943	0.8771	0.4468
GDD(MX)	0.6306	-0.2521	0.1881	0.2810	0.8254	0.7183	0.8943	1.0000	0.6695	0.4153
GDD(NI)	0.3197	-0.0868	-0.0158	0.1679	0.4631	0.8059	0.8771	0.6695	1.0000	0.5277
GDD(PE)	0.2220	0.4045	-0.4424	0.3546	0.0417	0.6294	0.4468	0.4153	0.5277	1.0000

Table A.14: Correlation matrix $\Gamma_{twofactors}$ of the ROEX and GDD

Appendix B. Figures

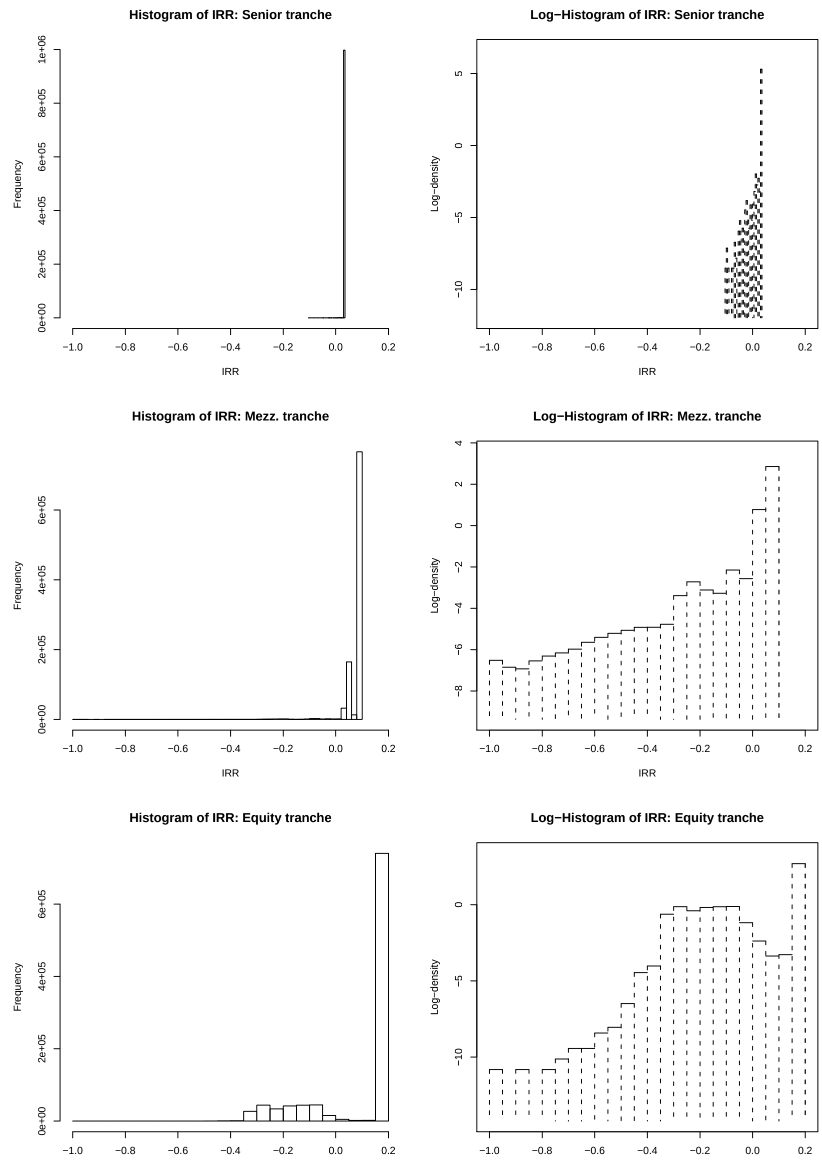

Figure B.3: Histograms and Log-Histograms of the IRR_{TR} (Basic Model using ROAX)

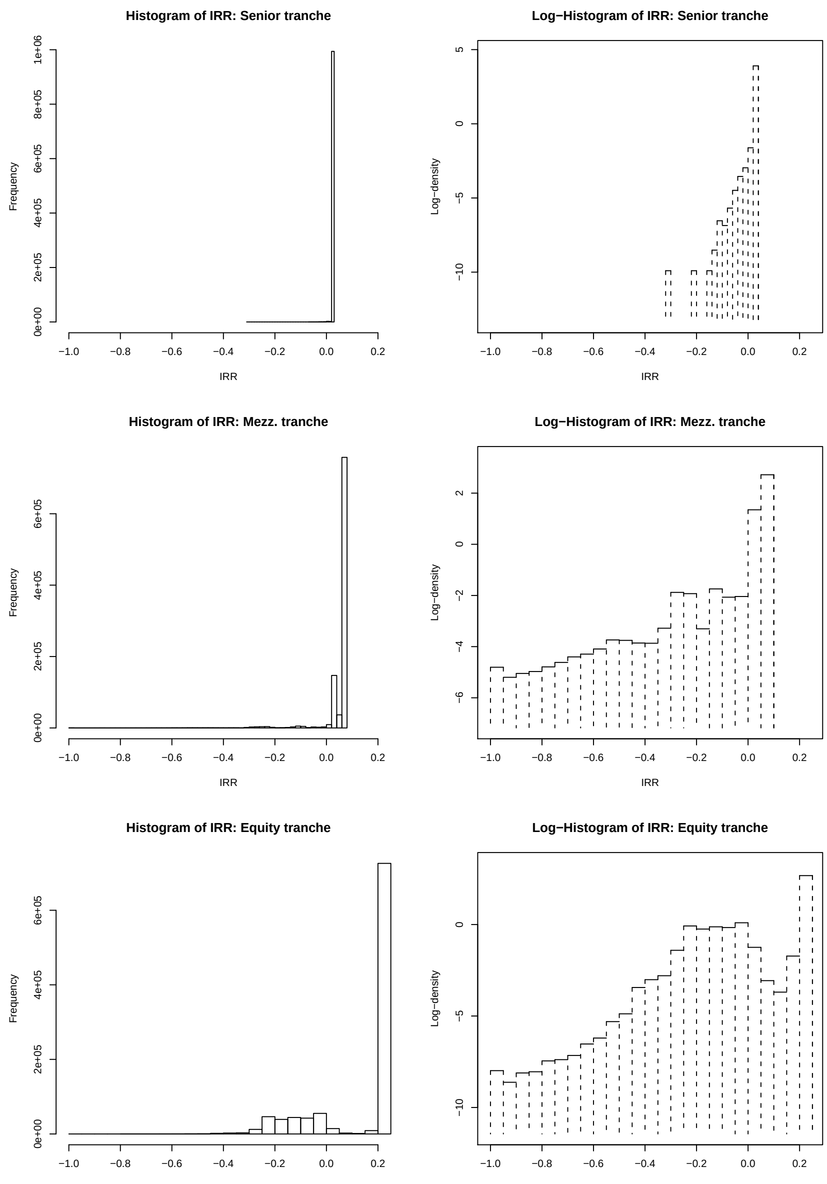

Figure B.4: Histograms and Log-Histograms of the IRR_κ (Two-Factor Extension)

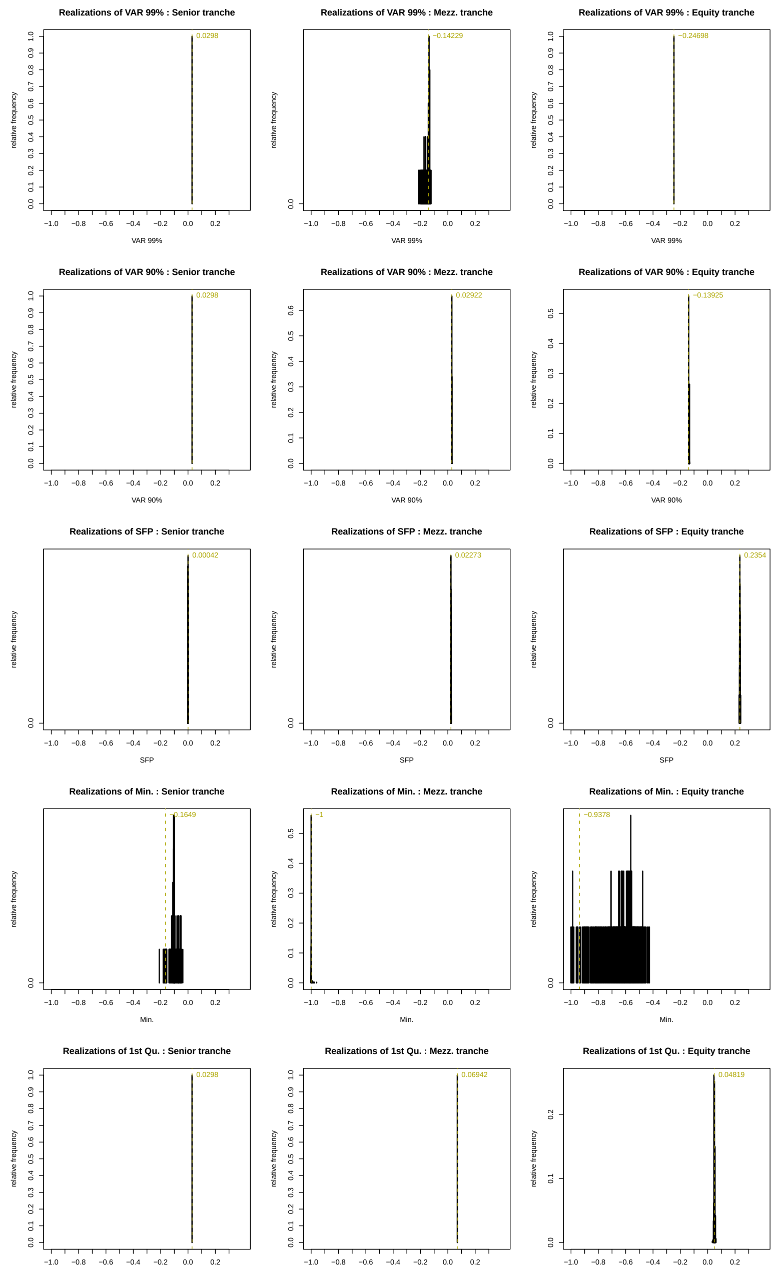

Figure B.5: Simulation results robustness check: Barplots of measure realizations (result of IRR analysis in color)

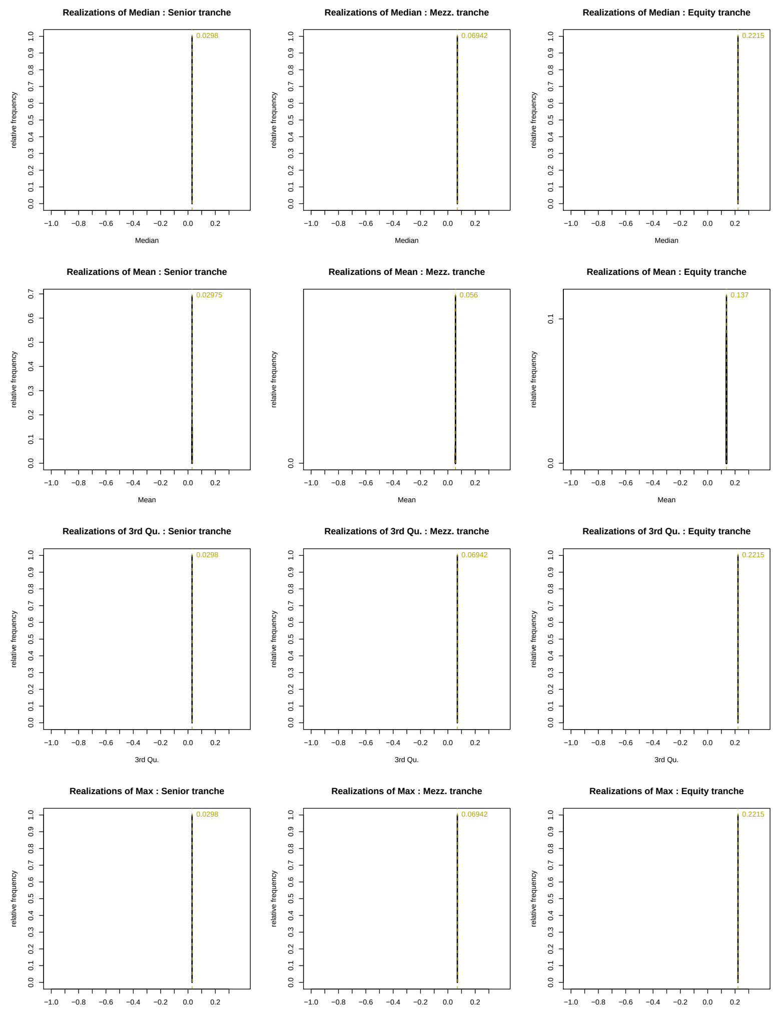

Figure B.6: Simulation results robustness check: Barplots of measure realizations (result of IRR analysis in color)