A SUM RULE DESCRIPTION OF GIANT RESONANCES AT FINITE TEMPERATURE

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A generalization of the sum rule approach to collective motion at finite temperature is presented. The m_1 and m_{-1} sum rules for the isovector dipole and the isoscalar monopole electric modes have been evaluated with the modified SkM force for the ^{208}Pb nucleus. The variation of the resulting giant resonance energies with temperature is discussed.

Recent experimental developments in the domain of light projectile induced reactions (e.g. pγ [1]) as well as heavy ion reactions [2] make it important to quantitatively describe the behaviour of nuclear giant resonances when increasing the excitation energy. As a natural extension of the zero excitation energy case one may describe relevant strength functions in the statistical approximation through finite temperature RPA calculations. This has been performed for the dipole mode in the schematic force model and recently with more realistic forces but within a non fully self-consistent approach [3]. The technical difficulties associated with such RPA calculations provide an incentive to develop a tractable yet sufficiently accurate method to grasp the bulk of the phenomenon under study.

In the zero temperature case the sum rule approach has been shown to be well suited to that purpose (see e.g. ref. [4]). Moreover it has appeared that a semi-classical determination of some sum rules, such as the inverse energy weighted sum rule, yielded very good approximations to quantal (i.e. Hartree–Fock) results [5,6]. The finite temperature generalisation of such an approach implies an extension of some basic results to the non zero excitation energy case. It is the aim of this letter to discuss such a generalisation and to provide some illustrations of the methods in the particular cases of isovector dipole and isoscalar monopole electric modes.

For a statistical mixture defined by its density matrix

$$D = \sum_n P_n |n\rangle \langle n|, \tag{1}$$

whose dynamics is governed by a hamiltonian H with eigenvectors $|n\rangle$ and eigenvalues E_n, the kth order moment of the strength function associated to a given operator Q may be defined as

$$m_k(Q) = \sum_{n,p\neq n} |\langle n|Q|p\rangle|^2 P_n (E_p - E_n)^k . \tag{2}$$

For all moments which can be expressed in the pure

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state case as expectation values of operators O_k involving a sum of products of powers of H and Q one still has in the non zero temperature case

$$m_k(Q) = \mathrm{tr}(DO_k) . \tag{3}$$

This is the case for positive odd k values. One finds for instance

$$\sum_{n,p} |\langle n|Q|p\rangle|^2 P_n(E_p - E_n) = \tfrac{1}{2}\mathrm{tr}\{D[Q,[H,Q]]\} , \tag{4}$$

In the pure case the m_{-1} moment is known to be proportional to the static polarisability α associated to the operator Q. For a statistical mixture (1) corresponding to a canonical (or grand canonical) thermodynamical equilibrium, i.e. with

$$P_n = Z^{-1} \exp(-\beta E_n) , \tag{5}$$

it can be shown [5] that the static polarisability α [‡1] is given by

$$\alpha = 2\sum_{n\neq p} P_n \frac{|\langle n|Q|p\rangle|^2}{E_p - E_n} + \beta\left[\sum_n P_n\langle n|Q|n\rangle^2 - \left(\sum_n P_n\langle n|Q|n\rangle\right)^2\right]. \tag{6}$$

The first term of the rhs in eq. (6) is equal to $2m_{-1}$. The two other terms are vanishing at zero temperature, leading thus back to the well-known result mentioned above. This is still the case however, even at finite temperature, whenever the operator Q has no diagonal matrix element between eigenstates of H, i.e. if Q breaks a symmetry of such states. These two extra terms are also vanishing if the set of states $|n\rangle$ can be considered as resulting from a harmonic oscillator in the "coordinate variable" $Q - \Sigma_n P_n \langle n|Q|n\rangle$, due to the constancy of $\langle n|Q|n\rangle$ in such a case. In so far as the RPA provides a reasonable estimate of the collective motion under study and the operator Q a satisfactory ansatz for the collective operator, it is expected that even when Q does not break any symmetry of the eigenstates of H, one has approximately

$$m_{-1} \simeq \tfrac{1}{2}\alpha . \tag{7}$$

‡1 Of course the polarisability must be understood in the finite temperature case as resulting from a constraint with respect to Q on the variational solutions corresponding to the Helmholtz free energy $\langle H\rangle$-TS and not to the energy $\langle H\rangle$.

We will discuss quantitatively below the validity of eq. (7) on a typical example.

In what follows we will evaluate for the ^{208}Pb nucleus the temperature dependence of the m_1 and m_{-1} sum rules for the isovector dipole and isoscalar monopole electric modes. Calculations will be performed in the independent particle (Hartree–Fock) approximation using the modified SkM force which has recently been shown [7] to provide an excellent description of both static and low energy dynamical properties. As for the operators Q we take with usual notation

$$D = \frac{1}{2}\sum_{i=1}^{A}\theta_i z_i - \frac{1}{2A}\left(\sum_{i=1}^{A}\theta_i\right)\sum_{i=1}^{A} z_i , \tag{8}$$

for the dipole mode (θ_i being the third isospin component) and

$$M = \sum_{i=1}^{A} r_i^2 , \tag{9}$$

for the monopole mode.

The m_1 sum rule is computed as indicated in eq. (4). One finds in the Hartree–Fock approximation

$$m_1(\mathrm{D}) = \frac{\hbar^2}{2m}\frac{NZ}{A} + \tfrac{1}{4}(t_1 + t_2)\int \rho_n(r)\rho_p(r)\mathrm{d}^3r , \tag{10a}$$

$$m_1(\mathrm{M}) = \frac{2\hbar^2}{m}\int [\rho_n(r) + \rho_p(r)]\, r^2\mathrm{d}^3r . \tag{10b}$$

In this approximation, the evaluation of m_1 moments necessitates only the knowledge of the one-body reduced diagonal (i.e. local) densities $\rho_n(r)$ and $\rho_p(r)$. Their variational determination (minimizing the Helmholtz free energy) has been performed with the energy density method using the extended Thomas–Fermi kinetic energy density functional $\tau[\rho]$ (up to fourth order in $\hbar$) with four-parameter "modified Fermi" density profiles [8]

$$\rho_q(r) = \rho_{q0}\{1 + \exp[(r - R_q)/\alpha_q]\}^{\gamma q} , \quad q = n, p. \tag{11}$$

The results of this semi-classical approximation have been shown to very well reproduce on the average those of full Hartree–Fock calculations [7,8]. At finite temperature, a suitable modification of the extended Thomas–Fermi method has also been shown recently [9] to yield a good reproduction of microscopic Hartree–

Table 1
Parameters of the variational semi-classical density parameters as functions of the temperature T (in MeV) with the notation of eq. (11). The ρ_{q0} ($q = p, n$) are reported in units of fm^{-3}, the α_q in fm while the γ_q are dimensionless. The Helmholtz free energy F (in MeV) and the energy TS (in MeV), where S is the entropy, are also reported.

T	ρ_{p0}	ρ_{n0}	α_p	α_n	γ_p	γ_n	F	TS
0	0.0622	0.0911	0.532	0.661	1.42	1.57	−1608.9	0.0
1	0.0621	0.0909	0.537	0.663	1.42	1.53	−1628.5	38.5
2	0.0615	0.0899	0.542	0.662	1.37	1.42	−1685.2	149.5
3	0.0605	0.0882	0.544	0.648	1.27	1.22	−1778.0	333.5
4	0.0591	0.0857	0.543	0.627	1.11	0.96	−1908.4	602.0

Fock results [10], in particular at temperatures above $T \sim 2.5$–3 MeV where the single-particle (shell) effects are washed out. The parameters of the densities eq. (11) are displayed in table 1 for temperatures varying from 0 up to 4 MeV. At a temperature higher than ~3 – 4 MeV the grand canonical approximation cannot be considered to be valid any longer due to the importance of the particle emission phenomenon as quantitatively checked in recent calculations [11]. For the dipole mode one observes a very small decrease of m_1(D) with an increasing temperature (~2% from 0 to 3 MeV). The variation of $\langle r^2 \rangle$, and thence of $m_1(M)$, is already known [10] to be also very slow (~5% from 0 to 3 MeV).

To evaluate the m_{-1} moments we have also computed the corresponding polarisabilities α in a semi-classical approximation. Indeed a fully quantal determination of α from constrained Hartree–Fock calculations is technically feasible a priori but is plagued by serious accuracy problems (e.g. at zero temperature the α value so calculated for the dipole mode has been found in ref. [12] to be at variance with a direct evaluation from the RPA strength function by ~15% whereas the Thouless theorem ensures their equivalence). It is our opinion that the numerical errors on α in constrained Hartree–Fock calculations are at least of the same order of possible shell effects on m_{-1}. For technical details concerning the evaluation of m_{-1} moments the reader is referred to refs. [13,14] where the zero temperature evaluation of such moments is discussed in detail. In the dipole case $\langle n|D|n \rangle$ is vanishing for symmetry reasons and thus the m_{-1} moment is exactly equal to the polarizability divided by 2. In the monopole case this is a priori no longer the case. However, the extra contribution to m_{-1} has been found to be negligible in the following approximate approach. The relevant one-dimensional Bohr hamiltonian has been evaluated in the scaling approximation (see, e.g. ref. [14]) and diagonalized in a large harmonic oscillator basis. The resulting matrix elements $\langle n|M|n \rangle$ have then to be calculated. From the spectrum E_n, the probabilities P_n have been computed leading for all temperatures under consideration to very small terms

$$\beta\left[\sum_n P_n(\langle n|M|n\rangle^2) - \left(\sum_n P_n\langle n|M|n\rangle\right)^2\right].$$

Resulting m_{-1} moments for both the dipole and the monopole modes (in the latter case the scaling approximation was used) are reported in table 2.

As a suggestive way of condensing the results for m_1 and m_{-1}, the E_1 energies

$$E_1 = (m_1/m_{-1})^{1/2} \qquad (12)$$

are displayed in fig. 1. These energies for the ^{208}Pb nucleus provide at zero temperature good estimates

Table 2
Inverse energy weighted sum rules m_{-1} for the dipole (D) and monopole (M) modes as functions of the temperature T (in MeV). Results are reported in units of fm^2 MeV^{-1} for D and fm^4 MeV^{-1} for M modes. For the breathing mode, the compressibility K_A^{scal} (in MeV) in the scaling approximation is also reported.

T	m_{-1}(D)	m_{-1}(M)	K_A^{scal}
0	7.11	2875	138.5
1	7.17	2930	136.8
2	7.31	3137	130.5
3	7.50	3491	121.5
4	7.96	4173	108.0

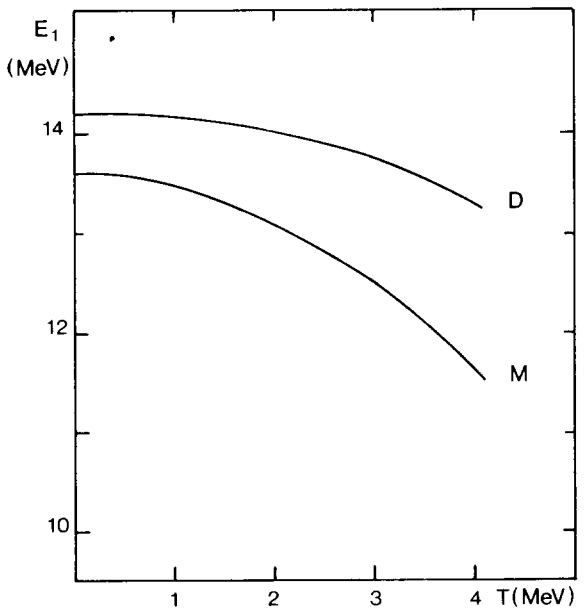

Fig. 1. Energies E_1 (in MeV) as functions of the temperature T (in MeV) for the dipole (D) and monopole (M) modes.

of the location of the giant resonance for both dipole [5] and monopole [14] modes. When increasing the temperature these energies are found to decrease. This result is qualitatively consistent with the schematic model results of ref. [3] (in our case, however, the variation is less marked). It is also in agreement with the conclusions of ref. [2] even though the possible occurrence of such giant resonances at rather high spin may lead to a deformation of the resonance structure due to various broadening effects as sketched in the phenomenological approach of ref. [15]. In our calculations, when the temperature increases, the variation of E_1 results from at least three effects: (i) the vanishing of the particle–hole residual interaction as noted in ref. [3], (ii) a decrease of the stiffness parameters, (iii) a possible side effect on m_1 from the increase of resonance widths with increasing temperature. In the monopole case the above variation (i) is not effective since the global effect of the particle–hole residual interaction on the giant resonance energy at zero temperature is rather small and the variation of the stiffness parameter (i.e. the incompressibility) seems to be dominating.

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