Temperature Models for Pricing Weather Derivatives*

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Abstract

We present four models for predicting temperatures that can be used for pricing weather derivatives. Three of the models have been suggested in previous literature, and we propose another model which uses splines to remove trend and seasonality effects from temperature time series in a flexible way. Using historical temperature data from 35 weather stations across the United States, we test the performance of the models by evaluating virtual heating degree days (HDD) and cooling degree days (CDD) contracts. We find that all models perform better when predicting HDD indices than predicting CDD indices. However, all models based on a daily simulation approach significantly underestimate the variance of the errors.

Keywords: Weather Derivatives, Stochastic Processes, Temperature Dynamics, Heating Degree Days, Cooling Degree Days, Daily Simulation

JEL Classification: C52, G13, Q40

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1. Introduction

Weather derivatives are derivative financial instruments, whose underlying is meteorological data such as temperature, wind, or precipitation. They enable corporations and other organisations to insure their business extensively against unfavourable weather.

A study of the US Department of Commerce (see Dutton, 2002) concluded that up to one third of the US Gross Domestic Product, i.e. approximately 3.8 trillion USD, are exposed to weather risks. However, the traded nominal volume of all weather derivatives between April 2007 and March 2008 has only been 32 billion USD (see Weather Risk Management Association, 2008). It appears that many firms consider the effects of weather as unavoidable constraints, although the profits of various industrial sectors depends heavily on the weather. Most of the corporations merely insure themselves at most against natural disasters such as hurricanes.

Generally, a weather derivative is defined by (1) the *measurement period*, usually given by the starting date τ_1 and finishing date τ_2, (2) a *weather station*, which measures (3) a *weather variable* during the measurement period, (4) an *index*, aggregating the weather variable during the measurement period, which is converted by (5) a *payoff-function* into a cash flow shortly after the end of the measurement period, and (6) possibly a *premium*, which the buyer has to pay to the seller (cf. Jewson and Brix, 2005).

Table 1: Trades by type of contract, notional value of contracts from April 2005 to March 2006 (PricewaterhouseCoopers, 2006).

Type of Contract	Percentage of Total Volume
HDD	79%
CDD	18%
other temperature	2%
other indices	1%

As table 1 shows, the vast majority of all weather contracts traded are written on temperature. Therefore, we constrain our further analysis to temperature derivatives. In the United States, these derivatives are usually written on heating degree days (HDD) and cooling degree days (CDD) indices, which are defined as follows: Let the temperature T_t be defined as the average of the maximal temperature $T_t^{\max}$ and the minimal temperature $T_t^{\min}$ at day t. The HDD index over a period $[\tau_1, \tau_2]$ is defined as HDD $= \sum_{t=\tau_1}^{\tau_2} \max(T^{\text{ref}} - T_t, 0)$, where T^{ref} is a reference temperature (typically 65 degrees Fahrenheit). Similarly, the CDD index over a period $[\tau_1, \tau_2]$ is defined as CDD $= \sum_{t=\tau_1}^{\tau_2} \max(T_t - T^{\text{ref}}, 0)$.

One serious barrier in the development of weather derivatives is the absent consensus of a pricing model. Whilst many market participants are using an Index Modelling approach to model the overall distribution of a derivative's underlying without regarding the daily changes of the underlying, this method cannot be used for classical delta-hedge option pricing (Wilmott, 2007). Since the latter requires information about the daily behaviour of the underlying, a variety of models for the daily temperature processes have been proposed in the literature over the past few years. It should be noted that these models are all statistical models that only depend on a single station's historical temperature and hereby differ from the models used by meteorological services.

In this paper we analyse the performance of these so-called daily simulation methods. Cao and Wei (2004) demonstrate numerically that the market price of risk associated with temperature is insignificant in most cases, which stresses the importance of a proper prediction for the expected index value. For this, we refer to two methods suggested in previous literature and introduce another method that captures the temperature dynamics in a flexible way. The goal of all models is to predict the distribution of the index for a specific weather contract. Applying the payoff function to the distribution yields the predicted distribution of the payoff of a derivative.

Our paper is structured as follows: In section 2, we commence with a brief literature review, which is followed by a detailed description of the specific models considered in this paper in section 3. In section 4, we use temperature data of 35 weather stations in the United States to evaluate the performance of the models. Section 5 concludes the paper.

2. Literature Review

Generally, we can distinguish between three different approaches for the valuation of weather derivatives (Jewson and Brix, 2005):

Burn Analysis. Using Burn Analysis, weather derivatives are valued using historical index values yielding the derivative's fair value. The price of a derivative is then calculated as its fair value plus a possible risk premium.

Index Modelling. This approach extends the Burn Analysis by estimating the distribution of the weather index. If the distribution can be estimated relatively well, the Index Modelling approach yields a more stable price estimation than the Burn Analysis.

Daily Simulation. Using stochastic methods, the development of temperatures are modelled on a daily basis.

The first occurrence of a daily simulation approach which we found in the scientific literature is Dischel (1998), which is refined in Dornier and Queruel (2000). These papers follow an approach similar to Hull and White (1990), who model future interest rates by a continuous Ornstein-Uhlenbeck type stochastic process. Whilst the former authors use the average historical temperatures of each day separately, Alaton et al. (2002) refines the approach by modelling the average historical temperature with a sine function. Brody et al. (2002) observe that temperature dynamics exhibit long-range temporal dependencies and suggest using an Ornstein-Uhlenbeck process driven by a *fractional* Brownian motion. Benth and Šaltytė-Benth (2005) show that for Norwegian temperature data an Ornstein-Uhlenbeck process driven by a generalised hyperbolic Lèvy process with time-dependent variance fits reasonable well and that there is no requirement for a fractional model. Recently, Zapranis and Alexandridis (2008) began proposing using a time dependent speed of mean reversion parameter in the Ornstein-Uhlenbeck type models and use neural networks to estimate the parameters.

Based on a more econometric point of view, Cao and Wei (2000) commenced working on another branch in the development of daily simulation models. Whilst the former authors use time continuous processes, Cao and Wei (2000) adjust the historical temperatures by their trend and seasonality components and suggest a discrete AR process to model the temperature residuals. Similarly to Brody et al. (2002) in the continuous case, Caballero et al. (2002) observe the long-range dependence of temperature time series and proposed modelling these with ARMA or ARFIMA processes. A special ARMA type process is introduced in Jewson and Caballero (2003) to facilitate the estimation of parameters. Campbell and Diebold (2005) show that seasonal ARCH processes can be used to model temperature data as well.

By suggesting the use of a continuous-time autoregressive (CAR) process, Benth et al. (2007) combine both the time continuous approach and the econometric approach and apply it to Swedish temperature data. Benth and Šaltytė-Benth (2007) claim that a standard Ornstein-Uhlenbeck process with seasonal volatility might suffice to price weather derivatives reasonably well and prove their statement with temperature data from Stockholm, Sweden.

Oetomo and Stevenson (2005) compare different temperature models. Our examination surpasses the work of Oetomo and Stevenson by several factors. Using a larger data basis allows us to examine the models for 35 different weather stations instead of ten weather stations, with a majority of more than 50 years of past temperature compared to ten years. This larger data basis allows us to state statistically sounder results and actually rate the models by their prediction quality. Moreover, Oetomo and Stevenson do not consider different evaluation times. Our work shows that the performance of the models varies widely depending on whether a contract is priced well before the start of the measurement period or in the middle of the measurement period. Finally, we do not only analyse the the quality of the models in predicting the first moment, but we also consider the prediction of the second moment. Since a lot of actual pricing is based on the expected value and the variance, a sound prediction of the variance plays an important role in the pricing of weather derivatives.

3. Methodology

Historical temperature data usually exhibits a trend. The reason may not only be attributed to the effects of global warming, but also urbanisation effects that have lead to local warming (Cotton and Pielke, 2007). It is well known that the average temperature in high-density areas is above the temperature in sparsely populated areas due to waste heat from the buildings and the reduced circulation of air. Hence, increasing building density around a weather station leads to a warming trend in the historical temperature data. For the valuation of weather derivatives this implies that a trend removal component should be embedded in each model.

In the subsequent part of this section, we describe the four models we are comparing in this paper. We chose an Index Modelling approach as a benchmark for three daily simulation methods: Firstly, the model introduced by Alaton et al. (2002) due to the fact that it is cited frequently in literature (subsequently called *Alaton model*). Secondly, the model introduced by Benth and Šaltytė-Benth (2007) due to the fact that the authors claim that despite its simplicity the model explained the basic statistical properties of temperature sufficiently well (subsequently called *Benth model*). Finally, we introduce a third daily simulation model, in which we use splines to remove trend and seasonality components from the temperature and follow Jewson and Caballero (2003) to model the residues (subsequently called *Spline model*).

3.1. Burn Analysis and Index Modelling

The Burn Analysis, which is also called *actuarial valuation*, is the simplest method to evaluate weather derivatives. Despite all simplifications it is used by many traders on the market (cf. Dorfleitner and Wimmer, 2010). The main idea of the Burn Analysis is to calculate the future payoff of a derivative by considering the payoffs as the same derivative yielded in the past. If for example a derivative for measurement period $[\tau_1, \tau_2]$ should be priced for the year $n+1$, we would calculate the fictive indices the same derivative had in the year n, $n-1$, $n-2$, etc. This yields a series $Y_1, Y_2, \dots, Y_n$ of n indices for the past n years. Using the linear model

$$Y_i = \beta_0 + \beta_1 \cdot i + \varepsilon_i, \quad i = 1, \dots, n, \tag{1}$$

we can estimate the constant (intercept) parameter β_0 and the trend (slope) parameter β_1 as[1]

$$\begin{aligned} \hat{\beta}_1 &= \frac{\sum_{i=1}^{n} \left(i - \frac{n+1}{2}\right) \left(Y_i - \bar{Y}\right)}{\sum_{i=1}^{n} \left(i - \frac{n+1}{2}\right)^2}, \\ \hat{\beta}_0 &= \bar{Y} - \tfrac{n+1}{2} \hat{\beta}_1, \end{aligned}$$

where $\bar{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_n$ is the mean of the calculated indices over the past n years. We establish three assumptions:

1. The expected error $\mathbb{E}(\varepsilon_i) = 0$ for all years $i = 1, \dots, n+1$.
2. The variance of the errors $\mathrm{Var}(\varepsilon_i) = \sigma^2$ is constant for all years $i = 1, \dots, n+1$.
3. The covariance of the errors $\mathrm{Cov}(\varepsilon_i, \varepsilon_j) = 0$ for all years $i \neq j$.

Under these assumptions, by the Gauss-Markov theorem, the estimator $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 i$ is a best linear unbiased estimator for Y_i. Hence, we can predict the index Y_{n+1} of the next year $n+1$ as

$$\hat{Y}_{n+1} = \hat{\beta}_0 + \hat{\beta}_1 (n+1).$$

[1] Notice that there are a few typos in the QF printed version for the two equations.

In order to derive a measure of the certainty of the prediction $\hat{Y}_{n+1}$ we need to establish a fourth assumption:

4. The errors ε_i, $i = 1, \dots, n+1$, are independent identically normally distributed.

In fact, this assumption extends the Burn Analysis to an Index Modelling approach, since $\varepsilon_i \sim \mathrm{N}(0, \sigma^2)$ implies $Y_i \sim \mathrm{N}(\beta_0 + \beta_1 i, \sigma^2)$. With this assumption we can use the well-known theory of linear models (cf. Rencher, 2008) to estimate the variance of the error of the prediction $\hat{Y}_{n+1}$:

$$\widehat{\mathrm{Var}}(\hat{Y}_{n+1} - Y_{n+1}) = \frac{(n+2)(n+1)(n-2)}{n(n-1)(n-4)} s^2, \tag{2}$$

where

$$s^2 = \frac{1}{n-2} \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$$

is the unbiased estimate for the variance σ^2 of the errors.

3.2. Alaton Model

Alaton et al. (2002) model the temperature time series T_t, $t = 1, \dots, n$, using the Hull and White (1990) type stochastic process

$$\mathrm{d}T_t = \left(a(\theta_t - T_t) + \frac{\mathrm{d}\theta_t}{\mathrm{d}t} \right) \mathrm{d}t + \sigma_t \mathrm{d}W_t, \quad t \geq 0, \tag{3}$$

where the parameter a represents the speed of mean reversion, the parameter σ_t the seasonality of the daily temperature change of the residues, and W_t a standard Wiener process. With the initial condition T_0, using Itô's formula, the SDE (3) yields the strong solution

$$T_t = \theta_t + (T_0 - \theta_0) \exp(-at) + \int_0^t \exp(-a(t-s)) \sigma_s \mathrm{d}W_s. \tag{4}$$

The seasonality θ_t of the temperature is modelled with a simple sine curve plus a linear trend:

$$\theta_t = A + Bt + C \sin(\omega t + \varphi). \tag{5}$$

Since the seasonality of the temperatures equals one year and the temperatures are modelled on a daily basis, $\omega = 2\pi/365$ (neglecting the effects of leap years[2]).

Alaton et al. (2002) claim that the variance σ_t^2 remains nearly constant during each month and give two estimators for the monthly variance $\bar{\sigma}_m^2$, $m = 1, \dots, 12$. In this paper we use the

[2]Technically, we have deleted all leap days from the temperature data for our analysis.

quadratic variation of T_t:

$$\hat{\hat{\sigma}}_m = \frac{1}{N_y}\sum_{y=1}^{N_y}\hat{\hat{\sigma}}_{m,y},$$
$$\hat{\hat{\sigma}}^2_{m,y} = \frac{1}{N_m-1}\sum_{y=1}^{N_m-1}\left(T_{t+1,m,y}-T_{t,m,y}\right)^2.$$

In this context, N_m denotes the number of days of month m, $T_{t,m,y}$ denotes the temperature at day t in month m in year y, and N_y denotes the number of years of past temperature data used.

To estimate the mean-reversion parameter a, we follow the approach of Alaton et al. (2002), who use the martingale estimation functions method of Bibby and Sørensen (1995) to derive

$$\hat{a} = -\log\left(\frac{\sum_{i=1}^{n}\frac{(T_{i-1}-\theta_{i-1})(T_i-\theta_i)}{\sigma^2_{i-1}}}{\sum_{i=1}^{n}\frac{(T_{i-1}-\theta_{i-1})^2}{\sigma^2_{i-1}}}\right). \tag{6}$$

Once the parameters of the model (3) have been estimated, it becomes straightforward to use Monte Carlo methods to simulate the process and therewith to predict the distribution of the temperatures of the measurement period of a weather derivative.

3.3. Benth Model

This model was recently published in Benth and Šaltytė-Benth (2007). In general, they use the same process (3) as Alaton et al. (2002). However, Benth and Šaltytė-Benth use different specifications for modelling the seasonality component θ_t and the variance component σ_t^2.

Let θ_t be specified as the truncated Fourier series with linear trend[3]

$$\theta_t = b + ct + \sum_{i=1}^{I_1} a_i \sin(2i\pi(t-f_i)/365) + \sum_{j=1}^{J_1} b_j \cos(2j\pi(t-g_j)/365), \tag{7}$$

and let σ_t^2 be specified as

$$\sigma_t^2 = d + \sum_{i=1}^{I_2} c_i \sin(2i\pi t/365) + \sum_{j=1}^{J_2} d_j \cos(2j\pi t/365). \tag{8}$$

Using Swedish temperature data from Stockholm, Benth and Šaltytė-Benth argue that setting $I_1 = 0$, $J_1 = 1$, $I_2 = 4$ and $J_2 = 4$ suffices to capture the seasonality of the temperature and its variance well enough.

Benth and Šaltytė-Benth approximate (4) by discretizing the process (4) and estimate the parameter a with a linear regression. Since the Benth and the Alaton model are close in nature,

[3] As in the Alaton model, we delete all leap days to obtain years of equal length.

we will use the same estimate (6) from the Alaton model for the Benth model in order to make the comparison of the models fair.

3.4. Spline Model

The main idea of the Spline model is to separate the daily temperature data T_t into a trend and seasonality component in the mean μ_t and a trend and seasonality component in the standard deviation σ_t:

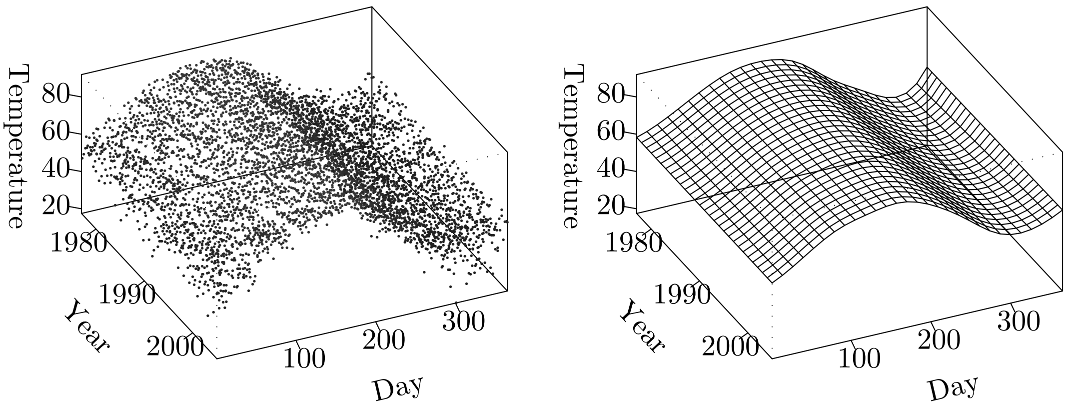
$$T_t = \mu_t + \sigma_t R_t.$$

Both μ_t and σ_t are modelled using splines. The remaining residues R_t can then be expected to have a mean close to zero and a variance close to one and are be modelled separately with an autoregressive process.

3.4.1. Modelling the Trend and Seasonality Components

Let $T_{d,y}$ denote the temperature at day $d \in \{1, \ldots, m\}$ of year $y \in \{1, \ldots, n\}$, where $T_{1,1}$ is the first day of the earliest year observed. Moreover, let $\mathbb{S}_{d,K}$ be the vector space of all splines of degree d and knot sequence K. Now, $T_{d,y}$ represents a bivariate surface of the temperatures, as shown in panel (a) of figure 1 for the temperatures of Houston, Texas. Note that figure 1 shows temperatures of the whole year. When pricing a derivative, it is only necessary to consider the temperature data of the measurement period of the derivative, and a few days ahead, as we will describe later.

Figure 1: Spline approximation of the temperature of Houston, TX.



(a) Daily average temperatures

(b) Spline approximation with $d_d = 4$, $K_d = \{1, 61, 122, 183, 244, 305, 365\}$, $d_y = 2$, $K_y = \{1, 30\}$.

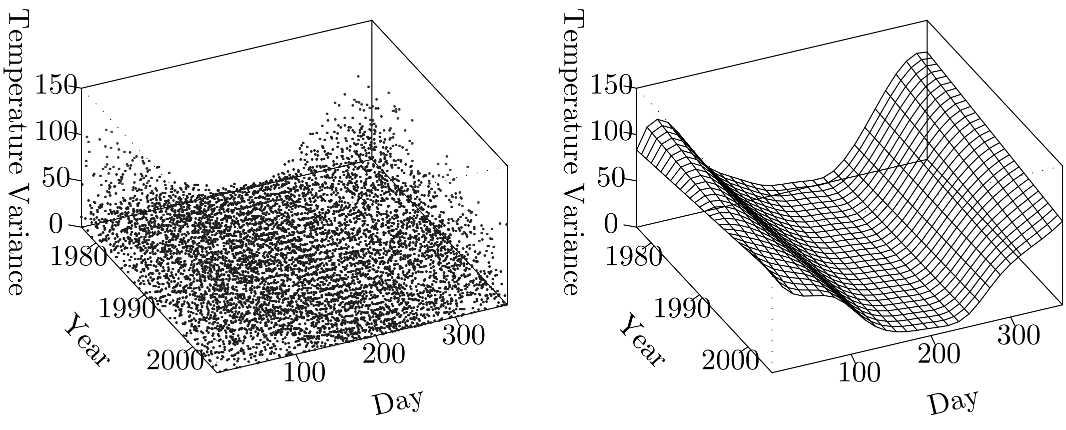
We restrict the approximation μ of this surface to the space of tensor product splines, i.e. $\mu \in \mathbb{S}_{d_d,K_d} \otimes \mathbb{S}_{d_y,K_y}$. Here, d_d and K_d denote the dimension and knot sequence of the splines that face in the direction of the days, and d_y and K_y denote the dimension and knot sequence of the splines that face in the direction of the years. The surface $\mu_{d,y}$ can then be computed from the temperature data $T_{d,y}$ using a least-squares approximation:

$$\min \sum_{y=1}^{n} \sum_{d=1}^{m} (T_{d,y} - \mu_{d,y})^2, \qquad \mu \in \mathbb{S}_{d_d,K_d} \otimes \mathbb{S}_{d_y,K_y} \tag{9}$$

The dimensions d_d and d_y, and also the knot sequences K_d and K_y require specification in advance. In the direction of the days, we use cubic splines with knots approximately every second month. Strictly, we are using $\lceil m/60 \rceil = \min(n \in \mathbb{N} : \ n > m/60)$ evenly distributed knots. The choice of cubic splines seems natural, since these are capable of building a smooth surface. Furthermore, we are convinced that the choice of one knot every second month is sufficient, since the mean temperature data does not exhibit too wild variations during a two months period that could not be caught by a order three polynomial.

In the direction of the years, we set $d_y = 2$ and $K_y = \{1, n\}$, which is the same as a linear regression. Panel (b) of figure 1 displays the spline approximation μ for the temperature data of Houston. Note that unlike modelling $\mu_t = a + bt + s(t)$, where $s(t)$ is a cubic spline function with knots every second month, repeating itself for every year, our model (9) allows different trends for each day of the year. This corresponds to the general findings that e.g. warming trends tend to be stronger in the winter than in the summer (cf. Intergovernmental Panel on Climate Change, 2007).

Figure 2: Spline approximation of the temperature variance of Houston, TX.



(a) Squared residues $(T_{d,y} - \mu_{d,y})^2$

(b) Spline approximation with $d_d = 4$, $K_d = \{1, 61, 122, 183, 244, 305, 365\}$, $d_y = 2$, $K_y = \{1, 30\}$.

After removing the trend and seasonality component μ from the temperatures, we now consider the variance of the temperatures. Since

$$\mathrm{Var}(T_{d,y}) = \mathrm{Var}(\tilde{T}_{d,y}) = \mathbb{E}(\tilde{T}_{d,y}^2) - (\mathbb{E}(\tilde{T}_{d,y}))^2,$$

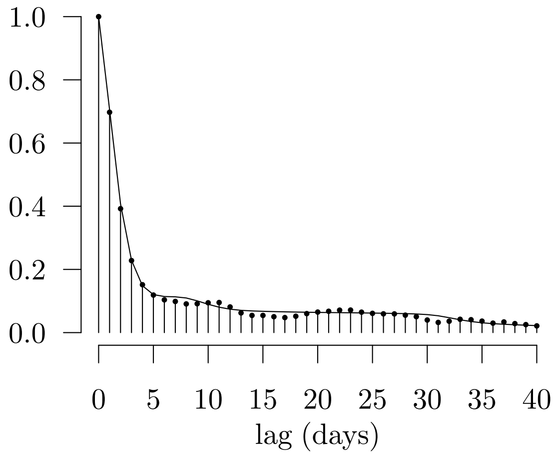
and $\mathbb{E}(\tilde{T}_{d,y})$ has been levelled off in the first step, it suffices to consider the squared residues $\tilde{T}_{d,y}^2 = (T_{d,y} - \mu_{d,y})^2$. Panel (a) of figure 2 shows the squared residues that again exhibit strong seasonal dependence. To remove the trend and seasonality from the squared residuals, we approximate another spline surface $\sigma_{d,y}^2 \in \mathbb{S}_{d_d,K_d} \otimes \mathbb{S}_{d_y,K_y}$ to the squared residuals. Using the same parameters for the knot sequences and spline dimensions as for $\mu_{d,y}$, the resulting spline surface $\sigma_{d,y}^2$ is shown in panel (b) of figure 2. Furthermore, we restrict $\sigma_{d,y}^2$ to be non-negative on $d = 1, \dots, m$ and $y = 1, \dots, n+1$.

In order to obtain the remaining residues $R_{d,y}$, we need to divide $\tilde{T}_{d,y}$ by the estimate of the standard deviation $\sigma_{d,y}$:

$$R_{d,y} = \frac{T_{d,y} - \mu_{d,y}}{\sigma_{d,y}}.$$

3.4.2. Modelling the Residues

Figure 3: Autocorrelation of the residues (spikes) and fitted autocorrelation of an AROMA-process (solid line) of Houston, TX.



In this section, we describe an adequate model for the residues $R_{d,y}$. The spikes in figure 3 show the autocorrelation of the residues, which is calculated as the average of the autocorrelation functions of the residues per year. Although the autocorrelation falls below 0.2 after four days, it remains strictly positive for more than 40 days. Since the autocorrelation of an AR(1)-process decreases exponentially with time, such a process is not able to capture the slow decay. Instead, we follow the path proposed by Jewson and Caballero (2003), who introduced the Autoregressive on Moving Average (AROMA) process, which is a subclass of ordinary AR-processes. However, instead of regressing on the past temperature data directly as an AR(p)-process would do, an AROMA process regresses today's temperature residues on several averages of the past temperature residues:

$$R_t = \phi_1 \bar{R}_{m_1,t} + \phi_2 \bar{R}_{m_2,t} + \ldots + \phi_r \bar{R}_{m_r,t} + W_t, \tag{10}$$

where

$$\bar{R}_{m,t} = \frac{1}{m} \sum_{i=1}^{m} R_{t-i} \tag{11}$$

is the average of the temperature residues of the m days before day t, and W_t is independent identically normally distributed.

Caballero et al. (2002) present a rationale for using an AROMA process to model temperature data. They argue that the slow decay of temperature data results from the aggregation of several processes with different timescales. Internal atmospheric variability affects short timescales, land-surface processes affect medium timescales, and the interaction of the atmosphere and oceans affects long timescales.

As Jewson and Caballero (2003) showed, four processes normally suffice to capture temperature data, and thus we set the parameter $r = 4$ in the model in (10). Moreover, as in Jewson and Caballero (2003), we limit $\max_{i=1,\ldots,r} m_i \leq 35$ and set $m_1 = 1$ and $m_2 = 2$. To estimate the AROMA process, we now cycle through all possible choices of m_3 and m_4, compute the parameters $\phi_1, \ldots, \phi_4$, and finally choose the AROMA process whose autocorrelation function fits best to the empirical autocorrelation function in a least-squares context.

Table 2: Estimated AROMA parameters for Houston, TX.

m_1	m_2	m_3	m_4	ϕ_1	ϕ_2	ϕ_3	ϕ_4
1	2	8	31	1.0343	−0.4280	0.0579	0.1092

The estimated parameters for Houston are given in table 2; the autocorrelation function of this AROMA process is the solid line in figure 3 and captures the empirical autocorrelation fairly well. The slow decay of the empirical autocorrelation in the figure also pinpoints that a similar fit using an AR-process would require estimating far more than four parameters. Although the variance of the white noise process W_t in (10) is not required for the estimation of the AROMA parameters, it is needed for Monte Carlo simulations when predicting the future temperature. By plugging in (11) into (10), the AROMA process can be rewritten as an ordinary AR(p)-process $R_t = \tilde{\phi}_1 R_{t-1} + \tilde{\phi}_2 R_{t-2} + \ldots + \tilde{\phi}_p R_{t-p} + W_t$, where $p = \max_{i=1,\ldots,r} m_i$. Then, the variance of W_t can be estimated as

$$\hat{\sigma}^2(W_t) = \gamma(0) - \tilde{\phi}_1 \gamma(1) - \ldots - \tilde{\phi}_p \gamma(p),$$

where γ denotes the empirical auto-covariance function of the residues.

3.4.3. Prediction

In order to calculate a prediction for the temperature data for the following year $n+1$, the calculation of the splines $\mu_{\cdot,n+1}$ and $\sigma_{\cdot,n+1}$ is straightforward. Since in our context an AROMA process requires data from up to the past 35 days (the maximum value allowed for m_i), we need an estimate for the residues of 35 days before the beginning of the measurement period. We take a random selection of 35 consecutive residues $R_{(i,i+1,\ldots,i+34),y}$, where i is chosen uniformly from $\{1,2,\ldots,m-34\}$ and y uniformly from $\{1,2,\ldots,n\}$. After simulating the residues $R_{d,n+1}$ for the measurement period, they can be transformed into temperatures by multiplying and adding the standard deviation and mean component, respectively:

$$T_{d,n+1} = \mu_{d,n+1} + \sigma_{d,n+1} R_{d,n+1}.$$

4. Results and Discussion

We have temperature data available from 35 weather stations across the United States. The data originated from the US National Weather Service and consists of daily minimum and maximum temperatures. Since the data contains gaps due to failures in measurement equipment or data transmission, and jumps due to changes in measurement equipment, it was pre-processed by Earth Satellite Corporation to fill in such gaps and remove such jumps (Boissonnade et al., 2002). Most temperatures series start in 1950. We have grouped the weather stations into four geographical regions: Midwest, Northeast, South and West. The exact locations of the weather stations are shown in figure 4 and listed in table 7 in appendix A.

Figure 4: Location of the weather stations in the United States. The four geographic regions are seperated by solid lines.

4.1. Backtesting

Using a backtesting analysis, we wish to examine how well the models described in the previous section would have performed in the past when predicting expected aggregated temperature indices for weather derivatives. To do so, we have chosen the twelve most common contracts and computed the error each model had made for each contract at each station for each year from 1983 until 2005. However, for some stations there is not enough temperature data available to calculate the years from 1983 onwards (cf. table 7). In these cases, we took fewer years accordingly. The contracts are winter seasonal contract, summer seasonal contract, the five summer months, and the five winter months. Their exact specifications are listed in table 3.

Table 3: Contract specifications.

		Measurement Period		
Name	Index	Starting Date	Finishing Date	Days
Summer	CDD	05/01	09/30	153
May	CDD	05/01	05/32	31
June	CDD	06/01	06/30	30
July	CDD	07/01	07/31	31
August	CDD	08/01	08/31	31
September	CDD	09/01	09/30	30
Winter	HDD	11/01	03/31	151
November	HDD	11/01	11/30	30
December	HDD	12/01	12/31	31
January	HDD	01/01	01/31	31
February	HDD	02/01	02/28	28
March	HDD	03/01	03/31	31

Moreover, we have evaluated each single contract at two different times: First, we have priced the contract 180 days prior the start of the measurement period. Secondly, we have re-evaluated the contracts in the middle of the measurement period. We expect it to yield more accurate results than the first time, since in the latter case the temperatures are available up to the middle of the measurement period. All calculations were performed on an out-of-sample basis with the past 30 years of temperature data. Clearly, the various models use different numbers of parameters. However, since our analysis is on an out-of-sample basis, the problem of overfitting should be mostly avoided (cf. Clark, 2004). For the daily simulation models, we have simulated 100,000 trajectories for each contract.

4.2. Data analysis

As the different contracts have different contract lengths and different index values, we use the mean relative error (MRE) and the mean squared relative error (MSRE) to compare the models. The relative error $\delta\hat{Y}$ is defined as the fraction of the error $\hat{Y} - Y$ and the real index value Y, and the squared relative error $(\delta\hat{Y})^2$ is the square thereof.

For a real index value $Y = 0$ the relative error and squared relative error are not defined. Since in the calculation of these errors the real index is in the denominator, the (squared) relative error becomes especially large when small real index values occur. Therefore, we have only considered derivatives with a real index value of at least 50 degree days[4]. At the end, we have 17,056 contract valuations for further analysis.

Table 4: Mean relative errors and mean square relative errors of the models over all HDD and CDD contracts.

		HDD		CDD	
		180 Days Ahead	Mid Measurement Period	180 Days Ahead	Mid Measurement Period
MRE	Index Modelling	2.58%	0.77%	2.68%	0.89%
	Alaton Model	5.89%	2.60%	11.83%	5.00%
	Benth Model	4.78%	2.79%	6.31%	2.93%
	Spline Model	2.52%	0.72%	2.62%	1.03%
MSRE	Index Modelling	3.31%	1.29%	11.43%	4.62%
	Alaton Model	4.31%	1.16%	16.74%	5.12%
	Benth Model	4.89%	1.53%	12.77%	4.68%
	Spline Model	3.25%	1.12%	11.47%	4.21%

Table 4 shows the MRE and MRSE of all contracts, sorted by the type of model used. In all cases, the Index Modelling approach and the Spline model yield similar results, with a small advantage for the Spline model in the HDD case. Both the Alaton model and the Benth model display higher errors in nearly all situations.

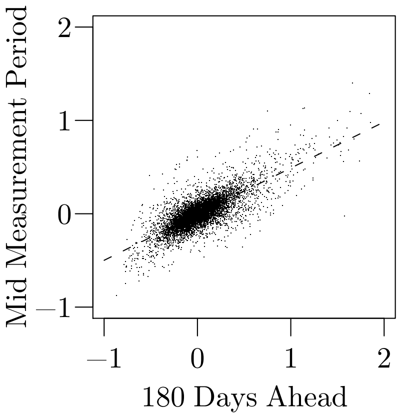
However, table 4 presents the data in a highly aggregated form. Although the errors of the contracts evaluated at the middle of the measurement period are smaller than the errors of the contracts evaluated ahead of the measurement period, it is not possible to draw any further conclusions from the data in this aggregated form.

Figure 5 shows scatterplots of the relative errors at the two different valuation times of the models. Since there is no significant difference between the HDD and CDD contracts, we have plotted both types into the same figure. A linear regression of the errors yields the following results for the different models:

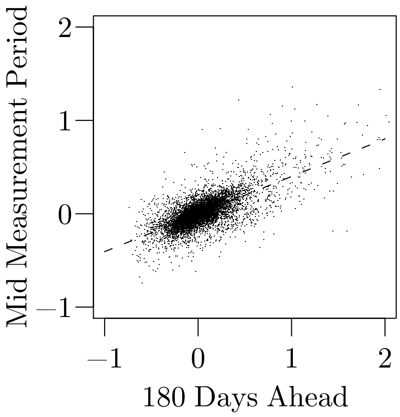
$$\begin{aligned}
\delta\hat{Z}_{\text{Index}} &= 0.4942^{***} \cdot \delta\hat{Y}_{\text{Index}} &-& \; 0.0047^{***}, \\
\delta\hat{Z}_{\text{Alaton}} &= 0.4197^{***} \cdot \delta\hat{Y}_{\text{Alaton}} &+& \; 0.0008, \\
\delta\hat{Z}_{\text{Benth}} &= 0.4794^{***} \cdot \delta\hat{Y}_{\text{Benth}} &-& \; 0.0022^{*}, \\
\delta\hat{Z}_{\text{Spline}} &= 0.4157^{***} \cdot \delta\hat{Y}_{\text{Spline}} &-& \; 0.0020,
\end{aligned}$$

[4] In fact, weather derivatives with such low expected indices are rarely traded.

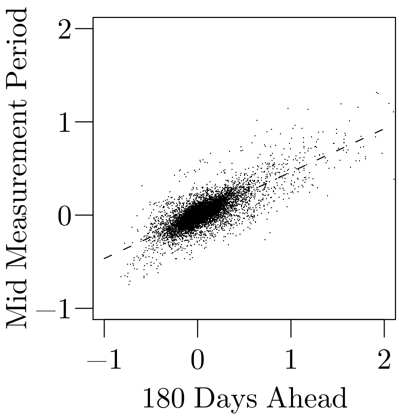
Figure 5: Comparison between the relative errors of the contracts evaluated 180 days ahead of and evaluated in the middle of the measurement period.



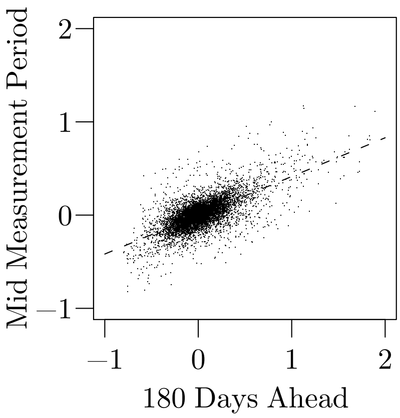
(a) Index Modelling



(b) Alaton model



(c) Benth model



(d) Spline model

where $\delta\hat{Z}$ and $\delta\hat{Y}$ denote the relative errors of the models evaluated in the middle of the measurement period and 180 days ahead, respectively and *, **, and *** indicate significance levels of 5, 1, and 0.1 percent, respectively. While the slopes of the regressions are highly significant for all models, the intercept of the Benth model and the Spline model are not significant on a 5% level. Apparently, the Index Modelling fulfils our expectation to yield quite accurately half of the original error when evaluated in the middle of the measurement period[5]. In particular, the Alaton model and the Spline model reduce the errors by a much larger degree, due to the fact that they are capable of using the given temperature data of the first half of the measurement period to forecast the temperatures of the second half.

Table 5: Mann-Whitney U test results (p-values). The rows and columns represent the Index Modelling approach (I), the Alaton model (A), the Benth model (B) and the Spline model (S).

	180 Days Ahead				Mid Measurement Period			
	I	A	B	S	I	A	B	S
I	—	0.0000	0.0002	0.9887	—	0.9994	0.0019	1.0000
A	1.0000	—	1.0000	1.0000	0.0006	—	0.0000	1.0000
B	0.9998	0.0000	—	0.9999	0.9981	1.0000	—	1.0000
S	0.0113	0.0000	0.0001	—	0.0000	0.0000	0.0000	—

In order to rank the models by their prediction performance, we perform a Mann-Whitney U test (cf. Mann and Whitney, 1947) between the MSRE of each pair of models. Table 5 shows the p-values of the test with the null hypothesis $H_0 : (\delta\hat{Y})^2_x = (\delta\hat{Y})^2_y$ and alternative hypothesis $H_1 : (\delta\hat{Y})^2_x < (\delta\hat{Y})^2_y$. The rejection of the null hypothesis on a 5% significance level for certain pairs of models yields the following preference order of the MSRE of the contracts evaluated 180 days ahead of the measurement period:

$$\text{Spline model} \prec \text{Index Modelling} \prec \text{Benth model} \prec \text{Alaton model.}$$

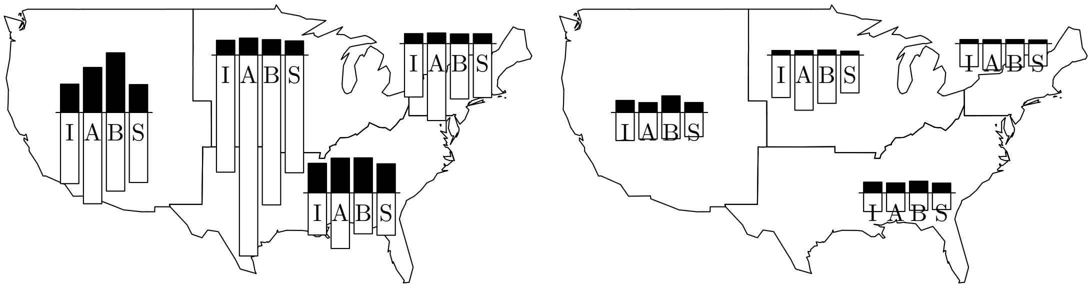
Clearly, the Spline model dominates all other models, the Index Modelling approach dominates the Alaton model and the Benth model, and the Benth model dominates the Alaton model. As we have indicated above, the daily simulation models improve compared to the Index Modelling approach in the case where the contracts are evaluated in the middle of the measurement period. Table 5 shows a change in the preference order in this case which becomes

$$\text{Spline model} \prec \text{Alaton model} \prec \text{Index Modelling} \prec \text{Benth model.}$$

It is clear that depending on its geographic region, the results vary for the different weather

[5]The fact that the error of the Index Modelling approach is slightly below 50% results from the fact that due to rounding we evaluate contract one half-day after the middle of the measurement period if the measurement period is uneven.

Figure 6: Mean squared relative errors grouped by geographical regions. The black bars indicate the winter (HDD) errors and the white bars the summer (CDD) errors, respectively. Bar I corresponds to Index Modelling, bar A corresponds to the Alaton model, bar B corresponds to the Benth model and bar S corresponds to the Spline model.



(a) 180 days ahead

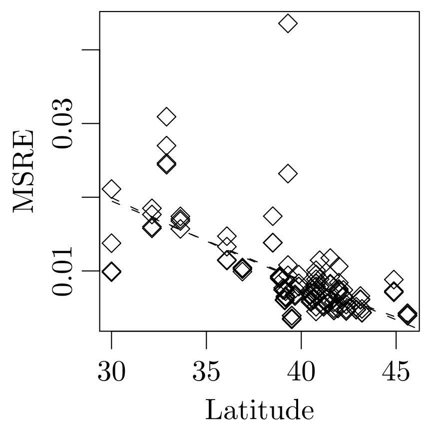
(b) Middle of measurement period

stations. In figure 6 we have plotted the MSRE of the different models for the different weather stations; table 8 in appendix A shows the values of the corresponding MSREs. We have grouped the weather stations into the geographic regions Midwest, Northeast, South and West. The black bars in figure 6 represent the MSREs of the HDD contracts, while the white bars represent the MSREs of the CDD contracts in the respective region. Note that while Northeast, Midwest and South also represent climatic regions of the United States, this is not the case for the Western region. As we have already indicated in table 4, the data clearly shows that the errors for the CDD contracts exceed the errors of the HDD contracts in all regions. This is especially true in the Midwest, which shows the highest CDD errors in all models. It is also apparent from the data, that the Index Modelling approach and the Spline Model outperform the other models in all regions.

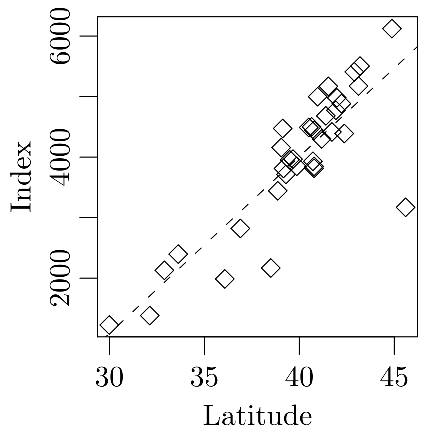
In general, figure 6 suggests that the MSREs depend on the latitude of the weather stations. To analyse this further, figures 7(a) and 7(c) show the mean values of the MSREs of the four models for the winter seasonal contract and summer seasonal contract, respectively. We have also added a linear and a quadratic regression line to the figures. It is apparent that the errors decrease with the latitude of the station in the HDD case and increase in the CDD case. A possible explanation for this is shown in figures 7(b) and 7(d): The realised HDD indices increase with the latitude of the station, while the realised CDD indices decrease with the latitude of the station. Since we are dividing the actual errors by the realised index when computing the MSRE, it is clear that the MSREs decrease with increasing realised indices.

Notice that basically all indices are calculated as the aggregated number of degrees when the temperature was above or below a 65 degrees Fahrenheit threshold over the measurement period. This implies that the random terms in the daily simulation models may not have a big influence when examining contracts with high index values, and that the advantage of the Spline

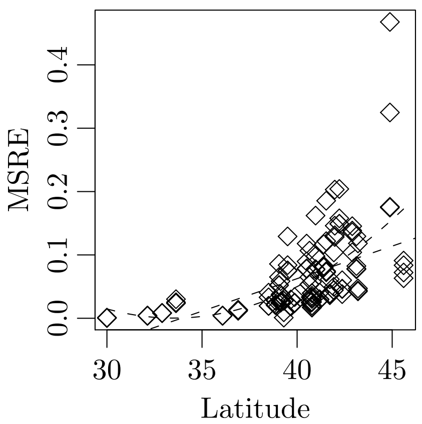
Figure 7: The dependence of the MSRE on the latitude of the weather station in the 180 days ahead case. Panels (a) and (c) show the mean value of the MSRE for each of the four models for the winter seasonal contract and summer seasonal contract, respectively. Panels (b) and (d) show the mean value of the realised indices for each station for the winter seasonal contract and summer seasonal contract, respectively.



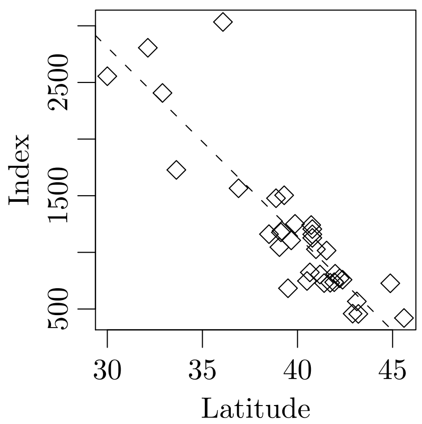
(a) MSRE HDD



(b) Realised indices HDD



(c) MSRE CDD



(d) Realised indices CDD

model in the analysis above might be mainly driven by the fact that the seasonal temperature component is more accurate in this model. In order to check this hypothesis, we perform the same backtesting analysis as above (with 180 days before the start of the measurement period), but for the plain temperatures instead of the temperature indices. Since the random terms play no role in this analysis, it shows which of the three daily simulation models captures the trend and seasonal temperature component (i.e., θ_t in the Alaton and Benth model, and μ_t in the Spline model) best. The analysis yields an MSRE of 1.49% for the Alaton model, an MSRE of 1.61% for the Benth model, and an MSRE of 1.57% for the Spline model. Apparently, the advantage of the Spline model is not driven by the trend and seasonal component, but by the way the residues are modelled. Basically, the Alaton model and Benth model use an AR(1) process for the residues, which can capture only exponentially decreasing autocorrelations, killing off variations from the mean quite quickly. However, the Spline model is able to capture more realistic autocorrelation functions, as diplayed in figure 3.

Table 6: Slope parameters for the relation between the realised standard deviation and the predicted standard deviation.

	Slope	95% Confidence Interval
Index Modelling	0.9976	(0.9821, 1.0131)
Alaton Model	1.2259	(1.1971, 1.2546)
Benth Model	1.0793	(1.0498, 1.1089)
Spline Model	1.1556	(1.1387, 1.1726)

Dubrovský et al. (2004) point out that many daily simulation methods underestimate the variability of the temperatures. In order to check this hypothesis, we have computed the standard deviation of the mean errors of each weather contract at each weather station and regressed it on the predicted standard deviation with fixed intercept zero. In the Index Modelling case, the predicted standard deviation is given in equation (2), and for the daily simulation models we use the standard deviation of the predictions in the Monte Carlo simulations. Table 6 shows the slope coefficients of the regression for the models together with their 95% confidence interval. Whilst we cannot reject the null hypothesis that the Index Modelling approach predicts the error standard deviation correctly, we can reject the same null hypothesis for the three daily simulation models on a 5% significance level. In particular, the Spline model and the Alaton model underestimate the standard deviation by more than 10% and 20%, respectively.

5. Conclusion

Weather derivatives are an interesting extension of the derivative market. In contrast to traditional derivatives, the underlying of weather derivatives, temperature, precipitation, wind, etc., cannot be traded solely. Since weather variables are mostly uncorrelated with the classical financial market, weather derivatives form the only possibility on the financial market to insure against unfavourable weather. The development of the weather derivatives market supposes that an increasing number of corporations take advantage of these new opportunities.

We have presented three daily simulation models for predicting temperature indices and compared them with the Index Modelling approach. Since all models include a linear detrending component, the comparison proceeded on a fair basis. The results in section 4 suggest that more complex mathematical models do not necessarily yield more accurate results. While the Benth model seems to be appropriate for Stockholm temperature data (Benth and Šaltytė-Benth, 2007), it does not capture relevant phenomena for US weather stations. Apparently, these stations require setting different truncation parameters I_1 and J_1 for the truncated Fourier series[6].

Our analysis of the models showed that the prediction errors depend strongly on the geographic location of the weather station. Whilst only the Spline model showed a significant improvement compared to the Index Modelling approach when evaluating contracts well before the start of the measurement period, the performance of all daily simulation models gained considerably compared with the Index Modelling approach when evaluating the contracts in the middle of the measurement period. Nevertheless, all daily simulation methods analysed in this paper underestimated the variance of the error.

Further research on this topic could investigate the observed underestimation of the variance of the errors of daily simulation models. Since we constrained our work to the prediction of the first two moments, the results can be adopted directly to weather futures. However, when pricing more complex derivative structures, the performance of the models might vary. Extending our research to more complex weather derivatives could be another interesting area of research. Also, we constrained our analysis to two different time instances (180 days prior and in the middle of the measurement period). Extending the analysis to a daily basis, one could analyse how well the models capture the autocorrelation of the index time series.

[6] In fact, we have also tested the Benth model with parameters $I_1 = 4$, $J_1 = 4$, which improved its overall performance in the expected value, albeit an increased error in the variance.

Appendix A. Tables

Table 7: The location of the 35 weather stations in the United States. We also state the first observation of the temperature data available.

City	State	Airport ID	Starting Date	Latitude	Longitude	Elevation (ft)
Allentown	PA	ABE	1/1/1950	40.65	−75.45	390.0
Atlanta	GA	ATL	1/1/1950	33.63	−84.43	1009.6
Baltimore	MD	BWI	1/1/1950	39.17	−76.68	147.9
Boston	MA	BOS	1/1/1950	42.37	−71.02	20.0
Bridgeport	CT	BDR	1/1/1950	41.18	−73.15	9.8
Casper	WY	CPR	1/1/1950	42.90	−106.47	5336.6
Chicago	IL	ORD	11/1/1958	41.98	−87.92	658.0
Cleveland	OH	CLE	1/1/1950	41.40	−81.85	769.8
Concord	NH	CON	1/1/1950	43.20	−71.50	346.0
Covington	KY	CVG	1/1/1950	39.05	−84.67	868.9
Dallas	TX	DFW	1/1/1950	32.90	−97.02	559.9
Des Moines	IA	DSM	1/1/1950	41.53	−93.67	956.8
Detroit	MI	DTW	1/1/1959	42.22	−83.35	637.0
Grand Island	NE	GRI	1/1/1950	40.97	−98.32	1839.4
Grand Junction	CO	GJT	1/1/1950	39.13	−108.53	4856.7
Hartford	CT	BDL	1/1/1950	41.93	−72.68	160.1
Houston	TX	IAH	6/1/1969	30.00	−95.37	95.1
Kansas City	MO	MCI	10/1/1972	39.30	−94.72	978.8
Las Vegas	NV	LAS	1/1/1950	36.08	−115.15	2126.4
Minneapolis	MN	MSP	1/1/1950	44.88	−93.23	893.8
New York City	NY	NYC	1/1/1950	40.78	−73.97	129.9
New York City	NY	LGA	1/1/1950	40.78	−73.88	11.2
Newark	NJ	EWR	1/1/1950	40.72	−74.18	9.8
Norfolk	VA	ORF	1/1/1950	36.90	−76.20	29.8
Philadelphia	PA	PHL	1/1/1950	39.87	−75.23	4.9
Pittsburgh	PA	PIT	1/1/1950	40.50	−80.23	1149.6
Portland	OR	PDX	1/1/1950	45.60	−122.62	19.0
Providence	RI	PVD	1/1/1950	41.72	−71.43	50.8
Reno	NV	RNO	1/1/1950	39.50	−119.78	4402.7
Sacramento	CA	SAC	1/1/1950	38.50	−121.50	15.1
Salt Lake City	UT	SLC	1/1/1950	40.78	−111.97	4224.0
Syracuse	NY	SYR	1/1/1950	43.12	−76.10	410.0
Tucson	AZ	TUS	1/1/1950	32.13	−110.95	2548.2
Washington	VA	DCA	1/1/1950	38.87	−77.03	9.8
Wilmington	DE	ILG	1/1/1950	39.67	−75.60	74.1

Table 8: Mean squared relative errors of the different models grouped by geographical regions.

		HDD		CDD	
		180 Days Ahead	Mid Measurement Period	180 Days Ahead	Mid Measurement Period
Midwest	Index Modelling	2.58%	0.80%	20.42%	7.40%
	Alaton Model	2.96%	0.78%	35.02%	9.61%
	Benth Model	2.69%	0.88%	26.12%	8.41%
	Spline Model	2.47%	0.69%	20.56%	6.59%
Northeast	Index Modelling	1.74%	0.71%	9.30%	4.02%
	Alaton Model	1.86%	0.69%	13.43%	4.57%
	Benth Model	1.69%	0.71%	9.64%	4.08%
	Spline Model	1.72%	0.64%	9.40%	3.90%
South	Index Modelling	5.15%	1.87%	7.34%	3.26%
	Alaton Model	6.05%	1.73%	9.70%	3.25%
	Benth Model	6.10%	2.03%	7.14%	3.05%
	Spline Model	5.06%	1.70%	7.39%	2.91%
West	Index Modelling	4.94%	2.10%	12.40%	4.89%
	Alaton Model	7.85%	1.72%	15.95%	4.63%
	Benth Model	10.40%	2.90%	13.77%	4.56%
	Spline Model	4.86%	1.74%	12.27%	4.21%

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