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# The Intraday ex ante Profitability of DAX-Futures Arbitrage for Institutional Investors in Germany - The Case of Early and Late Transactions

## 1. Introduction

The German futures market "Deutsche Terminbörse" started trading in DAX-futures contracts on November 23, 1990. The standardised delivery months are March, June, September, December. The German futures market simultaneously lists the three next delivery dates. The cash settlement always takes place at the third Friday of the delivery month.

Trading Organisation of the DAX-futures is similar to SOFFEX with the exception of market makers and construction of DAX-index. Table 1 lists some important specification features. In the first section we will introduce the mathematical construction of the DAX-index as a performance index.

In this paper we will show the impact of dividends on the value of the DAX-futures contract. Our valuation models include specifications of the German market with respect to income and corporate taxes, taxation of dividends and short sale restrictions. The arbitrage model assumes cash and carry trading without divisibility restrictions and ignores margin requirements. All investors face identical trading costs and reaction times. In the empirical analysis arbitrageurs are assumed to pay a 50% corporate tax rate corresponding to German tax law.

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**Table 1: DAX-Futures Contract Specifications**

| | |
|---|---|
| Quantity: | DM 100 times the DAX-index |
| Delivery: | Cash settlement |
| Trading: | No Market Maker |
| Tick size: | DM 50 |
| Future Trading time: | since Nov. 23, 1990<br>10.30 a.m. to 1.45 p.m.<br>since Feb. 15, 1991<br>10.30 a.m. to 3 p.m.<br>since June 3, 1991<br>9.30 a.m. to 4 p.m. |
| Index Trading time: | 10.30 a.m. to 1.30 p.m. |

See PRIGGE/SCHLAG (1992).

We take care of trading problems caused by opening and closing procedures. Our exante analysis based on intraday transaction data and minutely DAX values in 1991 and 1992 shows increasing futures market activity and falling arbitrage opportunities. Arbitragers with short sales possibilities can mainly profit from index arbitrage focusing on contracts with short time to maturity.

## 2. Construction and Replication of the DAX-index

The construction of the "Deutscher Aktienindex DAX" is different to other indexes as e. g. S&P 500, Dow Jones, Major Market or Value Line Index. The underlying of the German futures contract is a performance index which takes into account corrections for dividends or increase of capital. The DAX is calculated minutely during trading hours under the use of the prices of 30 Blue Chips listed at the Frankfurt Stock Exchange. The calculation equation of the DAX-index is:

$$DAX_t = K_{T_1} \cdot \frac{\sum_{i=1}^{30} p_{i,t} \cdot q_{i,T_1} \cdot c_{i,t}}{\sum_{i=1}^{30} p_{i,t_0} \cdot q_{i,t_0}} \cdot 1000\,, \tag{1}$$

where:

$t_0$ = 12/30/1987 as basis date.
$t$ = minutely calculation dates.
$T_1$ = last adjusting day for the stock capital of the companies.
$T_2$ = last day of dividend distribution ($T_2$ depends on $i$).
$p_{i,t}$ = stock price of company $i$ at time $t$.
$q_{i,T1}$ = stock capital of company $i$ at $T_1$.
$p_{i,t0}$ = closing price of company $i$ at 12/30/1987.
$q_{i,t0}$ = stock capital of company $i$ at 12/30/1987.
$c_{i,t}$ = correction factor for company $i$ at time $t$,

where: $$c_{i,t} = \begin{cases} 1 & \text{for} \quad t < T_2 \\ c_{i,T_2} & \text{for} \quad t \geq T_2 \end{cases},$$

$K_{T1}$ = constant linking factor. Once a year, at $T_1$, all correction factors $c_{i,T2}$ are set to 1 and the capital weights are actualised. The linking factor prevents the index from jumping.

In this paper we only take dividends into consideration. After dividend distribution at ex-dividend day $T_2$ the correction factor $c_{i,t}$ is adjusted from 1 to:

$$c_{i,T_2} = \frac{\text{stock price}_{i,cum}}{\text{stock price}_{i,cum} - \text{cash div}_i} \tag{2}$$

where:

*stock price*$_{i,cum}$ = cum-dividend closing price of company $i$.
*cash div*$_i$ = dividend after deduction of 36 % corporate tax per share.

The correction of the DAX implies the reinvestment of the cash dividend in the shares of the distributing company. Since the marginal tax rate of most of the investors differs from 36 %, they will have to adjust their investment in the index portfolio in order to duplicate the DAX.
The amount to be reinvested is given by the number of shares of the corresponding company in the index portfolio. For calculation reasons the Frankfurt Stock Exchange daily publishes the weighting factors $G_{i,t}$ of the 30 Blue Chips [1].

$$G_{i,t} = \frac{q_{i,T_1}}{\sum_{i=1}^{30} q_{i,t_0}} \cdot 100 \cdot K_{T_1} \cdot c_{i,t} \tag{3}$$

During trading hours the Frankfurt Stock Exchange calculates the DAX using the following formula:

$$DAX_t = \frac{\sum_{i=1}^{30} p_{i,t} \cdot G_{i,t}}{\sum_{i=1}^{30} p_{i,t_0} \cdot \frac{q_{i,t_0}}{\sum_{j=1}^{30} q_{j,t_0}} \cdot 100} \cdot 1000\,, \tag{4}$$

where:

$\sum_{j=1}^{30} q_{j,t_0}$ = stock capital sum of the 30 companies at 12/30/1987.

From (4) and the historical data we get:

$$DAX_t = \frac{\sum_{i=1}^{30} p_{i,t} \cdot G_{i,t}}{29356.73} \cdot 1000\,. \tag{5}$$

Arbitragers are mainly interested in the number of shares necessary to duplicate the DAX-index. Independent from actual stock prices the weighting factors $G_{i,t}$ give the numbers of shares to purchase:

$$\text{number}_i = G_{i,t} \cdot \text{multiplier}, \tag{6}$$

where:

number$_i$ = number of company $i$'s shares to construct the index portfolio.

multiplier = constant multiplier depending on the investment amount.

$$\text{multiplier} = \frac{\text{amount}}{\sum_{i=1}^{30} p_{i,t} \cdot G_{i,t}}. \tag{7}$$

where:

amount = amount to invest in the index portfolio.

An arbitrager, who is interested in replicating the underlying of the DAX-Futures contract, needs to invest hundred times the face value of the DAX-index. The amount is given by:

$$\text{amount} = 100 \cdot DAX_t = \frac{\sum_{i=1}^{30} p_{i,t} \cdot G_{i,t}}{29356.73} \cdot 100000. \tag{8}$$

Using equation (7) in case of the DAX-futures contract we get:

$$\text{multiplier} = \frac{\frac{\sum_{i=1}^{30} p_{i,t} \cdot G_{i,t}}{29356.73} \cdot 100000}{\sum_{i=1}^{30} p_{i,t} \cdot G_{i,t}} = 3.406373939 \tag{9}$$

In the case of dividend payment [2] the arbitrager has to reinvest 64 % of the gross dividends in the stocks of the distributing company. The distributed gross dividends depend on the company $i$'s number of shares belonging to the index portfolio.

$$\begin{aligned} DIV_i^{gr} &= number_i \cdot gross\ div_i \\ &= 3.406373939 \cdot G_{i,t} \cdot gross\ div_i \end{aligned} \tag{10}$$

where:

*gross div$_i$* = gross dividend of company $i$ per share.

$DIV_i^{gr}$ = gross dividend of company i corresponding to the number of shares in the index-portfolio.

Arbitragers are interested in the value of the index portfolio or the value of the index replicating portfolio at any time in comparison with the DAX-index. This value depends on the investor's personal tax rate $s$.

$$FW_T\left(S_t^{Pf}\right) = S_T + FW_T\left(\sum_{i=1}^{30} DIV_i^{\Delta}\right) = S_T + \Delta DIV_T, \tag{11}$$

where:

$FW_T(.)$ = final value at time $T$.

$S_t^{Pf}$ = market value of the index replicating portfolio at time $t$.

$DIV_i^{\Delta}$ = $(0.36\text{-}s) \cdot DIV_i^{gr}$

The investor has to reinvest 64 % of gross-dividends in the dividend distributing company's stocks. The after-tax dividend only equals this investment when the investor's tax rate reaches 36%. In order to maintain a risk-free arbitrage position the arbitrager has to invest or desinvest the difference into risk-free bonds. The final value of the index portfolio contains two parts: the index portfolio and the difference dividend account. The value of this account depends on the personal tax rate [3]:

$$\Delta DIV_T > 0 \Leftrightarrow \text{marginal tax rate} < 36\%$$

$$\Delta DIV_T = 0 \Leftrightarrow \text{marginal tax rate} = 36\%$$

$$\Delta DIV_T < 0 \Leftrightarrow \text{marginal tax rate} > 36\%.$$

We get the final value $\Delta DIV_T$ at time $T$ of all difference dividend cash flows between $t$ and $T$ :

$$\Delta DIV_T = FW_T\left(\sum_{i=1}^{30} DIV_i^{\Delta}\right) = \sum_{i=1}^{30}(DIV_i^{\Delta} \cdot e^{r(1-s)(T-t_i)}) \tag{12}$$

where: $t_i$ = ex dividend day ($T_2$) of company $i$.

## 3. Fundamental No-Arbitrage Equation

Stock index futures can be priced by arbitrage arguments. The fundamental equation for stock index futures was published by CORNELL/ FRENCH (1983) and MODEST/SUNDARESAN (1983). Under the assumptions that dividends and interest rates are nonstochastic, markets are perfect and there are no taxes, the fair value of a stock index futures contract is:

$$F_t^* = S_t \cdot e^{r(T-t)} - FW_t^T(D) \, , \tag{13}$$

where:

| | | |
|---|---|---|
| $F_t^*$ | = | futures (fair) value at time t. |
| $S_t$ | = | stock index at time t. |
| $r$ | = | risk-free continuously compounded interest rate. |
| $T$ | = | maturity day of the futures contract. |
| $FW_t^T(D)$ | = | time $T$ value of dividends paid on the component stocks in the index portfolio between $t$ and $T$. |

Some authors including the authors of this article indicated the difference to the DAX-futures contract [4]. Neglecting taxes on dividends leads to the valuation formula of the DAX-futures contract.

$$F_t^* = S_t \cdot e^{r(T-t)} \, , \tag{14}$$

In this paper we show the importance of dividends calculating the futures' fair value.

## 4. Taxes and Pricing of the DAX-Futures

### 4.1 Income Taxes

The German taxation system taxes all income cash flows with the same rate. Let $s$ denote the marginal tax rate of an arbitrager. In this section we assume private investors paying taxes only on dividends and interest earnings [5].

Suppose the futures price declines the time $t$ fair value. The arbitrager undertakes the following trades:

- borrow $S_t$ at the risk-free rate and buy the index portfolio $S_t$.
- sell one futures contract (short).

The net cash flow of these transactions is zero. The arbitrager closes this position at the maturity day of the futures and receives the following cash flow:

- $-S_t \cdot e^{r(1-s)(T-t)}$ redemption of debt and after tax interest on the risk-free debt.
- $S_T$ from the stock.
- $F_t - S_T$ from the cash settlement of the short futures position.
- $\Delta DIV_T$ value of the risk-free investment of the difference dividend at time $T$. ($\Delta DIV_T < 0$ when $s > 0.36$).

In an arbitrage-free economy the net cash flow of this closing out must equal to zero (or in this case: In order to get an arbitrage gain it must be greater than zero):

$$F_t - S_t \cdot e^{r(1-s)(T-t)} + \Delta DIV_T = 0 \, .$$

Hence the fair value $F_t^*$ of the DAX-futures contract is:

$$F_t^* = S_t \cdot e^{r(1-s)(T-t)} - \Delta DIV_T \tag{15}$$

If the futures price exceeds the value given by equation (15) the arbitrager has to realise the reverse transactions. This reverse cash and carry strategy implies the shorting of the DAX-index portfolio. Since in Germany private investors cannot short stocks, this strategy is only practicable if investors already own this portfolio and want to hold it until day $T$. We will discuss this problem in section 4.3. For some selected investor's tax brackets we get:

$$s=0\% \Leftrightarrow F_t^* = S_t \cdot e^{r(T-t)} - 0.36 \sum_{i=1}^{30} \left( DIV_i^{gr} \cdot e^{r(T-t_i)} \right)$$

$$s=36\% \Leftrightarrow F_t^* = S_t \cdot e^{r \cdot 0.64 \cdot (T-t)}$$

$$s=50\% \Leftrightarrow F_t^* = S_t \cdot e^{r \cdot 0.5 \cdot (T-t)} + 0.14 \sum_{i=1}^{30} \left( DIV_i^{gr} \cdot e^{r \cdot 0.5 \cdot (T-t_i)} \right)$$

## 4.2 Corporate Taxes

In Germany institutional investors and private investors are differently taxed. In this section we consider a tax rate $k$ of 50% on all gains belonging to index arbitrage. An arbitrager undertaking the cash and carry arbitrage transactions of the previous section has to face the following after tax cash flows at time $T$:

- $-S_t \cdot e^{r(1-k)(T-t)}$ — redemption of debt and after tax interest on the risk-free debt.
- $S_T$ — from the stock.
- $-(S_T - S_t) \cdot k$ — tax payment or tax shield caused by realised capital gains or losses.
- $F_t - S_T$ — from the cash settlement of the short futures position.
- $-(F_t - S_T) \cdot k$ — tax payment or tax shield caused by the cash settlement of the short futures position.
- $\Delta DIV_T$ — value of the risk-free investment of the difference dividend at time $T$. ($\Delta DIV < 0$ when $k = 0.5$).

The net after tax cash flow of the closed arbitrage portfolio is:

$$\begin{aligned} & F_t - S_t \cdot e^{r(1-k)(T-t)} - (S_T - S_t) \cdot k - (F_t - S_T) \cdot k + \Delta DIV_T \\ &= F_t - S_t \cdot e^{r(1-k)(T-t)} - k \cdot (F_t - S_t) + \Delta DIV_T \\ &= (1-k) \cdot F_t - S_t \cdot \left( e^{r(1-k)(T-t)} - k \right) + \Delta DIV_T \end{aligned}$$

In order to prevent from index arbitrage this cash flow has to be equal to zero. As fair value of the DAX-futures contract for institutional investors we obtain:

$$F_t^* = \frac{S_t \cdot \left( e^{r(1-k)(T-t)} - k \right) - \Delta DIV_T}{(1-k)}$$

or

$$F_t^* = \frac{S_t \cdot \left( e^{r(0.5)(T-t)} - 0.5 \right) + \sum_{i=1}^{30} \left( 0.14 \cdot DIV_i^{gr} \cdot e^{r(0.5)(T-t_i)} \right)}{0.5} \cdot \qquad (16)$$

## 4.3 Reverse Cash and Carry Arbitrage and Stock Short Sales

If the costs of purchasing the stock and delivering it at time $T$ exceed the futures price $F_t$ the arbitrager reverses his arbitrage position.

- short the index portfolio $S_t$ and invest the proceeds $S_t$ at the risk-free rate.
- buy one futures contract.

As mentioned in section 4.1 in Germany short sales are not possible. But there is the alternative to borrow the stocks from $t$ to $T$, sell it at $t$ and repurchase the stocks at time $T$. For example, the "Deutscher Kassenverein" offers as a security loan office a clearing service for stock loaning [6].
We assume that institutional investors can use this service without any restrictions and will receive the

full proceeds of the borrowed stocks [7]. We assume index portfolio lending costs of 2.5% p.a. of the market value of the index portfolio. The lending costs are paid at the end of the lending period $[t,T]$. Let $SL_T$ denote the total lending costs at $T$:

$$SL_T \cdot (1-k) = S_t \cdot l \cdot (1-k) \cdot (T-t) \qquad (17)$$

where:
$SL_T$ = total pre-tax lending costs at time $T$.
$l$ = annual lending rate.

At time $t$ the arbitrager undertakes the following transactions:
- Borrow the index portfolio $S_t$.
- Sell the index portfolio $S_t$ and invest the proceeds $S_t$ at the risk-free rate.
- Buy one futures contract.

The market value of this portfolio is zero. A dividend payment at time $t_i$ implies the following transactions:
- Pay the gross dividend $DIV_i^{gr}$ to the lender.
- Borrow (and sell short) additional stocks of company $i$ (amount $DIV_i^{cash} = 0.64 \cdot DIV_i^{gr}$).
- Invest or borrow the net cash flow of these additional transactions at the risk-free rate.

Since the investor additionally has to pay the lending fee at time $T$, the total after tax cash flow of the reverse cash and carry arbitrage strategy is:

- $S_t \cdot e^{r(1-k)(T-t)}$ proceeds from the risk-free investment.
- $-S_T$ rebuy the index portfolio.
- $FW_T\left(\sum_{i=1}^{30} DIV_i^{cash}\right)$ final value from the additional risk-free investment.
- $-FW_T\left(\sum_{i=1}^{30} DIV_i^{gr} \cdot (1-k)\right)$ final value of the after tax dividend payments to the stock lender.
- $-(S_t - S_T) \cdot k$ taxation of capital gains or tax reduction for capital losses.
- $S_T - F_t$ cash settlement of the futures contract.
- $-(S_T - F_t) \cdot k$ taxation of cash settlement gains or tax reduction for settlement losses.
- $-SL_T \cdot (1-k)$ after tax lending fee [8].

The net cash flow of these transactions is:

$$-F_t + S_t \cdot e^{r(1-k)(T-t)} + (S_T - S_t) \cdot k$$
$$+ (F_t - S_T) \cdot k - \Delta DIV_T - SL_T \cdot (1-k)$$

$$= -F_t + S_t \cdot e^{r(1-k)(T-t)} + k \cdot (F_t - S_t)$$
$$-\Delta DIV_T - SL_T \cdot (1-k)$$

$$= -(1-k) \cdot F_t + S_t \cdot \left(e^{r(1-k)(T-t)} - k\right)$$
$$-\Delta DIV_T - SL_T \cdot (1-k)$$

In an arbitrage-free market this cumulated cash flow matches zero. Therefore we can calculate the futures' value:

$$(1-k) \cdot F_t - S_t \cdot \left(e^{r(1-k)(T-t)} - k\right)$$
$$+\Delta DIV_T + SL_T \cdot (1-k) = 0 \quad \Leftrightarrow$$
$$F_t^* = \frac{S_t \cdot \left(e^{r(1-k)(T-t)} - k\right) - \Delta DIV_T - SL_T \cdot (1-k)}{(1-k)}$$

Inserting $k = 0.5$ we get:

$$F_t^* =$$
$$\frac{S_t \cdot \left(e^{r(0.5)(T-t)} - 0.5\right) + 0.14 \cdot \sum_{i=1}^{30}\left(DIV_i^{gr} \cdot e^{r(0.5)(T-t_i)}\right)}{0.5} - SL \qquad (18)$$

The institutional arbitrager receives an arbitrage signal if the actual futures price $F_t$ is less than the theoretical futures price $F_t^*$: $F_t < F_t^*$.

## 5. Empirical Results for Institutional Investors

### 5.1 Transaction Costs

It's not possible to determine the exact transaction costs belonging to the index arbitrage. Institutional investors face brokerage of 0.06% of the stock value. The futures market commissions are negligible. In fact, when neglecting the time value of the costs and the additional costs as market impact costs, the arbitrager has to pay minimum total transaction costs of 0.12% of stock value.
In this paper we assume proportional transaction costs of 0.5%, respectively 1%, for institutional investors. The total after tax-transaction costs are defined as [9]:

$$TK_t = S_t \cdot (1-k) \cdot \alpha \, , \qquad (19)$$

where:

$TK_t$ = total after-tax transaction costs of index arbitrage.

$\alpha$ = transaction costs factor.

Arbitrage is indicated when the absolute difference between the futures price and its fair value minus the total transaction costs exceeds zero.

$$|F_t - F_t^*| - TK_t > 0 \qquad (20)$$

### 5.2 Data

In this paper we focus on all trading days of the futures market between January 1991 and December 1992. We compare the results of two different samples. In the first sample we use all transactions between 10.35 a.m. and 1.30 p.m., the closing time of the Frankfurt Stock Exchange. In the second sample we exclude all transactions during the first fifteen minutes of stock trading before 10.45 a.m. and during the last five minutes after 1.25 p.m.
As the Frankfurt Stock Exchange publishes the opening notation of the DAX-index when the opening prices of at least 15 stocks and 70 % of index capitalisation are available (by using the last days closing prices of the other index stocks), the early index notation do not show the actual market value of the index portfolio. CHUNG (1991) has pointed out the problems corresponding to early trades in stock market. Arbitrage profits based on index date can overestimate the realisable arbitrage gains during the opening procedure of the stock market.
Since the stocks at the Frankfurt Stock Exchange are traded directly without the use of market makers the settlement of last trades of the day are not guaranteed; therefore we exclude the last five minutes of daily trading.
The results presented in this paper [10] are based on the DTB Financial Data Disseminator Service (DT-BFDD) and the "Deutsche Kapitalmarktdatenbank DFDB Karlsruhe - Mannheim - Aachen". We obtained the DAX values directly from the Frankfurt Stock Exchange. The daily listing of the 3 month FIBOR (new) is used to compute the risk-free interest rate.
We divided the data into two periods of the same length. One period is the year 1991, the other is the year 1992. We counted 241 trading days in 1991 and 246 trading days in 1992 [11]. The Frankfurt Futures Market simultaneously lists three contracts with different maturity. We divided the contracts into three groups. The first group always contains the contract with the shortest time to maturity. The second group consists of all transactions belonging to the contract with medium time to maturity. Finally, the transactions according to the contract with the longest time to maturity form the third group. Table 2 shows the frequency of transactions of these three groups.
Since the DAX-futures contract with the shortest time to maturity is the most actively traded and has most of liquidity, we exclusively calculate ex ante

**Table 2: Distribution of the transactions[a] according to the time to maturity**

| Contract | all (487 trading days) | | 1991 (241 trading days) | | 1992 (246 trading days) | |
|---|---|---|---|---|---|---|
| 1 (shortest maturity) | 220901 | (87.2%)[b] | 88359 | (84.9%)[b] | 132542 | (88.8%)[b] |
| 2 | 27463 | (10.8%)[b] | 13152 | (12.6%)[b] | 14311 | (9.6%)[b] |
| 3 (longest maturity) | 4939 | (1.9%)[b] | 2586 | (2.5%)[b] | 2353 | (1.6%)[b] |
| sum | 253303 | | 104097 | | 149206 | |

[a] transactions between 10.35 a.m. and 1.30 p.m..
[b] percentage of total transactions according to the time period.

**Table 3: Number of ex-post arbitrage signals neglecting dividends (cash and carry arbitrage vs. reverse cash and carry arbitrage)**

| Contract | all | transaction costs 1 % Cash and Carry | transaction costs 1 % Reverse | transaction costs 0.5 % Cash and Carry | transaction costs 0.5 % Reverse |
|---|---|---|---|---|---|
| 1 | 220901 | 112(0.1%)[a] | 29087(13.2%)[a] | 3641(1.6%)[a] | 79423(36.0%)[a] |
| 2 | 27463 | 1(0.0%)[a] | 13139(47.8%)[a] | 242(0.9%)[a] | 19194(69.9%)[a] |
| 3 | 4939 | 0(0.0%)[a] | 4299(87.0%)[a] | 1(0.0%)[a] | 4750(96.2%)[a] |
| sum | 253303 | 113(0.0%)[a] | 46525(18.4%)[a] | 3884(1.5%)[a] | 103367(40.8%)[a] |

[a] The percentage out of the total number of observations according to the time period.

**Table 4: Number of ex-post arbitrage signals corrected for dividend distribution (cash and carry arbitrage vs. reverse cash and carry arbitrage)**

| Contract | all | transaction costs 1 % Cash and Carry | transaction costs 1 % Reverse | transaction costs 0.5 % Cash and Carry | transaction costs 0.5 % Reverse |
|---|---|---|---|---|---|
| 1 | 220901 | 110(0.0%)[a] | 53339(24.1%)[a] | 1448(0.7%)[a] | 101219(45.8%)[a] |
| 2 | 27463 | 1(0.0%)[a] | 17685(64.4%)[a] | 34(0.9%)[a] | 22516(82.0%)[a] |
| 3 | 4939 | 0(0.0%)[a] | 4751(96.2%)[a] | 0(0.0%)[a] | 4931(99.8%)[a] |
| sum | 253303 | 111(0.0%)[a] | 75775(29.9%)[a] | 1482(0.6%)[a] | 128666(50.8%)[a] |

[a] The percentage out of the total number of observations according to the time period.

arbitrage profits corresponding to this contract in the next sections of this paper.

As table 2 shows the German futures market is strongly expanding. The importance of the contract 1 with respect to the other contracts increased from 1991 to 1992. The average number of transactions per trading day belonging to contract 1 increased from 367 to 539.

In table 3 we split the arbitrage signals into cash and carry and reverse cash and carry signals. When assuming no dividend payments, the reverse cash and carry strategy is dominant. We first noticed

considerable cash and carry arbitrage opportunities assuming 0.5 % transaction costs. Most of the arbitrage possibilities were signalled when focusing on the short maturity contract.
The results in table 4 were based on formula (16). As the dividend distribution in this case supports reverse cash and carry arbitrage and discourages cash and carry possibilities we notice with respect to table 3 a falling number of cash and carry signals and an increasing number of reverse cash and carry signals.

## 5.3 Reverse Cash and Carry Arbitrage

In this section we focus on the ex-ante index arbitrage gain of a reverse cash and carry strategy. We assume a 1 minute or 2 minutes time lag for realising the arbitrage transactions after observing ex-post mispricings based on:

$$F_t^* - F_t - TK_t > 0 \ . \tag{21}$$

The futures market is regarded as efficient if the expected value of ex-ante reverse cash and carry arbitrage is not positive:

$$E\left(F_{t+\Delta}^* - F_{t+\Delta} - TK_{t+\Delta} \;\middle|\; F_t^* - F_t - TK_t > 0\right) \leq 0 \ , \tag{22}$$

where:

$\Delta$ = reaction time lag after detecting a mispricing signal (one or two minutes).

### 5.3.1 Without Security Lending

The results of the ex-ante profit based on reverse cash and carry arbitrage are presented in table 5 and table 6. The following tables contain the ex ante arbitrage profits based on formula (22) per point of index. The figures have to be multiplied times 100 in order to calculate the arbitrage profit per DAX-futures contract.
Table 5 shows significant positive arbitrage profits, however falling from 1991 to 1992. The number of contract mispricings is also falling. We see that the effect of different reaction time is negligible. Only the high number of arbitrage transactions in the first contract indicates violations of market efficiency. The influence of excluding early and late transactions on the number of arbitrage possibilities and arbitrage profits in table 5 is negligible.
It is interesting that cutting the transaction costs in table 6 does not contribute to rising arbitrage profits but to an increasing number of arbitrage possibilities. We can see: Quantity does not imply quality. Again there is no effect of early transactions on arbitrage profits.

**Table 5: Ex-ante arbitrage profit (avg.). reverse cash and carry arbitrage, 1% transaction costs, correction for dividends**

| Sample[a] | Time Lag[b] | all Transactions 1991 to 1992 avg.[c] | t-st.[d] | min.[e] | max.[f] | no.[g] | 1991 avg.[c] | t-st.[d] | min.[e] | max.[f] | no.[g] | 1992 avg.[c] | t-st.[d] | min.[e] | max.[f] | no.[g] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| all | 1 | 6.32 | 292.41 | -7.15 | 24.29 | 46813 | 7.19 | 272.86 | -7.15 | 24.29 | 30024 | 4.77 | 138.28 | -5.71 | 15.78 | 16789 |
| | 2 | 6.27 | 285.34 | -7.15 | 24.29 | 46239 | 7.17 | 268.59 | -7.15 | 24.29 | 29634 | 4.68 | 132.77 | -7.15 | 15.78 | 16605 |
| no early or late | 1 | 6.31 | 289.07 | -7.15 | 24.29 | 45740 | 7.18 | 269.87 | -7.15 | 24.29 | 29317 | 4.77 | 136.62 | -5.71 | 15.78 | 16423 |
| | 2 | 6.28 | 282.55 | -7.15 | 24.29 | 45307 | 7.17 | 266.16 | -7.15 | 24.29 | 29064 | 4.68 | 131.20 | -7.15 | 15.78 | 16243 |

[a] Corresponding sample including all transactions per trading day or only transactions between 10.45 a.m. and 1.25 p.m..
[b] Assumed reaction time of ex ante index arbitrage. (One or two minutes).
[c] Arithmetic mean of ex ante arbitrage profit per index point. To get the profit per DAX-futures contract multiply avg. times 100.
[d] T-statistic corresponding to the hypothesis in Formula (22).
[e] Minimum ex ante arbitrage profit per index point.
[f] Maximum ex ante arbitrage profit per index point.
[g] Number of ex ante arbitrage possibilities.

**Table 6: Ex-ante arbitrage profit (avg.), reverse cash and carry arbitrage, 0.5% transaction costs, correction for dividends**

| Sample | Time Lag | all Transactions 1991 to 1992 avg. | t-st. | min. | max. | no. | 1991 avg. | t-st. | min. | max. | no. | 1992 avg. | t-st. | min. | max. | no. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| all | 1 | 6.30 | 339.29 | -3.90 | 28.12 | 88622 | 7.43 | 283.94 | -3.72 | 28.12 | 49520 | 4.88 | 202.26 | -3.90 | 20.09 | 39102 |
| | 2 | 6.27 | 333.29 | -4.12 | 28.12 | 87541 | 7.44 | 281.41 | -3.79 | 28.12 | 48738 | 4.80 | 196.24 | -4.12 | 20.09 | 38803 |
| no early or late | 1 | 6.30 | 335.29 | -3.90 | 28.12 | 86639 | 7.43 | 280.62 | -3.72 | 28.12 | 48303 | 4.87 | 199.98 | -3.90 | 20.09 | 38336 |
| | 2 | 6.27 | 329.76 | -4.12 | 28.12 | 85796 | 7.44 | 278.65 | -3.79 | 28.12 | 47778 | 4.79 | 193.94 | -4.12 | 20.09 | 38018 |

## 5.3.2 With Security Lending

In section 4.1 we referred to the short sale restrictions in Germany. In this part we correct the reverse arbitrage profits for stock lending. This implies higher transaction costs.
Compared with table 5, table 7 reports reduced but still positive arbitrage profits. We also see the decreased number of arbitrage transactions. An arbitrager who had taken advantage of every arbitrage possibility in 1992 buying one DAX-contract (1% transaction costs, reaction time one minute, contract 1) would have hypothetically earned about 8 million Marks [12] without lending stocks and 1 million Marks [13] using stock lending. In this case cutting the transaction costs leads to strongly increased arbitrage profits. An arbitrager who has the possibility to undertake short sales without incurring extraordinary transaction costs will profit from reverse index arbitrage in Germany. But the statistics indicate falling profits.

## 5.4 Cash and Carry Arbitrage

In analogy to section 5.3 of this paper we examine the ex-ante arbitrage profit of cash and carry index arbitrage. In this section we do not have to consider

**Table 7: Ex-ante arbitrage profit (avg.), reverse cash and carry arbitrage, 1% transaction costs, correction for dividends and security lending**

| Sample | Time Lag | all Transactions 1991 to 1992 avg. | t-st. | min. | max. | no. | 1991 avg. | t-st. | min. | max. | no. | 1992 avg. | t-st. | min. | max. | no. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| all | 1 | 3.15 | 220.16 | -6.82 | 14.55 | 26335 | 3.68 | 219.17 | -3.84 | 14.55 | 19658 | 1.60 | 99.25 | -6.82 | 6.51 | 6677 |
| | 2 | 3.10 | 211.23 | -6.93 | 14.55 | 26041 | 3.62 | 211.37 | -6.69 | 14.55 | 19460 | 1.54 | 88.25 | -6.93 | 6.51 | 6581 |
| no early or late | 1 | 3.12 | 204.35 | -6.82 | 14.55 | 22403 | 3.63 | 202.36 | -3.84 | 14.55 | 16789 | 1.59 | 95.74 | -6.82 | 5.78 | 5614 |
| | 2 | 3.07 | 195.51 | -6.93 | 14.55 | 22215 | 3.59 | 194.79 | -6.69 | 14.55 | 16677 | 1.54 | 83.59 | -6.93 | 5.78 | 5538 |

**Table 8: Ex-ante arbitrage profit (avg.), reverse cash and carry arbitrage, 0.5% transaction costs, correction for dividends and security lending**

| Sample | Time Lag | all Transactions 1991 to 1992 avg. | t-st. | min. | max. | no. | 1991 avg. | t-st. | min. | max. | no. | 1992 avg. | t-st. | min. | max. | no. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| all | 1 | 4.97 | 310.43 | -4.01 | 18.37 | 43636 | 5.50 | 275.74 | -4.01 | 18.37 | 30502 | 3.76 | 163.40 | -3.43 | 10.80 | 13134 |
| | 2 | 4.93 | 302.10 | -3.88 | 18.37 | 43095 | 5.46 | 270.52 | -3.84 | 18.37 | 30137 | 3.69 | 154.64 | -3.88 | 10.80 | 12958 |
| no early or late | 1 | 4.93 | 286.87 | -3.38 | 18.37 | 37273 | 5.41 | 254.02 | -3.38 | 18.37 | 26400 | 3.78 | 150.67 | -3.13 | 10.12 | 10873 |
| | 2 | 4.90 | 278.87 | -3.88 | 18.37 | 36919 | 5.39 | 249.09 | -3.59 | 18.37 | 26172 | 3.71 | 142.12 | -3.88 | 10.12 | 10747 |

**Table 9: Ex-ante arbitrage profit (avg.), cash and carry arbitrage, 1% transaction costs, correction for dividends**

| Sample | Time Lag | all Transactions 1991 to 1992 | | | | | 1991 | | | | | 1992 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | avg. | t-st. | min. | max. | no. | avg. | t-st. | min. | max. | no. | avg. | t-st. | min. | max. | no. |
| all | 1 | 2.03 | 4.44 | -7.03 | 12.05 | 110 | 2.73 | 4.32 | -7.03 | 12.05 | 77 | 0.40 | 1.79 | -3.38 | 1.74 | 33 |
| | 2 | -0.77 | -2.31 | -7.32 | 12.05 | 110 | -0.99 | -2.16 | -7.32 | 12.05 | 77 | -0.26 | -0.90 | -4.87 | 1.74 | 33 |
| no early or late | 1 | 2.51 | 4.11 | -7.03 | 12.05 | 81 | 2.73 | 4.32 | -7.03 | 12.05 | 77 | -1.70 | -1.73 | -3.38 | 0.39 | 4 |
| | 2 | -1.12 | -2.51 | -7.32 | 12.05 | 81 | -0.99 | -2.16 | -7.32 | 12.05 | 77 | -3.50 | -2.73 | -4.87 | 0.35 | 4 |

**Table 10: Ex-ante arbitrage profit (avg.), cash and carry arbitrage, 0.5% transaction costs, correction for dividends**

| Sample | Time Lag | all Transactions 1991 to 1992 | | | | | 1991 | | | | | 1992 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | avg. | t-st. | min. | max. | no. | avg. | t-st. | min. | max. | no. | avg. | t-st. | min. | max. | no. |
| all | 1 | 0.72 | 10.52 | -3.90 | 15.98 | 1374 | 1.30 | 9.83 | -3.90 | 15.98 | 614 | 0.25 | 4.40 | -3.62 | 5.99 | 760 |
| | 2 | 0.13 | 1.95 | -3.90 | 15.98 | 1371 | 0.62 | 5.06 | -3.90 | 15.98 | 601 | -0.26 | -3.89 | -3.77 | 5.99 | 770 |
| no early or late | 1 | 0.61 | 8.86 | -3.90 | 15.98 | 1302 | 1.34 | 9.87 | -3.90 | 15.98 | 594 | 0.00 | -0.04 | -3.62 | 4.65 | 708 |
| | 2 | 0.02 | 0.37 | -3.90 | 15.98 | 1302 | 0.68 | 5.47 | -3.90 | 15.98 | 583 | -0.51 | -9.20 | -3.77 | 4.65 | 719 |

short sales restrictions. In an arbitrage-free economy the expected value of the ex ante arbitrage profit should not exceed zero.

(23)

Table 9 reports no more noticeable arbitrage profits with respect to one minute reaction time. Halving the transaction costs in table 10 illustrates increasing arbitrage possibilities. Cutting off early and late transactions only leaves 4 ex ante arbitrage signals in 1992 including an arithmetic loss of 170 DM per contract.

We only mind arbitrage profits related to contract 1 assuming one minute reaction time. In contrast to the reverse cash and carry arbitrage table 10 displays the importance of the arbitrager's reaction time. The average profit reacting within one minute decreased from 1991 to 1992 from 130 DM per contract to 25 DM per contract.

When we exclude transactions before 10.45 a.m. and after 1.25 p.m. no more arbitrage profits in 1992 are left. As Table 10 shows, excluding only 52 early and late transactions of all 760 possibilities in 1992 leads to arbitrage losses. These 52 transactions must have been the most profitable ex ante arbitrage possibilities.

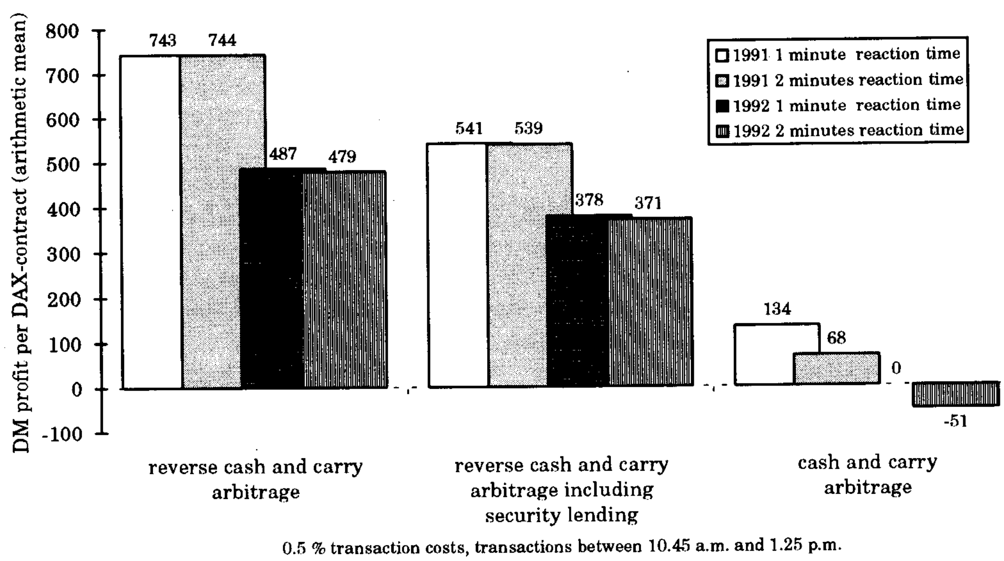
The analysis of the cash and carry arbitrage demonstrates market activity. One reason might be that arbitragers are not able to short stocks and therefore only focus on cash and carry arbitrage.

## 6. Summary of results

In this paper we introduced a valuation model of the DAX-index futures contract traded on the Frankfurt Futures Market. We corrected the futures value for dividend distributions assuming 50 % corporate tax rate.

We take care of short sale restrictions and corporate taxes. Our analysis confirms increasing futures market activity mainly according to contracts with short time to maturity and a falling number of arbitrage possibilities from 1991 to 1992. Figure 1 illustrates the main results of this paper.

**Figure 1: Comparing ex ante Index Arbitrage Profits**




The results of the ex-ante analysis of the reverse cash and carry arbitrage were:

- From 1991 to 1992 positive but decreasing arbitrage profits (see figure 1) and falling number of arbitrage possibilities.
- Transaction costs and reaction time have negligible effects on arbitrage profits.
- Security lending diminishes profits, but profits stay significantly positive.
- Excluding daily early and late transactions has negligible effect on arbitrage possibilities and profits.
- Arbitragers who can short stocks will profit from index arbitrage.

The results of the ex-ante analysis of the cash and carry arbitrage were:

- General lower profits from cash and carry arbitrage.
- From 1991 to 1992 positive but decreasing arbitrage profits.
- Transaction costs and reaction time have consequences on profits.
- Only arbitrage possibilities belonging to contracts with short time to maturity.
- Only arbitragers with 0.5 % transaction costs and reaction time of one minute in 1992 earn from index arbitrage.
- Excluding transactions before 10.45 a.m. and after 1.25 p.m. erases all arbitrage profits in 1992.

Generally we pointed out increasing synchronisation between the German cash and futures market.

**Footnotes**

[1] See JANSSEN/RUDOLPH (1992).
[2] In Germany usually dividends are payed once a year.
[3] We assume positive dividend payments.
[4] See BAMBERG/RÖDER (1992, 1993), BÜHLER/KEMPF (1992) or PRIGGE/SCHLAG (1992).
[5] For private investors earnings of the cash settlement and early unwinding of the DAX-futures contract are not taxed. We further assume no taxation on capital gains. In Germany only capital gains with holding periods under 6 months are taxed. See HOFFMANN/RAMKE (1990) p. 80 or BRÖKER (1991) or JUNG/REDANZ (1993).
[6] See DEUTSCHER KASSENVEREIN AG (1992), HOHMANN (1991), BÜHLER/KEMPF (1992) p. 22.
[7] This points out a critical part of the German stock borrowing system. For technical reasons it is in practice not possible to lend the stocks of the Allianz AG, one of the stocks in the DAX with the highest weight.
[8] We neglect additional lending fee caused by the dividend payment.
[9] Since private investors cannot shield transaction costs the corresponding formula for private investors is: $TK_t = S_t \cdot \alpha$.
[10] We would like to thank Prof. Dr. Bühler and Prof. Dr. Göppl and the Deutsche Börse AG providing us with the requested data.
[11] We had 10 days of missing data.
[12] $4.77 \cdot 16789 \cdot 100 = 8008353$ DM. See Table 5.
[13] $1.60 \cdot 6677 \cdot 100 = 1068320$ DM. See Table 7.

**References**

BAMBERG, G. und K. RÖDER (1992): "Arbitrage am DAX-Futures Markt unter Berücksichtigung von Steuern", Arbeitspapiere zur Mathematischen Wirtschaftsforschung, Heft 111/1992.

BAMBERG, G. und K. RÖDER (1993): "Arbitrage institutioneller Anleger am DAX-Futures Markt unter Berücksichtigung von Körperschaftssteuern und Dividenden", Arbeitspapiere zur Mathematischen Wirtschaftsforschung, Heft 113/1993.

BRÖKER, K. (1991): "Neue DTB-Produkte", Anlage Praxis, pp. 20-21.

BÜHLER, W. und A. KEMPF (1993): "Der DAX-Future: Kursverhalten und Arbitragemöglichkeiten", Zentrum für Europäische Wirtschaftsforschung GmbH, Discussion Paper no. 93-0.

CHUNG, P. (1991): "A Transactions Data Test of Stock Index Futures Market Efficiency and Index Arbitrage Profitability", Journal of Finance 66, pp. 1791-1809.

CORNELL, B. and K. FRENCH (1983a): "The Pricing of Stock Index Futures", Journal of Futures Markets 3, pp. 1-14.

CORNELL, B. and K. FRENCH (1983b): "Taxes and the Pricing of Treasury Bill Futures Contracts", Journal of Finance 38, pp. 675-694.

DEUTSCHER KASSENVEREIN AG (1992): "Technische Richtlinien für Leihgeschäfte - Banken", Frankfurt.

HOFFMANN, P. und R. RAMKE (1990): "Finanzinnovationen an der Deutschen Terminbörse (DTB): Chancen und Risiken des Handels mit Optionen und Finanzterminkontrakten", Erich Schmidt Verlag, Grundlagen und Praxis des Bank- und Börsenwesens, Bd. 23, Berlin.

HOHMANN, R. (1991): "Der Einfluss der Wertpapierleihe auf die Bewertung des DAX-Futures", Kapitalanlagen Wirtschaft Recht Steuern 4, pp. 574-582.

JANSSEN, B. und B. RUDOLPH (1992): "Der Deutsche Aktienindex DAX, Konstruktion und Anwendungsmöglichkeiten", Frankfurter Wertpapierbörse AG, Frankfurt.

JUNG, J. und U. REDANZ (1993): "Zur Besteuerung der DTB-Geschäfte von Privatanlegern im Gewerbebetrieb und in der privaten Vermögensverwaltung", Zeitschrift für Bankrecht und Bankwirtschaft, pp. 68-89.

KAWALLER, I. (1987): "A Note: Debunking the Myth of the Risk-Free Return", Journal of Futures Markets 7, pp. 327-331.

LOISTL, O. und M. KOBINGER (1992): "Index-Arbitrage insbesondere mit DAX-Futures", Beiträge zur Wertpapieranalyse (der DVFA), Nr. 28.

MODEST, D. and M. SUNDARESAN (1983): "The Relationship between Spot and Futures Prices in Stock Index Futures Markets: Some Preliminary Evidence", Journal of Futures Markets 3, pp. 15-41.

PRIGGE, J. und C. SCHLAG (1992): "Die Bewertung des DAX-Future-Kontrakts an der Deutschen Terminbörse", Zeitschrift für Bankrecht und Bankwirtschaft, pp. 299-307.