

# Angular dependence of the tunneling anisotropic magnetoresistance

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Based on general symmetry considerations we investigate how the dependence of the tunneling anisotropic magnetoresistance (TAMR) on the magnetization direction is determined by the specific form of the spin-orbit coupling field. By extending a phenomenological model, previously proposed for explaining the main trends of the TAMR in (001) ferromagnet/semiconductor/normal-metal magnetic tunnel junctions (MTJs) [J. Moser *et al.*, Phys. Rev. Lett. 99, 056601 (2007)], we provide a unified qualitative description of the TAMR in MTJs with different growth directions. In particular, we predict the forms of the angular dependence of the TAMR in (001), (110), and (111) MTJs with structure inversion asymmetry and/or bulk inversion asymmetry. The effects of in-plane uniaxial strain on the TAMR are also investigated.

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## I. INTRODUCTION

The tunneling anisotropic magnetoresistance (TAMR) effect refers to the dependence of the magnetoresistance of magnetic tunnel junctions (MTJs) on the absolute orientation(s) of the magnetization(s) in the ferromagnetic lead(s) with respect to the crystallographic axes.<sup>1,2,3,4</sup> Unlike the conventional tunneling magnetoresistance (TMR) effect, the TAMR is not only present in MTJs in which both electrodes are ferromagnetic but may also appear in tunneling structures with a single magnetic electrode.<sup>1,5</sup> Because of this remarkable property, if the major challenge of increasing the size of the effect at room temperature is solved, the TAMR could be an attractive candidate for applications in the design of new spin-valve based devices whose components could operate with a single magnetic lead. In what follows we focus our discussion on the case of MTJs in which only one of the electrodes is ferromagnetic.

The TAMR has been experimentally and theoretically investigated in a variety of systems under different configurations.<sup>1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18</sup> This diversity has made it difficult to build a unified theory of the TAMR. In fact, although there exists a general consensus in identifying the spin-orbit coupling (SOC) as the mechanism responsible for the TAMR, it has been recognized that the way the SOC influences the TAMR may depend on the considered system and configuration.

Two different configurations, the *in-plane* and *out-of-plane* configurations, have been considered for investigating the TAMR (for an extensive discussion see Ref. 18 and references therein). The in-plane TAMR refers to the changes in the tunneling magnetoresistance when the magnetization direction, defined with respect to a fixed reference axis  $[x]$ , is rotated in the plane of the ferromagnetic layer. The in-plane TAMR ratio is defined as<sup>18</sup>

$$\text{TAMR}_{[x]}^{\text{in}}(\phi) = \frac{R(\theta = 90^\circ, \phi) - R(\theta = 90^\circ, \phi = 0)}{R(\theta = 90^\circ, \phi = 0)}, \quad (1)$$

where  $R(\theta, \phi)$  denotes the tunneling magnetoresistance for the magnetization oriented along the direction defined

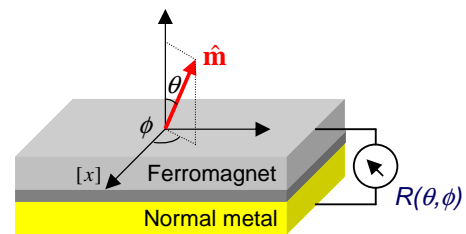


FIG. 1: Schematics of a MTJ composed of a normal-metal lead (bottom layer), a semiconductor or insulator barrier (central layer), and a ferromagnetic electrode (top layer). The vector  $\hat{\mathbf{m}}$  indicates the magnetization orientation, while  $[x]$  denotes a reference crystallographic axis.

by the unit vector  $\hat{\mathbf{m}} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$  (see Fig. 1).

In the out-of-plane configuration, the TAMR measures the changes in the tunneling magnetoresistance when the magnetization is rotated within the plane defined by the reference axis  $[x]$  and the direction normal to the ferromagnetic layer. The out-of-plane TAMR is given by<sup>18</sup>

$$\text{TAMR}_{[x]}^{\text{out}}(\theta) = \frac{R(\theta, \phi = 0) - R(\theta = 0, \phi = 0)}{R(\theta = 0, \phi = 0)}. \quad (2)$$

An important property of the TAMR is the form of its angular dependence. It has been experimentally shown that both the in-plane and out-of-plane TAMR exhibit rather regular, and relatively simple angular dependence with a well defined symmetry, in spite of the highly complicated band structure of the considered systems.<sup>4,5,10,16</sup> This suggests that although the size of the TAMR may depend on the detailed band structure of the system, its angular dependence is essentially determined by the symmetry properties of the SOC field. Here we investigate how the specific form of the TAMR angular dependence emerges from the properties of the SOC field.

A phenomenological model which incorporates the effects of the interference of Bychkov-Rashba and Dresselhaus SOC's was recently developed to explain the in-plane TAMR in (001) ferromagnet/semiconductor/normal-

metal (F/S/NM) MTJs.<sup>5,11,18</sup> In particular, it was shown that, in spite of its relative simplicity, the model was able to reproduce the two-fold symmetric angular dependence of the in-plane TAMR experimentally observed in (001) Fe/GaAs/Au MTJs.<sup>5,11</sup> In such heterojunctions all the involved materials are cubic in their bulk form. Therefore, the two-fold anisotropy of the in-plane TAMR must originate from the interfaces. Here we generalize the model and provide a unified qualitative description of the angular dependence of both the in-plane and out-of-plane TAMRs in (001), (110), and (111) MTJs. We consider systems in which the SOC originates from structure inversion asymmetry (Bychkov-Rashba-like SOC) and/or bulk inversion asymmetry (Dresselhaus-like SOC) and predict new forms of the TAMR angular dependence which could be tested in future experiments. The effects of uniaxial strain are also discussed.

## II. THEORETICAL MODEL

We consider a MTJ composed of a ferromagnetic electrode and a normal metal counter-electrode separated by an insulator or a semiconductor barrier. However, our conclusions are also valid for the case of MTJs with two ferromagnetic electrodes whose magnetizations are parallel to each other, since such systems are qualitatively similar to the case of MTJs with a single ferromagnetic lead.

The  $z$  direction is fixed along the normal to the ferromagnetic layer (i.e., parallel to the growth direction). The effective spin-orbit interaction corresponding to the  $n$ th band can be written as

$$H_{\text{SO}} = \mathbf{w}_n(\mathbf{k}) \cdot \boldsymbol{\sigma}, \quad (3)$$

where  $\mathbf{w}_n(\mathbf{k}) = [w_{nx}(\mathbf{k}), w_{ny}(\mathbf{k}), w_{nz}(\mathbf{k})]$  is the effective SOC field associated to the  $n$ th,  $\mathbf{k}$  is the wave vector, and  $\boldsymbol{\sigma}$  is a vector whose components are the Pauli matrices. Equation (3) is quite general, since by now we have not considered any specific form for the SOC field. The detailed form of the SOC field can be quite complicated as one goes away from the center of the Brillouin zone and quite different from band to band, as recently demonstrated by first-principles calculations<sup>19</sup>.

Due to the presence of the spin-orbit interaction, the transmissivity  $T_n(\mathbf{k}, \hat{\mathbf{m}})$  corresponding to the  $n$ th band becomes dependent on the magnetization direction  $\hat{\mathbf{m}}$ . Assuming that the strength of the SOC field is small relative to both the Fermi energy and the exchange splitting, one can expand the transmissivity in powers of  $\mathbf{w}_n(\mathbf{k})$ . For a given  $n$  and  $\mathbf{k}$  there are only two preferential directions in the system, defined by  $\hat{\mathbf{m}}$  and  $\mathbf{w}_n$ . Since the transmissivity is a scalar function, it can be written, to second order in the SOC field strength, in the form<sup>5,11,18</sup>

$$T_n(\mathbf{k}, \hat{\mathbf{m}}) \approx a_{1n}^{(0)}(\mathbf{k}) + a_{1n}^{(1)}(\mathbf{k})[\hat{\mathbf{m}} \cdot \mathbf{w}_n(\mathbf{k})] + a_{1n}^{(2)}(\mathbf{k})|\mathbf{w}_n(\mathbf{k})|^2 + a_{2n}^{(2)}(\mathbf{k})[\hat{\mathbf{m}} \cdot \mathbf{w}_n(\mathbf{k})]^2, \quad (4)$$

which represents the most general expansion (up to second order) of a scalar function (the transmissivity) in terms of two vectors ( $\hat{\mathbf{m}}$  and  $\mathbf{w}_n$ ). Note that the arguments used for obtaining Eq. (4) are also valid for MTJs with two ferromagnetic electrodes whose magnetizations are parallel to each other along the direction  $\hat{\mathbf{m}}$ . The expansion coefficients  $a_{in}^{(j)}$  ( $i = 1, 2; j = 0, 1, 2$ ) refer to the system in the absence of the SOC field and therefore do not depend on  $\hat{\mathbf{m}}$ . Since these coefficients reflect the cubic symmetry of the involved bulk materials they obey the relations  $a_{in}^{(j)}(k_x, k_y) = a_{in}^{(j)}(-k_x, -k_y)$ ,  $a_{in}^{(j)}(k_x, k_y) = a_{in}^{(j)}(-k_x, k_y)$ , and  $a_{in}^{(j)}(k_x, k_y) = a_{in}^{(j)}(k_y, k_x)$ . Cases in which the involved materials have other than cubic symmetry in their bulk form can be treated analogously.

Within linear response theory, the conductance  $G$  through the MTJ is determined by the states at the Fermi energy  $E_F$  and the  $\mathbf{k}$  dependence reduces to the in-plane  $\mathbf{k}_{\parallel}$  dependence at  $E = E_F$ . One can then write

$$G(\hat{\mathbf{m}}) = \frac{g_0}{8\pi^2} \sum_n \langle T_n(\mathbf{k}_{\parallel}, \hat{\mathbf{m}}) \rangle, \quad (5)$$

where  $g_0 = 2e^2/h$  is the conductance quantum and  $\langle \dots \rangle$  denotes evaluation at  $E = E_F$  and integration over  $\mathbf{k}_{\parallel}$ .

The time reversal symmetry implies that  $T_n(\mathbf{k}, \hat{\mathbf{m}}) = T_n(-\mathbf{k}, -\hat{\mathbf{m}})$  and  $\mathbf{w}_n(\mathbf{k}) = -\mathbf{w}_n(-\mathbf{k})$ . It follows then from Eq. (4) that the first-order term in the expansion must be an odd function of  $\mathbf{k}$  and will, therefore, vanish after integration over  $\mathbf{k}_{\parallel}$ . As a result the conductance can be rewritten as

$$G(\hat{\mathbf{m}}) = G^{(0)} + G_{\text{iso}}^{(2)} + G_{\text{aniso}}^{(2)}(\hat{\mathbf{m}}), \quad (6)$$

where  $G^{(0)}$  is the conductance in the absence of SOC,  $G_{\text{iso}}^{(2)} \propto \langle a_{1n}^{(2)}(\mathbf{k}_{\parallel} | \mathbf{w}_n(\mathbf{k}_{\parallel})|^2) \rangle$  and

$$G_{\text{aniso}}^{(2)}(\hat{\mathbf{m}}) = \frac{g_0}{8\pi^2} \sum_n \langle a_{2n}^{(2)}(\mathbf{k}_{\parallel}) [\hat{\mathbf{m}} \cdot \mathbf{w}_n(\mathbf{k}_{\parallel})]^2 \rangle, \quad (7)$$

are the isotropic and anisotropic SOC contributions, respectively. In terms of the components of  $\hat{\mathbf{m}}$  and the SOC field, Eq. (7) reduces to

$$G_{\text{aniso}}^{(2)}(\theta, \phi) = \frac{g_0}{8\pi^2} \text{Tr}[\mathbf{A}\mathbf{M}(\theta, \phi)], \quad (8)$$

where  $\mathbf{A}$  and  $\mathbf{M}(\theta, \phi)$  are matrices whose elements are given by

$$A_{ij} = \sum_n \langle a_{2n}^{(2)}(\mathbf{k}_{\parallel}) w_{ni} w_{nj} \rangle \quad (i, j = x, y, z) \quad (9)$$

and

$$M_{ij}(\theta, \phi) = m_i(\theta, \phi) m_j(\theta, \phi) \quad (i, j = x, y, z), \quad (10)$$

respectively.

Equations (8)-(10) are quite general and reveal how the symmetry of the SOC field can lead to the anisotropy of

the conductance. Further simplifications of these expressions can be realized by taking into account the properties of the specific form of the SOC field. In what follows we focus on the case in which the components of the SOC field can be written as  $w_{ni} = \mathbf{d}_{ni} \cdot \mathbf{k}_{\parallel}$ . As shown bellow, many relevant physical situations correspond to such a case. The matrix elements in Eq. (9) then reduce to

$$A_{ij} = \sum_n c_n (\mathbf{d}_{ni} \cdot \mathbf{d}_{nj}). \quad (11)$$

In obtaining Eq. (11) we took into account the four-fold symmetry of the expansion coefficients  $a_{2n}^{(2)}(\mathbf{k}_{\parallel})$  from which follows that the only non-vanishing averages are of the form  $c_n = \langle a_{2n}^{(2)}(\mathbf{k}_{\parallel}) k_x^2 \rangle = \langle a_{2n}^{(2)}(\mathbf{k}_{\parallel}) k_y^2 \rangle$ . By using the Eqs. (8), (10), and (11) the anisotropic part of the conductance can be rewritten as

$$G_{\text{aniso}}^{(2)} = \frac{g_0}{8\pi^2} \sum_{i,j,n} c_n m_i(\theta, \phi) m_j(\theta, \phi) (\mathbf{d}_{ni} \cdot \mathbf{d}_{nj}). \quad (12)$$

The dependence of the TAMR ratio on the magnetization direction is determined by the anisotropic part of the conductance. Thus, Eq. (12) is our starting formula for discussing important particular cases.

### III. RESULTS

We first neglect the effects of strain and focus on the particularly relevant case in which the SOC field results from the interference of the Bychkov-Rashba and Dresselhaus SOC. Later on we shall consider also MTJs with SOC induced by uniaxial strain.

The Bychkov-Rashba SOC originates from the structure inversion asymmetry (SIA) of the junction and is basically determined by the strong electric fields at the interfaces of the tunneling barrier. It is present, for example, in MTJs with the left and right electrodes made of different materials and therefore with broken inversion symmetry. The Dresselhaus SOC results from the bulk inversion asymmetry (BIA) of one or more of the constituent materials. Typical materials with BIA are the zinc blende semiconductors. Thus, the Dresselhaus SOC can be relevant for MTJs with non-centrosymmetric semiconductor barriers.

The specific form of the SOC field depends on the growth direction of the heterostructure. Below we analyze the most relevant cases, corresponding to MTJs grown in the [001], [110], and [111] crystallographic directions.

#### A. (001) MTJs with axes $\hat{x} \parallel [110]$ , $\hat{y} \parallel [\bar{1}10]$ , $\hat{z} \parallel [001]$

In this case the SOC corresponding to the  $n$ th band containing both Bychkov-Rashba and Dresselhaus terms is given by<sup>11,20,21</sup>

$$H_{\text{SO}} = (\alpha_n - \gamma_n) k_x \sigma_y - (\alpha_n + \gamma_n) k_y \sigma_x, \quad (13)$$

where  $\alpha_n$  and  $\gamma_n$  are the corresponding Bychkov-Rashba and Dresselhaus parameters, respectively.

One can extract the components of the SOC field by comparing Eqs.(3) and (13). It follows then from equation (12) that the angular dependence of the anisotropic conductance is given by

$$G_{\text{aniso}}^{(2)} = \frac{g_0 \sin^2 \theta}{8\pi^2} \sum_n c_n [(\alpha_n^2 + \gamma_n^2) + 2\alpha_n \gamma_n \cos(2\phi)]. \quad (14)$$

The expression above together with Eqs. (1) and (2) lead to the relations corresponding to case A in Table I.<sup>22</sup> The obtained TAMR coefficients, which are valid up to second order in the SOC field, reveal a clear distinction between the in-plane and out-of-plane configurations in [001] MTJs: while for a finite out-of-plane TAMR the presence of only one of the SOC's suffices (i.e., it is sufficient to have  $\alpha_n \neq 0$  or  $\gamma_n \neq 0$ ), the two-fold symmetric in-plane TAMR appears because of the interference of non-vanishing Bychkov-Rashba and Dresselhaus SOC's (i.e., both  $\alpha_n$  and  $\gamma_n$  have to be finite).<sup>23</sup> This explains why a finite out-of-plane TAMR appears in MTJs such as Fe(001)/vacuum/Cu(001) in which only the Bychkov-Rashba SOC is present<sup>9</sup> and is in agreement with the recent observation of the in-plane TAMR in epitaxial (001) Fe/GaAs/Au MTJs,<sup>5</sup> where due to the presence of the non-centrosymmetric zinc blende semiconductor GaAs as the barrier material not only the Bychkov-Rashba but also the Dresselhaus SOC become relevant. In both the in-plane and out-of-plane configurations, angular dependencies of the form  $\text{TAMR}_{[110]}^{\text{in}}(\phi) \propto [1 - \cos(2\phi)]$  and  $\text{TAMR}_{[110]}^{\text{out}}(\theta) \propto [\cos(2\theta) - 1]$  (see Table I) have been experimentally measured.<sup>4,5,10,16</sup>

The results displayed in Table I suggest the possibility of using different configurations and reference axes as complementary setups for TAMR measurements. In particular, our theoretical model predicts that in the regime  $\alpha_n \approx \gamma_n$  the out-of-plane TAMR with reference axis in the  $[\bar{1}10]$  is suppressed, while it remains finite if the complementary axis [110] is used as a reference.<sup>24</sup> The opposite behavior, i.e.,  $\text{TAMR}_{[110]}^{\text{out}} \neq 0$  and  $\text{TAMR}_{[\bar{1}10]}^{\text{out}} = 0$ , is expected when  $\alpha_n \approx -\gamma_n$ . Another relevant regime occurs when  $\alpha_n \approx 0$  for which the in-plane TAMR is expected to vanish (see case A in Table I). The existence of such a regime was previously invoked in Refs. 5,11 for explaining the suppression of the in-plane TAMR experimentally observed in (001) Fe/GaAs/Au MTJs.<sup>5</sup> Our theory predicts that although the in-plane TAMR vanishes, the out-of-plane TAMR should remain finite in such a regime. In fact, in the regime  $\alpha_n \approx 0$  the amplitude of the out-of-plane TAMR constitutes a direct measurement of the effects of BIA in the non-centrosymmetric barrier.

By combining the results shown in Table I one can find expressions such as

$$\text{TAMR}_{[110]}^{\text{in}}(90^\circ) = \text{TAMR}_{[110]}^{\text{out}}(90^\circ) - \text{TAMR}_{[\bar{1}10]}^{\text{out}}(90^\circ), \quad (15)$$

TABLE I: TAMR coefficients in units of  $g_0/[16\pi^2(G^{(0)} + G_{\text{iso}}^{(2)})]$  for different structures, reference axes  $[x]$ , and configurations.

Case	Structure	$[x]$	in-plane TAMR $_{[x]}^{\text{in}}(\phi)$	out-of-plane TAMR $_{[x]}^{\text{out}}(\theta)$
A	(001) MTJ	[110]	$4[1 - \cos(2\phi)] \sum_n c_n \alpha_n \gamma_n$	$[\cos(2\theta) - 1] \sum_n c_n (\alpha_n + \gamma_n)^2$
		$[\bar{1}10]$	$4[\cos(2\phi) - 1] \sum_n c_n \alpha_n \gamma_n$	$[\cos(2\theta) - 1] \sum_n c_n (\alpha_n - \gamma_n)^2$
B	(110) MTJ	$[\bar{1}10]$	$[1 - \cos(2\phi)] \sum_n c_n (\alpha_n^2 - \beta_n^2)$	$[\cos(2\theta) - 1] \sum_n c_n (\alpha_n^2 - \lambda_n^2)$
		[001]	$[\cos(2\phi) - 1] \sum_n c_n (\alpha_n^2 - \beta_n^2)$	$2 \sum_n c_n [\lambda_n^2 - (\beta_n \sin \theta + \lambda_n \cos \theta)^2]$
C	(111) MTJ	$[11\bar{2}]$	0	$[\cos(2\theta) - 1] \sum_n c_n (\alpha_n + \gamma_n)^2$
		$[\bar{1}10]$	0	$[\cos(2\theta) - 1] \sum_n c_n (\alpha_n + \gamma_n)^2$
D	strained (001) MTJ	[110]	$4[1 - \cos(2\phi)] \sum_n c_n \eta_n \mu_n$	$[\cos(2\theta) - 1] \sum_n c_n (\eta_n + \mu_n)^2$
		$[\bar{1}10]$	$4[\cos(2\phi) - 1] \sum_n c_n \eta_n \mu_n$	$[\cos(2\theta) - 1] \sum_n c_n (\eta_n - \mu_n)^2$

which correlate the in-plane and out-of-plane TAMR coefficients and can be experimentally tested.

### B. (110) MTJs with axes $\hat{x} \parallel [\bar{1}10]$ , $\hat{y} \parallel [001]$ , $\hat{z} \parallel [110]$

In this case the SOC is described by<sup>25,26</sup>

$$H_{\text{SO}} = \alpha_n k_y \sigma_x - \beta_n k_x \sigma_y - \lambda_n k_x \sigma_z. \quad (16)$$

Here  $\alpha_n$  and  $\beta_n$  are parameters related to the SIA-induced SOC, while  $\lambda_n$  characterizes the strength of the

SOC resulting from the BIA. Note that because of the reduced symmetry of the (110) structures with respect to the (001) MTJs, in the present case the usual SIA-induced SOC acquires, in addition to the usual Bychkov-Rashba SOC, an extra contribution which leads to  $\alpha_n \neq \beta_n$  in Eq. (16).<sup>25,26</sup>

Proceeding in the same way as in Sec. III A we obtain the following relation for the anisotropic contribution to the conductance

$$G_{\text{iso}}^{(2)} = \frac{g_0}{8\pi^2} \sum_n c_n [(\alpha_n^2 \cos^2 \phi + \beta_n^2 \sin^2 \phi) \sin^2 \theta + \lambda_n^2 \cos^2 \theta + \beta_n \lambda_n \sin \phi \sin(2\theta)]. \quad (17)$$

The corresponding TAMR coefficients are given in Table I. The relations for case B show that the angular dependence of the TAMR in both the in-plane and out-of-plane are similar to the ones obtained for the (001) MTJs [compare the cases A and B in Table I]. However, their physical origin is now different. In the present case the in-plane TAMR originates from the SIA induced SOC while the out-of-plane TAMR has contributions arising from both SIA- and BIA-like SOCs. Thus, our model predicts that in (110) MTJs it could be possible to observe the TAMR in the two configurations even if the tunneling barrier is composed of a centrosymmetric material. Another observation is that the out-of-plane TAMR with reference axis along the  $[\bar{1}10]$  direction could be suppressed if under some given conditions the regime  $\alpha_n = \pm \lambda_n$  (for the relevant to transport bands) is realized [see the out-of-plane TAMR in case B of Table I]. In such a case, however, the out-of-plane TAMR with  $[001]$  as the reference axis should remain finite.

### C. (111) MTJs with axes $\hat{x} \parallel [11\bar{2}]$ , $\hat{y} \parallel [\bar{1}10]$ , $\hat{z} \parallel [111]$

For this structure the SOC is given by<sup>25</sup>

$$H_{\text{SO}} = (\alpha_n + \gamma_n)(k_x \sigma_y - k_y \sigma_x), \quad (18)$$

where  $\alpha_n$  and  $\gamma_n$  are the parameters characterizing the strengths of the SIA- and BIA-like SOCs, respectively.

After computing the anisotropic part of the conductance we obtain

$$G_{\text{aniso}}^{(2)} = \frac{g_0 \sin^2 \theta}{8\pi^2} \sum_n c_n (\alpha_n + \gamma_n)^2. \quad (19)$$

This relation leads to the TAMR coefficients given in Table I for the case C).

In the present case the prediction of a vanishing in-plane TAMR is remarkable. We have checked that even if the cubic in  $k$  terms are included in the SOC field, the in-plane TAMR still vanishes. This could be used for experimentally exploring the origin of the TAMR. If a suppression of the in-plane TAMR is experimentally observed in (111) MTJs, it will be a strong indication

that indeed the mechanism behind the TAMR is the SIA-and/or the BIA-like SOCs. On the contrary, if no suppression of the in-plane TAMR is observed, the role of these spin-orbit interactions as the origin of the TAMR can be questioned.

Another interesting issue is the possibility of reaching the condition  $\alpha_n = -\gamma_n$  (for the bands relevant to transport), which leads to a vanishing out-of-plane TAMR (if only the linear in  $k$  terms in the SOC field are relevant), in addition to the above discussed suppression of the TAMR in the in-plane configuration.

#### D. Uniaxial strain in (001) MTJs with axes $\hat{x} \parallel [100]$ , $\hat{y} \parallel [010]$ , $\hat{z} \parallel [001]$

In our previous analysis we have disregarded the effects of the strain induced SOC which could be relevant for structures whose constituent materials have a sizable mismatch in their lattice constants. For a (001) MTJ the SOC induced by strain is, in general, given by<sup>27</sup>

$$H_{\text{SO}} = \alpha_n [(u_{zx}k_z - u_{xy}k_y)\sigma_x + (u_{xy}k_x - u_{yz}k_z)\sigma_y + (u_{yz}k_y - u_{zx}k_x)\sigma_z] + \gamma_n [k_x(u_{yy} - u_{zz})\sigma_x + k_y(u_{zz} - u_{xx})\sigma_y + k_z(u_{xx} - u_{yy})\sigma_z], \quad (20)$$

where  $u_{ij}$  are the components of the strain tensor and  $\alpha_n$  and  $\gamma_n$  are materials parameters. The SOC in Eq. (20) is quite rich and suggests the possibility of engineering the strain (see, for example, Ref. 28) in order to manipulate the behavior of the TAMR. Here we do not consider all the possibilities but focus, for the sake of illustration, on the case of an in-plane uniaxial strain such that the only non-vanishing components of the strain tensor are  $u_{xx} = u_{yy} \neq u_{xy} = u_{yx}$ . The existence of a similar strain was initially assumed for explaining TAMR experiments in (Ga,Mn)As/AlOx/Au MTJs.<sup>1,3</sup> For the in-plane uniaxial strain Eq. (20) reduces to

$$H_{\text{SO}} = \eta_n(k_x\sigma_y - k_y\sigma_x) + \mu_n(k_x\sigma_x - k_y\sigma_y), \quad (21)$$

where we have introduced the strain-renormalized SIA and BIA parameters  $\eta_n = \alpha_n u_{xy}$  and  $\mu_n = \gamma_n u_{xx}$ , respectively. The corresponding anisotropic contribution to the conductance is then given by

$$G_{\text{aniso}}^{(2)} = \frac{g_0 \sin^2 \theta}{8\pi^2} \sum_n c_n [(\eta_n^2 + \mu_n^2) + 2\eta_n \mu_n \sin(2\phi)]. \quad (22)$$

We note that Eq. (21) has the form of interfering Bychkov-Rashba and Dresselhaus SOCs in (001) structures with  $\hat{x} \parallel [100]$ .<sup>11,25</sup> Therefore, Eqs. (14) and (22) are similar. The angle  $\phi$  in Eq. (14) is measured with respect to the crystallographic direction [110] while in Eq. (22) it is defined with respect to the [100] axis. Thus, by making the transformation  $\phi \rightarrow \phi + \pi/4$  in Eq. (22) one recovers a relation similar to Eq. (14). Consequently, assuming the direction [110] as the reference axis for measuring the magnetization direction the results for the in-plane and out-of-plane TAMR coefficients in (001) MTJs with in-plane uniaxial strain [see case D in Table I] are essentially the same as in the case discussed in III A but with renormalized spin-orbit parameters which now account for the strain effects.

In our investigation we have assumed specific well-known forms for the SOC field. For some systems, however, the form of the SOC field may not be *a priori* known. In such a case one could use Eqs. (8)-(10) (which are general) and contrast them with complementary TAMR measurements in both the in-plane and out-of-plane configurations in order to deduce the symmetry properties of the SOC fields.

All the calculated TAMR coefficients, if not zero, show a two-fold symmetry in the  $(\theta, \phi)$ -space, which is the symmetry that has been observed in the experiments.<sup>4,5,8,10,16</sup> Our results are valid up to the second order in the SOC field. In particular, our predictions for vanishing TAMR under certain conditions, may change when higher orders in the SOC field become relevant. The next higher order contributions in the expansion in Eq. (4) which do not vanish after averaging are those containing the fourth order in the SOC field terms and terms of the fourth order in the cosine directions of  $\hat{\mathbf{m}}$  which describe the four-fold symmetry inherent to the involved bulk ferromagnet. These fourth order terms lead to four-fold symmetric corrections to the TAMR which may be finite even for (001) MTJs with centrosymmetric barriers for which the second order in-plane TAMR calculated here vanishes. Although the two-fold character of the TAMR is, in general unchanged by these corrections, they may influence the shape of its angular dependence. Thus, for the kind of systems here considered (see Table I) any deviation from the 8-*like* polar shape of the TAMR [see, for example, Fig.2 in Ref. 5] is interpreted in our theory as a manifestation of higher order contributions and/or strain effects.<sup>29</sup> Deviations from the 8-*like* polar shape of the TAMR have been experimentally observed.<sup>8,10</sup> In fact, it has been shown that these deviations may appear by increasing the bias voltage,<sup>10</sup> which within the present approach can be seen as an indication of higher order in the SOC field terms turning relevant at sufficiently high bias.

In all the above discussions small magnetic fields with negligible orbital effects were assumed. It has recently been observed in Fe/GaAs/Au MTJs that for high magnetic fields the orbital effects do influence the in-plane TAMR.<sup>30</sup> A version of the phenomenological model here presented which incorporates the orbital effects has recently been developed to qualitatively explain the magnetic field dependence of the in-plane TAMR experimentally observed in (001) Fe/GaAs/Au MTJs.<sup>30</sup>

#### IV. SUMMARY

We formulated a theoretical model in which the way the TAMR depends on the magnetization orientation of

the ferromagnetic electrode in MTJs is determined by the specific form and symmetry properties of the interface-induced SOC field. By using the proposed model, we deduced the angular dependence of the TAMR for various systems in dependence of their symmetry under spatial inversion and their growth direction. The effects of in-plane uniaxial strain were also investigated.

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  - <sup>23</sup> Here a finite TAMR is still possible if higher order terms in the SOC field are relevant.
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