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Forecasting Credit Portfolio Risk

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Abstract

The main challenge of forecasting credit default risk in loan portfolios is forecasting the default probabilities and the default correlations. We derive a Merton-style threshold-value model for the default probability which treats the asset value of a firm as unknown and uses a factor model instead. In addition, we demonstrate how default correlations can be easily modeled. The empirical analysis is based on a large data set of German firms provided by Deutsche Bundesbank. We find that the inclusion of variables which are correlated with the business cycle improves the forecasts of default probabilities. Asset and default correlations depend on the factors used to model default probabilities. The better the point-in-time calibration of the estimated default probabilities, the smaller the estimated correlations. Thus, correlations and default probabilities should always be estimated simultaneously.

Keywords: asset correlation, bank regulation, Basel II, credit risk, default correlation, default probability, logit model, probit model, time-discrete hazard rate

JEL classification: C23, C41, G21

Non-technical Summary

Forecasting credit portfolio risk poses a challenge for the banking industry. One important goal of modern credit portfolio models is the forecast of the future credit risk given the information which is available at the point of time the forecast is made.

Thus, the discussion paper “Forecasting Credit Portfolio Risk“ proposes a dynamic concept for the forecast of the risk parameters default probabilities and default correlations. The results are based on an extensive empirical analysis of a data set provided by Deutsche Bundesbank which contains financial statements for more than 50,000 German firms and a time period from 1987 to 2000.

Important results of this paper are:

1. The inclusion of macroeconomic risk drivers improves the forecast of default probabilities considerably. We included the macroeconomic variables business climate index, unemployment rate and systematic growth in new orders of the construction industry.

2. We find that a large part of co-movements can be attributed to lagged risk drivers. Thus, default rate or loss distributions can be forecasted given the values of the lagged risk drivers.

3. The model allows default probabilities to be forecasted for individual borrowers and to estimate correlations between those borrowers simultaneously. We show that asset and default correlations depend on the point in time calibration of the default probabilities. In addition a simultaneous estimation eases the validation of default probabilities. Thus, default probabilities and correlations should never be derived separately from each other.

4. The model is an empirical application of the model which is used for the calibration of risk weights by the Basel Committee on Banking Supervision. Hence, we are able to compare the estimated parameters from our model and Basel II directly.

Nichttechnische Zusammenfassung

Die Prognose von Kreditausfallrisiken stellt eine zentrale Herausforderung für Kreditinstitute und Finanzdienstleister dar. Ein wichtiges Ziel moderner Kreditrisikomodelle ist die Prognose zukünftiger Kreditrisiken auf Basis der im Prognosezeitpunkt zur Verfügung stehenden Information.

Vor diesem Hintergrund präsentiert der Diskussionsbeitrag "Forecasting Credit Portfolio Risk" ein dynamisches Konzept zur gemeinsamen Prognose der zentralen Risikoparameter Ausfallwahrscheinlichkeit und Ausfallkorrelation. Die empirischen Untersuchungen in dieser Arbeit basieren auf der Unternehmensbilanzdatenbank der Deutschen Bundesbank.

Wichtige Ergebnisse des Diskussionsbeitrags sind:

1. Die Berücksichtigung von makroökonomischen Einflußgrößen verbessert signifikant die Güte der Prognose von Ausfallwahrscheinlichkeiten. Als makroökonomische Einflußgrößen wurden der Ifo-Geschäftsklimaindex, die Arbeitslosenquote und die Auftragseingänge der Baubranche verwendet.

2. Ausfallwahrscheinlichkeiten und Ausfallkorrelationen können durch zeitverzögert wirkende Risikofaktoren erklärt werden. Resultierende Verlustverteilungen können deshalb bei Kenntnis der Ausprägungen der Risikofaktoren prognostiziert werden.

3. Der Modellansatz erlaubt erstmals die simultane Ermittlung von Ausfallwahrscheinlichkeiten und Ausfallkorrelationen. Mit der Point-in-Time-Kalibrierung der Ausfallwahrscheinlichkeiten nehmen die geschätzten Korrelationen ab. Des Weiteren erleichtert die simultane Schätzung die Validierung der Ausfallwahrscheinlichkeiten. Korrelationen und Ausfallwahrscheinlichkeiten sollten deshalb nicht getrennt voneinander ermittelt werden.

4. Das Modell entspricht dem des Baseler Ausschusses für Bankenaufsicht. Die geschätzten Parameter können deshalb unmittelbar mit den Basel II Vorgaben verglichen werden.

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Forecasting Credit Portfolio Risk*

1 Introduction

The main challenge of forecasting credit default risk in loan portfolios is forecasting the default probabilities and the default correlations. They are input parameters to a variety of credit risk models like CreditMetrics™, CreditRisk+, CreditPortfolioManager™ or CreditPortfolioView™. For outlines of these models see Gupton et al. [1997], Credit Suisse Financial Products [1997], Crosbie/Bohn [2002] and Wilson [1997a, 1997b].

The main direction of modeling credit risk has its origin in the seminal model of Merton [1974, 1977] and Black/Scholes [1973]. Extensions of the approach are described in Black and Cox [1976], Merton [1977], Geske [1977], Longstaff and Schwartz [1995] or Zhou [2001]. In this model it is assumed that a default event happens if the value of an obligor's assets falls short of the value of debt. Generally speaking, one of the model's major shortcomings is the assumption of available market prices for the asset value. This is not usually valid for retail or small and medium-sized obligors.

Chart 1 displays West German insolvency rates for the years 1980 to 2000. Insolvency rates are frequently taken as proxies for default rates. It can be seen that the rates fluctuate over time. An important object of modern credit risk management is the forecast of future credit risk given the available information at the point of time at which the forecast is made.

* We would like to thank Dr. Stefan Blochwitz, Dr. Klaus Düllmann and Dr. Daniel Rösch for stimulating discussions.

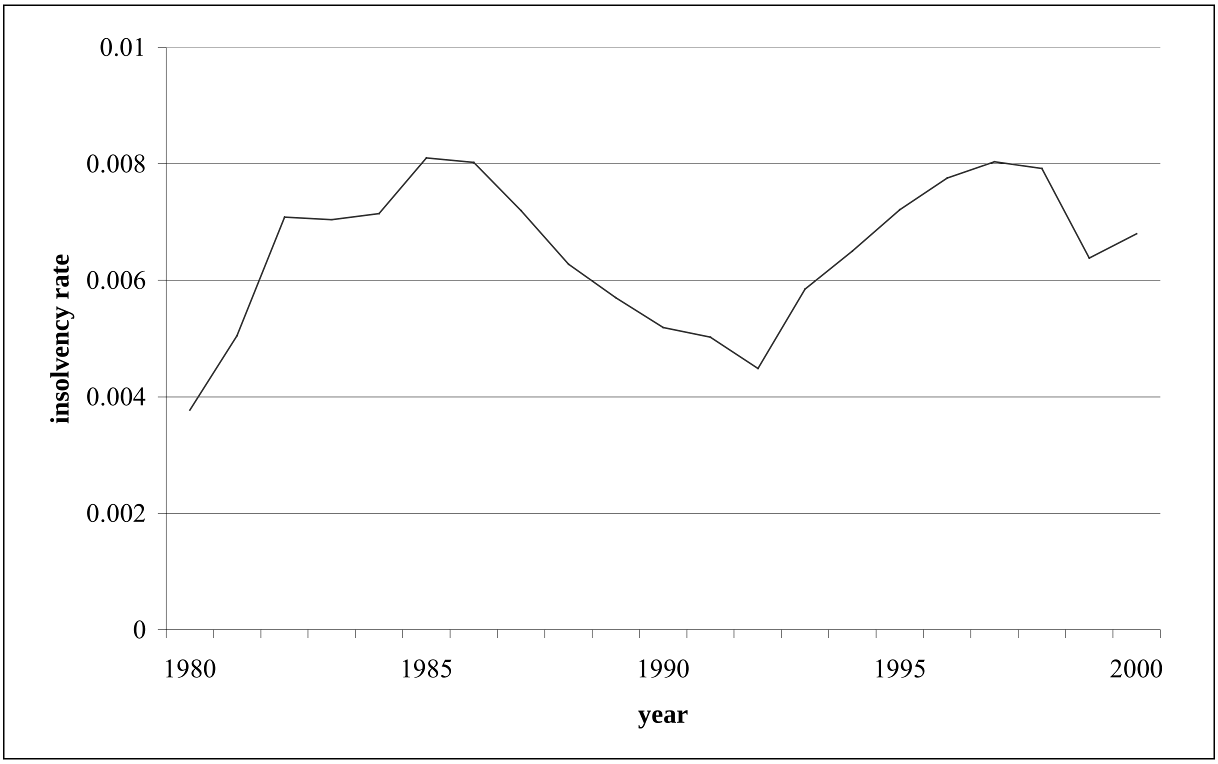

Chart 1: Insolvency rates of West Germany

In the present paper we use a model to forecast default probabilities and estimate default correlations based on the threshold model described above. The default probability measures the probability of an obligor's assets falling short of a threshold. In addition, asset correlations are modeled as a measure of co-movement of the asset values of two obligors. Default correlations can then be derived analytically.

Our approach differs from existing studies on forecasting default probabilities (such as Escott/ Glormann/ Kocagil [2001], Falkenstein [2000] and Shumway [2001]) and estimating default correlations (like Dietsch/ Petey [2002], Gupton/Finger/Bhatia [1997] and Lucas [1995]) in several ways and therefore leads to new important results. Firstly, we find that a large part of co-movements can be attributed to lagged risk drivers. Thus, default rate or loss distributions can be forecasted, given the values of the lagged risk drivers, and estimation uncertainty can be reduced. Secondly, the model we employ allows default probabilities to be forecasted for individual borrowers and to estimate correlations between those borrowers simultaneously. We show that asset and default correlations depend on the point in time calibration of the default probabilities. Thirdly, the model is an empirical application of the model which is used for the calibration of risk weights by the Basel Committee on Banking Supervision [2003]. Hence, we are able to compare the estimated parameters from our model and Basel II directly. As a matter of fact, we find significant differences. Fourthly, we use an extensive data set provided by Deutsche Bundesbank covering 221,684 observations of corporate balance sheet and default data. The observation period of 10 years spans more than one business cycle, which is an important requirement for the estimation of cyclical default probabilities and correlations.

The next section describes the modeling approach for default probabilities and the third section describes the modeling approach for asset and default correlations. Section 4 presents and interprets the empirical results for the data set of Deutsche Bundesbank. Section 5 provides a summary of the results and comments.

2 Modeling default probabilities

The event in which an obligor is unable to fulfill its payment obligations is defined as a default. The default event for obligor i in the time period t is random and modeled using the indicator variable y_{it}, i.e.

$$y_{it} = \begin{cases} 1 & \text{obligor } i \text{ defaults in } t \\ 0 & \text{otherwise} \end{cases}$$

($i = 1,...,N;\ t = 1,...,T$). The default event is assumed to be observable.

In addition, the continuous non-observable variable r_{it} is defined, which may be interpreted as the logarithmic return of an obligor's assets. For the relationship between r_{it} and the default event y_{it} a threshold-value model is assumed. Default is equivalent to the return of an obligor's assets falling below a threshold c_{it}, i.e.

$$r_{it} \le c_{it} \Leftrightarrow y_{it} = 1$$

($i = 1,...,N;\ t = 1,...,T$). Implicitly, a further assumption is made that no default has occurred in previous time periods. Therefore, the conditional default probability given that the obligor did not default until the beginning of the current time period

$$\lambda_{it} = P(y_{it} = 1) = P(r_{it} \le c_{it})$$

is also called a time-discrete hazard rate.

We now propose a linear panel model which includes time-lagged fundamental, macroeconomic and statistical risk drivers and a contemporary systematic random effect. The model can be written as

$$r_{it} = \beta_0 + \boldsymbol{\beta}'\boldsymbol{x}_{it-1} + \boldsymbol{\gamma}'\boldsymbol{z}_{t-1} + bf_t + \varpi u_{it}$$

($i = 1,\ldots,N;\ t = 1,\ldots,T$).

$\boldsymbol{x}_{it-1}$ denotes a vector of time-lagged obligor-specific risk factors such as the return on equity of the obligor's previous year's financial statement or the number of employees two years ago. Correspondingly, $\boldsymbol{z}_{t-1}$ denotes a vector of systematic risk factors, like the unemployment rate of the previous year or the money market rate two years ago. The time-lagged risk factors are known at the point of time at which the forecast is given. The subscript $t-1$ represents time lags of one and more periods.

In addition, a contemporary systematic factor f_t is included which explains the systematic risk components not captured by the model. Throughout this paper, we assume that f_t follows a standard normal distribution.

β_0, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ and b are suitably dimensioned parameter vectors. Note that the notation refers to a particular risk segment such as an industry. It is assumed that the obligors are homogenous within a risk segment regarding the relevant risk factors and the factor exposures. The parameters and risk factors are allowed to differ between risk segments like industries.

In practice, the realization of the risk drivers and the default indicator y_{it} are observable while the asset returns of the latent model are not. The link between the risk factors and the probability of default (PD) is described by a threshold model. Given that default has not happened before t, one obtains for the **conditional** probability of default given the realization of the random effect f_t (and given the values of the observable factors until time period $t-1$)

$$\begin{aligned}\lambda(\boldsymbol{x}_{it-1},\boldsymbol{z}_{t-1},f_t) &= P(y_{it}=1|\boldsymbol{x}_{it-1},\boldsymbol{z}_{t-1},f_t)\\ &= P(r_{it}\le c_{it}|\boldsymbol{x}_{it-1},\boldsymbol{z}_{t-1},f_t)\\ &= P\left(u_{it}\le\frac{c_{it}-\beta_0-\boldsymbol{\beta}'\boldsymbol{x}_{it-1}-\boldsymbol{\gamma}'\boldsymbol{z}_{t-1}-bf_t}{\varpi}|\boldsymbol{x}_{it-1},\boldsymbol{z}_{t-1},f_t\right),\\ &= F(\tilde{\beta}_0+\tilde{\boldsymbol{\beta}}'\boldsymbol{x}_{it-1}+\tilde{\boldsymbol{\gamma}}'\boldsymbol{z}_{t-1}+\tilde{b}f_t)\end{aligned}$$

where $\tilde{\beta}_{0it}=(c_{it}-\beta_0)/\varpi$, $\tilde{\boldsymbol{\beta}}=-\boldsymbol{\beta}/\varpi$, $\tilde{\boldsymbol{\gamma}}=-\boldsymbol{\gamma}/\varpi$ and $\tilde{b}=-b/\varpi$ and $F(.)$ denotes the distribution function of the error term u_{it}. Since the threshold c_{it} cannot be observed, we restrict the intercept to $\tilde{\beta}_0$.

Different assumptions about the error distribution function $F(.)$ lead to different models for the probability of default. In the empirical analysis we use the logistic distribution function (logit model) which leads to

$$\lambda(\boldsymbol{x}_{it-1},\boldsymbol{z}_{t-1},f_t)=\exp(\tilde{\beta}_0+\tilde{\boldsymbol{\beta}}'\boldsymbol{x}_{it-1}+\tilde{\boldsymbol{\gamma}}'\boldsymbol{z}_{t-1}+\tilde{b}f_t)/(1+\exp(\tilde{\beta}_0+\tilde{\boldsymbol{\beta}}'\boldsymbol{x}_{it-1}+\tilde{\boldsymbol{\gamma}}'\boldsymbol{z}_{t-1}+\tilde{b}f_t)),$$

whereas the standardnormal distribution function $\Phi(.)$ (probit model) leads to

$$\lambda(\boldsymbol{x}_{it-1},\boldsymbol{z}_{t-1},f_t)=\Phi(\tilde{\beta}_0+\tilde{\boldsymbol{\beta}}'\boldsymbol{x}_{it-1}+\tilde{\boldsymbol{\gamma}}'\boldsymbol{z}_{t-1}+\tilde{b}f_t).$$

Note that the probit model is assumed by the Basel Committee on Banking Supervision [2003] in its Internal-Rating-Based approach in order to calculate the regulatory capital (see Finger [2001]).

Since we do not know the value of f_t when the forecast is made we have to calculate the (expected) **unconditional** probability of default given by

$$\lambda(\boldsymbol{x}_{it-1}, \boldsymbol{z}_{t-1}) = \int_{-\infty}^{\infty} F\left(\tilde{\beta}_0 + \tilde{\boldsymbol{\beta}}' \boldsymbol{x}_{it-1} + \tilde{\boldsymbol{\gamma}}' \boldsymbol{z}_{t-1} + \tilde{b} f_t\right) \varphi(f_t)\, df_t ,$$

where $\varphi(f_t) = \left(1/\sqrt{2\pi}\right)\exp\left(-0{,}5\, f_t^{\,2}\right)$ denotes the density function of the standard normal distribution.

The Parameters $\tilde{\beta}_0$, $\tilde{\boldsymbol{\beta}}$, $\tilde{\boldsymbol{\gamma}}$ and $\tilde{b}$ can be estimated by the maximization of the expected value of the Likelihood $L\left(\tilde{\beta}_0, \tilde{\boldsymbol{\beta}}, \tilde{\boldsymbol{\gamma}}, \tilde{b}\right)$ with respect to the distribution of the random effect f_t over all obligors and periods of the data set:

$$E\left[L\left(\tilde{\beta}_0, \tilde{\boldsymbol{\beta}}, \tilde{\boldsymbol{\gamma}}, \tilde{b}\right)\right] =$$
$$= \prod_{t=1}^{T} \int_{-\infty}^{\infty} \left[\prod_{i=1}^{I} \left[F\left(\tilde{\beta}_0 + \tilde{\boldsymbol{\beta}}' \boldsymbol{x}_{it-1} + \tilde{\boldsymbol{\gamma}}' \boldsymbol{z}_{t-1} + \tilde{b} f_t\right)^{y_{it}} \left(1 - F\left(\tilde{\beta}_0 + \tilde{\boldsymbol{\beta}}' \boldsymbol{x}_{it-1} + \tilde{\boldsymbol{\gamma}}' \boldsymbol{z}_{t-1} + \tilde{b} f_t\right)\right)^{(1-y_{it})} \right] \varphi(f_t) \right] df_t .$$

This equation contains T integrals which can be solved approximately using adaptive Gauss-Hermite-quadrature (Pinheiro/ Bates [1995] or Rabe-Hesketh/ Skrondal/ Pickles [2002], pp. 5- 9). It follows from the general theory of Maximum-Likelihood estimation that the estimates exist asymptotically, are consistent and asymptotically normal distributed (Davidson/ MacKinnon [1993], pp. 243 et seq.).

3 Modeling correlations

Asset correlation for one risk segment

The correlation coefficient between the latent variables r_{it} and r_{jt} of two obligors i and j is called asset correlation $\rho(r_{it}, r_{jt})$:

$$
\begin{aligned}
\rho(r_{it}, r_{jt}) &= \frac{Cov(r_{it}, r_{jt})}{\sqrt{Var(r_{it})}\sqrt{Var(r_{jt})}} = \\
&= \frac{Cov(bf_t + \varpi u_{it}, bf_t + \varpi u_{jt})}{\sqrt{Var(bf_t + \varpi u_{it})}\sqrt{Var(bf_t + \varpi u_{jt})}} = \\
&= \frac{E[(bf_t + \varpi u_{it})(bf_t + \varpi u_{jt})]}{\sqrt{b^2 + \varpi^2 Var(u_{it})}\sqrt{b^2 + \varpi^2 Var(u_{jt})}} = \\
&= \frac{b^2 E(f_t f_t)}{b^2 + \varpi^2 Var(u_{it})} = \\
&= \frac{b^2 Var(f_t)}{b^2 + \varpi^2 Var(u_{it})} = \\
&= \frac{b^2}{b^2 + \varpi^2 Var(u_{it})}.
\end{aligned}
$$

We assumed that u_{it} and u_{jt} have the same distribution. If we assume the logistic distribution the variance of u_{it} and u_{jt} equals $\pi^2/3$ and the asset correlation is

$$
\rho(r_{it}, r_{jt}) = \frac{(b/\varpi)^2}{(b/\varpi)^2 + \pi^2/3} = \frac{\tilde{b}^2}{\tilde{b}^2 + \pi^2/3},
$$

whereas if we assume the standardnormal distribution the variance of u_{it} and u_{jt} equals 1 and the asset correlation is

$$\rho\left(r_{it}, r_{jt}\right) = \frac{\tilde{b}^2}{\tilde{b}^2 + 1}.$$

Asset correlation for multiple risk segments

Sometimes it is plausible to assume that the default probabilities are driven by different risk factors for different obligors, i.e. obligors belong to different risk segments. Let obligor i belong to risk segment l and obligor j to risk segment m. The model for the return on obligor i's assets is

$$r_{it}^{(l)} = \beta_0^{(l)} + \boldsymbol{\beta}^{(l)\prime} \mathbf{x}_{it-1}^{(l)} + \boldsymbol{\gamma}^{(l)\prime} \mathbf{z}_{t-1}^{(l)} + b^{(l)} f_t^{(l)} + \varpi^{(l)} u_{it}^{(l)},$$

while the model for the return on obligor j's assets is:

$$r_{jt}^{(m)} = \beta_0^{(m)} + \boldsymbol{\beta}^{(m)\prime} \mathbf{x}_{jt-1}^{(m)} + \boldsymbol{\gamma}^{(m)\prime} \mathbf{z}_{t-1}^{(m)} + b^{(m)} f_t^{(m)} + \varpi^{(m)} u_{jt}^{(m)}.$$

The correlation $\rho\left(r_{it}^{(l)}, r_{jt}^{(m)}\right)$ between the latent variables $r_{it}^{(l)}$ and $r_{jt}^{(m)}$ of two obligors is:

$$
\begin{aligned}
\rho\left(r_{it}^{(l)}, r_{jt}^{(m)}\right) &= \frac{Cov\left(r_{it}^{(l)}, r_{jt}^{(m)}\right)}{\sqrt{Var\left(r_{it}\right)}\sqrt{Var\left(r_{jt}\right)}} = \\
&= \frac{Cov\left(b^{(l)} f_t^{(l)} + \varpi^{(l)} u_{it}^{(l)}, b^{(m)} f_t^{(m)} + \varpi^{(m)} u_{jt}^{(m)}\right)}{\sqrt{Var\left(b^{(l)} f_t^{(l)} + \varpi^{(l)} u_{it}^{(l)}\right)}\sqrt{Var\left(b^{(m)} f_t^{(m)} + \varpi^{(m)} u_{jt}^{(m)}\right)}} = \\
&= \frac{E\left[\left(b^{(l)} f_t^{(l)} + \varpi^{(l)} u_{it}^{(l)}\right)\left(b^{(m)} f_t^{(m)} + \varpi^{(m)} u_{jt}^{(m)}\right)\right]}{\sqrt{b^{(l)2} + \varpi^{(l)2} Var\left(u_{it}\right)}\sqrt{b^{(m)2} + \varpi^{(m)2} Var\left(u_{jt}\right)}} = \\
&= \frac{b^{(l)} b^{(m)} E\left(f_t^{(l)} f_t^{(m)}\right)}{\sqrt{b^{(l)2} + \varpi^{(l)2} Var\left(u_{it}\right)}\sqrt{b^{(m)2} + \varpi^{(m)2} Var\left(u_{jt}\right)}} = \\
&= \frac{b^{(l)} b^{(m)} Cov\left(f_t^{(l)}, f_t^{(m)}\right)}{\sqrt{b^{(l)2} + \varpi^{(l)2} Var\left(u_{it}\right)}\sqrt{b^{(m)2} + \varpi^{(m)2} Var\left(u_{jt}\right)}}.
\end{aligned}
$$

Again, if a logit model is assumed for both risk segments the asset correlation $\rho\left(r_{it}^{(l)}, r_{jt}^{(m)}\right)$ is:

$$
\rho\left(r_{it}^{(l)}, r_{jt}^{(m)}\right) = \frac{\tilde{b}^{(l)} \tilde{b}^{(m)} Cov\left(f_t^{(l)}, f_t^{(m)}\right)}{\sqrt{\tilde{b}^{(l)2} + \pi^2/3}\sqrt{\tilde{b}^{(m)2} + \pi^2/3}}
$$

If a probit model is assumed for both risk segments the asset correlation $\rho\left(r_{it}^{(l)}, r_{jt}^{(m)}\right)$ is:

$$
\rho\left(r_{it}^{(l)}, r_{jt}^{(m)}\right) = \frac{\tilde{b}^{(l)} \tilde{b}^{(m)} Cov\left(f_t^{(l)}, f_t^{(m)}\right)}{\sqrt{\tilde{b}^{(l)2} + 1}\sqrt{\tilde{b}^{(m)2} + 1}}.
$$

Default correlation

The default correlation can be derived from the asset correlation. For simplicity we assume that the obligors i and j belong to the same risk segment and that the default probabilities can be explained by a probit model. The default indicators y_{it} and y_{jt} for different obligors i and j are binary random variables taking only the values 0 or 1. For binary random variables the correlation coefficient $\rho(y_{it}, y_{jt})$ for period t can be written as

$$\rho(y_{it}, y_{jt}) = \frac{\lambda(\boldsymbol{x}_{it-1}, \boldsymbol{x}_{jt-1}, \boldsymbol{z}_{t-1}) - \lambda(\boldsymbol{x}_{it-1}, \boldsymbol{z}_{t-1})\lambda(\boldsymbol{x}_{jt-1}, \boldsymbol{z}_{t-1})}{\sqrt{\lambda(\boldsymbol{x}_{it-1}, \boldsymbol{z}_{t-1})(1-\lambda(\boldsymbol{x}_{it-1}, \boldsymbol{z}_{t-1}))}\sqrt{\lambda(\boldsymbol{x}_{jt-1}, \boldsymbol{z}_{t-1})(1-\lambda(\boldsymbol{x}_{jt-1}, \boldsymbol{z}_{t-1}))}},$$

where $\lambda(\boldsymbol{x}_{it-1}, \boldsymbol{z}_{t-1})$ and $\lambda(\boldsymbol{x}_{jt-1}, \boldsymbol{z}_{t-1})$ are the unconditional default probabilities and $\lambda(\boldsymbol{x}_{it-1}, \boldsymbol{x}_{jt-1}, \boldsymbol{z}_{t-1})$ is the unconditional probability that both obligors i and j will default in period t given that neither obligor has defaulted before:

$$\lambda(\boldsymbol{x}_{it-1}, \boldsymbol{x}_{jt-1}, \boldsymbol{z}_{t-1}) = \int_{-\infty}^{\infty} F(\tilde{\beta}_0 + \tilde{\boldsymbol{\beta}}'\boldsymbol{x}_{it-1} + \tilde{\boldsymbol{\gamma}}'\boldsymbol{z}_{t-1} + \tilde{b} f_t) F(\tilde{\beta}_0 + \tilde{\boldsymbol{\beta}}'\boldsymbol{x}_{jt-1} + \tilde{\boldsymbol{\gamma}}'\boldsymbol{z}_{t-1} + \tilde{b} f_t) \varphi(f_t) df_t .$$

If we assume a probit model, it can be shown that

$$\lambda(\boldsymbol{x}_{it-1}, \boldsymbol{x}_{jt-1}, \boldsymbol{z}_{t-1}) = \Phi_2\left(\Phi^{-1}(\lambda(\boldsymbol{x}_{it-1}, \boldsymbol{z}_{t-1})), \Phi^{-1}(\lambda(\boldsymbol{x}_{jt-1}, \boldsymbol{z}_{t-1})), \rho(r_{it}, r_{jt})\right)$$

where $\Phi_2(.)$ symbolizes the standardized bivariate normal distribution and $\Phi^{-1}(.)$ the quantile of the standardnormal distribution (Gupton/ Finger/ Bhatia [1997], p. 89). In conclusion, the default correlation can be derived from the unconditional default probabilities and the asset correlation of the obligors i and j.

4 Empirical Analysis

4.1 Data

The empirical analysis is based on a data set of Deutsche Bundesbank which originally contains financial statements for up to 53,280 West German firms and a time period from 1987 to 2000. Compare Scheule [2003] for a more extensive analysis. The data is collected by Deutsche Bundesbank's branch offices in order to evaluate the credit quality of firms for refinancing purposes. The Bundesbank purchases them at the discount rate under its credit facility. An enterprise is deemed to have defaulted if insolvency proceedings have been initiated against it. The legal preconditions for the initiation of such proceedings are laid down in the German insolvency code, i.e. particularly the inability to meet due payments and over-indebtedness.

In addition, the data set is extended by macroeconomic risk factors for West Germany. They cover such fields as production, consumption, income, capital markets, employment, import and export, government activity and prices. All variables are assumed to be stationary. When they show a trend, rate of returns to the previous year are used. All macroeconomic variables are lagged by one or two years.

The resulting data set is modified in several ways. The data set is restricted to the years 1991 to 2000 in order to ensure a sufficient number of observations. In addition, only West German firms are included due to the different economic developments in West and East Germany during the last decade. The firms are seperated into the industries Manufacturing, Commerce and Others. Chart 2 shows the Manufacturing industry where the insolvency rates of the

Deutsche Bundesbank data differ from the insolvency rates of West Germany. The default rate is defined as the ratio between the number of defaulted and the total number of firms.

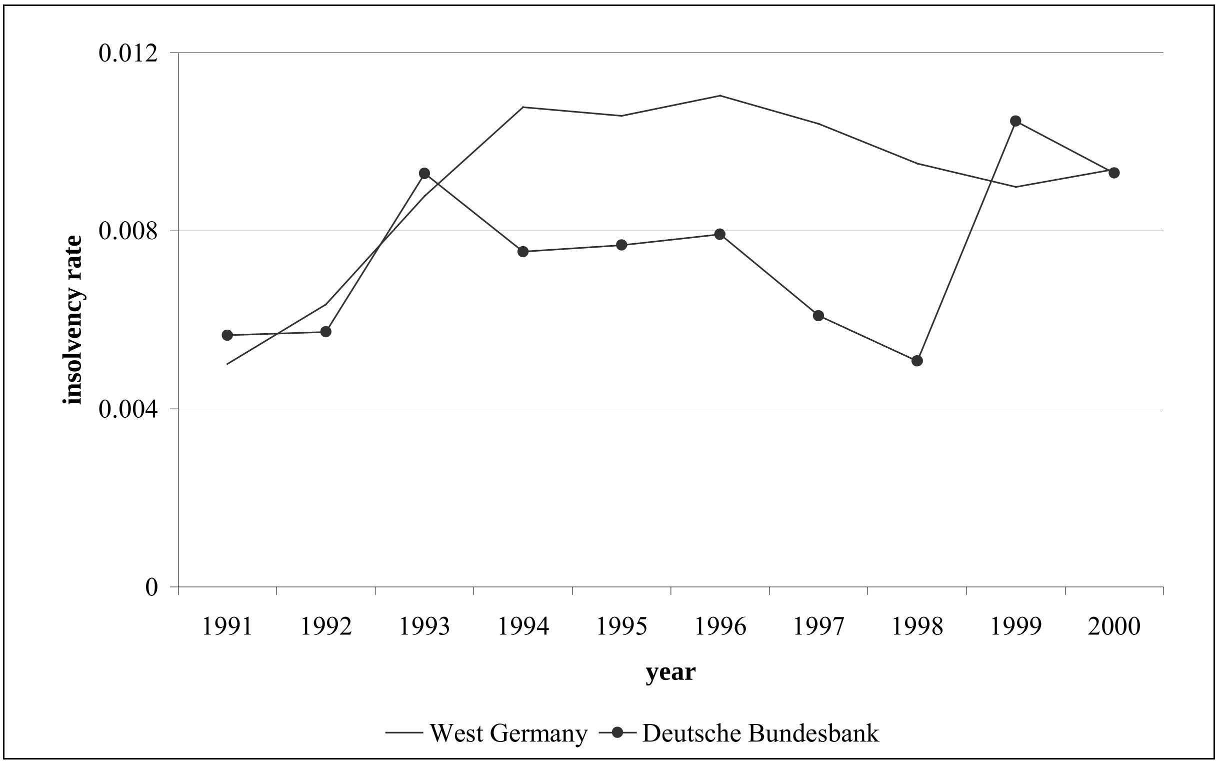

Chart 2: Default rates of the Manufacturing industry, Deutsche Bundesbank and West Germany

We assumed that the default rates for West Germany are more representative. Thus, the yearly default rates are adjusted for each industry according to the ones of West Germany by taking a random sample from either the defaults or non-defaults of each period. Table 1 shows that the resulting data set includes 221.684 observations with 1.570 defaults:

Industry	Observations	Defaults
Manufacturing	88,869	773
Commerce	71,827	406
Others	60,988	391
Total	221,684	1,570

Table 1: Data set Deutsche Bundesbank - number of observations and defaults for different industries

The data set is divided into an estimation period 1991 to 1999 and the forecast-year 2000. Table 2 shows the number of observations and defaults of the estimation and forecast periods:

Purpose	Period	Observations	Defaults
Estimation	1991- 1999	195,476	1,391
Forecast	2000	26,208	179
Total		221,684	1,570

Table 2: Data set Deutsche Bundesbank - number of observations and defaults for the estimation and forecast periods

4.2 Model-estimation for one risk segment

In a first step, we assume that the whole data set represents one risk segment, i.e. the default probabilities are driven by the same risk drivers and that the asset correlations are the same for all obligors. For this data set, two logit models with a random effect are estimated:

- model 1 includes only firm-specific risk drivers and
- model 2 includes firm-specific risk drivers and a systematic macroeconomic variable.

The highest p-value is 0.0015, i.e. all risk drivers are significant (α=0.05). The random effect represents an exception which will be explained below. Table 3 displays the estimated parameters for the two models:

Risk driver	**Model 1 (without macroeconomic risk driver)**	**Model 2 (with macroeconomic risk driver)**
Intercept	-7.7832	-7.8132
ART	0.0062	0.0062
APT	0.0123	0.0124
CRR	-0.0324	-0.0326
ETA	-0.0162	-0.0160
MAN	0.4045	0.4172
RIE	-0.0014	-0.0013
TTT	0.0308	0.0311
GOC	.	-0.0460
b	0.1205	0.0718

Table 3: Parameter estimates for logit models with random effects, without (model 1) and with systematic macroeconomic risk driver (model 2)

The risk drivers are the

- firm-specific ratio of trade accounts receivable to total turnover (ART), ratio of notes and trade accounts payable to total turnover (APT), the capital recovery rate (CRR), the equity to assets ratio (ETA), a dummy variable for the manufacturing industry (MAN), the return on interest expenses (RIE), the transformed total turnover (TTT), and the
- systematic growth in new orders of the construction industry (GOC).

All risk drivers were checked for economic plausibility. Let us take the equity to assets ratio as an example. The negative parameter estimate indicates that firms with a higher equity ratio have lower default probabilities. While most risk drivers show a monotone impact on the default probabilities, the default rates for small and large firms (low and high total turnover) are low and for medium firms (medium total turnover) are high. Since a logit model can only include risk drivers with a monotone impact, we use the default rates of five total turnover classes and their interpolated values (transformed total turnover) presented in Chart 3 as a risk driver. We applied a cubic spline-interpolation which uses third degree polynomials. Note that the interpolated values can be interpreted as the estimated default probability given the value of the total turnover.

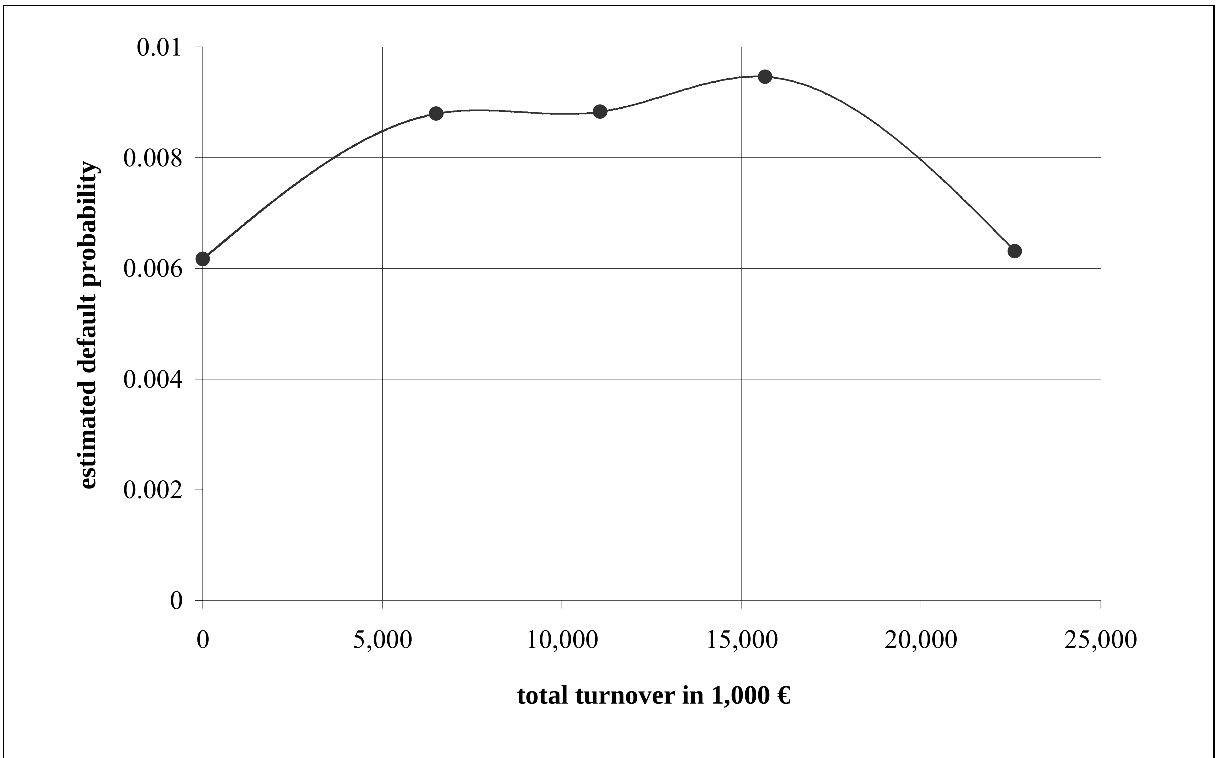

Chart 3: Estimated default probability given the total turnover

The dummy variable for the Manufacturing industry (1: Manufacturing industry, 0: other industries) indicates that firms of this industry show a higher default probability.

The risk drivers are determined by the use of forward-, backward- and stepwise-selection methods and are scanned for stationarity. In addition, outliers are adjusted by defining a lower and an upper boundary for every risk driver. Note that the firm-specific risk drivers are lagged on average by 1.5 years while the macroeconomic risk driver of model 2 is lagged by one year. The risk drivers are calculated in percentages. A more detailed definition of the risk drivers and descriptive statistics is provided in the appendix. Note that macroeconomic variables are usually recorded by national institutions and published earlier than the balance sheets of firms. It should be noted that we do not claim that the risk driver is responsible for the default probabilities themselves but rather that it represents the respective point in time of the business cycle.

Chart 4 compares the real default rate with the estimated default rates of model 1 and model 2. The estimated default rate is the average of the estimated default probabilities.

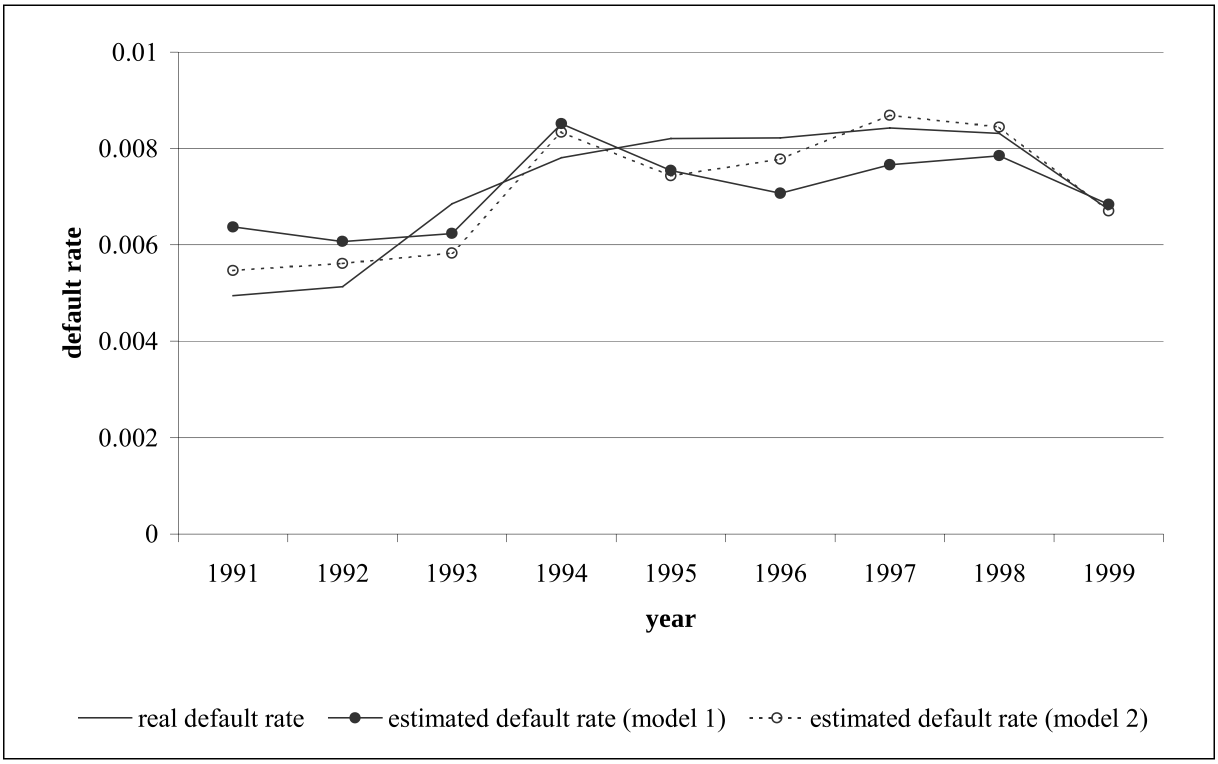

Chart 4: Real and estimated default rates for estimation period, model 1 and model 2

The calibration of the estimated to the real default rate is generally better for model 2 (with a macroeconomic variable) than for model 1 (without a macroeconomic variable). Another important property of a rating model is the power to discriminate between defaulted and non-defaulted obligors. The discrimination can be measured by the accuracy ratio (see Sobehart/ Keenan/ Stein [2000]). While the calibration of the two models differ considerably, the discrimination or accuracy ratio of model 1 (0.630) and model 2 (0.631) is very similar. Note that systematic macroeconomic risk drivers are the same for all obligors for a given year.

They change all default probabilities for a given year in the same direction. Thus, the estimated default rate fits better the real default rate if systematic risk drivers are included in the logit model. This result holds for all years except 1993 and 1995.

Section 3 showed that asset correlations can be estimated by a transformation of the parameter of the random effect. Table 4 contains the asset correlation estimates for model 1 and model 2:

Model	Parameter estimates	Standard error	P-value	Asset correlation
1	0.1205	0.0333	0.0010	0.0044
2	0.0718	0.0236	0.0833	0.0016

Table 4: Random effect parameter and asset correlation estimates, model 1 and model 2

The inclusion of the macroeconomic risk divers results in a decrease of the estimated parameter of the random effect and therefore the asset correlation. As a matter of fact, a likelihood ratio test shows that the random effect becomes insignificant ($\alpha = 0{,}05$). The estimated asset correlations are considerably lower than the ones assumed by the Basel Committee on Banking Supervision [2003]. Chart 5 shows that the Basel II asset correlation for corporate exposures is a decreasing function of the default probability with values between 12% and 24%:

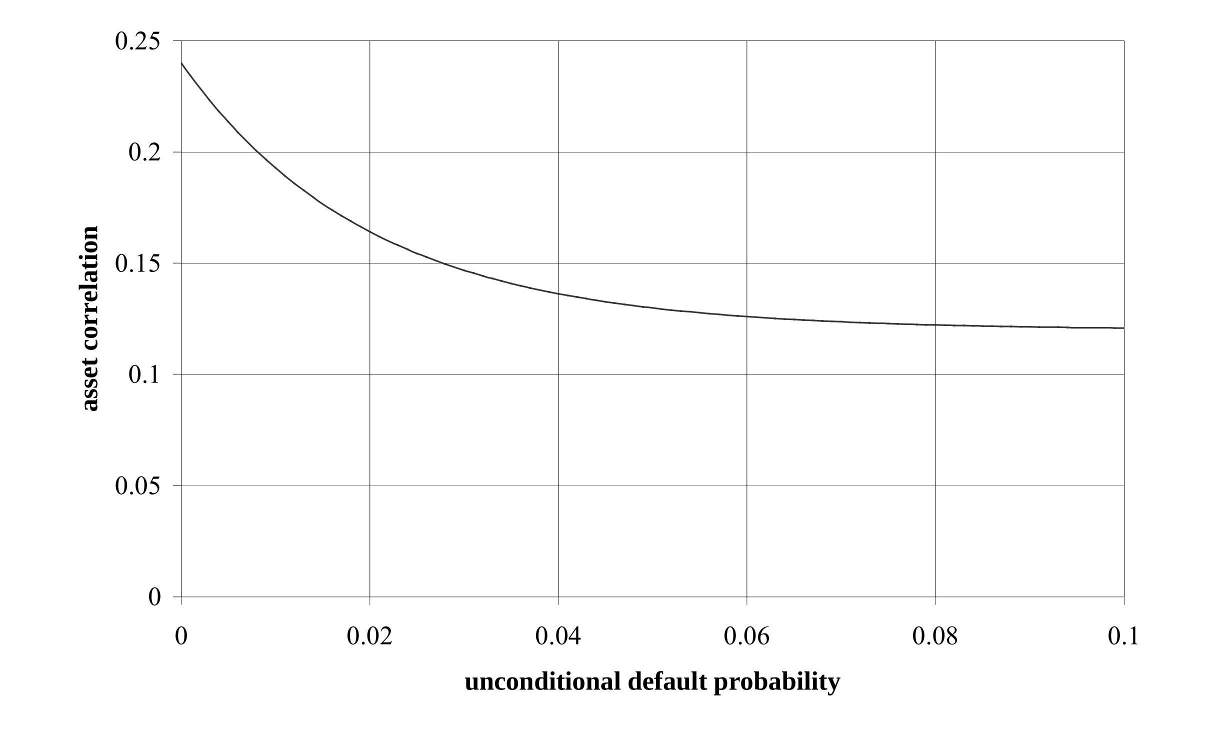

Chart 5: Basel II asset correlation for corporate exposures

4.3 Model-estimation for multiple risk segments

We will now assume that the industries Manufacturing, Commerce and Others define three risk segments. Again, the default probabilities within each risk segment are driven by the same risk drivers and the asset correlations are the same for all obligors. Two models are estimated which consist of one logit model with a random effect for each risk segment:

- model 3 includes firm-specific risk drivers only and
- model 4 includes firm-specific risk drivers and systematic macroeconomic variables.

Again, all risk drivers are significant ($\alpha = 0{,}05$). Table 5 displays the risk drivers for the two models. Note that no industry dummies are included because industry-models are estimated.

Risk segment	Model 3 (without macroeconomic risk driver)	Model 4 (with macroeconomic risk driver)
Manufacturing	ETA, APT, CRR, ITT, RIE, TTT	ETA, APT, CRR, ITT, RIE, TTT , BCI
Commerce	ART, ETA, APT, CRR, RIE, TTT	ART, ETA, APT, CRR, RIE, TTT
Others	ETA, CFT, APT, LD, RIE, TTT	ETA, CFT, APT, LD, RIE, TTT, UER

Table 5: Risk segment specific logit models with random effects, with (model 3) and without systematic macroeconomic risk driver (model 4)

In addition to the firm-specific risk drivers of model 1 and model 2 the cashflow to total turnover ratio (CFT) and the inventory to total turnover (ITT) are included. Again, the firm-specific risk drivers are lagged on average by 1.5 years. The risk drivers are calculated in percentages. A more detailed definition of the risk drivers and descriptive statistics is provided in the appendix. The macroeconomic risk drivers of model 4 are a business climate index (BCI) and the unemployment rate (UER). These systematic variables are lagged by one year.

Chart 6 compares the real default rates with the estimated default rates of model 3 and model 4. Note that the estimated default probabilities of all risk segments are aggregated.

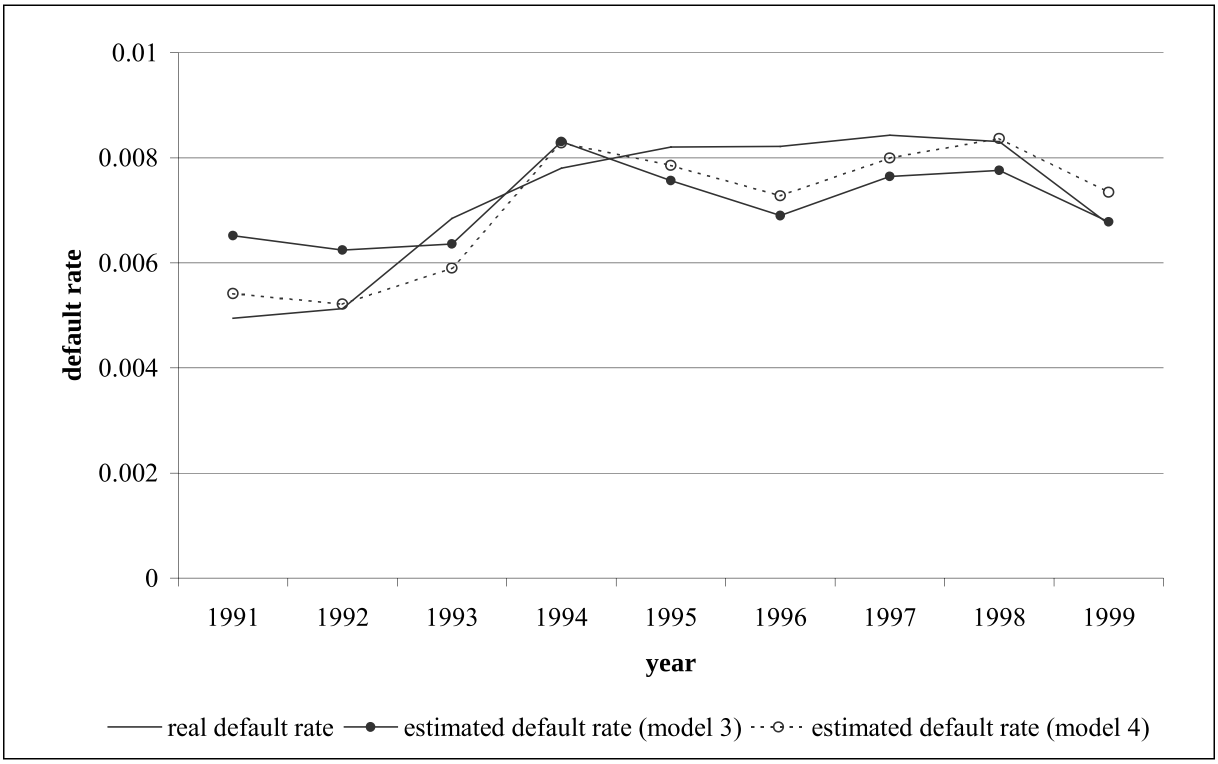

Chart 6: Real and estimated default rates for estimation period, model 3 and model 4

The results are comparable to the ones of model 1 and model 2 in the previous section. The calibration of model 3 and model 4 differ considerably while the accuracy ratio of model 3 (0.644) and model 4 (0.647) is very similar.

Table 6 (Table 7) displays the random effect estimates for model 3 (model 4) while Table 8 (Table 9) contains the asset correlation estimates for the three risk segments.

Risk segment	Parameter estimates	Standard error	P-value
Manufacturing	0.1531	0.0387	0.0013
Commerce	0.1234	0.0347	0.0495
Others	0.2130	0.0541	0.0009

Table 6: Random effect parameter estimates and significance, model 3

Risk segment	Parameter estimate	Standard error	P-value
Manufacturing	0.0796	0.0226	0.1167
Commerce	0.1234	0.0347	0.0495
Others	0.0990	0.0315	0.2033

Table 7: Random effect parameter estimates and significance, model 4

The inclusion of the macroeconomic risk drivers results in a decrease of the estimated parameter of the random effect and therefore the asset correlation. As a matter of fact, the random effect becomes insignificant ($\alpha = 0{,}05$). The asset correlation of the risk segment Commerce remains unchanged because no significant macroeconomic variable was found. Note that the asset correlation of this segment in model 3 is already lower than the ones of the other segments.

Table 8 summarizes the asset correlations for obligors of the same and different risk segments for model 3 and Table 9 for model 4.

	Manufacturing	Commerce	Others
Manufacturing	0.0071	0.0040	0.0038
Commerce		0.0046	0.0034
Others			0.0136

Table 8: Asset correlation estimates, model 3

	Manufacturing	Commerce	Others
Manufacturing	0.0019	0.0015	0.0003
Commerce		0.0046	0.0016
Others			0.0030

Table 9: Asset correlation estimates, model 4

4.4 Forecasting default probabilities

Time-varying variables enter the logit model with a time lag. Thus, given the estimated models 1 to 4 and the value of the risk drivers, default probabilities will now be forecasted for the year 2000. Chart 7 compares the empirical frequency distribution (class width: 0.001) of the forecasted default probabilities of model 1 and model 2:

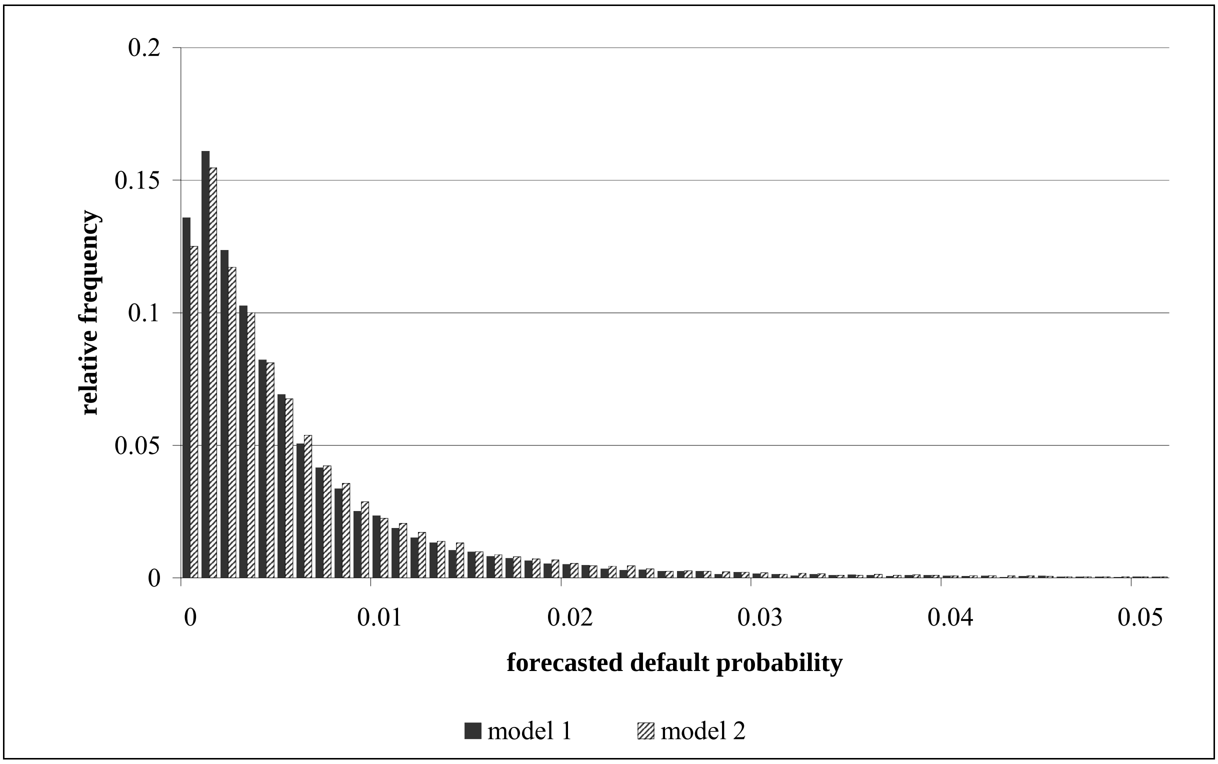

Chart 7: Frequency distribution of forecasted default probabilities, model 1 and model 2

Chart 8 compares the real and the mean forecasted default rates of model 1 to model 4. The forecasted default rate is the average of the forecasted default probabilities.

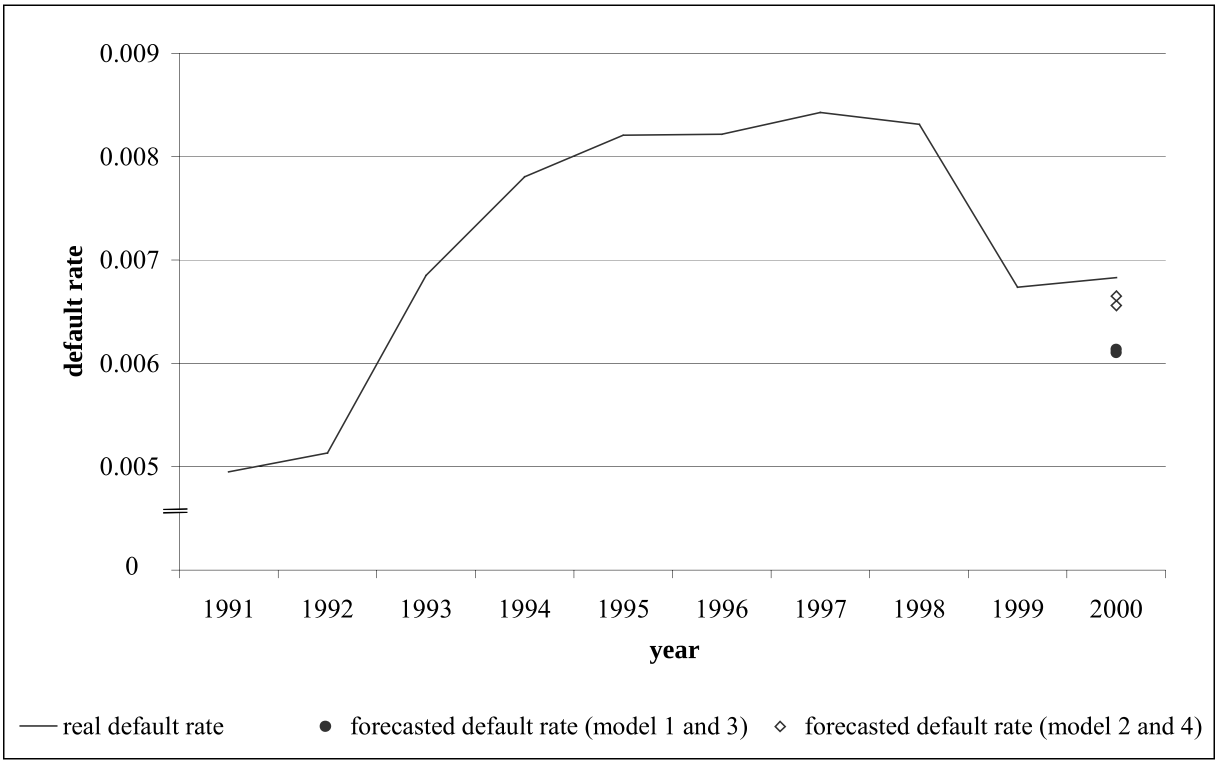

Chart 8: Real and forecasted default rates for year 2000, model 1 to model 4

Model 2 and model 4 which include macroeconomic variables, forecast the default rate more accurately than model 1 and model 3 which do not include macroeconomic variables. Note that the forecasted default rate for model 1 and 3 are very close to each other and therefore can not be differentiated in the chart. In other words, the calibration of the forecasted default probabilities would have been better if macroeconomic variables had been included in the respective model.

4.5 Forecasting the default rate distribution

The forecasted default probabilities and the estimated asset correlations can be aggregated to the forecasted default rate distribution. The forecasted default rate distribution can be interpreted as a loss distribution if the exposure at default and the loss given default equal one.

Chart 9 compares the forecasted default rate distribution of model 1 without macroeconomic variables and model 2 with macroeconomic variables. Table 10 shows the respective mean forecasted default rate and the quantiles of the forecasted default rate distribution.

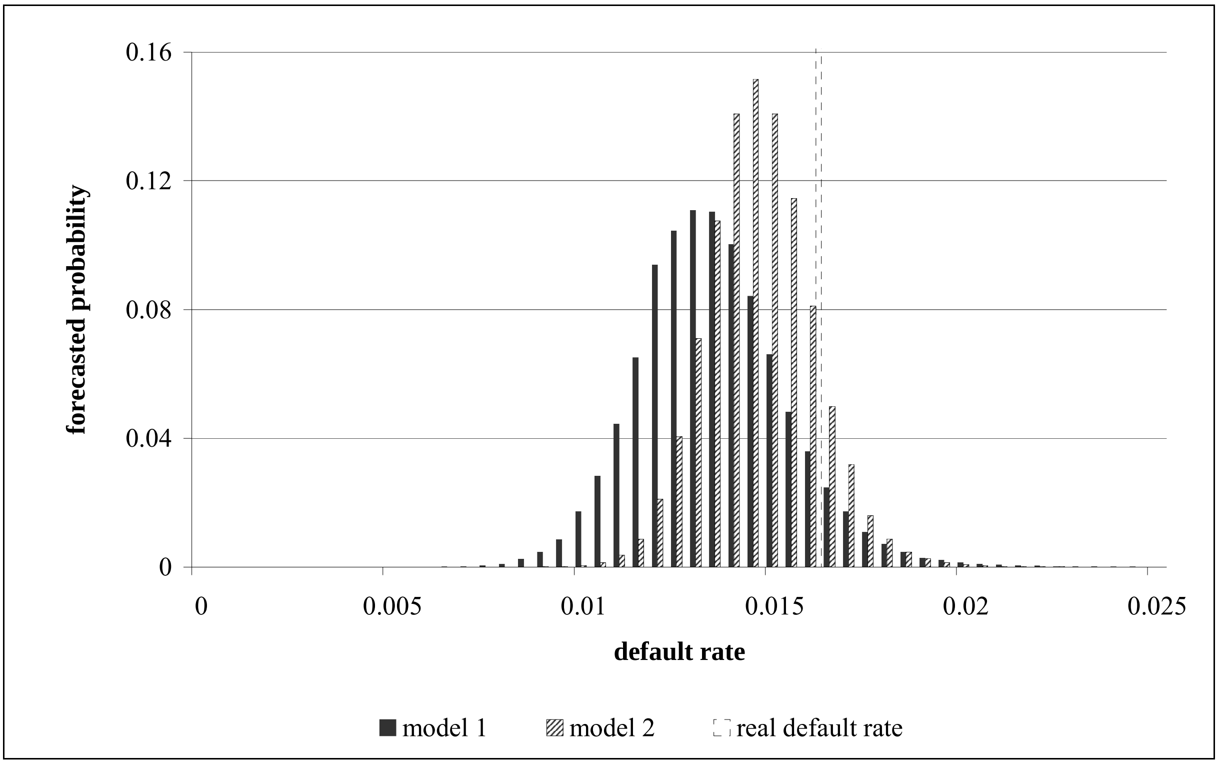

Chart 9: Forecasted default rate distribution, model 1 and model 2

	Mean forecasted default rate	0,95-Q.	0,99-Q.	0,999-Q.
Model 1	0.0134	0.0167	0.0187	0.0217
Model 2	0.0146	0.0170	0.0183	0.0201

Table 10: Mean forecasted default rate and quantiles of forecasted default rate distribution, model 1 and model 2

Again, it can be seen that the mean forecasted default rate for 2000 of model 2 is closer to the real default rate than that of model 1. In addition, model 2 estimates a lower asset correlation which leads to a lower variance of the forecasted default rate. Hence, the portfolio credit risk is forecasted more accurately. Similar results are observed for model 3 and model 4 when multiple risk segments are assumed.

Chart 10 compares the forecasted default rate distribution of one risk segment model 1 and multiple risk segment model 3. Table 11 shows the respective mean forecasted default rate and the quantiles of the forecasted default rate distribution:

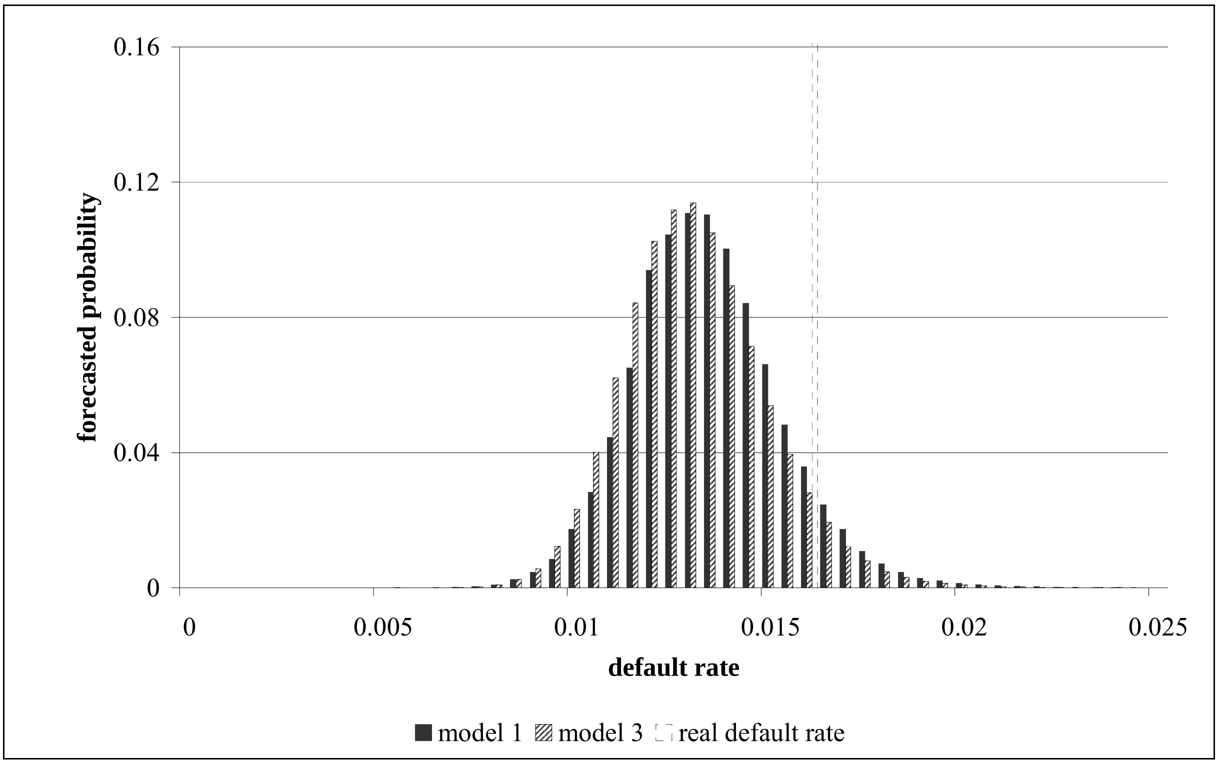

Chart 10: Forecasted default rate distribution, model 1 and model 3

	Mean forecasted default rate	0,95-Q.	0,99-Q.	0,999-Q.
Model 1	0.0134	0.0167	0.0187	0.0217
Model 3	0.0131	0.0163	0.0181	0.0206

Table 11: Mean forecasted default rate and quantiles of forecasted default rate distribution, model 1 and model 3

The forecasted default rate distribution of model 1 is broader than that of model 3. Since default rate distributions generally broaden with a higher mean forecasted default rate, we cannot conclude that the assumption of multiple risk segments leads to more accurate credit portfolio risk forecasts. An examination of further periods is advisable.

5 Summary

The present paper describes an alternative methodology for forecasting credit portfolio risk. We showed within this framework that

- individual default probabilities can be forecasted and asset (or default) correlations can be estimated, given the values of risk drivers that are observable in the point of time the forecasts are made.

- the inclusion of variables which are correlated with the business cycle improves the forecasts of default probabilities. The variance of the forecasted default rate decreases, i.e. the uncertainty of the forecasts is diminished.

- asset and default correlations depend on the factors used to model default probabilities. The better the point-in-time calibration of the estimated default probabilities, the smaller the estimated correlations. Thus, correlations and default probabilities should always be estimated simultaneously.

Appendix

- **Descriptive statistics firm-specific risk drivers**

Ratio	Mean	Median	Standarddev.	Min	Max
ART	34.5973	30.7526	25.8769	0	100
APT	34.8201	25.9981	30.1343	0	110
CFT	6.5248	4.7671	9.3863	-15	25
CRR	12.2592	10.0036	16.2009	-25	50
ETA	12.1587	9.7701	20.6234	-35	60
ITT	48.1980	38.0972	44.0308	0	160
RIE	387.7100	218.4480	486.0730	-650	1,400
TTT	0.0076	0.0074	0.0011	0.0062	0.0096

Table 12: Data set Deutsche Bundesbank - summary statistics of firm-specific risk drivers

	ART	APT	CFT	CRR	ETA	ITT	RIE	TTT
ART	1.0000	0.2212	-0.1255	-0.0949	-0.0400	0.0860	-0.0333	0.0675
APT		1.0000	-0.0757	-0.1780	-0.2418	0.2460	-0.2225	-0.0584
CFT			1.0000	0.7423	0.1812	-0.2627	0.3303	-0.1666
CRR				1.0000	0.1476	-0.2923	0.5290	-0.0227
ETA					1.0000	-0.0351	0.3423	-0.0588
ITT						1.0000	-0.1924	0.0121
RIE							1.0000	-0.0174
TTT								1.0000

Table 13: Data set Deutsche Bundesbank - Pearson correlations between firm-specific risk drivers

- **Descriptive statistics macroeconomic risk drivers**

Variable	Mean	Median	Standarddev.	Min	Max
BCI	88.311	85.028	7.483	82.29	103.36
GOC	0.0013	0.0054	0.0206	-0.0270	0.0369
Insolvency rate	0.0072	0.0073	0.0013	0.0050	0.0084
UER	9.0537	9.3259	1.8706	6.3000	11.4830

Table 14: Data set Deutsche Bundesbank - summary statistics of macroeconomic risk drivers

	BCI	GOC	Insolvency Rate	UER
BCI	1.0000	0.7734	-0.8278	-0.7685
GOC		1.0000	-0.7851	-0.7662
Insolvency Rate			1.0000	0.6347
UER				1.0000

Table 15: Data set Deutsche Bundesbank - Pearson correlations between macro-economic risk drivers

References

Basel Committee on Banking Supervision, The New Basel Capital Accord, Consultative Document, April 2003, Bank for International Settlement, Basel.

Black, F./ Cox, J.C., Valuing Corporate Securities: Some Effects of Bond Indenture Provisions, 1976, Journal of Finance, 31, pp. 351- 367.

Black, F./ Scholes, M., The Pricing of Options and Corporate Liabilities, 1973, Journal of Political Economy, 81, pp. 637- 654.

Credit Suisse Financial Products, CreditRisk+ - A Credit Risk Management Framework, 1997, London.

Crosbie, P.J./ Bohn, J.R., Modeling Default Risk, 2002, KMV, San Francisco.

Davidson, R./ MacKinnon, J.G., Estimation and Inference in Econometrics, 1993, Oxford University Press, New York.

Dietsch, M./ Petey, J., The credit risk in SME loans portfolios: Modeling issues, pricing, and capital requirements, 2002, Journal of Banking and Finance, 26, pp. 303- 322.

Engelmann, B./ Hayden, E./ Tasche, D., Testing for Rating Accuracy, 2003, Risk, 16, January, pp. 82-86.

Escott, P./ Glormann, F./ Kocagil, A.E., Moody's RiskCalc™ for Private Companies: the German Model, 2001, Moody's Investors Service, New York.

Falkenstein, E., RiskCalc™ for Private Companies: Moody's Default Model, 2000, Moody's Investors Service, New York.

Finger, C.C., The One-Factor CreditMetrics Model in the New Basle Capital Accord, 2001, RiskMetrics Journal, pp. 9- 18.

Geske, R., The Valuation of Corporate Liabilities as Compound Options, 1977, Journal of Financial and Quantitative Analysis, 12, pp. 541- 552.

Gupton, G.M./ Finger, C.C./ Bhatia, M., CreditMetrics Technical Document, 1997, J.P. Morgan & Co., New York.

Longstaff, F.A./ Schwartz, E.S., A Simple Approach to Valuing Risky Fixed and Floating Rate Debt, 1995, Journal of Finance, 50, pp. 789- 819.

Lucas, D.J., Default Correlation and Credit Analysis, 1995, Journal of Fixed Income, pp. 76- 87.

Merton, R. C., On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, 1974, Journal of Finance, 29, pp. 449- 470.

Merton, R. C., On the Pricing of Contingent Claims and the Modigliani-Miller Theorem, 1977 Journal of Financial Economics, 5, pp. 241- 249.

Pinheiro, J.C./ Bates, D.M., Approximations to the Log-Likelihood Function in the Nonlinear Mixed-Effects Model, 1995, Journal of Computational and Graphical Statistics, 4, pp. 12- 35.

Rabe-Hesketh, S./ Skrondal, A./ Pickles, A., Estimation of generalized linear mixed models, 2002, Stata Journal, 2, pp. 1- 21.

Scheule, H., Prognose von Kreditausfallrisiken, 2003, Dissertation, University of Regensburg, Uhlenbruch Verlag.

Shumway, T., Forecasting Bankruptcy More Accurately: A Simple Hazard Model, 2001, The Journal of Business, 74, pp. 101- 124.

Sobehart, J.R./ Keenan, S.C./ Stein, R.M., Benchmarking Quantitative Default Risk Models: A Validation Methodology, 2000, Moody's Investors Service, New York.

Wilson, T.C., Portfolio Credit Risk I, 1997a, Risk, 10, September, pp. 111- 117.

Wilson, T.C., Portfolio Credit Risk II, 1997b, Risk, 10, October, pp. 56- 61.

Zhou, C., An Analysis of Default Correlations and Multiple Defaults, 2001, Review of Financial Studies, 14, pp. 555- 576.

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