

Formation Types of a Flock of 1-Trailer Mobile Robots

¹K. Raghuwaiya, ¹S. Singh, ¹B. Sharma, ²G. Lingam

¹School of Computing, Information and Mathematical Sciences,

²School of Education,

University of the South Pacific, Suva, Fiji

Abstract – *In this paper, we control the motion of a flock of 1-trailer systems. A set of artificial potential field functions is proposed for split/rejoin of the flock of 1-trailer robots via the Lyapunov-based control scheme for the avoidance of obstacles and attraction to their designated targets. A leader follower strategy is used to accomplish the desired formation and reformation of the flock. The flock maintains a prescribed formation, splits and maneuvers around obstacles and then returns to its original position in the prescribed formation. The various formations shapes that we shall consider are the line, column, arrowhead and the double platoon. The effectiveness of the proposed control laws are demonstrated through computer simulations.*

Keywords: *Lyapunov based control scheme, split/rejoin, flocking, 1-trailer system.*

1 Introduction

Formation behaviors seen in nature, like flocking and schooling, benefit individuals like animals that use them in various ways. Flocking is a coordinated and cooperative motion of groups (flocks) of entities or beings, ranging from simple bacteria to mammals [1]. Common examples include schools of fishes, flocks of birds, and herds of land animals, to name a few. This outstanding behavior is based on the principle that there is safety and strength in numbers [2], [3]. Conversely, if a flock is attacked, the members can disperse, thus avoid being captured and rejoin later at a safe distance. At the same time flocking behavior can contribute to safer long range migration [4]. The control mechanism of formation control can be divided into three layers; formation shape, formation type and robotic control.

The basic flocking model consists of three simple steering behaviors; separation, alignment and cohesion, which describe how an individual member or boid maneuvers, based on the positions and velocities of its nearby flockmates [5]. There are various approaches found in literature in relation to the strict observance of a prescribed formation of a flock during motion [6], [7]. One is the *split/rejoin* maneuver in flock of birds, swarms of insects and ants, and herds of animals. There are various applications of split/rejoin maneuvers in the field of robotics, for example, reconnaissance, sampling and surveillance. The other considers *tight formations* as can be required in many

engineering applications, for example parallel and simultaneous transportation of vehicles or delivery of payloads [8], [9], [6], [7], [10].

In recent years the split/rejoin maneuvers in robotic applications have become progressively more popular [8], [10], [11]. Many researchers have utilized various techniques to create swarming and flocking behaviors for multi agent systems. Recently, Sharma *et al.* [12], [7], [13] and Vanualailai *et al.* [14] proposed control algorithms that considered motion planning and control of mobile robots within a constrained environment cluttered with obstacles. In [15], the authors considered the autonomous control of a flock of six 1-trailer robots in an arrowhead formation. Continuous acceleration control laws were derived from the Lyapunov-based control scheme. The Lyapunov-based control scheme requires the design of target attractive functions and obstacle avoidance functions [16]. In this paper, we adopt this control scheme to control the flocking motion of a group of 1-trailer robots.

The attractive potential field functions enable the flock of tractor-trailer robots to move towards their designated target. The repulsive potential field functions, on the other hand, ensure a collision free avoidance in the workspace. One of the common reference types widely used in formation control is the *leader-follower* strategy. The split/rejoin maneuver for a flock of n 1-trailer system in this research is guided by the above strategy. The flock maintains a prescribed formation, splits and maneuvers around obstacles and then returns to its original position in the prescribed formation.

This paper is organized as follows: in Section 2 the robot model is defined; in Section 3 and 4 the artificial potential field functions are defined; in Section 5 the dynamic constraints are defined; in Section 6 the acceleration-based control laws are derived; in Section 7 we illustrate the effectiveness of the proposed controllers via simulations and in Section 8 we conclude the paper and outline future work in the area.

2 Vehicle Model

Two different trailer systems can be distinguished from literature; standard and the general trailer systems, grouped into two different categories based upon their different hooking schemes [13]. The standard 1-trailer system embodies a car-like robot and an on-axle hitched two wheeled passive trailer. The authors will consider n standard 1-trailer system, in Euclidean plane. With reference to Figure 1, (x_{im}, y_{im}) for $m=1,2$ represents the Cartesian coordinates and gives the reference point of each solid body of the articulated robot, θ_{i1} and θ_{i2} give the orientations with respect to the z_1 axis of the i th

tractor and trailer, respectively, while ϕ_i gives the i th tractor's steering angle with respect to its longitudinal axis. For simplicity, the dimensions of the n members are kept the same. Therefore, L_1 is the distance between the center of the rear and front axles of the i th tractor, L_2 is the distance from the midpoint of the rear axle of the tractor robot to the midpoint of the rear axle of the attached trailer and w is the length of each axle. The connections between the two bodies give rise to the following holonomic constraints on the system:

$$x_{i2} = x_{i1} - \frac{L_1}{2} \cos \theta_{i1} - \frac{L_2+2d}{2} \cos \theta_{i2}; \quad y_{i2} = y_{i1} - \frac{L_1}{2} \sin \theta_{i1} - \frac{L_2+2d}{2} \sin \theta_{i2}$$

We define $d := \varepsilon_i + c$, where c is a small offset (see Fig 1). The model of the i th tractor-trailer, adopted from [13], is

$$\left. \begin{aligned} \dot{x}_{i1} &= v_i \cos \theta_{i1} - \frac{L_1}{2} \omega_i \sin \theta_{i1}; & \dot{\theta}_{i1} &= \omega_i; & \dot{y}_{i1} &= v_i \sin \theta_{i1} + \frac{L_1}{2} \omega_i \cos \theta_{i1}; \\ \dot{\theta}_{i2} &= \frac{v_i}{L_2} \sin(\theta_{i1} - \theta_{i2}) \end{aligned} \right\} \quad (1)$$

where v_i and ω_i are, respectively, the instantaneous translational and rotational velocities, while σ_{i1} and σ_{i2} are the instantaneous translational and rotational accelerations of the i th tractor. Without loss of generality, we assume $\phi_i = \theta_{i1}$. The state of the i th tractor-trailer robot is then described by $\mathbf{x}_i := (x_{i1}, y_{i1}, \theta_{i1}, \theta_{i2}, v_i, \omega_i) \in \mathbb{R}^6$ for $i = 1, 2, \dots, n$. For the n members in the flock, we define $\mathbf{x} := (\mathbf{x}_1, \dots, \mathbf{x}_n) \in \mathbb{R}^{6 \times n}$.

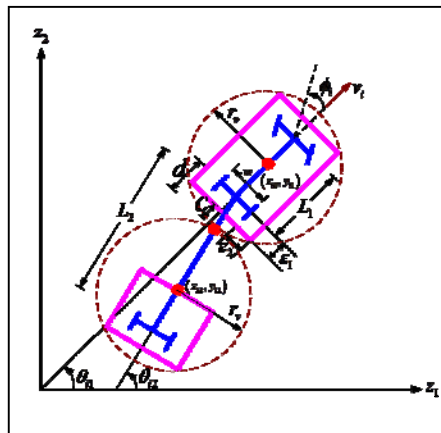


Fig. 1. Kinematic model of the i th 1-trailer boid.

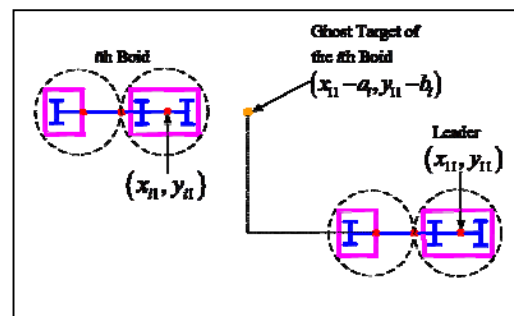


Fig. 2. Positioning of a mobile target relative to the position of the leader.

3 Attractive Potential Field Functions

3.1 Attraction to Target

This section formulates collision free trajectories of the robot system under kinodynamic constraints in a fixed and bounded workspace. It is assumed that the tractor-trailer robots have *priori* knowledge of the whole workspace. We want to design the acceleration controllers, σ_{i1} and σ_{i2} , so that the flock moves safely towards their respective targets. The leader-follower strategy utilized here is adopted from [17]. The flock of tractor-trailer robots follows one particular robot which acts as the leader. This is attained by fixing mobile ghost targets relative to the position of the leader (Figure 2).

Each of the follower robots have a different ghost target designated to it. While the leader moves towards its defined target, the ghost targets move relative to its position and the follower robots move towards their designated ghost target. We affix a target for each tractor-trailer robot to reach after some time t . For the i th tractor-trailer, we define a target

$$T_i = \left\{ (x, y) \in \mathbb{R}^2 : (z_1 - t_{i1})^2 + (z_2 - t_{i2})^2 \leq r_{t_i}^2 \right\}$$

with center (t_{i1}, t_{i2}) and radius r_{t_i} . The leader moves towards its target (t_{11}, t_{12}) . For the followers, the mobile ghost targets are positioned relative to the position of the leader whose center is given by $(t_{i1}, t_{i2}) = (x_{11} - a_i, y_{11} - b_i)$, for $i = 2, \dots, n$. For the attraction to these targets, we consider an attractive potential function

$$V_i(\mathbf{x}) = \frac{1}{2} \left[(x_{i1} - t_{i1})^2 + (y_{i1} - t_{i2})^2 + v_i^2 + \omega_i^2 \right] \quad (2)$$

for $i = 1, \dots, n$. This function is a measure of the distance between the first body of the i th tractor-trailer robot and its target T_i .

3.2 Auxiliary Function

To guarantee the convergence of the tractor-trailer mobile robot to its designated target, we design an auxiliary function defined as

$$G_i(\mathbf{x}) = \frac{1}{2} \left[(x_{i1} - t_{i1})^2 + (y_{i1} - t_{i2})^2 + (\theta_{i1} - t_{i3})^2 + (\theta_{i2} - t_{i4})^2 \right] \quad (3)$$

for $i = 1, \dots, n$ where t_{i3} is the desired final orientation of the i th tractor and t_{i4} is the desired final orientation of the i th trailer. This potential function is then multiplied to the repulsive potential functions to be designed in the following sections.

4 Repulsive Potential Field Functions

We desire the members of the flock to avoid all stationary and moving obstacles intersecting their paths. For this, we construct the obstacle avoidance functions that merely measure the distances between each body of the articulated robot and the obstacles in the workspace. To obtain the desired avoidance, these potential functions appear in the denominator of the repulsive potential field functions. This creates a repulsive field around the obstacles.

4.1 Fixed Obstacles in the Workspace

Let us fix q solid obstacles within the workspace and assume that the l th obstacle is circular with center (o_{l1}, o_{l2}) and radius ro_l . For the m th body of the i th articulated robot with a circular avoidance region of radius r_v to avoid the l th obstacle, we adopt

$$W_{iml}(\mathbf{x}) = \frac{1}{2} \left[(x_{im} - o_{l1})^2 + (y_{im} - o_{l2})^2 - (ro_l + r_v)^2 \right] \quad (4)$$

for $i = 1, \dots, n$, $m = 1, 2$ and $l = 1, 2, \dots, q$.

4.2 Moving Obstacles

To generate feasible trajectories, we consider moving obstacles of which the system has *priori* knowledge. Here, each member of the flock becomes a moving obstacle for all the other members. Therefore, for the m th body of the i th tractor trailer to avoid the u th body of the j th tractor-trailer, we have

$$M_{mij}(\mathbf{x}) = \frac{1}{2} \left[(x_{im} - x_{ju})^2 + (y_{im} - y_{ju})^2 - (2 \times r_v)^2 \right] \quad (5)$$

for $i, j = 1, \dots, n$ with $i \neq j$ and $m, u = 1, 2$.

4.3 Dynamic Constraints

Practically, the steering and bending angles of an articulated robot is limited due to mechanical singularities while the translational speed is restricted due to safety reasons. Subsequently, we have; $(\mathbf{i}) \quad |v| \leq v_{max}$, where

v_{max} is the maximal speed of the tractor; **(ii)** $|\phi_i| \leq \phi_{max} < \frac{\pi}{2}$, where ϕ_{max} is the maximal steering angle of the tractor; and **(iii)** $|\theta_{i1} - \theta_{i2}| \leq \theta_{max} < \frac{\pi}{2}$, where θ_{max} is the maximum bending angle of the trailer with respect to the orientation of the tractor. This prevents a jack knife situation. Thus, the trailer is free to rotate within $(-\frac{\pi}{2}, \frac{\pi}{2})$. Considering these constraints as artificial obstacles, we have the following potential field functions:

$$DC_{i1}(\mathbf{x}) = \frac{1}{2}[(v_{max} - v_i)(v_{max} + v_i)] \quad (6)$$

$$DC_{i2}(\mathbf{x}) = \frac{1}{2} \left[\left(\frac{v_{max}}{|\rho_{min}|} - \omega_i \right) \left(\frac{v_{max}}{|\rho_{min}|} + \omega_i \right) \right] \quad (7)$$

$$DC_{i3}(\mathbf{x}) = \frac{1}{2} [(\theta_{max} - (\theta_{i2} - \theta_{i1}))(\theta_{max} + (\theta_{i2} - \theta_{i1}))] \quad (8)$$

for $i = 1, \dots, n$. These potential functions guarantee the adherence to the above restrictions placed upon the translational velocity v_i , steering angle ϕ_i , and the rotation angle θ_{i2} of the i th trailer, respectively.

5 Design of Control Laws

Combining all the potential functions (2–8), and introducing constants, denoted as the control parameters, $\alpha_{iml} > 0$, $\beta_{muj} > 0$ and $\gamma_{is} > 0$ for $i, j, l, m, s, u \in \mathbb{N}$, we define a candidate Lyapunov function

$$L_{(1)}(\mathbf{x}) = \sum_{i=1}^n \left\{ V_i(\mathbf{x}) + G_i(\mathbf{x}) \left[\sum_{m=1}^2 \sum_{l=1}^q \left(\frac{\alpha_{iml}}{W_{iml}(\mathbf{x})} \right) + \sum_{s=1}^3 \frac{\gamma_{is}}{DC_{is}(\mathbf{x})} + \sum_{\substack{j=1 \\ i \neq j}}^n \sum_{m=1}^2 \sum_{u=1}^2 \left(\frac{\beta_{muj}}{M_{muj}(\mathbf{x})} \right) \right] \right\} \quad (9)$$

Clearly, $L(\mathbf{x})$ is locally positive and continuous on the domain $D(L) = \{\mathbf{x} \in \mathbb{R}^{6 \times n} : W_{iml}(\mathbf{x}) > 0, DC_{is}(\mathbf{x}) > 0, M_{muj}(\mathbf{x}) > 0\}$. We define $\mathbf{x}_e := (t_{i1}, t_{i2}, t_{i3}, t_{i4}, 0, 0) \in \mathbb{R}^{6 \times n}$ for $i = 1, \dots, n$ as an equilibrium point of system (1). Thus, we have $L(\mathbf{x}_e) = 0$.

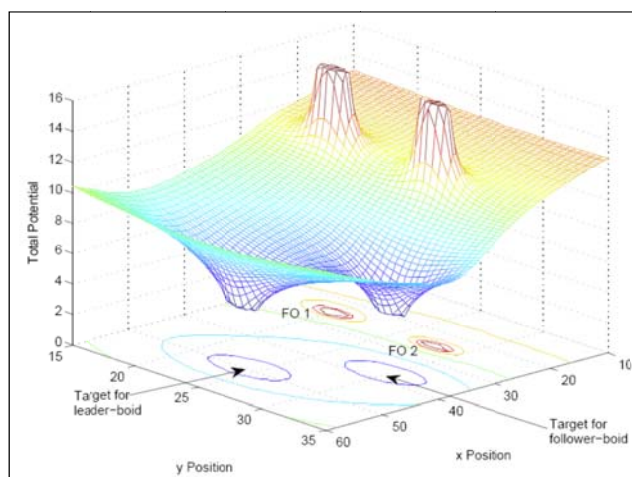


Fig. 3. Total Potential

To extract the control laws, we differentiate the various components of $L(\mathbf{x})$ separately and carry out the necessary substitutions from (1). We begin with the following theorem.

Theorem 1. *The equilibrium point \mathbf{x}_e of system (1) is stable in the sense of Lyapunov provided*

$$\sigma_{i1} = -\frac{1}{f_{5i}} \left[\delta_{i1} v_i + f_{1i} \cos \theta_{i1} + f_{2i} \sin \theta_{i1} + f_{4i} \frac{1}{L_2} \sin(\theta_{i1} - \theta_{i2}) \right]$$

and

$$\sigma_{i2} = -\frac{1}{f_{6i}} \left[\delta_{i2} \omega_i + \frac{L_1}{2} (-f_{1i} \sin \theta_{i1} + f_{2i} \cos \theta_{i1}) + f_{3i} \right]$$

for $i = 1, \dots, n$, where $\delta_{i1}, \delta_{i2} > 0$ are constants commonly known as convergence parameters.

Proof: The time derivative of our Lyapunov function $L(\mathbf{x})$ along a particular trajectory of system (1) is then:

$$\begin{aligned} \dot{L}_{(1)}(\mathbf{x}) = & \sum_{i=1}^n \left[f_{1i} \cos \theta_{i1} + f_{2i} \sin \theta_{i1} + f_{4i} \frac{1}{L_2} \sin(\theta_{i1} - \theta_{i2}) + f_{5i} \sigma_{i1} \right] v_i \\ & + \sum_{i=1}^n \left[\frac{L_1}{2} (-f_{1i} \sin \theta_{i1} + f_{2i} \cos \theta_{i1}) + f_{3i} + f_{6i} \sigma_{i2} \right] \omega_i \end{aligned}$$

where the functions f_{1i} to f_{6i} are defined as (upon suppressing \mathbf{x}):

$$\begin{aligned}
 f_{11} = & \left[1 + \sum_{m=1}^2 \sum_{l=1}^q \frac{\alpha_{1ml}}{W_{1ml}} + \sum_{s=1}^3 \frac{\gamma_{1s}}{DC_{1s}} + \sum_{j=2}^n \sum_{m=1}^2 \sum_{u=1}^2 \frac{\beta_{mu1j}}{M_{mu1j}} \right] (x_{11} - t_{11}) \\
 & - \sum_{i=2}^n \left[1 + \sum_{m=1}^2 \sum_{l=1}^q \frac{\alpha_{iml}}{W_{iml}} + \sum_{s=1}^3 \frac{\gamma_{is}}{DC_{is}} + \sum_{j=1}^n \sum_{m=1}^2 \sum_{u=1}^2 \frac{\beta_{muj}}{M_{muj}} \right] (x_{i1} - t_{i1}) \\
 & - G_1 \sum_{m=1}^2 \sum_{l=1}^q \frac{\alpha_{1ml}}{W_{1ml}} (x_{1m} - o_{11}) - G_1 \sum_{j=1}^n \sum_{m=1}^2 \sum_{u=1}^2 \frac{\beta_{mu1j}}{M_{mu1j}} (x_{1m} - x_{ju}) + \sum_{j=1}^n \sum_{m=1}^2 \sum_{u=1}^2 G_j \frac{\beta_{muj}}{M_{muj}} (x_{jm} - x_{iu})
 \end{aligned}$$

$$\begin{aligned}
 f_{21} = & \left[1 + \sum_{m=1}^2 \sum_{l=1}^q \frac{\alpha_{1ml}}{W_{1ml}} + \sum_{s=1}^3 \frac{\gamma_{1s}}{DC_{1s}} + \sum_{j=2}^n \sum_{m=1}^2 \sum_{u=1}^2 \frac{\beta_{mu1j}}{M_{mu1j}} \right] (y_{11} - t_{12}) \\
 & - \sum_{i=2}^n \left[1 + \sum_{m=1}^2 \sum_{l=1}^q \frac{\alpha_{iml}}{W_{iml}} + \sum_{s=1}^3 \frac{\gamma_{is}}{DC_{is}} + \sum_{j=1}^n \sum_{m=1}^2 \sum_{u=1}^2 \frac{\beta_{muj}}{M_{muj}} \right] (y_{i1} - t_{i2}) \\
 & - G_1 \sum_{m=1}^2 \sum_{l=1}^q \frac{\alpha_{1ml}}{W_{1ml}} (y_{1m} - o_{12}) - G_1 \sum_{j=1}^n \sum_{m=1}^2 \sum_{u=1}^2 \frac{\beta_{mu1j}}{M_{mu1j}} (y_{1m} - y_{ju}) + \sum_{j=1}^n \sum_{m=1}^2 \sum_{u=1}^2 G_j \frac{\beta_{muj}}{M_{muj}} (y_{jm} - y_{iu})
 \end{aligned}$$

$$\begin{aligned}
 f_{1i} = & \left[1 + \sum_{m=1}^2 \sum_{l=1}^q \frac{\alpha_{iml}}{W_{iml}} + \sum_{s=1}^3 \frac{\gamma_{is}}{DC_{is}} + \sum_{j=1}^n \sum_{m=1}^2 \sum_{u=1}^2 \frac{\beta_{muj}}{M_{muj}} \right] (x_{i1} - t_{i1}) - G_i \sum_{m=1}^2 \sum_{l=1}^q \frac{\alpha_{iml}}{W_{iml}} (x_{im} - o_{11}) \\
 & - G_i \sum_{j=1}^n \sum_{m=1}^2 \sum_{u=1}^2 \frac{\beta_{muj}}{M_{muj}} (x_{im} - x_{ju}) + \sum_{j=1}^n \sum_{m=1}^2 \sum_{u=1}^2 G_j \frac{\beta_{muj}}{M_{muj}} (x_{jm} - x_{iu})
 \end{aligned}$$

$$\begin{aligned}
 f_{2i} = & \left[1 + \sum_{m=1}^2 \sum_{l=1}^q \frac{\alpha_{iml}}{W_{iml}} + \sum_{s=1}^3 \frac{\gamma_{is}}{DC_{is}} + \sum_{j=1}^n \sum_{m=1}^2 \sum_{u=1}^2 \frac{\beta_{muj}}{M_{muj}} \right] (y_{i1} - t_{i2}) - G_i \sum_{m=1}^2 \sum_{l=1}^q \frac{\alpha_{iml}}{W_{iml}} (y_{im} - o_{12}) \\
 & - G_i \sum_{j=1}^n \sum_{m=1}^2 \sum_{u=1}^2 \frac{\beta_{muj}}{M_{muj}} (y_{im} - y_{ju}) + \sum_{j=1}^n \sum_{m=1}^2 \sum_{u=1}^2 G_j \frac{\beta_{muj}}{M_{muj}} (y_{jm} - y_{iu})
 \end{aligned}$$

$$\begin{aligned}
 f_{3i} = & \left[\sum_{m=1}^2 \sum_{l=1}^q \frac{\alpha_{iml}}{W_{iml}} + \sum_{s=1}^3 \frac{\gamma_{is}}{DC_{is}} + \sum_{j=1}^n \sum_{m=1}^2 \sum_{u=1}^2 \frac{\beta_{muj}}{M_{muj}} \right] (\theta_{i1} - t_{i3}) - G_i \frac{\gamma_{i3}}{DC_{i3}} (\theta_{i2} - \theta_{i1}) \\
 & - G_i \frac{L_1}{2} \sum_{l=1}^q \frac{\alpha_{i2l}}{W_{i2l}} [(x_{i2} - o_{11}) \sin \theta_{i1} - (y_{i2} - o_{12}) \cos \theta_{i1}] \\
 & - G_i \frac{L_1}{2} \sum_{j=1}^n \sum_{u=1}^2 \frac{\beta_{2uj}}{M_{2uj}} [(x_{i2} - x_{ju}) \sin \theta_{i1} - (y_{i2} - y_{ju}) \cos \theta_{i1}] \\
 & + \frac{L_1}{2} \sum_{j=1}^n \sum_{m=1}^2 G_j \frac{\beta_{m2ji}}{M_{m2ji}} [(x_{jm} - x_{i2}) \sin \theta_{i1} - (y_{jm} - y_{i2}) \cos \theta_{i1}]
 \end{aligned}$$

$$\begin{aligned}
 f_{4i} = & \left[\sum_{m=1}^2 \sum_{l=1}^q \frac{\alpha_{iml}}{W_{iml}} + \sum_{s=1}^3 \frac{\gamma_{is}}{DC_{is}} + \sum_{j=1}^n \sum_{m=1}^2 \sum_{u=1}^2 \frac{\beta_{muj}}{M_{muj}} \right] (\theta_{i2} - t_{i4}) + G_i \frac{\gamma_{i3}}{DC_{i3}^2} (\theta_{i2} - \theta_{i1}) \\
 & - G_i \frac{L_2 + 2d}{2} \sum_{l=1}^q \frac{\alpha_{i2l}}{W_{i2l}^2} [(x_{i2} - o_{i1}) \sin \theta_{i2} - (y_{i2} - o_{i2}) \cos \theta_{i2}] \\
 & - G_i \frac{L_2 + 2d}{2} \sum_{j=1}^n \sum_{u=1}^2 \frac{\beta_{2uj}}{M_{2uj}^2} [(x_{i2} - x_{ju}) \sin \theta_{i2} - (y_{i2} - y_{ju}) \cos \theta_{i2}] \\
 & + \frac{L_2 + 2d}{2} \sum_{j=1}^n \sum_{m=1}^2 \sum_{i \neq j} G_j \frac{\beta_{m2ji}}{M_{m2ji}^2} [(x_{jm} - x_{i2}) \sin \theta_{i2} - (y_{jm} - y_{i2}) \cos \theta_{i2}]
 \end{aligned}$$

for $i = 2, \dots, n$, and

$$f_{5i} = 1 + G_i \frac{\gamma_{i1}}{DC_{i1}^2}, \quad f_{6i} = 1 + G_i \frac{\gamma_{i2}}{DC_{i2}^2}$$

for $i = 1, \dots, n$.

We note that $\dot{L}_{(1)}(\mathbf{x}) = -\sum_{i=1}^n (\delta_{i1} v_i^2 + \delta_{i2} \omega_i^2) \leq 0$ for all $\mathbf{x} \in D(L)$, and $\dot{L}_{(1)}(\mathbf{x}_e) = 0$.

A careful scrutiny of the properties of our scalar function reveals that \mathbf{x}_e is an equilibrium point of system (1) in the sense of Lyapunov and $L(\mathbf{x})$ is a legitimate Lyapunov function guaranteeing stability. This is in no contradiction with Brockett's result [18] as we have not proven asymptotic stability.

6 Simulation

To illustrate the effectiveness of the proposed controllers, we present a split/rejoin maneuver of a flock of six, 1-trailer robots. The robots are clustered at the starting line, then get into a prescribed formation and move in the direction of their targets. Upon encountering an obstacle, the formation of the articulated robots split. The members move around the obstacle and later rejoin the group into their prescribed formation.

TABLE 1
NUMERICAL VALUES OF INITIAL STATES, CONSTRAINTS AND
PARAMETERS FOR THE ARROWHEAD FORMATION.

Initial Conditions	
Rectangular positions	$(x_{11}, y_{11}) = (10, 14), (x_{21}, y_{21}) = (10, 17); (x_{31}, y_{31}) = (10, 11), (x_{41}, y_{41}) = (4, 14);$ $(x_{51}, y_{51}) = (4, 17), (x_{61}, y_{61}) = (4, 11)$
Angular positions and velocities	$v_1 = 0.5$ and $v_i = 9$ for $i = 2$ to $6, \omega_i = 0.8, \theta_{i1} = \theta_{i2} = 0$ for $i = 1$ to 6
Control and Convergence Parameters	
Obstacle avoidance	$\alpha_{ml} = 0.5$ for $i = 1$ to $6, m = 1, 2$ and $l = 1, 2$
Boid avoidance	$\beta_{muj} = 0.001$ for $i = 1$ to $6, j = 1$ to $6, i \neq j, m = 1, 2$ and $u = 1, 2$
Dynamic constraints	$\gamma_{is} = 0.1$ for $i = 1$ to 6 and $s = 1$ to 4
Convergence	$\sigma_{11} = 92, \sigma_{i1} = 10$ for $i = 2$ to 6 and $\sigma_{i2} = 10$ for $i = 1$ to 6
Constraints and Parameters	
Final orientations	$t_{i3}, t_{i4} = 0$ for $i = 1$ to 6
Leader Target	$(t_{11}, t_{12}) = (80, 14), rt_1 = 0.2$
Dimensions of boids	$L_1 = 1.3, L_2 = 1.9, w = 0.5$
Position of ghost targets relative to position of leader	$(a_2, b_2) = (3, -3), (a_3, b_3) = (3, 3), (a_4, b_4) = (6, 0),$ $(a_5, b_5) = (9, -3), (a_6, b_6) = (9, 3)$
Fixed obstacles (o_{11}, o_{12})	$(o_{11}, o_{12}) = (40, 20), (o_{21}, o_{22}) = (40, 8); ro_1 = ro_2 = 2.5$
Max. translational speed	$v_{\max} = 10$
Min. turning radius	$\rho_{\min} = 0.75$
Clearance parameters	$\varepsilon_1 = 0.1, \varepsilon_2 = 0.2$

The leader is chosen arbitrarily from the flock. The members of the flock move towards their ghost targets positioned relative to their leader. The corresponding initial and final states and other details for the simulation are listed in Table 1 (assuming that appropriate units have been taken into account). From the initial configuration, the members of the flock quickly move into a desired formation. Upon encountering an obstacle, the members split from their designated formation and maneuver around the obstacles. While avoiding the two obstacles, the members maintain a collision free avoidance with another member of the flock. After avoidance, the members rejoin the flock into the same prescribed formation. Several formations of a team of six 1-trailer robots are considered (Fig 8):

- Line – the robots travel line-abreast.
- Column – the robots travel one after the other.
- Arrowhead – the robots travel in an arrowhead formation.
- Double Platoon – the robots travel in double platoon.

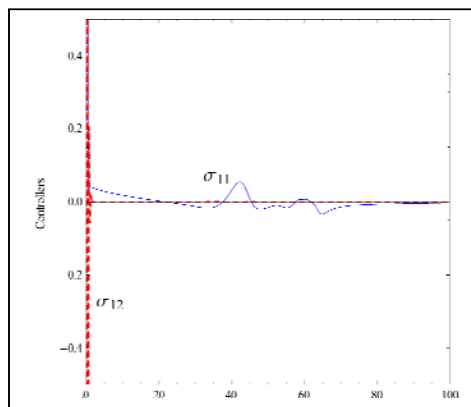


Fig. 4. Accelerations: σ_{11} and σ_{12} .

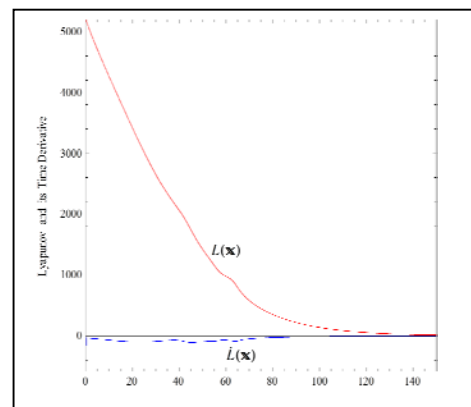


Fig. 5. Evolution of $L(\mathbf{x})$ and its time derivative.

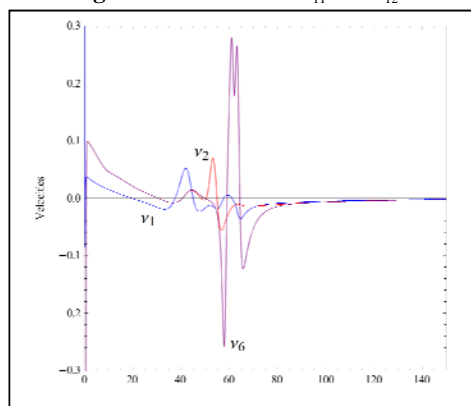


Fig. 6. Translational Velocities: v_1 , v_2 and v_6 .

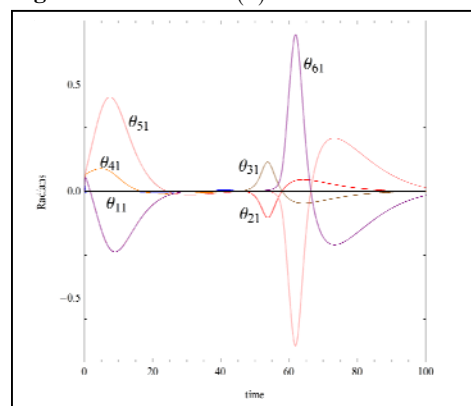
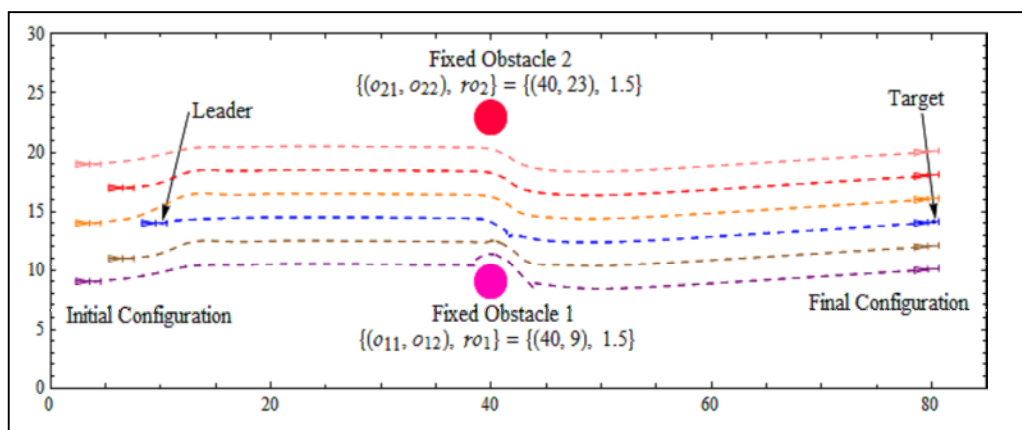
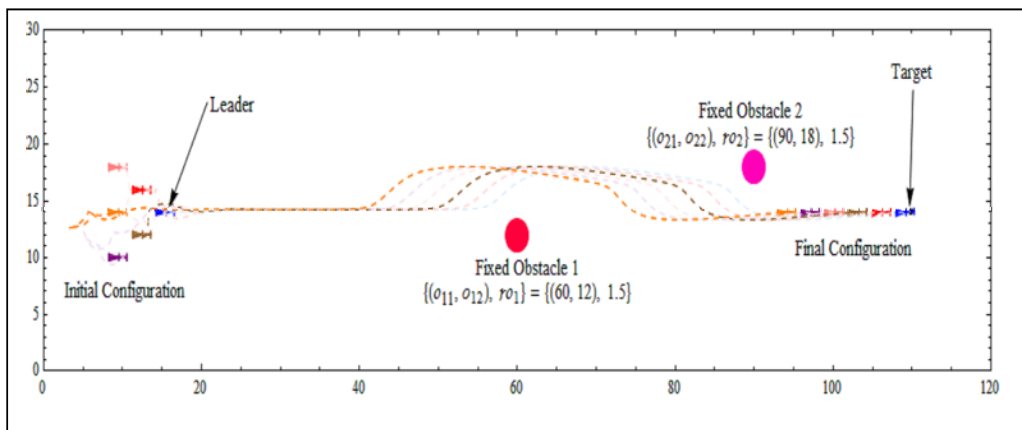


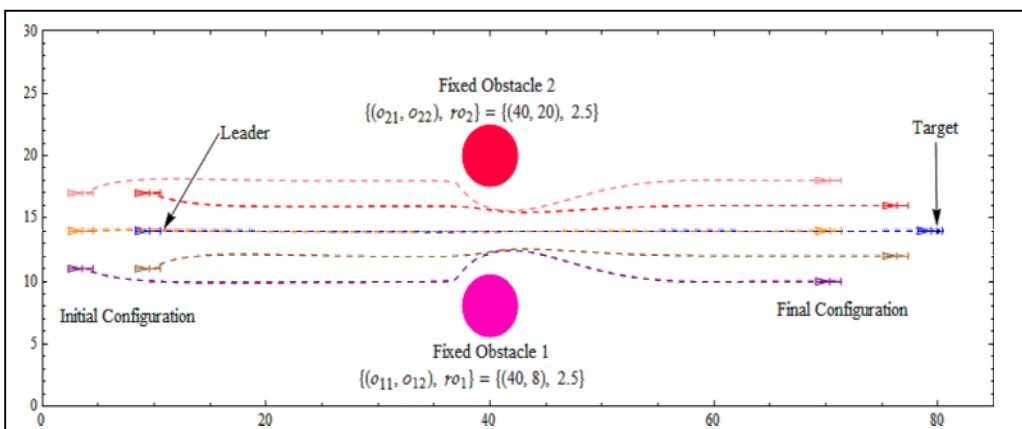
Fig. 7. Orientations of the 1st body of the leader and its followers.



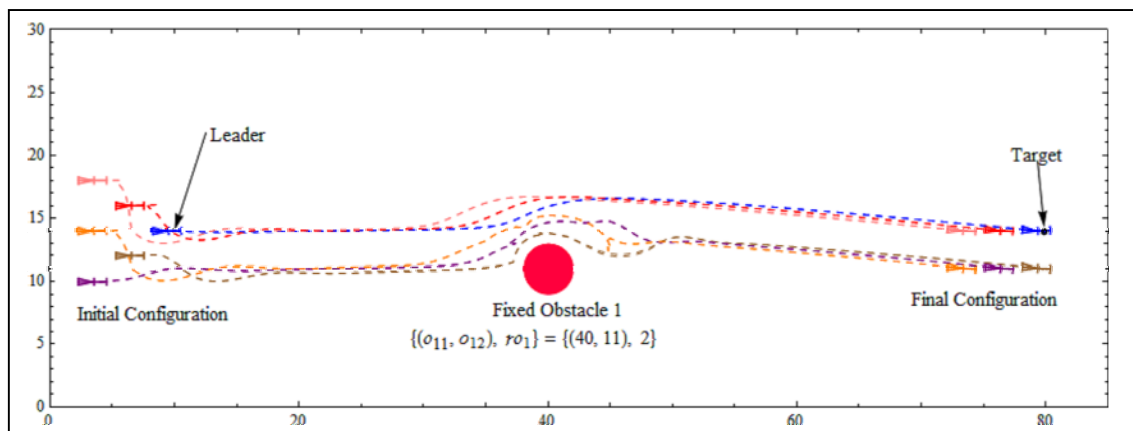
(a) Line



(b) Column



(c) Arrowhead



(d) Double-Platoon

Fig. 8: The line, column, double platoon and arrowhead formation, split/rejoin maneuvers for a flock of six 1-trailer robots and reformation before reaching its final state.

Fig. 4 and Fig.5 show the convergence and boundedness of the variables at the final state, respectively, implying the effectiveness of the control laws. Fig. 6 and Fig.7 show the time evolution of the translational and rotational accelerations of the leader.

7 Conclusion

This paper presents a set of artificial field functions derived from a Lyapunov-based control scheme. We were able to generate a prescribed formation, split/rejoin when necessary to maneuver around obstacles and re-group to the original formation. The derived controllers produced feasible trajectories and ensured a nice convergence of the system to its equilibrium state while satisfying the necessary kinematic and dynamic constraints. We note here that convergence is only guaranteed from a number of initial states of the system.

The prescribed formations of the flock were possible by the leader-follower strategy that involved having mobile ghost targets positioned relative to the position of the leader. Future research will address the general tractor-trailer systems and extending the results to steerable trailer systems.

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