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Magnetic field effect on tunnel ionization of deep impurities by far-infrared radiation

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Abstract

The probability of electron tunneling from a bound to a free state in an alternating electric and a constant magnetic field is calculated in the quasiclassical approximation. It is shown that the magnetic field reduces the probability of electron tunneling. The application of the external magnetic field perpendicular to the electric field reduces the ionization probability at high magnetic fields, when cyclotron resonance frequency becomes larger than reciprocal tunneling time. The increase of electric field frequency to values larger than the same reciprocal tunneling time enhances the influence of magnetic field.

Keywords: Magnetic field; Far infrared; Ionization rate; Deep centers

1. Introduction

Recently, it has been demonstrated theoretically and experimentally that the ionization of deep impurities in semiconductors in the presence of static as well as high-frequency electric fields occurs via the multiphonon-assisted tunneling Refs. [1–4]. In contrast to tunneling ionization of atoms, where only electron tunneling takes place, ionization of impurities in solids is accomplished by two simultaneous processes: electron tunneling and the redistribution of the vibrational system by defect tunneling [1,4]. Recently, using results [5] for the probability of electron tunneling, it has been shown that an external magnetic field applied perpendicular to the electric field, suppresses the multiphonon-assisted tunneling in a static electric field at $\omega_c\tau_2 > 1$, where ω_c is the cyclotron frequency and τ_2 the defect tunneling time [6].

In this work we study theoretically the influence of an external magnetic field on the thermally activated ionization of impurities in the presence of an alternating electric field. The interest to this problem is caused by the observation of an enhancement of multiphonon-assisted tunnel ionization of deep impurities in various semiconductors in terahertz electric fields as compared to static fields [3]. As shown in Ref. [3], this process is also controlled by the defect tunneling time τ_2. At $\Omega\tau_2 > 1$, where Ω is the frequency of electric field the probability of tunnel ionization increases drastically with rising frequency.

The work consists of two parts. In the first part an expression for the probability of tunneling of an electron through an alternating barrier is obtained. In this part we use the method similar to used in Ref. [8]. In the second part we study the thermally activated tunneling in the presence of an alternating electric field and an external magnetic field in semiclassical approximation and neglecting preexponential factors. We show that in this approximation the logarithm of the ionization rate linearly depends on the square of the electric field amplitude, as it has already been obtained for phonon-assisted tunneling without magnetic field [7,3]. The application of the external magnetic field perpendicular to the electric field reduces the ionization probability at $\omega_c\tau_2 > 1$. At

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frequencies of the electric field high enough to achieve the regime $\Omega\tau_2 > 1$, the influence of the magnetic field drastically increases.

2. Quasiclassical wave function of tunneling electron

The quasiclassical wave function in quasiclassical approximation and neglecting preexponential factors is given by the expression

$$\Psi(\boldsymbol{r},t) \sim \mathrm{e}^{(i/\hbar)\cdot \tilde{S}(\boldsymbol{r},t)}, \tag{1}$$

where the action $\tilde{S}(\boldsymbol{r},t)$ must confirm to the Hamilton–Jacobi equations

$$\frac{\partial \tilde{S}}{\partial t} = -\mathscr{H}(\boldsymbol{P},\boldsymbol{r},t), \quad \nabla\tilde{S} = \boldsymbol{P} \tag{2}$$

with the boundary condition $\tilde{S}(\boldsymbol{r},t) = -\varepsilon t$ at $\boldsymbol{r} = 0$, where ε is the energy of an electron bound to the center ($\varepsilon < 0$). In Eq. (2) $\mathscr{H}$ is the Hamilton function, $\boldsymbol{P}$ is the generalized momentum. In our case

$$\mathscr{H}(\boldsymbol{P},\boldsymbol{r},t) = \frac{(\boldsymbol{P} - e/c\boldsymbol{A})^2}{2m} - ZF(t), \quad P = m\dot{\boldsymbol{r}} + \frac{e}{c}\boldsymbol{A}, \tag{3}$$

where $F(t) = F\cos\Omega t$ is the force applied to an electron from an electric field of a wave, $F = eE$. We consider that the electric field $\boldsymbol{E}$ is directed along the axis Z, and a constant magnetic field H along the axis Z. Below we use the calibration $A_x = A_z = 0$; $A_y = -Hz$.

The action $\tilde{S}$, which confirms to required conditions can be written

$$\tilde{S} = S_0 - \varepsilon t_0, \quad S_0 = \int_{t_0}^{t} \mathscr{L}(\boldsymbol{r}',\dot{\boldsymbol{r}}',t')\,\mathrm{d}t', \tag{4}$$

where $\mathscr{L}$ is the Lagrange function

$$\mathscr{L}(\boldsymbol{r}',\dot{\boldsymbol{r}}',t') = \frac{m\dot{\boldsymbol{r}}'^2}{2} - z'\left[\frac{eH}{c}\dot{y}' - F(t')\right]. \tag{5}$$

The radius-vector $\boldsymbol{r}'(t')$ confirms to the motion laws

$$\ddot{x}' = 0, \qquad \ddot{y}' = \omega_c\dot{z}', \qquad \ddot{z}' = \frac{F}{m}\cos\Omega t' - \omega_c\dot{y}' \tag{6}$$

with conditions

$$\boldsymbol{r}'(t_0) = 0, \quad \boldsymbol{r}'(t) = r. \tag{7}$$

In Eq. (4) t_0 is a function $\boldsymbol{r}$ and t determined by the condition

$$\left(\frac{\mathrm{d}\tilde{S}}{\mathrm{d}t_0}\right)_{r,t} = 0. \tag{8}$$

We want to emphasize that due to the sense of wave functions values of $\boldsymbol{r}$ and t are real, while $\boldsymbol{r}'$, t' and t_0 can be complex. Using equation of Ref. [9]

$$\left(\frac{\mathrm{d}S_0}{\mathrm{d}t_0}\right)_{r,t} = \mathscr{H}|_{t=t_0}$$

and the first boundary condition of Eqs. (7) and (8) can be written as

$$\frac{m}{2}(v_{0x}^2 + v_{0y}^2 + v_{0z}^2) = \varepsilon, \tag{9}$$

where $\boldsymbol{v}_0 = \dot{\boldsymbol{r}}'(t_0)$ is the velocity at the beginning of motion t_0. The value of this velocity is purely imaginary due to the condition $\varepsilon < 0$ in Eq. (9). This is natural because an electron is under the barrier.

3. Probability of direct electron tunneling

After solving of the motion equation (6) with conditions (7), it is possible to get $\boldsymbol{v}_0$ as a function of $\boldsymbol{r}$, t, t_0. Then Eq. (9) determines t_0 as a function of $\boldsymbol{r}$, t. Finally, we find the action $\tilde{S}$ as a function $\boldsymbol{r}$, t, and therefore wave function $\Psi(\boldsymbol{r},t)$.

To find the density current from the center, which is proportional to $|\Psi|^2$, it is enough to find $\mathrm{Im}\tilde{S}$ in the area of space, where it reaches its maximum, e.g. with the values of $\boldsymbol{r}$, where $\nabla\mathrm{Im}\tilde{S} = 0$. The probability of ionization P_e can be written as

$$P_e \sim \exp[-2S_e(\varepsilon)], \tag{10}$$

where $S_e(\varepsilon) = \mathrm{Im}\,\tilde{S}/\hbar$ in the area of space, which determines the result of tunneling. Then $S_e(\varepsilon)$ can be found as

$$S_e(\varepsilon) = \frac{m}{2\hbar}\int_0^{\tau_e}(\dot{z}'^2 - \dot{y}'^2)\,\mathrm{d}\tau - \frac{\varepsilon\tau_e}{\hbar}, \tag{11}$$

where τ_e is defined by the equation

$$\mathrm{sh}^2\Omega\tau_e\left[1 - \left(\frac{\Omega\omega_c}{\Omega^2 - \omega_c^2}\right)^2\left(\mathrm{cth}\Omega\tau_e - \frac{\omega_c}{\Omega}\,\mathrm{cth}\omega_c\tau_e\right)^2\right] = \frac{\Omega^2}{F^2}2m|\varepsilon|, \tag{12}$$

and

$$\dot{y}' = \frac{F\omega_c}{m(\Omega^2 - \omega_c^2)}\left[-\mathrm{ch}\Omega\tau + \frac{\omega_c}{\Omega}\frac{\mathrm{sh}\Omega\tau_e}{\mathrm{sh}\omega_c\tau_e}\mathrm{ch}\omega_c\tau\right], \tag{13a}$$

$$\dot{z}' = \frac{F\omega_c}{m(\Omega^2 - \omega_c^2)}\left[\frac{\Omega}{\omega_c}\mathrm{sh}\Omega\tau - \frac{\omega_c}{\Omega}\frac{\mathrm{sh}\Omega\tau_e}{\mathrm{sh}\omega_c\tau_e}\mathrm{sh}\omega_c\tau\right]i. \tag{13b}$$

Thus, the probability of ionization, neglecting pre-exponential factors, is given by Eq. (10), where $S_e(\varepsilon)$ and $\tau_e(\varepsilon)$ are defined by Eqs. (11) and(12). Note that the parameter:

$$\tau_e = -\hbar\frac{\partial S_e}{\partial \varepsilon}, \tag{14}$$

and thus can be named as time of electron tunneling. Without magnetic field (with $\omega_c = 0$) Eqs. (10)–(13) coincide with the result of Ref. [8] for the tunnel ionization in an alternating electric field and when $\Omega = 0$ with the result of Ref. [5] for the tunnel ionization in a constant electric field in the presence of the magnetic field.

The probability of the ionization decreases and tunneling time increases when the magnetic field increases.

4. Thermally activated tunnel ionization of deep center

The probability of tunnel ionization with participation of phonons can be considered as a result of three processes: (i) thermal excitation of the system to a vibrational level $\mathscr{E}_1$ in the adiabatic potential $U_1(x)$, corresponding to an electron bound to the center (x is the vibrational coordinate), (ii) tunnel transition of the vibrational system to the adiabatic potential $U_{2\varepsilon}(x)$, corresponding to free electron with potential energy ε and (iii) tunnel transition of an electron under influence of an electric field out of the well to the free state with negative energy ε.

In quasiclassical approximation the probability of ionization with fixed ε and $\mathscr{E}_1$ is proportional to the expression:

$$\exp\left(-\frac{\mathscr{E}_1}{kT}\right)\exp[-2(S_{2\varepsilon} - S_{1\varepsilon})]\exp[-2S_e(\varepsilon)], \tag{15}$$

where the three factors are probabilities of the three processes enumerated above,

$$S_{1\varepsilon} = \frac{\sqrt{2M}}{\hbar}\int_{a_1}^{x_{c\varepsilon}} \sqrt{U_1(x) - \mathscr{E}}\,dx, \quad \mathscr{E} = \mathscr{E}_1 - \varepsilon_T, \tag{16a}$$

$$S_{2\varepsilon} = \frac{\sqrt{2M}}{\hbar}\int_{a_{2\varepsilon}}^{x_{c\varepsilon}} \sqrt{U_{2\varepsilon}(x) - \mathscr{E}}\,dx, \quad U_{2\varepsilon}(x) = U_2(x) + \varepsilon, \tag{16b}$$

where M is a mass, corresponding to the vibrational mode, which plays the main role in the process. $\mathscr{E}$ and $\mathscr{E}_1$ are local vibration energies, counted from the bottom of potentials U_2 and U_1, respectively. The scheme of adiabatic potentials and the tunneling trajectories are shown in Fig. 1.

The total probability of ionization is obtained by integrating product (15) over $\mathscr{E}_1$ and ε. Calculation of this integral by the saddle-point method yields two equations for the optimal values $\mathscr{E}_{1m}$ and ε_m:

$$\tau_{2\varepsilon} = \tau_{1\varepsilon} + \hbar/2kT, \quad \tau_{2\varepsilon} = \tau_e, \tag{17}$$

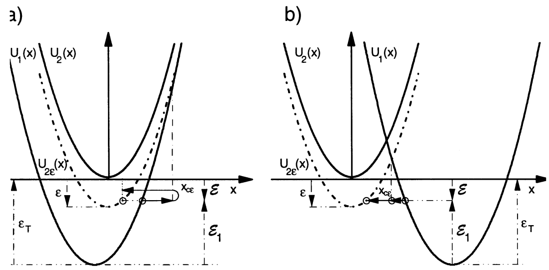

Fig. 1. Adiabatic potential configurations for: (a) weak electron–phonon coupling and (b) strong electron–phonon coupling (autolocalization). Potentials plotted in broken lines correspond to the electron with negative kinetic energy tunneling in electric fields of two different strengths. Solid arrows show the tunneling trajectories of vibrational system.

where the "tunneling times" of the vibrational system $\tau_{n\varepsilon}$ are determined by the expression

$$\tau_{n\varepsilon} = -\hbar\frac{\partial S_{n\varepsilon}}{\partial \mathscr{E}}.$$

For sufficiently weak fields the value of ε_m is small and in the first Eq. (17) we can consider $\varepsilon = 0$. Then it determines the energy $\mathscr{E}_{1m}$, which does not depend on the electric field. In Eq. (15) the difference $S_{2\varepsilon} - S_{1\varepsilon}$ can be expanded in a power series of ε and only the first term in the expansion need be retained

$$S_{2\varepsilon} - S_{1\varepsilon} = S_2 - S_1 + \frac{\varepsilon\tau_2}{\hbar}, \tag{18}$$

where S_2, S_1, τ_2 are the values of $S_{2\varepsilon}$, $S_{1\varepsilon}$ and $\tau_{2\varepsilon}$ at $\varepsilon = 0$. Then using Eq. (11) at $\tau_e = \tau_2$ we can get for the ionization probability $e(F)$:

$$e(F) = e(0)\exp\frac{F^2}{F_c^{*2}}, \tag{19}$$

where it is convenient to write

$$F_c^{*2} = \frac{3m\hbar}{\tau_2^{*3}}, \tag{20a}$$

$$\tau_2^{*3} = \frac{3\omega_c^2}{(\Omega^2 - \omega_c^2)^2}\int_0^{\tau_2}\left\{\left[-\mathrm{ch}\Omega\tau + \frac{\omega_c}{\Omega}\frac{\mathrm{sh}\Omega\tau_2}{\mathrm{sh}\omega_c\tau_2}\mathrm{ch}\omega_c\tau\right]^2 + \left[\frac{\Omega}{\omega_c}\mathrm{sh}\Omega\tau - \frac{\omega_c}{\Omega}\frac{\mathrm{sh}\Omega\tau_2}{\mathrm{sh}\omega_c\tau_2}\mathrm{sh}\omega_c\tau\right]^2\right\}d\tau. \tag{20b}$$

The dependence of τ_2^* on the magnetic field is illustrated in Fig. 2.

Without magnetic field ($\omega_c \to 0$) Eqs. (19) and (20) coincide with expression for the probability of multiphonon thermally activated tunneling under the influence of an alternating electric field obtained in Ref. [3], and at $\Omega \to 0$ with results of Ref. [6].

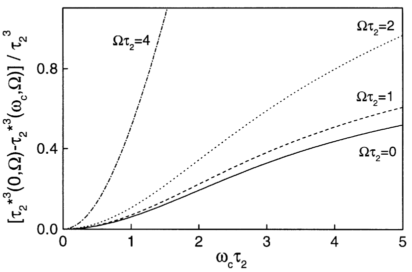

Fig. 2. The ratio $(\tau_2^{*3}(0,\Omega) - \tau_2^{*3}(\omega_c,\Omega))/\tau_2^3$ versus $\omega_c\tau_2$ calculated after Eq. (20b) for various values of the $\Omega\tau_2$. Note, that the only parameter in Eq. (20b) which determines a scale for the frequency is the tunneling time τ_2.

Note, that we can use Eqs. (19) and (20) when $F \gg F_c^*$ and $|\varepsilon_m| \ll \varepsilon_T$, where $|\varepsilon_m|$ is determined by Eq. (12) at $\tau_e = \tau_2$. Corresponding to the first formula (17) at $\varepsilon = 0$

$$\tau_2 = \tau_1 + \hbar/2kT, \tag{21}$$

where τ_1 practically does not depend on temperature. At adiabatic potentials (autolocalization) $\tau_1 < 0$ (Fig. 1b).

5. Summary

In this work an expression for the probability of electron tunneling from a bound to a free state under an alternating electric field in the presence of constant magnetic field is obtained. The result is used for the calculation of the rate of thermally activated ionization of impurities by alternating electric fields. It is shown that the application of the external magnetic field perpendicular to the electric field declines carriers which increases the tunneling trajectory. Thus, magnetic field reduces the ionization probability at $\omega_c\tau_2 > 1$. This effect is enhanced at frequencies, when $\Omega\tau_2$ becomes larger than unity.

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