Ehrenfest time dependent suppression of weak localization

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The Ehrenfest time dependence of the suppression of the weak localization correction to the conductance of a *clean* chaotic cavity is calculated. Unlike in earlier work, no impurity scattering is invoked to imitate diffraction effects. The calculation extends the semiclassical theory of K. Richter and M. Sieber [Phys. Rev. Lett. **89**, 206801 (2002)] to include the effect of a finite Ehrenfest time.

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The average conductivity of a disordered metal is reduced with respect to the classical value by quantum interference. This phenomenon, known as weak localization, has been understood long ago [1, 2, 3] in terms of the constructive interference of time-reversed diffusive trajectories. Weak localization exists also in quantum dots, which are so small and clean that impurity scattering can be neglected [4]. In such ballistic cavities, quantum interference effects develop only after a time scale on which a minimal wave packet has spread to cover the entire cavity. This time scale, known as the Ehrenfest time [5], is of order $\tau_{\rm E} = \lambda^{-1} \ln k_F L$, with λ the Lyapunov exponent of the chaotic dynamics, k_F the Fermi wavevector, and L the linear size of the cavity. The time scale $\tau_{\rm E}$ becomes important if it is larger than the mean dwell time $\tau_{\rm D}$ of an electron in the quantum dot, coupled via two point contacts to electron reservoirs.

Suppression of weak localization in the Ehrenfest regime $\tau_{\rm D} < \tau_{\rm E}$ was first proposed and studied by Aleiner and Larkin [6]. Their calculation played a seminal role in the development of the subject, but it was unsatisfactory in one key aspect: A small amount of impurity scattering was introduced by hand to imitate the effects of diffraction in a ballistic system. The main aim of our work is to provide a derivation of the weak localization correction in the Ehrenfest regime without recourse to impurity scattering. To our knowledge no such derivation exists.

The theoretical framework that we shall adopt is the semiclassical theory of Richter and Sieber [7], which is a well-understood and controlled approximation scheme. In Ref. [7] the effects of finite $\tau_{\rm E}$ were not considered, so there the weak localization correction was given by the value known from random matrix theory [8, 9]. We find that the absence of interfering trajectories when $\tau_{\rm D} < \tau_{\rm E}$ leads to the exponential suppression of the weak localization correction $\propto \exp(-\tau_{\rm E}/\tau_{\rm D})$, in agreement with Ref. [6].

Apart from the setting of weak localization, effects of a finite Ehrenfest time have received much attention recently: The excitation gap in an Andreev billiard [10] as well as the shot noise [11] of a ballistic cavity are predicted to be suppressed when $\tau_{\rm E} > \tau_{\rm D}$. The latter effect have received experimental support [12]. For these problems there now exist semiclassical theories, which do not invoke impurity scattering. However, all these theories

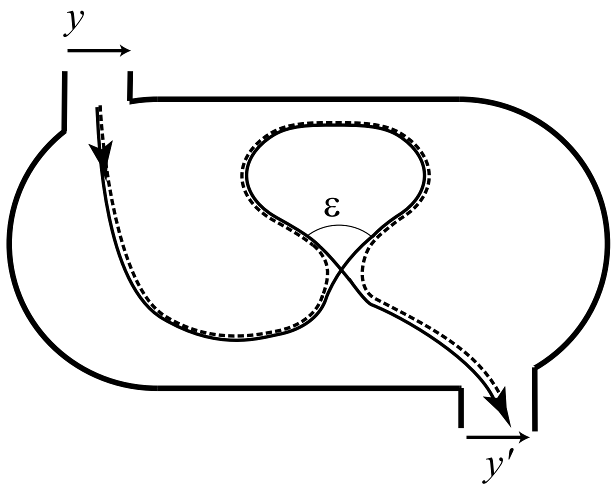

FIG. 1: The Richter-Sieber pair. The weak localization correction to the transmission amplitudes comes from selfcrossing angles $\epsilon \lesssim \sqrt{\lambda\hbar/E_F}$. The characteristic time of such orbits is the Ehrenfest time $\tau_E = (1/\lambda)\ln(E_F/\lambda\hbar)$

deal only with leading order effects. Quantum corrections such as weak localization are beyond their reach. That is why in this work we follow an altogether different approach.

Following Richter and Sieber, we consider a two-dimensional ballistic quantum dot to which two leads of width w and w' are attached. We assume that the classical dynamics of this dot is chaotic, with Lyapunov exponent λ. The Landauer formula for the conductance is given by

$$G = 2\frac{e^2}{h}\sum_{n=1}^{N}\sum_{m=1}^{N'} |t_{nm}|^2, \tag{1}$$

where t_{nm} is the transmission amplitude between incoming and outgoing channels m and n at the Fermi energy E_F and $N(N')$ is the number of channels of width $w(w')$. The semiclassical expression for t_{nm} is given as a sum over classical trajectories γ joining two leads [7, 13]:

$$t_{nm} = -\sqrt{\frac{\pi\hbar}{2ww'}} \sum_{\gamma(\bar{n},\bar{m})} \frac{\exp\left((i/\hbar)S_\gamma\right)\Phi_\gamma}{|\cos\theta_{\bar{n}}\cos\theta_{\bar{m}} M_{21}^{\gamma}|^{1/2}}. \tag{2}$$

Here $\sin\theta_{\bar n} = \bar n\pi/k_F w$ and $\sin\theta_{\bar m} = \bar m\pi/k_F w'$, $\bar n = \pm n$ and $\bar m = \pm m$, and $\Phi_\gamma = \mathrm{sgn}(\bar m)\mathrm{sgn}(\bar n)\exp\big(i\pi(\bar m y/w - \bar n y'/w' - \mu_\gamma/2 + 1/4)\big)$. The term S_γ is the classical action, M_{21}^γ is an element of the monodromy matrix, and μ_γ is the Maslov index. The trajectory γ starts at transverse coordinate y in lead w with an angle $\theta_{\bar n}$ and ends at the transverse coordinate y' in lead w' with angle $\theta_{\bar m}$.

When calculating $|t_{nm}|^2$ the double sum over trajectories γ and γ' is approximated to leading order by the diagonal approximation $\gamma = \gamma'$ [13]. The first order quantum correction to the transmission amplitudes (responsible for the weak localization effect [14]) is due to Richter-Sieber pairs [7]: γ is exponentially close to γ' everywhere except in the vicinity of a crossing point of γ where γ' avoids that crossing. This is illustrated in Fig. 1. The action difference between γ and γ' is: $\Delta S = E_F\epsilon^2/\lambda$, where ϵ is the angle at the crossing. In the diagonal approximation, the sum over trajectories can be evaluated via the sum rule [7]

$$\sum_{\gamma(y',\theta_n;y,\theta_m)} \frac{\delta(T-T_\gamma)}{|M_{21}^\gamma|} = \frac{\cos\theta_n\cos\theta_m}{2\pi m A} dy\, dy'\,\rho(T), \quad (3)$$

where the sum is over all trajectories that begin in interval dy' around y and end in interval dy around y, $\rho(T) \propto \exp(-T/\tau_\mathrm{D})$ is the dwell time distribution and

$$\tau_\mathrm{D} = mA/\hbar(N+N') \quad (4)$$

is the mean dwell time, we denote by m the effective electron mass, by A the area of the cavity, and by $N = k_F w/\pi$, $N' = k_F w'/\pi$ the number of channels in the two leads. The weak localization correction from Richter-Sieber pairs is given by

$$\delta|t_{nm}|^2 = \frac{2E_F\hbar}{\pi m^2A^2}\int_0^\pi d\epsilon \int_{T_\epsilon}^\infty dT \mathrm{e}^{-T/\tau_\mathrm{D}}(T-T_\epsilon)^2 \times \cos(E_F\epsilon^2/\lambda\hbar)\sin\epsilon, \quad (5)$$

where $T_\epsilon = -(2/\lambda)\ln\epsilon$. The lower bound in the integral over T signifies that there are no orbits shorter than T_ϵ with a selfcrossing angle ϵ.

So far we have followed the calculation of Richter and Sieber [7]. Now we depart from it. We first evaluate the T integral,

$$\delta|t_{nm}|^2 = \frac{4E_F\hbar\tau_\mathrm{D}^3}{\pi m^2A^2}\int_0^\pi d\epsilon\,\mathrm{e}^{-T_\epsilon/\tau_\mathrm{D}}\cos(E_F\epsilon^2/\lambda\hbar)\,\sin\epsilon. \quad (6)$$

In the semiclassical limit, the main contribution to this integral comes from $\epsilon \lesssim \sqrt{\lambda\hbar/E_F} \ll 1$. Thus we may approximate $\sin\epsilon \approx \epsilon$ and extend the upper limit of the integral to infinity. The result is

$$\begin{aligned}\delta|t_{nm}|^2 &= \frac{4E_F\hbar}{\pi m^2A^2}\tau_\mathrm{D}^3\int_0^\infty d\epsilon\,\epsilon^{1+2/\lambda\tau_\mathrm{D}}\cos(E_F\epsilon^2/\lambda\hbar) \\ &= -\left(\frac{\hbar\tau_\mathrm{D}}{mA}\right)^2\frac{2\lambda\tau_\mathrm{D}}{\pi}\sin\left(\frac{\pi}{2\lambda\tau_\mathrm{D}}\right)\Gamma\left(1+\frac{1}{\lambda\tau_\mathrm{D}}\right) \\ &\quad\times\exp(-\tau_\mathrm{E}/\tau_\mathrm{D}),\end{aligned} \quad (7)$$

where $\tau_\mathrm{E} = (1/\lambda)\ln(E_F/\lambda\hbar)$ is the Ehrenfest time of this problem. In the relevant regime $\lambda\tau_\mathrm{D} \gg 1$ we have

$$\delta|t_{nm}|^2 \simeq \left(\frac{\hbar\tau_\mathrm{D}}{mA}\right)^2 \mathrm{e}^{-\tau_\mathrm{E}/\tau_\mathrm{D}}. \quad (8)$$

Finally, using Eq. (4) and the sum rule (3), we find the weak localization correction to the conductance

$$\delta G_= -\frac{2e^2}{h}\frac{NN'}{(N+N')^2}\exp(-\tau_\mathrm{E}/\tau_\mathrm{D}), \quad (9)$$

in agreement with Ref. [6].

Up to this point we have rederived a known result. Now we shall apply this technology to the magnetic field dependence of the weak localization correction in the Ehrenfest regime. This is done via the calculation of the magnetic field dependence of the density of self crossings [7]. Accordingly, Eq.(5) is modified as follows:

$$\begin{aligned}\delta|t_{nm}|^2 &= \frac{4E_F\hbar\tau_\mathrm{B}^2}{\pi m^2A^2}\int_0^\pi d\epsilon\int_{T_\epsilon}^\infty dT\cos(E_F\epsilon^2/\lambda\hbar)\sin\epsilon \\ &\quad\times\mathrm{e}^{-T/\tau_\mathrm{D}}\left(\mathrm{e}^{(T_\epsilon-T)/\tau_\mathrm{B}} - 1 + \frac{T-T_\epsilon}{\tau_\mathrm{B}}\right),\end{aligned} \quad (10)$$

where $\tau_\mathrm{B} = \phi_0^2/(8\pi^2\beta B^2)$ is the magnetic time, ϕ_0 is the flux quantum, B is the magnetic field, and β is a system dependent parameter [7, 13]. As before, we first evaluate the T integral exactly and then evaluate the ϵ integral in stationary phase approximation. This produces the B dependent transmission matrix elements $\delta|t_{nm}(B)|^2 = \delta|t_{nm}(0)|^2(1+\tau_\mathrm{D}/\tau_\mathrm{B})^{-1}$. Finally, summing over all channels we obtain the magnetic field dependence of the weak localization correction to the conductance,

$$\delta G(B) = -\frac{2e^2}{h}\frac{NN'}{(N+N')^2}\frac{\mathrm{e}^{-\tau_\mathrm{E}/\tau_\mathrm{D}}}{1+\tau_\mathrm{D}/\tau_\mathrm{B}} \quad (11)$$

We see that the Lorentzian lineshape of the weak localization peak is preserved in the Ehrenfest regime, while its size is exponentially suppressed.

In conclusion, we have presented a derivation of the Ehrenfest time dependence of the weak localization correction in a two dimensional chaotic billiard. All interference effects are fully accounted for within the framework of a controlled semiclassical approximation [7], without requiring the artificial inclusion of impurity scattering [6]. Interesting extensions include the appearance of a second

Lyapunov exponent in three dimensions, and the coexistence of chaotic and mixed regions of phase space. It would also be of interest to extend the method to describe universal conductance fluctuations in the Ehrenfest regime.

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