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The Exotic Baryon $\Theta^+(1540)$ on the Lattice

Dieter Hierl[*]

University of Regensburg
E-mail: `dieter.hierl@physik.uni-regensburg.de`

Christian Hagen

University of Regensburg
E-mail: `christian.hagen@physik.uni-regensburg.de`

Andreas Schäfer

University of Regensburg
E-mail: `andreas.schaefer@physik.uni-regensburg.de`

We report on a study of the pentaquark $\Theta^+(1540)$, using a variety of different interpolating fields. We use Chirally Improved fermions in combination with Jacobi smeared quark sources to improve the signal and get reliable results even for small quark masses. The results of our quenched calculations, which have been done on a $12^3 \times 24$ lattice with a lattice spacing of $a = 0.148$ fm, do not provide any evidence for the existence of a Θ^+ with positive parity. We do observe, however, a signal compatible with nucleon-kaon scattering state. For the negative parity the results are inconclusive, due to the potential mixture with nucleon-kaon and N^*-kaon scattering states.

[*]Speaker.

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1. Introduction

The possible discovery of the $\Theta^+(1540)$ by the LEPS Collaboration at SPring-8 [1] has initiated great interest in exotic baryons. Since then, there has been a large number of experiments that have confirmed this result, e.g. [2, 3], but also about the same number that could not confirm it, e.g. [4, 5].

To confirm or disprove the existence of the Θ^+, if a clear conclusion of the lattice community could be reached and would eventually be experimentally confirmed, this gave a substantial boost to lattice QCD. Therefore, many groups have started to work on this problem. Here we present our first results.

In our calculations we perform a qualitative study using different types of spin-$\frac{1}{2}$ operators with the quantum numbers of the Θ^+. We compute all cross correlators and use the variational method [6, 7] to extract the lowest lying eigenvalues. These are used to create effective mass plots for a comparison to the *N-K* scattering state which we computed separately on the the same lattice.

2. Details of the calculation

We considered the following interpolating fields as basis for our correlation matrix:

- Currents suggested by Sasaki [8]:

$$\Theta^1_+ = \varepsilon_{abc}\varepsilon_{aef}\varepsilon_{bgh}(u_e^T C d_f)(u_g^T C\gamma_5 d_h)C\bar{s}_c^T \tag{2.1}$$

$$\Theta^2_{+,\mu} = \varepsilon_{abc}\varepsilon_{aef}\varepsilon_{bgh}(u_e^T C\gamma_5 d_f)(u_g^T C\gamma_5\gamma_\mu d_h)C\bar{s}_c^T \tag{2.2}$$

$$\Theta^3_{-,\mu} = \varepsilon_{abc}\varepsilon_{aef}\varepsilon_{bgh}(u_e^T C d_f)(u_g^T C\gamma_5\gamma_\mu d_h)C\bar{s}_c^T \tag{2.3}$$

- A current which is a suggestion by L. Ya. Glozman [9], however, using only s-wave quarks instead of a mixture of s-wave and p-wave quarks in (2.4):

$$I_\mu = (\delta_{ae}\delta_{bg} + \delta_{be}\delta_{ag})\varepsilon_{gcd} \tag{2.4}$$

$$\times \begin{pmatrix} u_a^T C u_b \\ \frac{1}{\sqrt{2}}(u_a^T C d_b + d_a^T C u_b) \\ d_a^T C d_b \end{pmatrix}$$

$$\times \begin{pmatrix} d_c^T (C\gamma_\mu)d_d \\ \frac{1}{\sqrt{2}}(u_c^T (C\gamma_\mu)d_d + d_c^T (C\gamma_\mu)u_d) \\ u_c^T (C\gamma_\mu)u_d \end{pmatrix} C\bar{s}_e^T$$

- An other current related to suggestions by L. Ya. Glozman [9], using only s-wave quarks and a factor $(\delta_{ae}\delta_{bg} - \delta_{be}\delta_{ag})$ instead the factor $(\delta_{ae}\delta_{bg} + \delta_{be}\delta_{ag})$ in (2.5):

$$I_\nu = (\delta_{ae}\delta_{bg} - \delta_{be}\delta_{ag})\varepsilon_{gcd} \times \tag{2.5}$$

$$\times \begin{pmatrix} u_a^T(C\sigma_{\mu\nu})u_b \\ \frac{1}{\sqrt{2}}(u_a^T(C\sigma_{\mu\nu})d_b + d_a^T(C\sigma_{\mu\nu})u_b) \\ d_a^T(C\sigma_{\mu\nu})d_b \end{pmatrix}$$

$$\times \begin{pmatrix} d_c^T(C\gamma_\mu)d_d \\ \frac{1}{\sqrt{2}}(u_c^T(C\gamma_\mu)d_d + d_c^T(C\gamma_\mu)u_d) \\ u_c^T(C\gamma_\mu)u_d \end{pmatrix} C\bar{s}_e^T$$

In order to get only spin-$\frac{1}{2}$ pentaquarks, we have to project the spin. This is done using the spin projection operator for a Rarita-Schwinger field [10]. We also apply a parity projection to be able to distinguish both parity channels. The parity channel of the Θ^+ is not known. There are conflicting theoretical predictions and no conclusive experimental data.

For the interpolators (2.4) and (2.5) p-wave quarks are required. Since we do not have p-wave sources we had to adjust the color structure in the interpolator (2.5) to obtain a signal at all.

The interpolators (2.4) and (2.5) are linear combinations of two diquarks with $I = 1$. Thus they are a mixture of isospin $I = 0$ and $I = 2$ states. The interpolators (2.1), (2.2) and (2.3) do not have any visible isospin projection, but hidden ones.

Then we use the five interpolators to calculate a cross correlation matrix $C_{ij}(t)$ which is then inserted into the generalized eigenvalue problem

$$C_{ij}(t)\vec{v}_i^{(k)} = \lambda^{(k)}(t)C_{ij}(t_0)\vec{v}_i^{(k)}. \tag{2.6}$$

The solutions of this equation behave like

$$\lambda^{(k)}(t) \propto \exp\left(-m^{(k)}(t-t_0)\right). \tag{2.7}$$

These eigenvalues are used to compute effective masses according to

$$m_{eff}(t) = \ln\left(\frac{\lambda(t)}{\lambda(t+1)}\right). \tag{2.8}$$

Ordering the five eigenvalues according to their absolute value the largest eigenvalue in the positive parity channel should give the Θ^+ mass if Θ^+ is a positive parity particle. The second largest eigenvalue in the negative parity channel should give the Θ^+ mass, where the largest eigenvalue corresponds to the a *N-K* scattering state at rest.

In our quenched calculation we use the Chirally Improved Dirac operator [11, 12]. It is an approximate solution of the Ginsparg-Wilson equation [13], with good chiral behavior [14, 15]. The gauge fields are generated with the Lüscher-Weisz gauge action [16, 17] at $\beta = 7.90$. The corresponding value of the lattice spacing is $a = 0.148$ fm as determined from the Sommer parameter in [18]. The strange quark mass is fitted using the pseudoscalar K meson. The error bars are computed using the jackknife method. The parameters of our calculation are collected in Table 1.

3. Results

The results of our calculations are shown in Fig. 1, where we plot the effective masses of the two lowest lying states of both parity channels obtained with the cross-correlation technique. These

size $L^3 \times T$	$12^3 \times 24$
a [fm]	0.148
L [fm]	≈ 1.8
#conf N	100
quark masses am_q	0.02, 0.03, 0.04, 0.05, 0.06, 0.08, 0.10, 0.12, 0.16, 0.20
smearing parameters:	$n = 18$, $\kappa = 0.210$
s-quark mass am_s	0.0888(17)

Table 1: Parameters of our calculations.

states are approaching a possible plateau very slowly as we expected, since states consisting of five quarks are very complicated and therefore should contain a large number of excited states which have to die out before the effective mass reaches a plateau. We use in addition to the cross-correlation technique Jacobi smeared Gaussian quark sources for all our quarks to improve the signal for the lowest lying states.

The lower horizontal line in the negative parity channel is the sum of the nucleon and kaon mass at rest in the ground state obtained from a separate calculation on the same lattice. Since we project the final state to zero momentum a scattering state can also be a two particle state where the two particles have the same but antiparallel momentum, i.e. $\vec{p}_N = -\vec{p}_K$. We use the relativistic E-p-relation to calculate the energy of such states,

$$E = \sqrt{p^2 + m_N^2} + \sqrt{p^2 + m_K^2},\tag{3.1}$$

where the smallest momentum is $2\pi/L \approx 700$ MeV on our lattice. In Fig. 1 this energy is represented by the upper horizontal line.

We find effective mass plateaus which are consistent with *N-K* scattering states in the negative parity channel as we expected. We find that the second state is noisy but within errors consistent with the energy in (3.1). Therefore, it is most likely that we do not observe a Θ^+ state in the negative parity channel. However, this conclusion is not completely certain, if the Θ^+ is broad, because such a Θ^+ state would mix strongly with the continuum states.

In the positive parity channel one expects to find either a bound Θ^+ or an excited *N-K* scattering state. For such an excited state there are several possibilities, e.g., *N*-K*, or *N-K* with a relative angular momentum, and so on.

On the positive parity side, we also show the two lowest lying states obtained from our calculation. Both of them are too heavy to describe a Θ^+ state. They probably correspond to excited *N-K* scattering states. If there were a signal belonging to the Θ^+ it is supposed to lie below the red line assuming that the chiral extrapolation of the Θ^+ does not lead to dramatic effects below our smallest quark mass.

4. Conclusion

In this article we present the results of a pilot study of the Θ^+ using different types of operators.
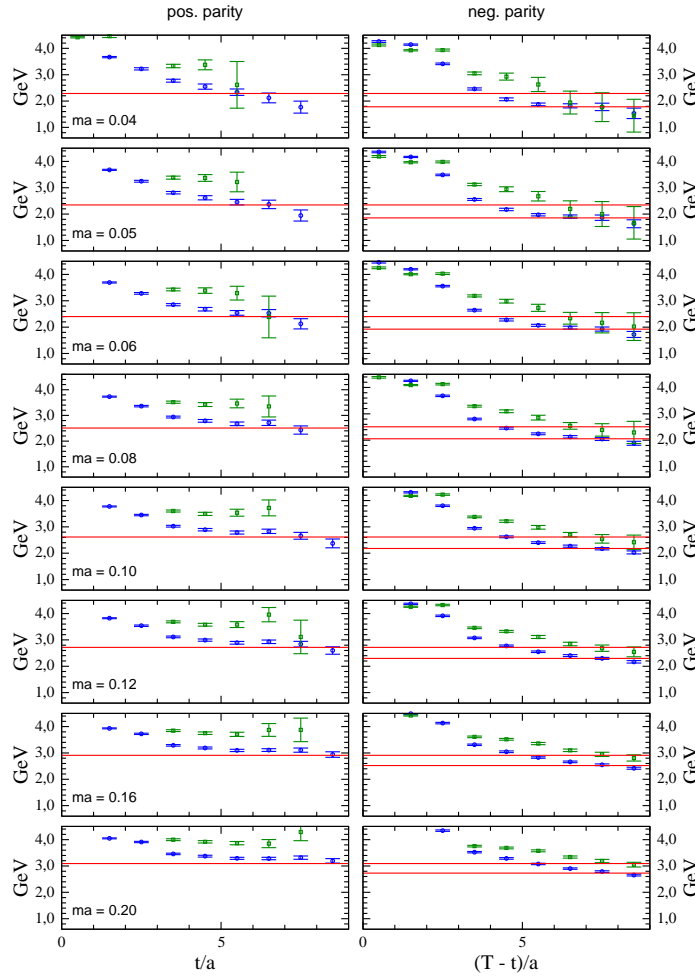
Figure 1: Results from cross-correlation of five interpolators. We use Jacobi smeared Gaussian quark sources for all our quarks. Only the effective masses of the two largest eigenvalues are plotted.

We find that for negative parity our results are in good agreement with a *N-K* scattering state in the ground state and a quite noisy signal for the first excited state. For positive parity we find states which are typically more than 500 MeV heavier than the Θ^+ and thus not compatible with a Θ^+ mass of 1540 MeV.[1]

Thus our calculation do not show any hints for a Θ^+ in the quenched approximation with chiral fermions for the positive channel. In the negative parity channel we would need smaller errors to be able to make a really firm statement for the existence of a Θ^+ state, but it is most likely that there is no such state in our data.

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[1]We assume that the Θ^+ extrapolates smoothly in both, the chiral and the continuum limit.

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