

# Spin Filter Effects in Mesoscopic Ring Structures

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We study the spin-dependent conductance of ballistic mesoscopic ring systems in the presence of an inhomogeneous magnetic field. We show that, for the setup proposed, even a small Zeeman splitting can lead to a considerable spin polarisation of the current. Making use of a spin-switch effect [1] we propose a device of two rings connected in series that in principle allows for both creating and coherently controlling spin polarized currents at low temperatures.

Since the proposal of the Datta-Das transistor [2] over a decade ago, much effort has been spent on finding an effective mechanism for achieving spin-polarized electron injection into semiconductors [3]. The ability to inject and detect spin-polarized currents in a semiconducting material widens the field of usual magneto-electronics in metals and opens up the intriguing program of performing spin electronics [4] based on nonmagnetic semiconductor devices. Due to the obstacle of the conductivity mismatch [5] it has so far proved difficult to demonstrate polarized spin injection from a ferromagnetic metallic contact into a semiconductor. An alternative approach, which is based on magnetic semiconductors [6,7], gives excellent results concerning the injection efficiencies but has the drawback that it is, at least up to now, restricted to low temperatures. The mechanisms commonly used for creating spin-polarized currents rely on magnetic materials or a large Zeeman splitting in a homogeneous magnetic field [8]. In this work we study ballistic mesoscopic rings with *inhomogeneous* magnetic fields and show that even a small Zeeman splitting (compared to the Fermi energy) can lead to a considerable spin-polarisation of the current. Usually the Zeeman splitting is exploited to align the spin to the energetically favourable lower Zeeman level. Here we demonstrate that for the presented field texture the electron spin, which occupies the *higher* Zeeman level, is more likely to align itself with the local field direction and thus contributes to a larger degree to the current. The described effect is most pronounced in the adiabatic regime of strong fields where the electron spin follows the spatially varying direction of the magnetic field. But our numerical calculations show that there also exists a region of moderate field strengths for which a spin-polarisation of about 30% can be achieved. Such spin-injection efficiencies could be realised using GaAs/AlGaAs heterostructures with low carrier densities. This has the advantage that both the injector and a possible spin controlling device, such as the spin-switch [1], can be fabricated from the same material and negative interface effects would be avoided.

To be more specific we consider symmetric 1d and 2d ballistic mesoscopic rings with two attached leads as

shown in Fig. 1. The chosen magnetic field texture is illustrated in Fig. 2.

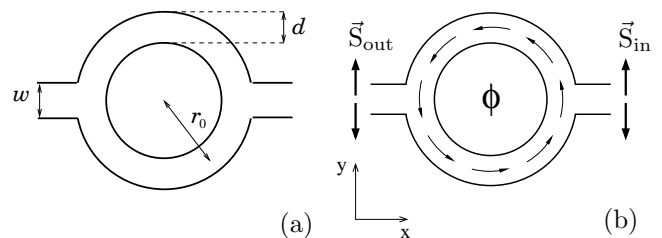


FIG. 1. (a) Example of a quasi 1d ring structure with mean radius  $r_0$  and width  $d$  as used for the numerical calculations. (b) Illustration of the ring with circular in-plane field and flux  $\Phi$  through the ring. We assume electrons coming from the right and define the spin polarisation with respect to the  $y$ -axis.

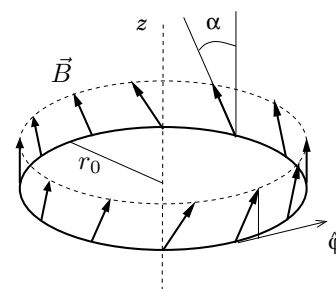


FIG. 2. The total magnetic field  $\vec{B}(r_0)$  in the ring of Fig. 1 is composed of an inhomogeneous in-plane field  $\vec{B}_i = B_i(r_0) \hat{\phi}$  and a perpendicular homogeneous field  $\vec{B}_0 = B_0 \hat{z}$ .

For the inhomogeneous magnetic field we assume a circular configuration  $\vec{B}_i(\vec{r}) \sim 1/r \hat{\phi}$  (in polar coordinates) centered around the inner disk of the microstructure. Such a field can be viewed as being generated by a perpendicular electrical current through the disk [9]. The magnetic flux through the ring can be varied via a homogeneous field  $\vec{B}_0 = B_0 \hat{z}$  pointing in the  $z$ -direction. The Hamiltonian for noninteracting electrons with effective mass  $m^*$  and charge  $-e$ , in the presence of a magnetic field  $\vec{B}(\vec{r}) = \nabla \times \vec{A}(\vec{r})$ , reads

$$H = \frac{1}{2m^*} \left[ \vec{p} + \frac{e}{c} \vec{A}(\vec{r}) \right]^2 + V(\vec{r}) + \mu \vec{B}(\vec{r}) \cdot \vec{\sigma}. \quad (1)$$

The coupling of the electron spin to the magnetic field enters via the Zeeman term  $\mu \vec{B}(\vec{r}) \cdot \vec{\sigma}$ , where  $\vec{\sigma}$  is the Pauli spin vector and  $\mu = g^* e \hbar / (4m_0 c)$  the magnetic moment with  $g^*$  the gyromagnetic ratio. The electrostatic potential  $V(\vec{r})$  defines the confinement of the ballistic conductor. At  $T = 0$  the spin-dependent conductance of a mesoscopic system with two attached leads is given by the Landauer formula [10]

$$G(E, B) = \frac{e^2}{h} \sum_{m', m=1}^M \sum_{s', s=\pm 1} |t_{s's}^{m'm}|^2. \quad (2)$$

The coefficient  $t_{s's}^{m'm}$  denotes the transmission amplitude from an incoming quantum channel  $m$  with spin  $s$  to an outgoing channel with respective quantum numbers  $m'$  and  $s'$ . We calculate  $t_{s's}^{m'm}$  by projecting the corresponding Green function matrix onto the asymptotic spinors in the leads. As shown in Fig. 1 we define the spin directions with respect to the y-axis. In the case of zero flux through the ring the asymptotic spinors are therefore eigenstates of the Hamiltonian when entering and exiting the ring structure. In the following we focus on the case, where the two leads of width  $w$  support only one open channel ( $M = \text{Int}[k_F w / \pi] = 1$ ). We compute the Green function for the Hamiltonian (1) numerically, using a generalized version of the recursive Green function technique based on a tight-binding model [10] including spin [1].

For a ballistic 1d ring it is feasible to obtain the eigenstates of the Hamiltonian (1) analytically [11,12]. They read  $\Psi_{n,s} = \exp(in\varphi) \otimes \psi_n^s(\varphi)$  where the first factor describes the motion along the ring and the second refers to the spin state  $s = \uparrow, \downarrow$ . The Zeeman term causes a slight difference in the kinetic energy of spin- $\uparrow$  and spin- $\downarrow$  electrons travelling clockwise and counter-clockwise around the ring so that we must distinguish four different orbital quantum numbers  $n : n_j^\uparrow, n_j^\downarrow$  ( $j = 1, 2$ ). They are given by  $n' \equiv n + \phi e/h$  where the  $n'$  are the solutions of the equation

$$\begin{aligned} \tilde{E}_F = n'^2 + n' + 1/2 \\ \pm \sqrt{(n' + 1/2)^2 + (2n' + 1)\tilde{\mu}B \cos \alpha + (\tilde{\mu}B)^2}. \end{aligned} \quad (3)$$

Here,  $\tilde{E}_F = (2m^* r_0^2 / \hbar^2) E_F$  is the scaled Fermi energy,  $\tilde{\mu} = (2m^* r_0^2 / \hbar^2) \mu$ , and  $\alpha$  the tilt angle of the textured magnetic field with respect to the z-axis (Fig. 2). For the purpose of this paper we can specialize on very small homogeneous fields so that the total field is almost in-plane ( $\alpha \approx \pi/2$ ). In that case the  $\cos \alpha$ -term under the square root is negligible, and we get for counter-clockwise travelling waves [13] the eigenvalues

$$n_1^\uparrow = -\frac{1}{2} + \frac{1}{2} \sqrt{1 + 4\tilde{E} - 4\sqrt{(\tilde{\mu}B)^2 + \tilde{E}}}, \quad (4)$$

$$n_1^\downarrow = -\frac{1}{2} + \frac{1}{2} \sqrt{1 + 4\tilde{E} + 4\sqrt{(\tilde{\mu}B)^2 + \tilde{E}}}. \quad (5)$$

The spin indices are chosen in such a way that spin- $\uparrow$  ( $\downarrow$ ) labels the smaller (larger) quantum number  $n_1$ . In the adiabatic limit this corresponds to a spin being parallel (antiparallel) to the local field axis. In the general, nonadiabatic case the electron spin is not aligned with the local field direction. Instead, two angles  $\gamma_1^\uparrow, \gamma_1^\downarrow \leq \alpha$  take the role of  $\alpha$  and characterize the spin eigenstates which read

$$\psi_{n_1}^\uparrow = \begin{pmatrix} \sin \frac{\gamma_1^\uparrow}{2} \\ ie^{i\varphi} \cos \frac{\gamma_1^\uparrow}{2} \end{pmatrix}, \quad \psi_{n_1}^\downarrow = \begin{pmatrix} \cos \frac{\gamma_1^\downarrow}{2} \\ -ie^{i\varphi} \sin \frac{\gamma_1^\downarrow}{2} \end{pmatrix} \quad (6)$$

with

$$\tan \gamma_1^{\uparrow(\downarrow)} = \frac{\tilde{\mu}B}{n_1^{\uparrow(\downarrow)} + 1/2}. \quad (7)$$

In the nonadiabatic limit of a weak magnetic field the polarisation of the electron spin remains unchanged and it is  $\gamma_1^{\uparrow(\downarrow)} \rightarrow 0$ . In the opposite limit of strong coupling between the spin and the field the solutions (6) become eigenstates of the Zeeman term in the Hamiltonian (1). Hence, the adiabatic limit can be defined by the requirement  $\gamma_1^{\uparrow(\downarrow)} \rightarrow \alpha = \pi/2$  or  $\tan \gamma_1^{\uparrow(\downarrow)} \rightarrow \infty$ . Expanding Eq. (4) and (5) in  $\tilde{\mu}B/\tilde{E}_F$  and inserting them in Eq. (7) gives in zeroth order a criterion for the adiabatic regime which is independent of the spin direction:

$$Q \equiv \frac{\tilde{\mu}B}{\sqrt{\tilde{E}_F}} \gg 1. \quad (8)$$

Eq. (8) coincides with the condition for adiabaticity in 1d rings, which was previously deduced by decomposing the Hamiltonian (1) into an adiabatic and nonadiabatic part [11]. Using the first order approximation

$$n_1^{\uparrow(\downarrow)} \approx -\frac{1}{2} + \sqrt{\tilde{E}_F} \mp \frac{1}{2} \sqrt{Q^2 + 1} \quad (9)$$

we recognize that the condition for adiabaticity now becomes *spin-dependent* and reads, for  $\sqrt{Q^2 + 1} \approx Q$ ,

$$\text{spin-}\uparrow: \quad \frac{Q}{1 - \frac{1}{2} \tilde{\mu}B/\tilde{E}_F} \gg 1, \quad (10)$$

$$\text{spin-}\downarrow: \quad \frac{Q}{1 + \frac{1}{2} \tilde{\mu}B/\tilde{E}_F} \gg 1. \quad (11)$$

This means that spin-up electrons can reach the adiabatic regime already for lower magnetic fields compared to spin-down electrons. The physical origin of this behavior can be understood as follows. In the weak adiabatic regime  $Q \gtrsim 1$  the electron spin is to a certain extent aligned with the local field direction ( $0 \ll \gamma_1 < \alpha$ ) and nonadiabatic spin flips are rare. Spin- $\uparrow$  ( $\downarrow$ ) electrons,

which occupy the higher (lower) Zeeman level, have a smaller (larger) kinetic energy  $E_{kin}^{\uparrow(\downarrow)} \approx E_F \mp \mu B$  (Fig. 3). But an electron that transverses the ring with a smaller velocity has more time to align its spin with the local field direction and hence will reach the adiabatic regime more easily.

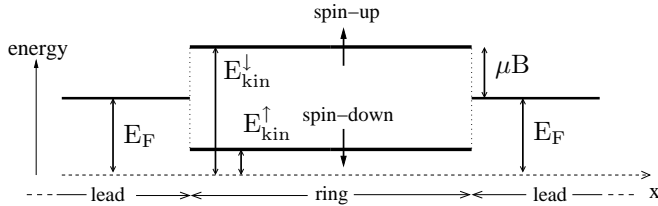


FIG. 3. An electron with fixed Fermi energy in the leads has different kinetic energies in the ring depending on its spin state  $E_{kin}^{\uparrow} < E_F < E_{kin}^{\downarrow}$ .

We now show how this spin-splitting in the adiabaticity parameter can lead to a spin polarized current in mesoscopic quasi 1d rings as illustrated in Fig. 1. For this purpose we calculate numerically the spin-resolved transmission  $T_{s,s'} = |t_{s,s'}|^2$  ( $s, s' = \uparrow, \downarrow$ ). It has been shown [1] (see also Fig. 6) that in ballistic symmetric ring structures supporting only one open transverse mode in the leads the nonadiabatic components  $T_{\uparrow\uparrow}$  and  $T_{\downarrow\downarrow}$  vanish if the magnetic flux is  $\phi = 0.5\phi_0$  (flux quantum  $\phi_0 = h/e$ ). The remaining adiabatic components  $T_{\uparrow\downarrow}$  and  $T_{\downarrow\uparrow}$ , with the spin following the field, are functions of the adiabaticity parameter and increase when increasing  $Q$ .

Our numerical results for the spin-resolved transmission as a function of  $Q$  for a ring with aspect ratio  $d/r_0 = 0.33$  and flux  $\phi = 0.5\phi_0$  are depicted in Fig. 4 (a). We have performed an energy average over half of the first channel ( $k_F W/\pi = 1.41 - 1.91$ ) to avoid strong energy dependencies due to the quantization of the angular momentum.

The adiabatic transmission components show a clear spin splitting  $T_{\uparrow\uparrow} > T_{\downarrow\downarrow}$  for  $Q \leq 9$ . As discussed before this behavior can qualitatively be understood using Eq. (10) and (11). Electrons occupying the upper Zeeman level travel with smaller velocity and hence reach the adiabatic regime of maximum transmission  $T_{\uparrow\uparrow}$  at smaller  $Q$ -values compared to spin-down electrons which have transmission  $T_{\downarrow\downarrow}$ . This effect leads to a spin polarisation  $P = (T_{\uparrow\uparrow} - T_{\downarrow\downarrow}) / (T_{\uparrow\uparrow} + T_{\downarrow\downarrow} + T_{\uparrow\downarrow} + T_{\downarrow\uparrow})$  of up to 45 % (for the geometry used) at an energy ratio  $\mu B \approx 0.25 E_F$  (Fig. 4 (b)). Note that in Fig 4 (b) one does observe another, sharply peaked polarisation maximum in the nonadiabatic regime  $Q < 1$ , which cannot exclusively be attributed to the spin-dependent scaling of the adiabaticity parameter  $Q$ , as elucidated above. However, the numerical calculations show a maximum value of 30 % spin polarisation at a very small energy ratio  $\mu B \approx 0.025 E_F$ .

Energy ratios of that order of magnitude might also become of experimental relevance. For a 2DEG built from a GaAs/AlGaAs heterostructure with carrier densities as low as  $n_s = 1.8 \cdot 10^{10} \text{ cm}^{-2}$  [14] this energy ratio translates into a required in-plane magnetic field of  $B \approx 500 \text{ mT}$ . Circular inhomogeneous fields of that magnitude are well within the means of present experimental setups.

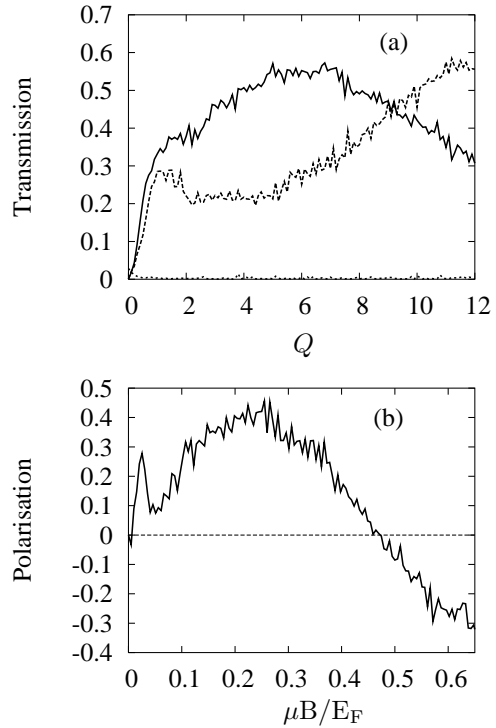


FIG. 4. (a) Energy averaged transmission components  $T_{\downarrow\uparrow}$  (solid) and  $T_{\uparrow\downarrow}$  (dashed) as a function of the scaled magnetic field  $Q$  for a ring with aspect ratio  $d/r_0 = 0.33$ ,  $\phi = 0.5\phi_0$  and one open channel  $k_F W/\pi = 1.41 - 1.91$ . The nonadiabatic components  $T_{\uparrow\uparrow}$  and  $T_{\downarrow\downarrow}$  (dotted) almost vanish. (b) Resulting polarisation of the current as a function of the energy ratio  $\mu B/E_F$ .

On the other hand, reaching the extended region of about 40 % spin-polarisation (Fig. 4 (b)) would require magnetic field strengths which are an order of magnitude larger. This regime could be reached using diluted magnetic semiconductors (DMS) which exhibit a huge g-factor and therefore lead to a large Zeeman splitting [15].

The depicted spin filter effect does not continue to exist for arbitrary high Zeeman splitting. In Fig. 4 (b) we observe a crossover to negative values of the polarisation at  $\mu B \approx 0.45 E_F$ , a phenomenon which becomes more pronounced with increasing field strength  $B$ . This counteracting effect results from the fact that for  $\mu B \gtrsim E_F$  the Zeeman splitting acts as a barrier for spin-up electrons leading to enhanced backscattering and finally filtering of this spin state.

We note that the nonadiabatic components  $T_{\uparrow\uparrow}$  and  $T_{\downarrow\downarrow}$  are symmetric ( $T_{\uparrow\uparrow} = T_{\downarrow\downarrow}$ ) and do not show spin splitting behavior. In accordance with the theoretical expectation  $T_{\uparrow\uparrow}$  and  $T_{\downarrow\downarrow}$  vanish for almost any  $Q$ -value.

We now study whether the spin filtering effect depends on the chosen geometry (size and aspect ratio  $d/r_0$ ) of the ring. Numerical results for the spin-dependent transmission through a ring with  $d/r_0 = 0.22$  are presented in Fig. 5 (a). Plotting the resulting polarisation of the current (Fig. 5 (b)) we obtain a spin filter behavior which is pretty much alike the previous one for the smaller ring. The second polarisation maximum is now at  $\mu B \approx 0.4E_F$  and reaches only a value of  $P \approx 0.3$ . The first maximum is also smaller compared to Fig. 4 (b) but is now found at  $\mu B \approx 0.01E_F$  thus reducing the required field strength by a factor of two. Further numerical simulations show that the described spin filter effect exists independent of the size of the ring with a maximum spin polarisation ranging between 30 % and 45 %.

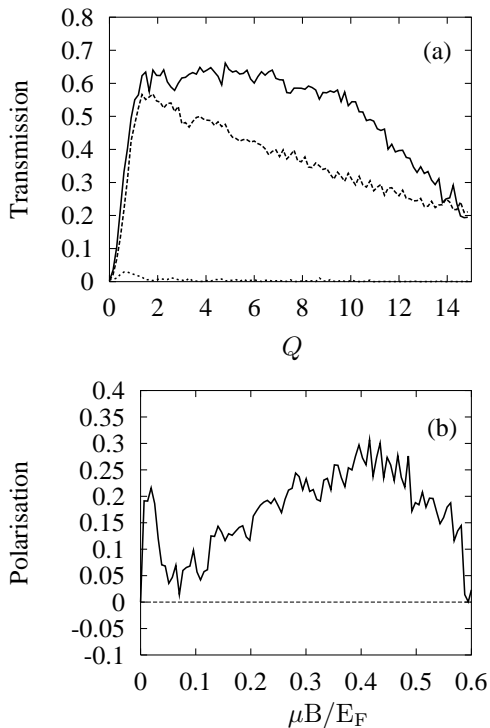


FIG. 5. (a) Energy averaged transmission components  $T_{\downarrow\downarrow}$  (solid) and  $T_{\uparrow\uparrow}$  (dashed) as a function of the scaled magnetic field  $Q$  for a ring with aspect ratio  $d/r_0 = 0.22$ . The nonadiabatic components  $T_{\uparrow\uparrow}$  and  $T_{\downarrow\downarrow}$  (dotted) almost vanish. (b) Resulting polarisation of the current as function of the energy ratio  $\mu B/E_F$ .

Symmetric 1d rings do not only show a *spin filter* effect for Zeeman energies  $\mu B < E_F$ . Together with the *spin-switch* effect described in Ref. [1] a serial connection of these structures allows both for spin filtering and tuning of the polarisation of the current. The spin-switch

mechanism of Ref. [1] is most efficient in a region of moderate magnetic fields described by  $Q \approx 1$ . This coincides approximately with the  $Q$  values for which the first polarisation maximum of up to 30 % arises. As illustrated in Fig. 6 for spin-up incoming electrons, applying zero flux leaves the spin polarisation unchanged. For half a flux quantum the polarisation is reversed and the electron exits in a spin-down state. Analogous results hold for spin-down incoming electrons.

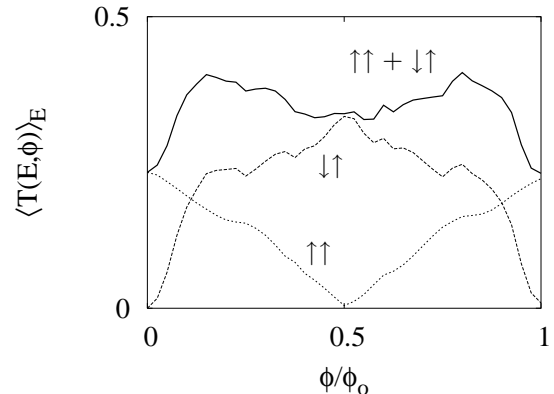


FIG. 6. Energy averaged transmission  $T_{\downarrow\downarrow}$  (dashed) and  $T_{\uparrow\uparrow}$  (dotted) for spin-up incoming electrons through a 1d ring (Fig. 1) as a function of magnetic flux. The polarisation of the exiting current can be tuned from spin-up ( $\phi = 0$ ) to spin-down ( $\phi = 0.5$ ). The total transmission  $T_{\downarrow\downarrow} + T_{\uparrow\uparrow}$  (solid) remains almost constant (from [1]).

In Fig. 7 we propose a configuration of two ballistic 1d rings as in Fig. 1 that could serve as a device to coherently create and control spin-polarized currents. The inhomogeneous magnetic field in both rings can be chosen approximately equal and in such a way that  $Q \approx 1$ . Applying a fixed flux  $\phi = 0.5\phi_0$  to the ring on the right hand side will create a partially spin-down polarized current. The flux through the second ring is tunable between  $\phi = 0$  and  $\phi = 0.5\phi_0$  and controls the direction of the polarisation.

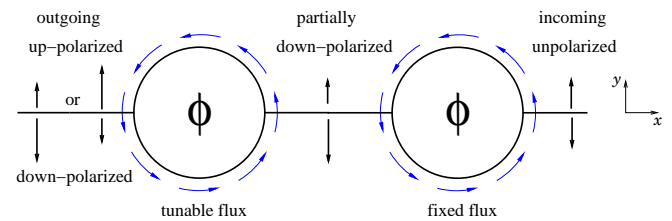


FIG. 7. Scheme for a serial combination of two 1d rings that could serve as integrated system for both creating and controlling spin-polarized currents at low temperatures. The flux through the ring on the right hand side should be fixed at  $\phi = 0.5\phi_0$  leading to a partially down polarized current. The direction of the polarisation can be changed in the second ring by applying a tunable flux between  $\phi = 0$  and  $\phi = 0.5\phi_0$ .

We note that an analogous scheme could also be realized using radial inhomogeneous magnetic fields ( $\vec{B} \sim \hat{r}$ ) instead of circular ones. Such fields can be created by placing a micromagnet in the center of the disk [16]. In that case the polarisation of the current would be defined with respect to the x-axis.

However, we do not expect the described spin filter effect to exist for inhomogeneous effective fields resulting from a strong spin-orbit Rashba coupling [17]. The Rashba effect leads to a spin splitting in the dispersion relation but electrons occupying the two spin branches travel with equal group velocity and hence have the same adiabaticity parameter  $Q$ .

Finally we summarize the conditions which have to be met to make the scheme of Fig. 7 work. Both the filter and switching mechanism are based on coherent transport and hence exist only at low temperatures. The ballistic ring structures have to be symmetric with respect to the x-axis and may only support one open channel in the leads. There is no restriction on the number of transverse modes in the ring arms themselves.

Although our proposed setup is based on a ballistic system we have numerical evidence that the observed spin filter effect also exists in disordered rings, however, it is less pronounced. Since the random distribution of the impurity potential breaks the symmetry of the ring, the nonadiabatic transmission components no longer vanish independently of the value of the adiabaticity parameter  $Q$ . Especially for  $Q < 1$  they give a strong contribution to the transmission and thus decrease the polarisation considerably. Given that the mean free path  $\ell$  depends on the (spin-dependent) travelling velocity of the electrons (weak scattering limit) the maximum value of the spin polarisation in the adiabatic limit is also lowered. The reason is that the adiabaticity parameter in the diffusive regime scales with the inverse square root of the mean number of scattering events ( $Q/\sqrt{N} \sim Q \ell/L$ ) [18] which counteracts the spin-dependent scaling of  $Q$  described above.

To summarize we have studied the spin-dependent transport properties of ballistic symmetric 1d ring structures subject to an inhomogeneous magnetic field. Our numerical calculations show that a moderate Zeeman splitting compared to the Fermi energy can lead to an enhanced transmission probability for electrons occupying the *upper* Zeeman level, which results in a spin polarization of the current of up to 45%. This effect exists in the adiabatic regime and can qualitatively be understood in terms of a spin-dependent adiabaticity parameter  $Q$ . Further increase of the strength of the inhomogeneous field leads finally to a reversal of the effect, because it is counteracted by the usual spin filtering of the charge carriers at a large Zeeman barrier.

For very small Zeeman splitting our numerical calculations indicate the existence of another, sharply peaked polarisation maximum of up to 30%. This effect, which is

not yet fully understood, could be observed for a strength of the inhomogeneous field that is well within the means of present experimental setups.

Moreover we make use of the predicted spin-switch effect in such a system and propose an integrated device consisting of two rings connected in series that in principle can serve both as spin filter and spin controller for currents at low temperatures.

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