Sains Malaysiana 42(6)(2013): 855–862

Heat Transfer Analysis for Falkner-Skan Boundary Layer Flow Past a Stationary Wedge with Slips Boundary Conditions Considering Temperature-dependent Thermal Conductivity

(Analisis Pemindahan Haba bagi Aliran Lapisan Sempadan Falkner-Skan Melintasi Suatu Baji Pegun dengan Syarat Sempadan Gelinciran serta Keberaliran Haba Bersandar Suhu)

A.A. MUTLAG*, MD. JASHIM UDDIN, M.A.A. HAMAD & AHMAD IZANI ISMAIL

ABSTRACT

We studied the problem of heat transfer for Falkner-Skan boundary layer flow past a stationary wedge with momentum and thermal slip boundary conditions and the temperature dependent thermal conductivity. The governing partial differential equations for the physical situation are converted into a set of ordinary differential equations using scaling group of transformations. These are then numerically solved using the Runge-Kutta-Fehlberg fourth-fifth order numerical method. The momentum slip parameter δ leads to increase in the dimensionless velocity and the rate of heat transfer whilst it decreases the dimensionless temperature and the friction factor. The thermal slip parameter leads to the decrease rate of heat transfer as well as the dimensionless temperature. The dimensionless velocity, rate of heat transfer and the friction factor increase with the Falkner-Skan power law parameter m but the dimensionless fluid temperature decreases with m. The dimensionless fluid temperature and the heat transfer rate decrease as the thermal conductivity parameter A increases. Good agreements are found between the numerical results of the present paper with published results.

Keywords: Falkner-Skan; momentum slip; thermal slip; scaling group of transformation; temperature dependent thermal conductivity

ABSTRAK

Kami mengkaji permasalahan pemindahan haba bagi aliran lapisan sempadan Falkner-Skan melintasi suatu baji pegun dengan syarat sempadan gelinciran momentum dan haba serta kekonduksian haba bersandar-suhu. Persamaan pembezaan separa menakluk bagi situasi fizik dijelmakan kepada suatu set persamaan pembezaan biasa menggunakan penjelmaan kumpulan penskalaan. Set persamaan pembezaan biasa tersebut kemudiannya diselesaikan secara berangka menggunakan kaedah berangka Runge-Kutta-Fehlberg keempat-kelima. Parameter gelinciran momentum δ didapati meningkat terhadap halaju tak berdimensi dan kadar pemindahan haba. Parameter gelinciran momentum didapati berkurang terhadap suhu tak berdimensi dan juga tegasan ricih. Halaju tak berdimensi, kadar pemindahan haba dan pekali tegasan ricih meningkat terhadap parameter hukum kuasa Falkner-Skan m, tetapi suhu bendalir menurun dengan m. Suhu bendalir tak berdimensi dan kadar pemindahan haba menurun apabila parameter konduktiviti haba A meningkat. Didapati keputusan berangka dalam kertas ini menepati keputusan yang telah diterbitkan sebelum ini.

Kata kunci: Falkner-Skan; gelinciran haba; gelinciran momentum; kekonduksian haba bersandar suhu; penjelmaan kumpulan penskalaan

INTRODUCTION

Falkner and Skan (1931) studied the flow over a static wedge which is immersed in a viscous fluid and introduced the Falkner-Skan equation. They used similarity transformations to reduce the boundary layer equations with the associated boundary conditions to a nonlinear third-order ordinary differential equation with corresponding boundary conditions. In the past few years some researchers investigated Falkner-Skan flow considering different effects. Liu and Chang (2008) presented an accurate and simple method without using any iteration procedure to estimate unknown initial conditions by applying the Lie-group shooting method on the Blasius and Falkner–Skan equations. Alizadeh et al. (2009) studied the boundary layer equations of laminar flow past a wedge with different angles by the Adomian decomposition method. Afzal (2010) discussed the effects of the suction and injection on the laminar boundary layer flow of a viscous and incompressible fluid. Parand et al. (2011) applied Hermite functions pseudospectral method to find an approximate solution for the third order nonlinear ordinary differential laminar boundary layer Falkner–Skan equations. Postelnicu and Pop (2011) studied the steady two-dimensional laminar boundary layer flow of a power-law fluid past a permeable stretching wedge considering variable free stream. Bachok and Ishak (2011) presented a numerical solution for a stagnation-point flow towards a stretching/shrinking sheet. Chen et al. (1981) reported that the slip boundary condition is often used in numerical computation of flow problems. They examined the influence of slip on the solution when there is a rotational body force field. Some works have been done recently on the flow of a Newtonian and non-Newtonian fluid with heat transfer considering slip conditions. Hayat et al. (2007) studied the steady flows of a non-Newtonian fluid in the case when the slippage between the plate and the fluid is valid. The solvability, regularity and vanishing viscosity limit of the 3D viscous magnetohydrodynamic system in a class of bounded domains with a slip boundary condition was investigated by Xiao et al. (2009). Rahman and Eltayeb (2011) investigated the convective slip flow of slightly rarefied fluids over a wedge with thermal jump in the case of the temperature dependent transport properties. The two-level pressure projection stabilized finite element methods for Navier–Stokes equations with nonlinear slip boundary condition has been investigated by Li and An (2011).

In many engineering problems, the thermal conductivity is reported as a function of the temperature as the value of thermal conductivity changes with temperature especially if the region of temperature change is large (Mierzwiczak & Kolodzie 2011). Some recent papers that considered the temperature-dependent thermal conductivity includes Abel et al. (2009) who studied the flow of a power-law fluid past a vertical stretching sheet and Ahmad et al. (2010) who studied the boundary layer flow of viscous incompressible fluid over a stretching plate. According to Shang (2010), the effect of variable thermal conductivity on heat transfer coefficient is greater than the variable absolute viscosity. Mierzwiczak and Kołodzie (2011) studied the non-iterative inverse determination of temperature-dependent thermal conductivity in two-dimensional steady-state heat conduction problem. They modeled the thermal conductivity in the form of a polynomial function of temperature with unknown coefficients.

In this paper we consider the problem of Falkner-Skan boundary layer flow over a stationary wedge. The effect of the momentum and thermal slip boundary conditions and variable thermal conductivity are taken into account. Scaling group of transformations is used to present the similarity representations of the problem. Similarity equations are then solved numerically to show the effects of the parameters we are considering. Namely, the Falkner–Skan power law, momentum slip, thermal slip and thermal conductivity parameters on the dimensionless flow velocity, friction factor and heat transfer characteristics. The present study may find applications in lubrication for microfluid flow.

PROBLEM FORMULATION

Consider a two dimensional steady Falkner-Skan boundary layer laminar flow past a static wedge in the moving free stream. The physical model is depicted in Figure 1. We consider the effects of the momentum and thermal slip boundary conditions and temperature dependent thermal conductivity. It is further assumed that the velocity of the free stream is of the form $\bar{u}_e = U_\infty \left(\bar{x}/L\right)^m$ (Yacob et al. 2011). A Cartesian coordinate system ($\bar{x}$, $\bar{y}$), where $\bar{x}$ and $\bar{y}$ are the coordinates along the surface of the wedge and normal to it. Under the above assumptions, the partial differential equations and the corresponding boundary conditions govern the problem are given by (White 1991):

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \bar{u}_e \frac{\partial \bar{u}_e}{\partial \bar{x}} + \upsilon \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}, \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{1}{\rho c_p}\frac{\partial}{\partial \bar{y}}\left[k(T)\frac{\partial T}{\partial y}\right], \tag{3}$$

subject to the boundary conditions (Hayat et al. 2007, Uddin et al. 2012),

$$\bar{u} = N(\bar{x})\upsilon\frac{\partial \bar{u}}{\partial \bar{y}},\ \bar{v} = 0,\ T = T_w + D_1(\bar{x})\frac{\partial T}{\partial \bar{y}} \quad \text{at} \quad y = 0,$$
$$\bar{u} = \bar{u}_e(\bar{x}),\ \ T = T_\infty \qquad \text{as} \qquad \bar{y} \to \infty. \tag{4}$$

Here $\bar{u}$ and $\bar{v}$ are the velocity components along the $\bar{x}$ and $\bar{y}$ axes, ρ is the fluid density, c_p is the specific heat, υ is the kinematic viscosity, $N(\bar{x})$ is the variable slip factor with dimension (velocity)$^{-1}$, $D_1(\bar{x})$ is the variable thermal slip factor with dimension length, T is the temperature inside boundary layer, T_∞ is the free stream temperature, T_w is the wall temperature and k is the thermal conductivity. The following relations for k are introduced (Aziz et al. 2012),

$$k(T) = k_\infty[1+c(T - T_\infty)], \tag{5}$$

where c and k_∞ are constants.

For lubricating fluids, heat is generated by internal friction and the increase in the temperature affects the viscosity and thermal conductivity of the fluid and hence

Thermal boundry layer

Momentum boundry layer

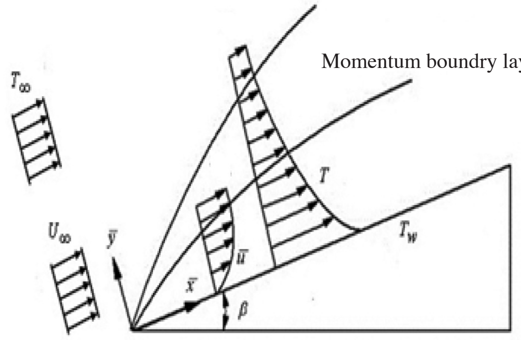

FIGURE 1. Physical model and coordinate system

the fluid properties should no longer be assumed to be constant (Prasad et al. 2010). Therefore, in order to predict the flow characteristics in an accurate and reliable manner it is necessary to consider the variation of thermal conductivity with the temperature.

We now introduce the following dimensionless variables to reduce the number of independent variables and the number of equations,

$$x=\frac{\bar{x}}{L},\quad y=\frac{\bar{y}}{L}\sqrt{\text{Re}},\quad u=\frac{\bar{u}}{U_\infty},\quad v=\frac{\bar{v}}{U_\infty}\sqrt{\text{Re}},$$
$$u_e=\frac{\bar{u}_e}{U_\infty},\ \theta=\frac{T-T_\infty}{T_w-T_\infty},\ \text{Re}=\frac{U_\infty L.}{\nu} \tag{6}$$

Here Re is the Reynolds number, L is the characteristics length and U_∞ is some reference velocity.

The dimensionless forms of the governing equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{7}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}+\frac{d^2u_e}{\partial y^2}, \tag{8}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\text{Pr}^{-1}\frac{\partial}{\partial y}\left[\left(1+A\theta\right)\frac{\partial \theta}{\partial y}\right], \tag{9}$$

here $A=c(T_w-T_\infty)$ is the parameter of thermal conductivity, α is the thermal diffusivity and $\text{Pr}=\frac{\upsilon}{\alpha}$ is the Prandtl number.

The boundary conditions become,

$$u=N\left(x\right)\upsilon\frac{\partial u}{\partial y}\frac{\sqrt{\text{Re}}}{L},\ v=0,\ \theta=1+\frac{D_1\left(x\right)\sqrt{\text{Re}}}{L}\frac{\partial \theta}{\partial y}\quad \text{at}\ \ y=0,$$
$$u=u_e\left(x\right),\quad \theta=0\qquad \text{as}\qquad y\to\infty. \tag{10}$$

We introduce stream function ψ which is defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$ to reduce the number of equations and number of dependent variables. Then (7)- (9) with the boundary conditions in (10) transform as follows:

$$\psi_y\psi_{xy}-\psi_x\psi_{yy}=u_e\frac{du_e}{dx}+\psi_{yyy}, \tag{11}$$

$$\psi_y\theta_x-\psi_x\theta_y=\text{Pr}^{-1}\frac{\partial}{\partial y}\left[\left(1+A\theta\right)\frac{\partial \theta}{\partial y}\right], \tag{12}$$

with the boundary conditions,

$$\psi_y=N\upsilon\frac{\sqrt{\text{Re}}}{L}\psi_{yy},\ \psi_x=0,\ \ \theta=1+\frac{D_1\sqrt{\text{Re}}}{L}\theta_y\ \ \text{at}\ y=0,$$
$$\psi_y=u_e(x),\ \theta=0\quad \text{as}\quad y\to\infty. \tag{13}$$

A closed-form solution of the set of partial differential (11) - (13) may not exist. So we transform this system to an ordinary system using scaling group transformations (Aziz et al. 2012; Khan et al. 2012; Uddin et al. 2012),

$$\Gamma{:}x^*=e^{\varepsilon c_1}x, y^*=e^{\varepsilon c_2}y, \psi^*=e^{\varepsilon c_3}\psi, \theta^*=e^{\varepsilon c_4}\theta. \tag{14}$$

Here ε is the parameter of the group Γ and c_i's, (i = 1,2,3,4) are arbitrary real numbers. The system of (11)-(13) will remain invariant under the group transformations in (14) if the following relationships hold,

$$c_2=\frac{1}{2}(1-m)c_1,\ c_3=\frac{1}{2}(1+m)c_1,\ c_4=0. \tag{15}$$

In terms of differential,

$$dx=c_1x,\ dy=\frac{1}{2}(1-m)c_1y,\ d\psi=\frac{1}{2}(1+m)c_1\psi,\ d\theta=0. \tag{16}$$

Solving (16) we obtain,

$$\eta=x^{\frac{m-1}{2}}y,\ \psi=x^{\frac{m+1}{2}}f(\eta),\ \theta=\theta(\eta). \tag{17}$$

Here η, $f(\eta)$, $\theta(\eta)$ are similarity independent and dependent variables.

Substituting (17) into (11)- (13), we get,

$$f'''+\frac{m+1}{2}ff''+m\left(1-f'^2\right)=0, \tag{18}$$

$$(1+A\theta)\theta''+\frac{m+1}{2}\text{Pr}\,f\theta'+A\theta'^2=0, \tag{19}$$

$$f(0)=0, f'(0)=\delta f''(0),\ \theta(0)=1+b\,\theta'(0),$$
$$f(\infty)=1,\ \theta(\infty)=0. \tag{20}$$

It is worth noting that if at this stage of our analysis we put $A=\delta=0$, then our problem reduces to Bachok & Ishak (2011) if we put $\varepsilon=0$ in their paper. This supports the validity of our group analysis.

For further investigation, we use the following minor modification:

$$\eta=\sqrt{\frac{m+1}{2}}x^{\frac{m-1}{2}}y,\ \psi=\sqrt{\frac{2}{m+1}}x^{\frac{m+1}{2}}f(\eta),\ \theta=\theta(\eta). \tag{21}$$

Substituting (21) into (18) - (19), we get,

$$f'''+ff''+\frac{2m}{m+1}\left(1-f'^2\right)=0, \tag{22}$$

$$(1+A\theta)\theta''+\text{Pr}\,f\theta'+A\theta'^2=0. \tag{23}$$

The boundary conditions become,

$$f(0)=0, f'(0)=\delta f''(0),\ \theta(0)=1+b\,\theta'(0),$$
$$f'(\infty)=1,\ \theta(\infty)=0. \tag{24}$$

where prime is the derivative with respect to η, $\delta=\frac{v\sqrt{\text{Re}}}{L}N(x)$ $x^{(m-1)/2}$ is the velocity slip parameter and $b=\frac{D_1\sqrt{\text{Re}}}{L}\sqrt{\frac{m+1}{2}}$ $x^{(m-1)/2}$ is the thermal slip parameter. Note that for true similarity solutions we have $N(x)\propto x^{\frac{1-m}{2}}$, $D_1(x)\propto x^{\frac{1-m}{2}}$.

Expressions for the quantities of physical interests, the skin friction factor and the rate of heat transfer can be found from the following definitions:

$$C_{f\bar{x}} = \frac{\mu}{\rho \bar{u}_e^2}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0}, \; Nu_{\bar{x}} \frac{-\bar{x}}{T_w - T_\infty}\left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0}. \tag{25}$$

Using (6) and (21) into (25) we get,

$$\left(\frac{2\,\mathrm{Re}_{\bar{x}}}{m+1}\right)^{\frac{1}{2}} C_{f\bar{x}} = f''(0), \left(\frac{\mathrm{Re}_x\,(m+1)}{2}\right)^{\frac{-1}{2}} Nu_{\bar{x}} = -\theta'(0), \tag{26}$$

where $\mathrm{Re}_{\bar{x}} = \frac{\bar{u}_e \bar{x}}{\upsilon}$ is the local Reynolds number.

Results and Discussion

The ordinary differential equations (22) and (23) with the boundary conditions in (24) have been solved numerically using the Runge–Kutta–Fehlberg fourth-fifth order numerical method. The Runge-Kutta-Fehlberg (RKF) method is a well established adaptive numerical method for solving a scalar or system of ordinary differential equations with associated conditions (Butcher 2008). The RKF we shall use, utilizes a fourth order and fifth order accurate method to provide estimation of the solution in the next step and assists in the adaptive proven.

The numerical results were obtained to study the effect of the parameters *A, mm,* Pr, *b* and δ on the dimensionless velocity, dimensionless temperature, friction factor and the rate of heat transfer (Figures 2-11). Our results were compared to those of the corresponding published data by Watanabe (1990), Yacob et al. (2011) and Yih (1998) for different *m*. Also our present results was compared to those of the corresponding published data by Rajagopal et al. (1983) and Bararnia et al. (2012) and White (1991). The present results are in compatible with those of the published data. The comparison results are given in Tables 1 to 3.

From Figure 2 it is found that friction factor reduces with the momentum slip δ. This is because for positive values of slip parameter and slip velocity, friction factor decreases with the increase of positive values of slip parameter. This is what we observed from second boundary condition of (24) and Figure 2. It is further found that

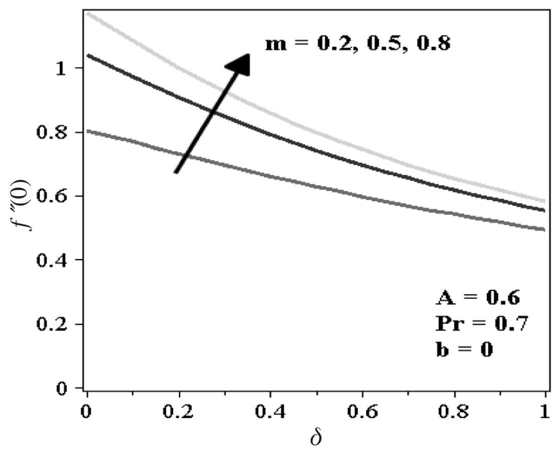

FIGURE 2. Variation of friction factor with *δ*, *m*

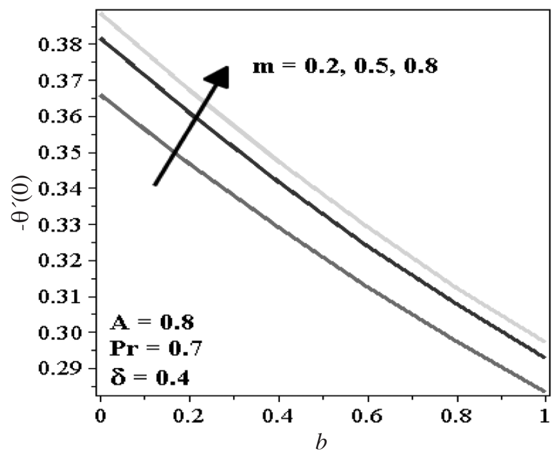

FIGURE 4. Variation of Nusselt number with *b* and *m*

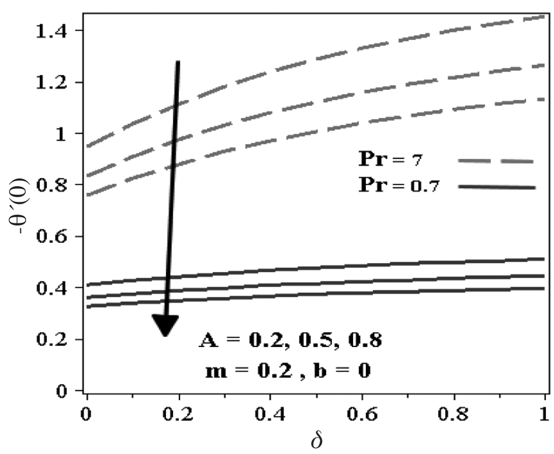

FIGURE 3. Variation of Nusselt number with *δ*, *A*, Pr and $m = 0.2$

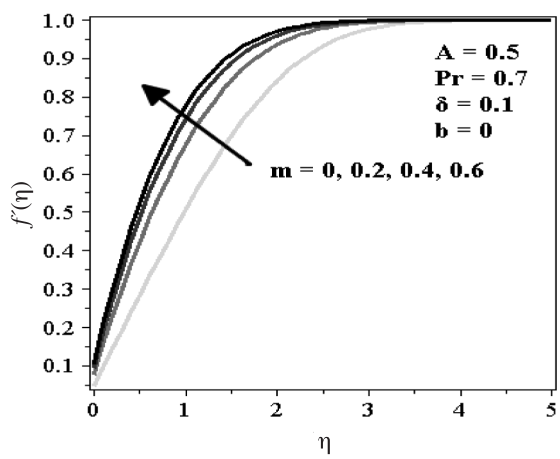

FIGURE 5. Effects of *m* on the dimensionless velocity, when $A = 0.5$, $\mathrm{Pr} = 0.7$, $b = 0$ and $\delta = 0.1$

friction factor rises with the thermal conductivity parameter A. From Figure 3 we notice that the rate of heat transfer increases with δ, Pr. This is because increasing δ causes more amount of fluid to flow through the boundary layer due to the slipping effect. From Figure 4 we notice that the rate of heat transfer increases with *m*. It is further found that the rate of heat transfer decreases with the thermal slip parameter. The effect of the Falkner–Skan power law parameter *m* on the dimensionless velocity and the dimensionless temperature profiles are shown in Figures 5 and 6, respectively. We noticed that the dimensionless velocity increases as *m* increases while the dimensionless temperature profiles decreases with increasing values of *m*. Figure 7 illustrates the effect of the thermal conductivity parameter *A* on the dimensionless temperature. We note that the dimensionless temperature increases when *A* increases and there is no change in the dimensionless velocity profiles in this case. This is because for increasing A leads to increasing the wall temperature for uniform bulk fluid temperature. Increasing wall temperature in turn leads to increase in the dimensionless temperature and consequently reduces the heat transfer rate. This is what we observe from Figure 3.

Figures 8 and 9 depict the effects of the momentum slip parameter δ on the dimensionless velocity and dimensionless temperature profiles, respectively. It is clear from these figures that the dimensionless velocity increases with the increase of the slip parameter. Physically, with the increase of momentum slip parameter, the penetration of the stagnant surface via the fluid domain decreases yields a reduction in the hydrodynamics boundary layer. It is found from Figure 9 that dimensionless temperature reduces with δ. The reason is, more flow will penetrate via the thermal boundary layer due to slipping effect with the increasing of δ. Hence hot plate heats more amounts of fluid and this yields to reduce in the temperature profiles.

Figure 10 illustrates the effects of Prandtl number Pr on the profile of dimensionless temperature. We see that the dimensionless temperature decreases when Pr increases. This is because for fixed dynamic viscosity, Pr increases means thermal conductivity decreases. Thermal conductivity is a linear function of temperature

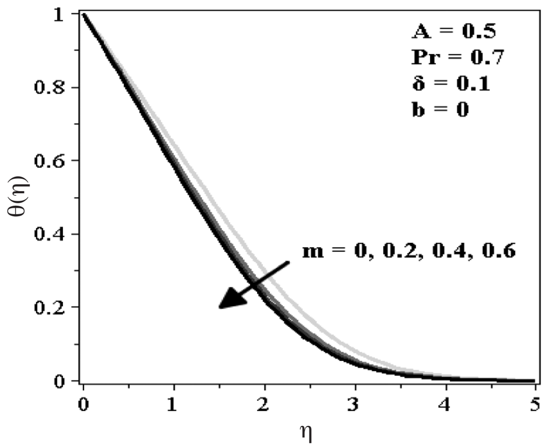

FIGURE 6. Effects of *m* on the dimensionless temperature, when $A = 0.5$, $\mathrm{Pr} = 0.7$, $b = 0$ and $\delta = 0.1$

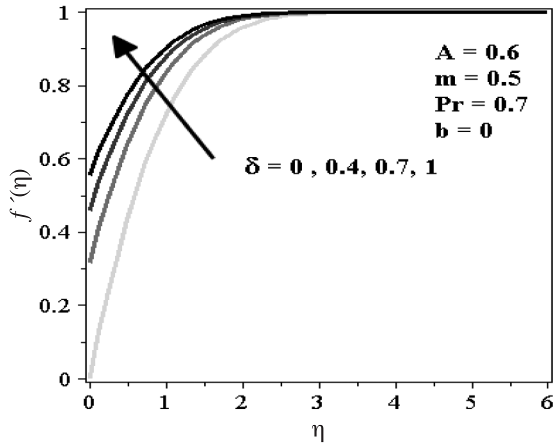

FIGURE 8. Effects of δ on the dimensionless velocity when $m = 0.5$, $\mathrm{Pr} = 0.7$, $b = 0$ and $A = 0$ .6

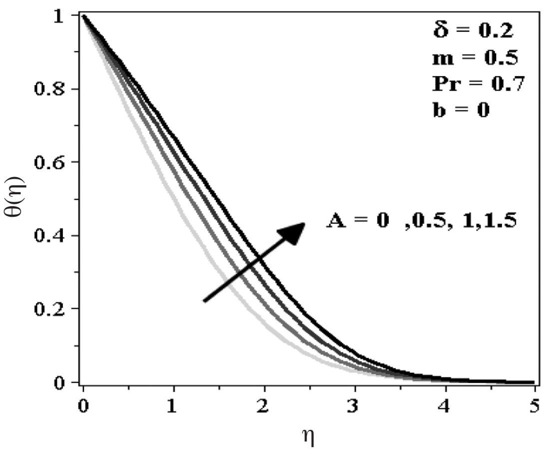

FIGURE 7. Effects of *A* on the dimensionless temperature, when $m = 0.5$, $\mathrm{Pr} = 0.7$, $b = 0$ and $\delta = 0.2$

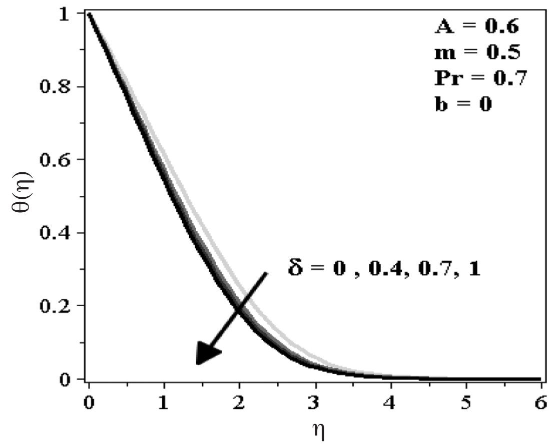

FIGURE 9. Effects of δ on the dimensionless temperature when $m = 0.5$, $\mathrm{Pr} = 0.7$, $b = 0$ and $A = 0.6$

as indicated in (5). Hence increasing Pr leads to decrease in temperatures which in turn lead to increase in heat transfer rates (Figure 3). Finally, Figure 11 illustrates the effects of the thermal slip parameter b on the profile of dimensionless temperature. We see that the dimensionless temperature decreases when b increases. This is because with the increase in thermal slip parameter, the fluid will not sense the heating effects of the hot plate and less amount of heat will be transferred from the hot plate to the cold fluid.

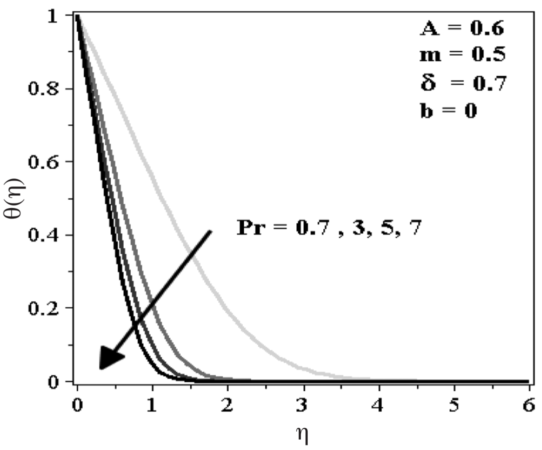

FIGURE 10. Effects of Pr on the dimensionless temperature when $m = 0.5$, $\delta = 0.7$, $b = 0$ and $A = 0.6$

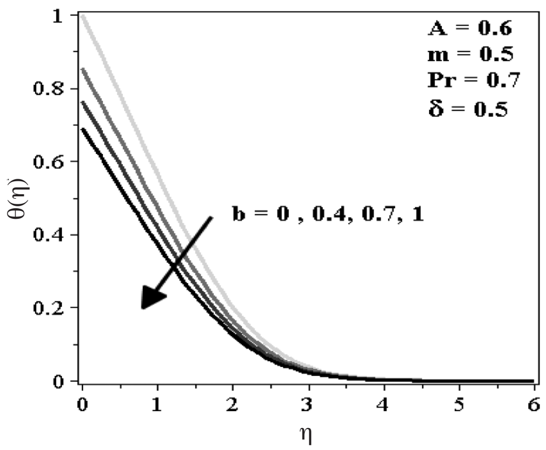

FIGURE 11. Effects of b on the dimensionless temperature when $m = 0.5$, $\delta = 0.7$, $\delta = 0.5$ and $A = 0.6$

TABLE 1. The values of $f''(0)$ for various m when $A = 0$, Pr = 1 and $\delta = b = 0$

Present results	Yacob (2011)	Yih (1998)	Watanabe (1990)	m
0.4696452	0.4696	0.649600	0.46960	0
0.6550006	0.6550	0.654979	0.65498	1/11
0.8021272	0.8021	0.802125	0.80213	0.2
0.9276804	0.9277	0.927653	0.92765	1/3
1.0389035	1.0389	……….	………	0.5
1.2325876	1.2326	1.232588	………	1

TABLE 2. The values of $-\theta'(0)$ for various $\beta = \frac{2m}{m+1}$ and Pr when $A = \delta = b = 0$

Pr	White (1991)	Present results	White (1991)	Present results	White (1991)	Presen results
	$\beta = 0$		$\beta = 0.3$		$\beta = 2.0$	
0.1	0.1980	0.198033	0.2090	0.209076	0.2260	0.226002
0.3	0.3037	0.303717	0.3278	0.327829	0.3668	0.366808
0.6	0.3916	0.391675	0.4289	0.428924	0.4913	0.491302
0.72	0.4178	0.418091	0.4592	0.459551	0.5292	0.529607
1.0	0.4690	0.469600	0.5195	0.519518	0.6052	0.605197
2.0	0.5972	0.597233	0.6690	0.669044	0.7959	0.795991
3.0	0.6859	0.685961	0.7739	0.773436	0.9303	0.930351
6.0	0.8672	0.867277	0.9872	0.987267	1.2069	1.206924
10.0	1.0297	1.029747	1.1791	1.179129	1.4557	1.455749
30.0	1.4873	1.487319	1.7198	1.719841	2.1577	2.157737
60.0	1.8746	1.874594	2.1770	2.177565	2.7520	2.751962
100.0	2.2229	2.222905	2.5892	2.589233	3.2863	3.286249
400.0	3.5292	3.529230	4.1331	4.133069	5.2890	5.289017
1000.0	4.7901	4.790063	5.6230	5.623036	7.2212	7.221172
4000.0	7.6039	7.604410	8.9481	8.948084	11.532	11.53203
10000.0	10.320	10.366391	12.157	12.161747	15.692	15.69277

TABLE 3. The values of $f''(0)$ for various $\beta = \frac{2m}{m+1}$ when $A = 0$, Pr = 1 and $\delta = b = 0$

β	Rajagopal et al. (1983) (Block-tridiagonal factorization technique)	Bararnia et al. (2012) (Homotopy perturbation method)	Present results (Runge–Kutta–Fehlberg fourth- fifth)
0.0	0.4696	0.46964	0.469600
0.05	0.5311	0.53119	0.531129
0.1	0.5871	0.58716	0.587035
0.2	0.6867	0.68672	0.686708
0.3	0.7747	0.77475	0.774754
0.4	.85440	.854420	0.854421
0.6	0.9958	0.99589	0.995836
0.7	……..	1.05985	1.059807
0.8	1.1202	1.12020	1.120267
0.9	……..	1.17699	1.177727
1.0	1.2325	1.23150	1.232587
1.2	1.3357	1.33559	1.335720
1.6	1.5215	1.52141	1.521513
2.0	……..	1.68462	1.687218

CONCLUSION

A heat transfer analysis for the Falkner-Skan boundary layer flow past a stationary wedge with momentum and thermal slip boundary conditions considering the temperature-dependent thermal conductivity was investigated numerically. Using scaling group of transformations, the partial differential equations governing the problem were converted to a non-linear system of ordinary differential equations, which was then solved numerically by the Runge–Kutta–Fehlberg fourth- fifth order numerical method. From the numerical results, the following conclusion can be drawn. The friction factor reduces with δ and rises with A; the heat transfer rates increases with δ, Pr, m but it decreases with A and b.; the dimensionless velocity, rate of heat transfer at the wall and friction factor increase with m, while the dimensionless temperature decreases; the increase of the slip parameter increases the dimensionless velocity and the wall heat transfer rate, while it decreases the dimensionless temperature and the friction factor; the dimensionless temperature decrease with the thermal conductivity parameter, the wall heat transfer rates is just opposite and the dimensionless temperature decreases with Pr and b.

ACKNOWLEDGMENT

Authors acknowledge the financial support of USM.

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A.A. Mutlag*, Md. Jashim Uddin & Ahmad Izani Ismail
School of Mathematical Sciences
Universiti Sains Malaysia
11800 Penang
Malaysia

A.A. Mutlag*
Mathematics Department
College of Education for Pure Science
AL- Anbar University, AL- Anbar
Iraq

Md. Jashim Uddin
Mathematics Department
American International University-Bangladesh
Banani, Dhaka 1213
Bangladesh

M.A.A. Hamad[4]
Mathematics Department
Faculty of Science
Assiut University, Assiut 71516
Egypt

*Corresponding author; email: alassafi2005@yahoo.com

Received: 13 June 2012
Accepted: 27 September 2012