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Group Decision via Usage of Analytic Hierarchy Process and Preference Aggregation Method

(Keputusan Berkumpulan Menggunakan Proses Hierarki Analisis dan Kaedah Pengagregatan Keutamaan)

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ABSTRACT

The Analytic Hierarchy Process (AHP) is a recognised modern approach to solve decision making problems. Initially introduced by Saaty in 1971 as a tool for handling individual decision making situation, the method has since been extended to incorporate groups. In this paper, a new method for AHP group decision making is proposed. The method integrates AHP with a Data Envelopment Analysis (DEA)-based preferential aggregation method. It manipulates the preferential weights and ranking aspect of each decision maker in coming up with an optimisation model that determines the best efficiency score of each alternatives. These efficiency scores are then used to rank the alternatives and determine the group decision weights. A comparative analysis of the method with another AHP group decision making method indicates a similar 'satisfactory index' level.

Keywords: Analytic hierarchy process; data envelopment analysis; group decision making; preference aggregation

ABSTRAK

Proses Hierarki Analisis (PHA) adalah kaedah moden yang digunakan untuk penyelesaian masalah pembuatan keputusan. Ia diperkenalkan oleh Saaty pada tahun 1971 sebagai alat untuk mengendali situasi membuat keputusan individu. Kaedah itu telahpun dikembangkan untuk mengendalikan situasi membuat keputusan berkumpulan. Dalam kertas ini, satu kaedah baru pembuatan keputusan berkumpulan PHA dicadangkan. Kaedah ini menggabungkan PHA dengan kaedah pengagregatan keutamaan yang berasaskan kepada Analisis Penyampulan Data (APD). Ia menggunakan pemberat keutamaan dan aspek taraf kedudukan setiap pembuat keputusan dalam menghasilkan suatu model pengoptimuman untuk menentukan skor kecekapan terbaik setiap alternatif. Skor kecekapan ini kemudiannya digunakan untuk menaraf kedudukan setiap alternatif dan menentukan pemberat keputusan kumpulan. Satu analisis perbandingan di antara kaedah yang dicadangkan dengan satu kaedah pembuatan keputusan berkumpulan PHA yang lain menghasilkan tahap 'indeks kepuasan' yang setara.

Kata kunci: Analisis penyampulan data; membuat keputusan berkumpulan; pengagregatan keutamaan; proses hierarki analisis

INTRODUCTION

Analytic Hierarchy Process (AHP) is a flexible decision making tool for multiple criteria problems (Saaty 1980). In the last two decades, AHP has gained significant popularity and there are many reported real life applications in business, energy, health, transport and housing (Vaidya & Kumar 2006). This is mainly due to its mathematical and methodological simplicity and its ability to handle both quantitative and qualitative data. AHP is also supported by the availability of good computer software.

The essence of the process is the decomposition of a complex problem into a hierarchy with goal (objective) at the top of the hierarchy, criteria and sub-criteria at levels and sub-levels of the hierarchy, followed by decision alternatives at the bottom of the hierarchy as in Figure 1.

Besides being used as a stand-alone tool, AHP has been integrated with various other tools for many real applications. For example, Ozdemir and Gasimov (2004) studied a faculty course assignment problem using binary non linear programming model. They reduced the multiple objective functions to a single objective function and used AHP to determine the relative importance weightings of the objectives or the preferences of the instructors and administrators. The objective was to select the best assignment that maximised the satisfaction of instructors and administrators. Another example was the combined used of AHP and Data Envelopment Analysis (DEA) for selecting internet company stocks (Ho & Oh 2010).

In a survey by Ho (2008) on integrated AHP models and its applications, it was reported that the five tools that are commonly combined with AHP are mathematical

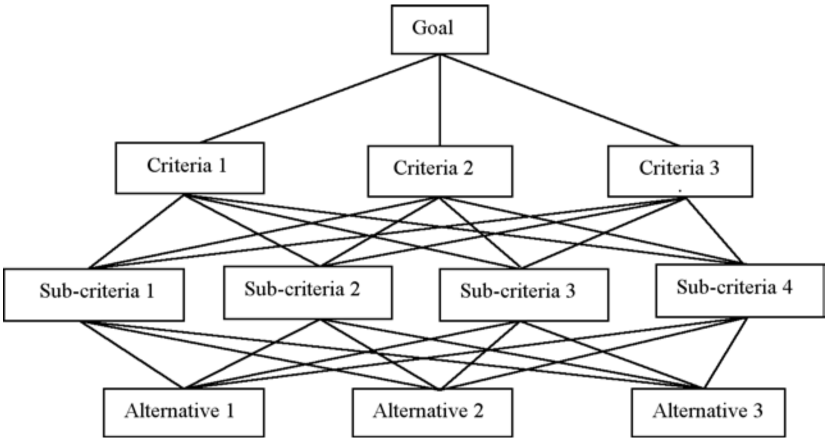

FIGURE 1. Decision hierarchy of AHP

programming, quality function deployment (QFD), meta-heuristics, SWOT (strength, weaknesses, opportunities, and threats) analysis and data envelopment analysis (DEA). In this paper, another integrated AHP model is described. The model combines AHP with a DEA-based preference aggregation method for group decision.

The purpose of this study was similar to the purpose set out by Huang et al. (2009) which was to develop a group AHP approach which embraces the effects in dealing with multiple criteria group decision problems in a more realistic and rational fashion. As in Huang et al. (2009), it also considered the preferential weights and preferential ranks aspects in the construction of a group AHP decision model. However, the two aspects were manipulated in a different manner.

BACKGROUND

GROUP DECISIONS IN AHP

Two approaches commonly used to handle group decision making in AHP are the aggregation of individual judgments (AIJ) and the aggregation of the individual priorities (AIP) (Forman & Peniwati 1998). AIJ is usually performed using the geometric mean, while AIP is usually performed using the arithmetic mean. However, there are a number of shortcomings associated with the use of geometric mean and arithmetic mean. Among them are the influence of extreme values and the simplicity of the technical manipulation that is used to combine the judgments of decision makers which affect the ability of capturing the group preferences. To overcome these shortcomings, Huang et al. (2009) proposed a method that considers the aspect of preferential differences and preferential ranks in the construction of a group AHP model. Preferential differences denote the differences of preferential weights among alternatives for each decision maker and preferential ranks denote the ranks of the alternatives for each decision maker. The use of the two factors aggregates preferences of decision makers in the sense of compromise rather than optimisation. This differs from the method proposed in this paper which aggregates preferences of decision makers based on preferential weights and preferential ranks in the sense of optimisation.

DATA ENVELOPMENT ANALYSIS

Data Envelopment Analysis (DEA) is a linear programming technique that was first introduced in Charnes, Cooper and Rhodes' paper of 1978 (Charnes et al. 1978). This technique evaluates the relative efficiency of Decision Making Units (DMUs) which contain some non homogeneous input and output. In the traditional DEA models like the CCR model (Charnes et al. 1978) and the BCC model (Banker et al. 1984), efficiency scores of efficient DMUs are 1 and efficiency scores of inefficient DMUs are less than 1.

In mathematical terms, consider a set of n DMUs, in which x_{ij} $(i = 1, 2, \ldots, m)$ and y_{rj} $(r = 1, 2, \ldots, s)$ are input and output of DMU $(j = 1, 2, \ldots, n)$. A standard DEA model for assessing DMU_p which is known as the CCR model (Charnes et al. 1978), is formulated in Model (1).

$$\begin{aligned} \max \quad & z_p = \sum_{r=1}^{s} u_r y_{rp} \\ s.t. \quad & \sum_{i=1}^{m} v_i x_{ip} = 1 \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \\ & u_r, v_i \ge 0 \quad \forall r, i. \end{aligned} \tag{1}$$

In Model (1), the optimal value (z^*_p) demonstrates the relative efficiency score associated with DMU_p which is under evaluation. v_i and u_r are the associated input and output weights. In this model, DMU_p is efficient if z^*_p = 1. Basically, the model finds the input and output weights that will give DMU_p the best possible efficiency score.

PREFERENCE AGGREGATION

Preference aggregation problem, in the context of a ranked voting system is a group decision making problem of selecting m alternatives from a set of n alternatives ($n > m$). Hence, each decision maker ranks the alternatives from the most preferred (rank = 1) to the least preferred (rank = n). Obviously, due to different opinions of the decision makers, each alternative may be placed in a different ranking position. Some studies suggest a simple aggregation method by finding the total score of each alternative as the weighted sum of the votes received by each alternative according to different decision makers. In this method, the best alternative is the one with the largest total score. The key issue of the preference aggregation is how to determine the weights associated with different ranking positions. Perhaps, Borda–Kendall method (Hashimoto 1997) is the most commonly used approach for determining the weights due to its computational simplicity.

Cook and Kress (1990) proposed a method that is based on DEA to aggregate the votes from a preferential ballot. For this purpose, they used the following DEA model (2) in which output are number of first place votes, second place votes and so on that a DMU obtained and a single input with value 1.

$$\max \beta_p = \sum_{k=1}^{n} \mu_k \omega_{pk}$$

$$\begin{aligned} s.t. \quad & \sum_{k=1}^{n} \mu_k \omega_{jk} \le 1 \qquad j = 1,2,\ldots,m \\ & \mu_k - \mu_{k+1} \ge d(k,\varepsilon) \quad k = 1,2,\ldots,n-1 \\ & \mu_n \ge d(n,\varepsilon), \end{aligned} \tag{2}$$

where ω_{jk} is the number of rank k vote that DMU_j obtained and μ_k is the weight of rank k calculated by Model (2). It is clear that $\mu_k \ge \mu_{k+1}$, so the extra constraint $\mu_k - \mu_{k+1} \ge d(k,\varepsilon)$ indicates how much vote $k+1$ is preferred to vote k. The notation $d(k,\varepsilon)$ is a function which is non-decreasing in $\mathring{a}$ and is referred to as a discrimination intensity function. Model (2) is solved for each candidate $j = 1, 2 \ldots, m$.

THE PROPOSED MODEL

Suppose that we have a group decision making situation where p decision makers are individually assessing n alternatives.

Stage 1. Develop an AHP model of the decision making problem and perform the traditional pair-wise assessment of the problem individually on each decision maker. This will results in the following weight matrix:

$$W = (w_{ij})_{p\times n} \; i = 1,2,\ldots,p; \; j = 1,2,\ldots,n,$$

where w_{ij} is the decision weight of alternative j resulted from the AHP assessment by decision maker i.

Stage 2. Convert the weight matrix that was obtained in Stage 1 into a rank matrix $R = (r_{ij})_{p\times n}$ where r_{ij} is the ranking order of w_{ij} in row i of matrix W.

Stage 3. Construct matrix $S = (s_{jk})_{n\times n}$ with reference to matrix R where s_{ij} is the number of times that alternative j is placed in rank k.

Stage 4. Construct matrix $\Omega = (\hat{\theta}_{jk})_{n\times n}$ where $\hat{\theta}_{jk}$ is the summation of the decision weights in matrix W which corresponds to alternative j being placed in rank k.

Stage 5. Obtain an efficiency score β_j of each alternative by constructing and solving the following modified Cook and Kress (1990) model:

$$\begin{aligned} & \max \beta_p = \sum_{k=1}^{n} u_k \theta_{pk} \\ & s.t. \\ & \quad \sum_{k=1}^{n} u_k \theta_{jk} \le 1 \qquad j = 1,2,\ldots,n \\ & \quad \mu_k - \mu_{k+1} \ge d(k,\varepsilon) \quad k = 1,2,\ldots,n-1 \\ & \quad u_n \ge d(n,\varepsilon), \end{aligned} \tag{3}$$

where θ_{jk} is as defined in Stage 4. Note that we have used θ_{jk} instead of ω_{jk}. Alternative with a higher efficiency score is the better alternative.

Stage 6. Determine the group decision weight w_j^G for alternative j by normalising the efficiency scores obtained in Stage 5 as follows:

$$w_j^G = \frac{\beta_j}{\sum_{i=1}^{n} \beta_i} \tag{4}$$

NUMERICAL EXAMPLE AND COMPARATIVE ANALYSIS

Consider a case study presented in Huang et al. (2009) in which three travel spots are evaluated by seven decision makers. Spot A is close to the equator and is famous for its beach scene and local costumes, Spot B has special landscapes of a volcano and exotic lodges with relaxing hot springs and Spot C is a tropical island which has irresistibly beautiful ocean sightseeing with spectacular underwater views. The criteria in this evaluation were lodging, dining, transportation, expenses, schedule and weather. The decision weights of the three travel spots for the seven decision makers after performing the traditional AHP process are shown in Table 1. Table 1 is actually the tabular form of matrix W in Stage 1.

TABLE 1. Tabular form of matrix *W* (decision weights of travel spots for decision makers)

Decision maker	Spot A	Spot B	Spot C
1	0.434	0.259	0.307
2	0.364	0.330	0.307
3	0.227	0.565	0.208
4	0.330	0.316	0.354
5	0.314	0.301	0.385
6	0.329	0.349	0.322
7	0.197	0.599	0.204

Note that the weight for Spot C is the same for decision makers 1 and 2, *i.e.* 0.307. But in term of rank, Spot C is ranked second by decision maker 1 and third by decision maker 2. In Stage 2, we obtained Table 2 (the tabular form of matrix *R*) which demonstrates the rank place of each travel spot by each decision maker. From this stage onwards, the problem is treated as a preferential voting situation whereby the decision makers have voted for the travel spots and the task now is to aggregate these votes into a group decision.

TABLE 2. Tabular form of matrix *R* (rank place of travel spots for decision makers)

Decision maker	Spot A	Spot B	Spot C
1	1	3	2
2	1	2	3
3	2	1	3
4	2	3	1
5	2	3	1
6	2	1	3
7	3	1	2

In Stage 3, a summary of the votes that the seven decision makers have made on the three travel spots is shown in Table 3 (the tabular form of matrix *S*). For example, Spot A has been voted twice in rank place 1, four times in rank place 2 and once in rank place 3. This is basically how the preferential rank aspect is handled in the model. The number of votes for each rank place that an alternative receives is determined.

TABLE 3. Tabular form of matrix *S* (number of rank place of each travel spot)

	Rank place 1	Rank place 2	Rank place 3
Spot A	2	4	1
Spot B	3	1	3
Spot C	2	2	3

Next, in Stage 4, the summations of the decision weights which corresponds to an alternative being placed in certain rank resulted in Table 4 (the tabular form of matrix *Ω*). For instance, in Table 3, Spot B received three rank 1 votes. By referring to Table 2, we notice that these votes came from decision makers 3, 6 and 7. In Table 1, the corresponding weights are 0.565, 0.349 and 0.599. The summation of these weights gives us a value of 0.565 + 0.349 + 0.599 = 1.513. This is basically how the preferential weight aspect is handled in the model. Weights which correspond to the same rank positions are added together. An advantage of adding the weights is a better discrimination of the alternatives. For example, Spots A and C both received two rank 1 vote. However, in term of total weight, the values are different.

TABLE 4. Tabular form of matrix *Ω*

	Rank place 1	Rank place 2	Rank place 3
Spot A	0.798	1.200	0.197
Spot B	1.513	0.330	0.876
Spot C	0.739	0.511	0.837

In Stage 5, a linear programming model is constructed and solved for each spot. For Spot A the model is as follows:

$$
\begin{aligned}
&\max 0.798\mu_1 + 1.200\mu_2 + 0.197\mu_3 \\
&s.t. \\
&\quad 0798\mu_1 + 1.200\mu_2 + 0.197\mu_3 \le 1 \\
&\quad 1.513\mu_1 + 0.330\mu_2 + 0.876\mu_3 \le 1 \\
&\quad 0.739\mu_1 + 0.511\mu_2 + 0.837\mu_3 \le 1 \\
&\quad \mu_1 - \mu_2 \ge 0.2993 \\
&\quad \mu_2 - \mu_3 \ge \frac{0.2993}{2} \\
&\quad \mu_3 \ge \frac{0.2993}{3}.
\end{aligned}
$$

As suggested in Cook and Kress (1990), $d(i,\varepsilon) = \frac{\varepsilon}{i}$ and ε_{max} = 0.2993 is used and the optimal value 0.757 is obtained.

Note that an optimisation method is used at this stage to find the efficiency scores of the alternatives. The efficiency scores, together with the normalisation of the scores and the final ranking of the travel spots are listed Table 5. Spot B is the best alternative followed by Spot A and finally Spot C. Compared to the results in Huang et al. (2009), the ranking is the same but the group decision weights are different. The group decision weights in Huang et al. (2009) were [0.305, 0.496, 0.199].

TABLE 5. Results

	Efficiency Score	Group Decision Weight	Rank
Spot A	0.757	0.319	2
Spot B	1.000	0.421	1
Spot C	0.617	0.260	3

COMPARATIVE DISCUSSION

Huang et al. (2009) introduced the concept of 'satisfactory index' to measure the satisfactory level of the final group decision that was obtained. The concept is based on: (1) the differences between the final weights integrated by the group decision makers and the original weights assessed by an individual decision maker and (2) the differences between the final ranks obtained by the group decision makers and the original ranks placed by an individual decision maker.

The first matter is referred to as the differentiation coefficient of the i^{th} alternative for the l^{th} decision maker and is defined as

$$d(i,\varepsilon) = \frac{\varepsilon}{i} \quad (5)$$

where w_i is the final weight of the i^{th} alternative by some group technique and w_{li} is the original weight of the i^{th} alternative for l^{th} the decision maker.

The second matter is referred to as the ranking coefficient of the i^{th} alternative for the l^{th} decision maker and is defined as

$$\zeta_{li} = |k_{li} - k_i|, \quad (6)$$

where k_i is the final rank of the i^{th} alternative by some group technique and k_{li} is the original rank of the i^{th} alternative for l^{th} the decision maker.

The two coefficients, (5) and (6), are integrated to obtain ρ_l , the satisfactory index for the l^{th} decision maker as follows:

$$\rho_l = \frac{1}{n}\sum_{i=1}^{n}\eta_{li}\zeta_{il}. \quad (7)$$

Finally, the overall satisfactory index for the decision group is obtained as

$$\rho = \rho_p\sqrt{\prod_{l-1}^{\rho}\rho_l}. \quad (8)$$

The differentiation, ranking coefficients and the satisfactory index for each decision maker for the method proposed in this paper is given in Table 6. The overall satisfactory index is 0.523. This value is almost the same as the overall satisfactory index obtained for the group decision method proposed by Huang et al. (2009), which is 0.522. This indicates that the satisfactory level of the decision using the method proposed in this paper is the same as the satisfactory level of the decision proposed by Huang et al. (2009).

TABLE 6. η_{li}, ζ_{li} and ρ_l for the proposed group AHP

	η_{li}			ζ_{li}			
Decision maker	Spot A	Spot B	Spot C	Spot A	Spot B	Spot C	ρ_l
1	0.241	0.171	0.588	1.000	2.000	1.000	0.286
2	0.408	0.202	0.390	1.000	1.000	0.000	0.537
3	0.293	0.187	0.520	0.000	0.000	0.000	1.000
4	0.818	0.086	0.096	0.000	2.000	2.000	0.339
5	0.924	0.039	0.037	0.000	2.000	2.000	0.334
6	0.769	0.107	0.124	0.000	0.000	0.000	1.000
7	0.259	0.177	0.564	1.000	0.000	1.000	0.618

CONCLUSION

A new method for AHP group decision making has thus been proposed. The method, based on preferential voting, is similar to a recently proposed method by Huang et al. (2009) as it also considers the preferential weights and preferential ranks aspects in the construction of a group AHP decision model. However, the two aspects are manipulated in a different manner. A comparative analysis of the two methods indicates a similar 'satisfaction index' level.

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