Sains Malaysiana 40(1)(2011): 1–3

Determination of Moment of Inertia for $^{162-168}$Hf and $^{164-176}$Yb Deformed Nuclei

(Penentuan Momen Inersia dan Tenaga untuk Nukleus Cangga $^{162-168}$Hf dan $^{164-176}$Yb)

A.A OKHUNOV*, HASAN ABU KASSIM & PH.N. USMANOV

ABSTRACT

In this paper, a method of defining the even-even deformed nuclei inertial parameters is suggested. Calculations for isotopes $^{162-168}$Hf and $^{164-176}$Yb are listed. The parameters of inertia of rotational nuclei are also defined. Dependence of the parameters of inertia on the nucleons number is shown.

Keywords: Collective level; nuclear structure models and methods; properties of nuclei

ABSTRAK

Dalam kertas ini, dicadangkan satu kaedah untuk menakrifkan parameter inersia bagi nukleus tercangga genap-genap. Penghitungan untuk isotop $^{162-168}$Hf dan $^{164-176}$Yb disenaraikan. Parameter untuk inersia bagi nukleus putaran juga ditakrifkan. Kebersandaran parameter inersia ke atas nombor nukleon ditunjukkan.

Kata kunci: Aras kolektif; model struktur nuklear dan kaedah; sifat nukleus

INTRODUCTION

Collective properties of low-lying states in even-even nuclei in the regions of deformed nuclei give rise to two types of important collective motion: nuclear surface vibration and nuclear rotation. There are a number of deformed nuclei that exhibit rotational bands, starting with light up to transuranium elements, which have important role in developing the study of nuclear structure. Nuclear rotation is a simple phenomenon to study. The nuclear response to rotation gives reliable information. Therefore, the nuclear rotation motion is one of the mechanisms in nuclear structure investigations. In this research, it is to calculate the nuclear moment of inertia parameters and energy spectra of $^{162-168}$Hf and $^{164-176}$Yb.

MOMENT OF INERTIA OF DEFORMED NUCLEI

The research study the low excitation states of even-even deformed nuclei which is the result of nuclear rotation (Bohr & Mottelson 1998). Nuclear moment of inertia appear as one of the main physical characteristics of deformed nuclei. There are various methods in the definition of the moment of inertia of nuclei (Harris 1965; Usmanov & Mikhaylov 1997). In classical electrodynamics, angular frequency in rotating nuclear core is given by the frequency of the electromagnetic radiation. One may define also the energy and angular frequency on the basis of the stretched quadrapole or dipole transitions where the choice depends on the symmetry (Frauendorf 2001). Bohr-Mottelson (1998) and Bengtsson-Frauendorf (1979) consider the definition of core inertia parameters using Harris parameterization for the energy and angular momentum, respectively (Harris 1965):

$$E_{rot}(I)=\frac{1}{2}J_0\omega_{rot}^2(I)+\frac{3}{4}J_1\omega_{rot}^4(I), \tag{1}$$

$$\sqrt{I(I+1)}=J_0\omega_{rot}(I)+J_1\omega_{rot}^3(I), \tag{2}$$

where J_0 and J_1 are the inertia parameters of nuclear rotational core and $\omega_{rot}(I)$ the angular frequency of rotational core. Hence, the energy of rotational core $E_{rot}(I)$ is in agreement with the energy of the ground state of rotational bands of even-even deformed nuclei in the lower value of spin I.

Given below is a method of defining the core parameters of the moment of inertia for the even-even deformed nuclei in the rare-earth region. Nuclear rotational angular frequency is given as follow:

$$\omega_{eff}(I)=\frac{E^{exp}(I+1)-E^{exp}(I-1)}{2}. \tag{3}$$

Moment of inertia for states $J_{eff}(I)$ in terms of the angular frequency of rotation $\omega_{eff}(I)$ is:

$$J_{eff}(I)=\frac{d\sqrt{I(I+1)}}{\omega_{eff}(I)}. \tag{4}$$

RESULTS AND DISCUSSION

From equation (4), this research calculate the effective moment of inertia $J_{eff}(I)$. Nuclear angular frequency of rotation $\omega_{eff}(I)$, is given by equation (3), with $E^{exp}(I)$ the energy from experiment (Begzhanov et al. 1989). Dependency of moment of inertia $J_{eff}(I)$ on the square of angular frequency of rotation $\omega^2_{eff}(I)$ for the isotopes is $^{162-168}$Hf illustrated in Figure 1.

At low angular frequency of rotation, i.e. in low spin $I \leq 10\,\hbar$ the dependency is linear, as can be seen from the Figure 1. This dependency parameterize as follows:

$$J_{eff}(I) = J_0 + J_1\omega^2_{eff}(I). \tag{5}$$

Equation (5) defines the parameters of inertia J_0 and J_1, for the effective moment of inertia $J_{eff}(I)$ when $I \leq 10\hbar$. Numerical values for J_0 and J_1 are determined using the least square method in equation (5). These results are shown in Table 1 for the isotopes $^{162–168}$Hf and $^{164–176}$Yb.

By using the values of J_0 and J_1 in Table 1, the moments of inertia $J_{rot}(I)$ of the nuclear core is calculated by using:

$$J_{rot}(I) = J_0 + J_1\omega^2_{rot}(I), \tag{6}$$

where the parameter J_0 characterizes the nuclear moment of inertia in $I = 0$ ($\omega_{rot}(0) = 0$) and appears as the nuclear moment of inertia of ground states. Also, Figure 1 (a-d) illustrates the results for $J_{rot}(I)$ by using equation (6). and $\omega_{rot}(I)$ appears cubic consistent with equation (2). This equation has two imaginary roots and one real solution. The value of angular frequency of rotation $\omega_{rot}(I)$ appears as a real solution of a given spin I, which is

$$\omega_{rot}(I) = \left\{ \frac{\tilde{I}}{2J_1} + \left[\left(\frac{J_0}{3J_1} \right)^3 + \left(\frac{\tilde{I}}{2J_1} \right)^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{3}} + \left\{ \frac{\tilde{I}}{2J_1} \left[\left(\frac{J_0}{3J_1} \right)^3 + \left(\frac{\tilde{I}}{2J_1} \right)^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{3}} \tag{7}$$

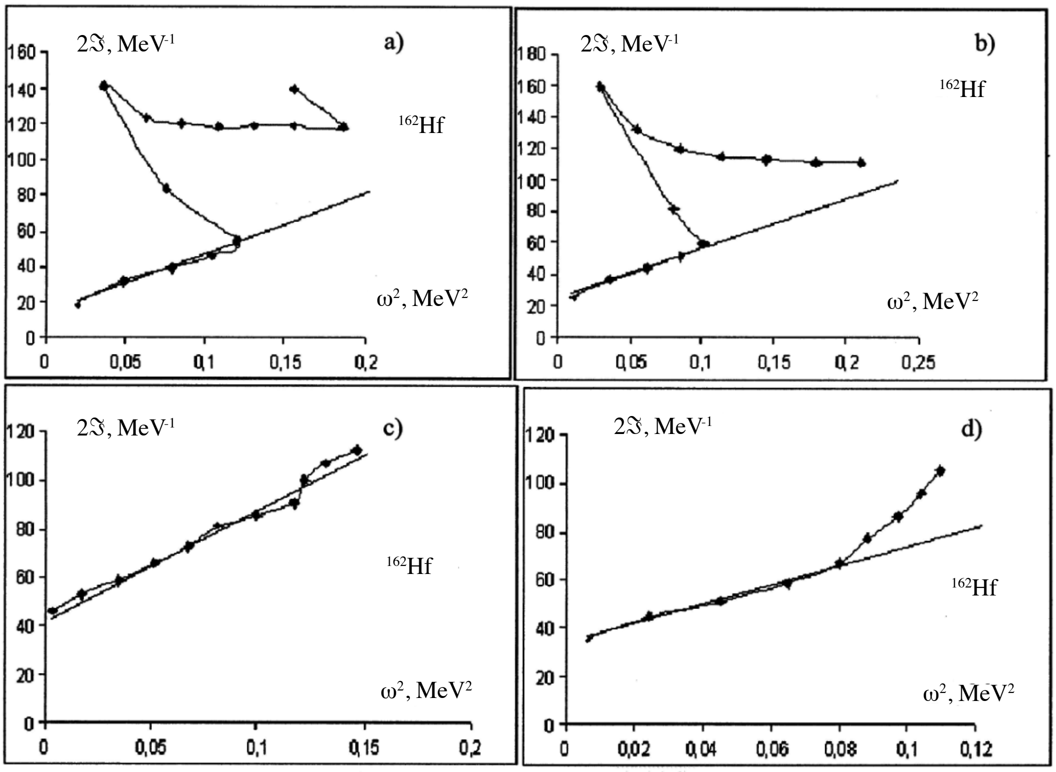

FIGURE 1. Moment of inertia as a function of $\omega^2_{eff}(I)$ for (a) ^{162}Hf, (b) ^{164}Hf, (c) ^{166}Hf and (d) ^{168}Hf

TABLE 1. The values of the parameters for the moment of inertia of Yb

A	J_0(MeV^{-1})	J_1(MeV^{-3})	E_{2+}(MeV)	Q_0 (Begzhanov et al. 1989)
164	22.47	192.85	0.123	6.79 (13)
166	27.57	166.60	0.102	7.26 (14)
168	32.31	198.22	0.087	7.62 (14)
170	33.91	129.89	0.084	7.80 (30)
172	36.35	128.02	0.079	7.91 (18)
174	37.47	122.65	0.077	7.82 (24)
176	34.87	104.22	0.082	7.59 (3)

where $\tilde{I}=\sqrt{I(I+1)}$. $\omega_{rot}(I)$ has deviated from equation (4) for large spin (Figure 1). Thus nonlinearity in large spin has bind with the mixture of ground bands with other rotation bands, which have vibrational character.

The parameter J_0 is proportional to the nuclear intrinsic quadrupole moment Q_0 and energy E_{2+} (Table 1). The nuclei are large if intrinsic quadrupole moment Q_0 in the ground state has large moment of inertia and hence the first vibration E_{2+} has the lowest excitation energy. The parameters J_0 and Q_0 increasing with the growing number of nucleons beginning from $A = 172$ until 174, and beginning with $A = 176$ onwards these parameters decrease. Therefore, the parameter J_0 describes the moment inertia of nuclear ground state having maximum value in $A = 172$ and $A = 174$.

CONCLUSION

In the present calculation, this research introduced the method of defining the core moment of inertia of even-even deformed nuclei in the rare-earth region. For the large values of angular frequency in the rotation, the core moment of inertia decline. The decline was explained by the fact that the nuclear core under rotation with the large mixture frequency of ground-state bands with other rotational bands that have vibrational characters. The calculation takes into account the Coriolis mixing of positive parity states which has good agreement with experimental data.

ACKNOWLEDGMENTS

The authors would like to thank University of Malaya and Fundamental Research Grant Scheme, FP036/2008C for financial support.

REFERENCES

Begzhanov, R.B., Belinkiy V.M., Zalyubovskiy, I.I. & Kuznichenko, A.B. 1989. *Handbook on Nuclear Physics Vol. 1 and 2*. Tashkent: FAN.

Bengtsson, R. & Frauendorf, S. 1979. An interpretation of backbending in terms of the crossing of the ground state band with an aligned two-quasiparticle band. *Nucl. Phys. A* 314: 27-36.

Bohr, A. & Mottelson, B.R. 1998. *Nuclear Structure*. Singapore: World Scientific.

Frauendorf, S. 2001. Spontaneous symmetry breaking in rotating nuclei. *Rev. Mod. Phys*. 73: 463–514.

Harris, S.M. 1965. Higher order corrections to the cranking model. *Phys. Rev.* 138: B509-B513.

Usmanov, Ph. N. & Mikhaylov, I.N. 1997. Non-adiabatic effects of collective motion in even-even deformed nuclei. *Physics of Elementary Particles and Atomic Nuclei (Fiz. Elem. Chastits At. Nucl.)* 28: 887-950.

A.A Okhunov* & H. Abu Kassim
Department of Physics
University of Malaya
50603 Kuala Lumpur
Malaysia

Ph.N. Usmanov
Institute for Nuclear Physics
Academy of Science of Uzbekistan
100214, Tashkent
Uzbekistan

*Corresponding author; email: abdurahim@um.edu.my

Received: 7 December 2009
Accepted: 13 July 2010