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Procedures for vibration serviceability assessment of high-frequency floors

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ABSTRACT

Manufacturing plants that produce micro-electronic components, and facilities for extreme-precision experimental measurements have strict vertical vibration serviceability requirements due to sub-micron feature size or optical/target dimensions. Failure to meet these criteria may result in extremely costly loss of production or failure of experiments. For such facilities floors are massive but stiff, generally have first mode natural frequencies above 10Hz and are typically classed as 'high frequency floors'. The process of design to limit in-service vibrations to within specific or generic vibration criteria is termed 'vibration control'.

Several guidance documents for vibration control of high frequency floors have been published, for different applications. These design guides typically propose simplifications of complex floor systems and use of empirical predictive design formulae. A recently published guide uses a more rigorous approach based on first-principle modal analysis and modeling footfalls as effective impulses, but there remain unresolved issues about its application, and this paper addresses these in order to develop an improved methodology.

First, the significant but conventionally discounted contribution of resonance well above the conventionally accepted boundary between low and high frequency floors is examined. The level of necessary modeling detail is then considered along with the effect of accounting for adjacent bays in simulation of a regular multi-bay floors. Finally, while it is assumed that contributions of higher

modes to impulsive response decrease so that a cut-off frequency can be prescribed, simulations demonstrate that with both effective impulse and real footfall forces, there is not necessarily asymptotic response with rising floor mode frequency.

The conclusion is that there are no shortcuts to predicting response of high frequency floors to footfall excitation. Simulations must consider resonant response due to high order harmonics, provide adequate detail in finite element models and adopt a cutoff frequency that depends more on usage than on features of the floor or of the walking.

INTRODUCTION

Vibration serviceability has become a high-profile research topic thanks to pedestrian and public assembly structures such as footbridges and grandstands that have failed, in full public view, to perform adequately under human dynamic loading [1]. These failures have been so judged according to qualitative or quantitative vibration tolerance assessments by the human occupants dynamically exciting the structure.

Such structures represent the visible part of the vibration serviceability problem, but there is a class of structures for which vibration serviceability failure may not be publicly evident yet for which the financial costs of being unfit for purpose are likely to be far greater. This class of structure includes laboratories using sensitive optical equipment, wafer fabrication plants (generically referred to here as ‘fabs’) housing vibration-sensitive manufacturing, testing and measurement equipment [2] and experimental research facilities housing instrumentation such as precision lasers or linear accelerators [3]. For fabs the component feature sizes are now typically less than one micron while for lasers and accelerators, the beam line and targeting tolerances are also well below one micron.

For experimental facilities, application-specific vibration tolerances are likely to apply and these will primarily be in respect of structural response to ground borne vibration. For fabs (and research labs) external vibration sources such as wind and ground motions need to be dealt with, but the primary sources of dynamic load are usually machinery and pedestrians causing vertical vibrations of floors in critical areas. Acceptance levels for floor vibrations are then usually judged according to well specified and stringent criteria evolved largely by North American experience. Correct operation of equipment in these facilities requires an extremely low level of motion at equipment supports and the design procedure to restrict these vibration levels is termed vibration control.

Rather than adopting equipment-specific vibration limits, the accepted approach for vibration control is typically specification of a generic vibration criterion or VC at one of five levels, VC-A through VC-E [4] or equivalent forms (ASHRAE [5]). The VCs are formally defined for vibrations down to 4Hz and specify root mean square (RMS) vibration levels which should not be exceeded in any one-third octave band, with VC-A having an upper limit of 50μm/sec RMS per one-third octave band and classes B through E successively halving that value.

Part of the logic behind using velocity as the measure of vibration is that for the short duration of the specific manufacturing operation or measurement, displacements must be of the same order as the feature sizes of the components being manufactured or measured. The use of narrow bands, whose width and centre frequencies follow a geometric progression, eliminates arguments about vanishing vibration levels evaluated at exact frequencies of a power spectral density function describing a response quantity.

Having first identified the goals of a vibration control procedure in a structural design, the design procedure then involves identifying the inputs to the structure and executing an analysis procedure which culminates in a presentation of responses as one-third octave velocity spectra. This is the classical source-path-receiver process.

The last step in the process, which is comparison with the generic vibration criteria, is relatively well defined, leaving the process of quantifying the inputs and executing the analysis to be determined. To this end there are a number of possibilities for characterizing the inputs and applying them in variants of dynamic structural analysis, and a number of proprietary design guides have been published.

DESIGN GUIDANCE FOR FLOOR VIBRATIONS

The applicable British Standard for floor vibrations is BS6399 [6] which is concerned primarily with safety of pedestrian structures subject to synchronized loads such as dancing or jumping. It provides no guidance for designers and this is left to a number of industry-related organizations who issue guidance for design using particular construction forms or materials for particular applications. Within the UK, guidance has been provided by the Steel Construction Institute [7] for all types of structure and more recently, for hospitals only [8]. The Concrete Society [9] and the Timber Research and Development Association [10] have also issued material-specific guidance while the latest guidance, published by The Concrete Centre [11] is aimed to apply to floors and bridges using any construction material.

Outside the UK, design guidance is provided by the Canadian Standards Association [12], in the USA by the Amick et al. [2] and then in more detail by the American Institute of Steel Construction (AISC) [13]. There have also been numerous technical papers written on the subject and the reader is directed to existing reviews [14] and [15].

Dynamic loads due to pedestrians

The cited guidance documents relate almost exclusively to performance of floors under human dynamic loading, and while ground-borne vibration may be the governing input for precision scientific instruments (e.g. linear accelerators, synchrotrons and lasers) our experience is that where pedestrian activity is present the majority of difficulties arise due to the ground reaction forces (GRFs) they generate hence a good representation of GRFs is essential.

Figure 1a shows GRFs for a 640N pedestrian walking with a pacing rate of 1.71Hz i.e. a cadence of 102.6 steps per minute. The figure shows time histories of separate left and right footfalls and a combined GRF for both feet, all recorded using an instrumented treadmill. The Fourier amplitude spectrum of the combined GRF, shown in Figure 1b leads to a number of observations:

1) The individual footfall resembles an elongated impulse.

2) The combined GRF has a mean value corresponding to the weight of the pedestrian.

3) It appears to have a periodic form, that is features repeat themselves at similar intervals.

4) Neither the features nor the intervals of the combined GRF are constant, but have a stochastic nature.

5) The Fourier amplitude spectrum shows predominant components at frequencies at or close to the pacing rate, taken as a fundamental frequency.

6) There are frequency components clustered around multiples (harmonics) of the pacing rate.

7) The energy around the harmonic lines shows greater spread and lower amplitudes as the harmonic order increases, in this example there is little energy above 10Hz, i.e. beyond the fifth harmonic.

The final row shows the Fourier amplitudes for a 870N pedestrian (first author) walking at 2.5Hz, also determined from a 10-second GRF time history. This shows strong high order harmonic components at frequencies approaching 50Hz and both frequency plots suggest that neglecting harmonics beyond the third or fourth may be unrealistic.

The narrowband characteristic of GRFs is due to the stochastic nature of footfall forces, which has been investigated by the first author [16]. Spread of energy away from exact harmonic components is due to imperfection of human gait.

Assuming a linear structure that does not interact with the load [17], the response of a floor is ideally predicted using a numerical model representing the complete dynamic character of the floor, to which is applied the pedestrian GRF as a moving load. Such an approach is rather too sophisticated for design processes and for this reason design guides strive to simply the procedure while retaining adequate accuracy. To this end GRFs are traditionally represented as either a single constant fundamental component with or without weaker harmonics, or by using impulses equivalent to individual footfalls. These different representations lead to two approaches to analysis of floor response.

Low Frequency Floors and High Frequency Floors.

Wyatt [7] was one of the first to differentiate between low frequency floors and high frequency floors. His criterion was that 'where the natural frequency of the floor exceeds that of the third harmonic of the walking pace' it is a high frequency floor. Floors with first natural below this cutoff can be assumed to develop resonances, reaching a proportion of steady state value as the pedestrian traverses the floor for a limited duration. Correct analysis approaches would need to consider both the duration of the transit and the modulating effect of the relevant mode shape along the walking path, but the mode shape modulation is not usually accounted for, resulting in response overestimation and conservative design.

The definition of the cutoff frequency varies according to the authority, for example the fourth harmonic of pacing rate is commonly used, setting the threshold frequency as approximately 10Hz. However it has been suggested to conduct low frequency analysis up to 12Hz and high frequency analysis from 10Hz [9]. The new Concrete Centre guide [11] defines a high frequency floor as one where the first vertical mode that is active at both response and excitation locations is at least 4.2 times the fastest walking frequency of around 2.5Hz.

For floors with fundamental frequencies higher than this threshold, the hypothesis is that due to the high frequencies of vibration modes, the transient response generated by an individual footfall decays to practically zero by the time the next footfall begins, so that no resonant buildup is possible. Hence loading models that consider resonant excitation by the fundamental and higher components are deemed inappropriate and the most appropriate simplification is to reduce a footfall to an equivalent impulse and then calculate the response of the floor to such an impulse.

Figure 2 shows typical measured responses for a high frequency floor (having a first mode frequency greater than 20Hz) and a low frequency floor (with first mode at 7.8Hz), in both cases due to the same pedestrian walking at a pacing rate close to 2Hz. The distinction between high frequency (transient) and low frequency (resonant) response is, in these two examples, clear. In both cases there is a modulation of response levels due to mode shape along the walking path but for the low frequency floor there is a distinct and asymmetric tail due to decay of the built-up resonant response.

The notion of transient footfall excitation for high frequency floors has formed the basis of the most rational approach to the problem, published by the Concrete Society [9], but the simpler North American approach [2] is widely used. These two approaches are summarized in the next section.

Specific design guidance for high frequency floors

Willford et al.[18] provide an authoritative critique of design methodologies including their own 'effective impulse' approach as published by the Concrete Society. Other than this the so-called 'kf' approach [2] is recommended by the American Institute of Steel Construction (AISC)[13].

The effective impulse approach described by the Concrete Society in Technical Report 43 or CSTR43 [9] was derived using a database of over 800 individual footfalls recorded using a force plate [19]. The footfall forces obtained for 40 individuals and a range of pacing rates f were used as inputs to dynamic response simulations for single degree of freedom (SDOF) oscillators with frequencies f_n. The peak velocities of the impulse-like responses obtained from the simulations were compared with peak velocities from unity-valued Newton-second impulses. The ratio of responses for each combination of f_n and f were used to a formula for an effective impulse I_{eff} as a function of the two frequencies.

The CSTR43 [9] formula for effective impulse I_{eff} derived from the data is given as

$$I_{eff} = Af^{1.43} / f_n^{1.3} \quad (1)$$

where the mean value of the coefficient A is 42Ns, the upper quartile value (used for design) is 54Ns and the dimensions of the frequencies are ignored. The velocity response of a floor vibration mode is then given by

$$V(t) = \mu_i \mu_j \frac{I_{eff}}{m} \exp^{-\zeta\omega t} \sin \omega_D t \quad (2)$$

Where $\omega = 2\pi f_n$ is the circular frequency of the mode and $\omega_D = \omega\sqrt{1-\zeta^2}$ is the damped circular frequency. The mode shape ordinates corresponding to the point i where the impulse (footfall) is applied and the point j where the response is measured are μ_i and μ_j, and m is the modal mass determined using the same scaling as the mode shape.

Peak velocity is given by the first half of the expression, while third octave RMS values depend on the averaging time of the calculation which is recommended to be the 'worst 1 second of largest vibration' as opposed to the footfall interval $1/f$.

The procedure recommends including all vibration modes with frequencies up to twice the fundamental frequency and the total response is the sum of all the individual modal contributions.

The 'kf' approach is far simpler and has its origins in the ratio of dynamic plus static response to static displacement response for an idealised footfall waveform derived from early research by Galbraith [20]. Ungar and White [21] showed that this ratio decreases with square of floor frequency f_n, and simple conversion to velocity leads to an inverse relationship of response with frequency. Applying this factor to static response that depends on midpoint stiffness k and including a scale factor C calibrated against measured data to include the effect of damping, type of structure and pedestrian parameters leads to a simple formula for RMS velocity V:

$$V = C/kf_n \tag{3}$$

Figure 3, derived from [22] compares predicted maximum floor velocity using the two approaches for typical footfall values and the same (SDOF) mode properties. The contours of constant mass are spaced at equal mass increments, moving further apart as the mass decreases. Figure 3a shows the CSTR43 [9] effective impulse approach with increasing response for decreasing mass, as would be expected. Figure 3b shows the 'kf' approach predicting response that goes up as both k and f_n

decrease, regardless of mass so that apparently lower response is obtained as modal mass is reduced.

VALIDATION AND EXTENSION OF EFFECTIVE IMPULSE APPROACH

The CSTR43 [9] effective impulse approach has been shown to be the more rational and it has also been validated numerically by its authors, so it is the right approach, but the details and range of applicability of the approach should be scrutinized.

Figure 4 shows a simulation based on the footfall data shown in Figure 1; the left and right columns respectively correspond to the 640N and 870N pedestrians with respective 1.71Hz and 2.4Hz pacing rates. The first row (Figure 4a and 4b) show individual footfalls and the second (Figure 4c and 4d) show the resulting velocity response for a 15Hz SDOF oscillator with stiffness 1GN/m and 3% damping. The response bears a resemblance to the classical exponential decay and the peak velocity usually occurs at the first cycle. The simulation (in this case) stops after a single footfall.

Picking the peak velocity in the duration of each footfall leads to a single value, and repeating the exercise for different oscillator frequencies and stiffnesses leads to surface plots shown as Figure 4e and 4f. The variation of response is inversely proportional to stiffness so the third (stiffness) axis is strictly unnecessary. Also for a given frequency and stiffness the oscillator mass is fixed, with contours of constant mass increasing from 125 tonne (1.25×10^5 kg) at 125 tonne increments.

The purpose of using a surface is to show clearly the combined effect of stiffness, frequency and mass. For an oscillator with fixed mass, only the frequency f_n can be varied independently and as suggested by equation (1) the other independent variable is pacing rate f. Figure 4 shows simulations for only two footfall forces out of a database of 100 time series of continuous walking obtained from 9 test subjects so the validity of equation (1) could be tested, in the spirit of equation (1), by varying both f and f_n and applying some measure to the simulated response.

All the individual footfalls collected in a single 60 second measurement of treadmill walking for a single pedestrian at one walking speed can be used to determine a set of peak or RMS velocities responses corresponding to the individual footfalls. Then the mean or maximum of values in this set can be found.

Each of the 100 GRF time histories has its own average pacing rate f . Average values span the range 1.3Hz to 2.55Hz, and using a range of oscillator frequencies f_n a surface plot of a response measure for each pair of frequencies (f, f_n) can be derived and the validity of equation 1 tested.

Figure 5 shows (in order from left) the mean of each set of peak velocities as function of (f, f_n), the surface representing the best fit function of the form $V_{pk} = Af^a \big/ f_n^b$ to the data and surface representing the CSTR43 [9] effective impulse formula, all evaluated for a floor with (nominal) modal mass of 10^6 kg.

The difference between the coefficients in Figure 5b and 5c is intended to illustrate the variation among individuals and also to show the non-uniform variation with respect to pacing rate even for the same individual, so that identifying best fit parameters is potentially an ill-conditioned problem.

For high frequency floors the vibration criteria are conventionally specified in terms of RMS rather than peak velocities and assuming the worst case that response for a single mode will fall within a single one-third octave band, the relevant measure is the simple RMS of the velocity signal.

When calculating RMS values, the damping ratio also needs to be considered. While peak velocity is almost independent of damping ratio if the maximum occurs at the beginning of the footfall, for RMS there is a weak dependence on damping ratio. For consistency a value of 3% is adopted here which is reasonable for concrete floors which are likely to form the majority of cases.

Calculation of an RMS for a transient also depends rather strongly on the averaging time, for which CSTR43 [9] recommends 1 second. However, if response to a single footfall is considered it may be more appropriate to take as averaging time the interval between footfalls i.e. $1/f$. The results are shown in Figure 6, this time taking the maxima of each set of footfall RMS velocities.

Finally, rather than considering a single footfall, the response to a sequence of left and right footfalls (i.e. an actual continuous GRF) can be considered and the RMS evaluated, for example, as the largest RMS value calculated over the $1/f$ durations of all the individual footfalls. The results are shown in Figure 7 where two effects can be seen. First, the RMS levels are increased approximately 20% with respect to single footfall results. Second, an oscillation in the RMS with varying floor frequency is clearly visible at the low frequency end of the range, and persists up to (and a little beyond) 20Hz.

This is thus strong evidence that some form of resonant amplification can have a significant effect well above even the highest of the variously defined high/low frequency floor thresholds.

The raggedness of the plot also shows that a far larger dataset of GRFs is required for characterizing the effect. Even with normalization to an average body mass, the large inter-subject variability remains, i.e. the surface will never be smooth in the direction of pacing rates, but mean, lower and upper quartile values can be established more reliably.

USING FINITE ELEMENT ANALYSIS FOR EVALUATION OF HIGH FREQUENCY FLOOR RESPONSE

A rigorous analysis of high frequency floor response would require a continuous GRF time history that can be used as direct input to a finite element simulation. The walking force should move or rove over that part of the floor expected to have either greatest response or lowest vibration tolerance and the ‘walking path’ should represent typical scenarios e.g. walking from one side of the floor to the other.

Roving the GRF to simulate a pedestrian walking path is impractical for most finite element codes and is unnecessary if there is no chance of resonant buildup, as is assumed for a high frequency floors, and has been shown here to be reasonable for floor frequencies above 20Hz. In practice for a high frequency floor it usually has to be enough to simulate walking on the spot at the critical location.

If the effect of a moving GRF must be considered, the simplest approach for analysis would be to extract modal frequencies and mode shapes (containing modal mass information) from an eigenvalue analysis in a finite element program and provide these to a spreadsheet or mathematical software package. The external software can account for walking path in a modal superposition analysis by converting a GRF time series to a position-dependent modulated modal force, an approach adopted (for example) in the software VSATS [23] implemented in MATLAB [24].

If the GRF remains at the same position the methodology of CSTR43 [9] can be applied directly. The modal effective impulse is determined and the modal response contributions summed based on the mode shape ordinates at the (fixed) point of excitation and response. Then RMS or peak velocity, as required can be determined. If response is only to be determined where the footfall is applied the calculation can be carried out for an entire floor by considering each node in turn.

Application of effective impulse approach to borderline high frequency floor

Figure 8 shows, from below, the reinforced concrete floor of the auditorium of Singapore Polytechnic, and the first mode of its finite element model, at 11.5Hz, making it by most definitions a high frequency floor. A forced vibration test of the structure provided a first mode frequency of 10.6Hz and close to 3% damping, which does not change the usual classification.

The CSTR43 [9] effective impulse approach was applied for all nodes using a pacing rate of 2Hz and summing modal responses for all ten analysed modes up to 30Hz (although only modes up to 23Hz should in principle be considered). The RMS values are presented in Figure 9. This approach does not take account of any resonant buildup and shows the worst vibration levels to be 155μm/sec. This is three times the VC-A limit if applied in a single one-third octave band.

By comparison, Figure 10a shows velocity response with the 870N pedestrian crossing the prototype floor 36 times with prompted pacing rates increasing in steps from 1.65Hz to 2.45Hz. The largest RMS velocities are four times the VC-A limit, and the spectrogram of Figure 10b shows this to occur when fifth multiple of the pacing rate is around the frequency of the fifth mode shape, just after 300 seconds.

Figure 10c zooms in on this part of the response and shows the RMS trend for a 1-second averaging time. The shape of the envelope of this trace is largely due to the mode shape of the floor along the walking path but it is clear that response to successive footfalls does not decay enough that the responses can be considered as distinct responses to individual footfalls.

To show how relevant the resonant build up is, the low frequency floor procedure of CSTR43[9] has also been used beyond its normal range. Since the fifth harmonic forcing is not dealt with, the fourth harmonic of the GRF due to walking at a very brisk 2.65Hz is used. The predicted RMS *acceleration* levels reach 0.04m/sec^2, approximately double the maximum recorded response of 0.021m/sec^2. There would seem to be a valid case to consider using harmonics beyond the fourth to avoid the possibility of significantly underestimating of floor response, but there are few takers for a design approach including fifth or higher harmonics, and very limited data on harmonic force amplitudes.

Finite element modeling detail

Even when it is very clear that a floor falls into the high frequency category and it is clear that the non-resonant effective impulse approach is completely valid, decisions need to be made about the level of detail e.g. whether to model more than one floor panel and how to model fixity at beams/columns. Additionally, where a mode superposition approach is used, the number of modes to be included needs to be defined rationally.

For an irregular floor such in Figure 8b the entire area of floor to be loaded by pedestrians needs to be modeled to identify the location and value of strongest response. High frequency floors often occur in regular arrangements of bays, for example fabs with up to a dozen bays in each direction, and simulating the entire floor may be unnecessary. Hence the question is, if panels (i.e. 1 bay wide by 1 bay long) are identical or very similar, is it enough to model one in order to characterize the whole multi-bay floor, and if not, how much of the floor –and beyond- should be modeled?

Figure 11 shows three vibration modes for three panels of a hypothetical 1 bay wide waffle-slab floor system based on a real structure. The floor has three vibration modes from 18.1Hz to 18.2Hz having no nodes (of zero response) in the transverse, narrow direction, then three modes from 22.4Hz to 22.5Hz having a single node in the transverse direction; Figure 11 shows the first two modes (with zero nodes in transverse direction) and the sixth mode (with one node in transverse direction).

Table 1 summarises the mode characteristics. The three mode (i.e. one per bay) sequence repeats, with frequency jumps between the 3-mode bands reducing as frequencies increase. For N bays there are 5N modes below 50Hz.

The RMS velocity responses of the same floor with between N=1 and N=10 bays in the long direction have been determined using the CSRT43 [9] effective impulse approach with a pacing rate of 2Hz, including the first 5N modes, i.e. up to approximately 50Hz. The simulations have been carried out for all nodes, producing data such as in Figure 9, from which the worst case total RMS velocities have been obtained. Figure 12 shows that the worst case response changes very little as more bays are added, while the average modal mass (a simple average over all 5N modes) steadily increases. For this example it seems clear that the effect of adding more modes to the summation of impulse responses is balanced by the increasing modal mass values. The largest responses are always found at the mid bay of the floor.

Hence it may be a fallacy to judge a single mode and to argue that including more panels will increase modal mass and hence improve performance. The repetition in a multi-panel floor leads to a great density of modes in a frequency band so the net effect is approximately neutral.

The main reason for the complexity of a finite element model covering several bays of a repetitive floor would be to reproduce appropriate boundary conditions for the critical bay. In addition [25] it is always advisable to include columns in the model for (at least) one storey above and below to allow for finite rotational stiffness rather than assuming full fixity.

Frequency limits of modal summation

Modal analyses produce a number of vibration modes according to a set maximum number or maximum frequency. In either case guidance is required on the number of modes or upper frequency for the summation of impulse responses. For earthquake engineers a calculation of participating mass as a function of number of modes serves very effectively to show how many modes to include, without consideration of the ground motion signal. For floors, more consideration is given to the character of the impulsive excitation signal, and it is clear from Figure 1 that significant energy is available to excite modes even up to 30Hz. Hence some form of rational guidance is required to determine an upper limit for modal summation.

Figure 13a shows a hypothetical but realistic 4x3 bay floor comprising 14m span deep ribbed-slabs supported on transverse main beams, the floor extending each way beyond the three-bay width and each panel having a total mass of approximately 120 tonnes. The floor is typical of a design for a fab, with different classes of production area on each half of the floor, as indicated by the asymmetric mode shown in Figure 13a. Node-by node estimates of VC for 2.5Hz footfalls are presented Figure 13b, including the first 50 modes up to 34Hz, more than twice the fundamental frequency. The high response areas at the edges are unrealistic as there would either be adjacent bays or supporting walls. 2.5Hz is used because it is around the upper bound of pacing rates; in reality in vibration-sensitive environments such as fabs, cumbersome protective clothing would limit practical walking to much lower pacing rates.

Figure 13b indicates the hot spots, indicated by node numbers 350, 541, 1790 and 1981 in their centres, and in Figure 14 the simulations for these locations are reproduced, but as overall RMS velocities as modes are added up to a limit of the 100^{th} mode just below 50Hz. Figure 14a uses the CSTR43 [9] effective impulse formula, which is intended to include modes up to twice the fundamental frequency, Figure 14b uses the effective impulse formula with coefficients derived from Figure 5b (obtained from the independent continuous GRF dataset), and Figure 14c is for a single recorded footfall, all plots being for 2.5Hz pacing rate.

There are two surprising features. First, the response does not level off at an asymptotic value, rather it begins to increase again after 35Hz i.e. beyond the 50^{th} mode. Second, the sudden jump in RMS as mode 97 is added, close to 50Hz, is apparently due to the excitation being at the principal (i.e. largest) anti-node of a mode with very small modal mass. That the methods provide different predictions is not surprising, although the CSTR43[9] method is least conservative.

These figures derive from numerical simulations on a single example and naturally validation is required. Experimental determination of such a large number of vibration modes would in fact be very challenging, so the simpler approach would be walking tests.

An alternative approach to fix the number of modes to include has been suggested by NAFEMS [26] using a participation factor and spectrum, similar to that used in seismic analysis. The method involves calculating response u for a single mode using Equation (4). The summation of each mode gives the total response.

$$u = \underbrace{Q_i Q_i}_{A} \underbrace{\max \left| \int_0^t h(t-\tau) p(\tau) d\tau \right|}_{B} \qquad (4)$$

Part A is a form of participation factor representing a spatial variation of the force, part B represents a time variation of the force, *h* is an impulse response function and *p* is a force time history. Q_i is the modal force, defined by $\phi^T F$ where ϕ is the mode shape and *F* is the force vector.

The method proposes that the summation of part A over all modes will converge to a certain value. A sufficient convergence of part A would be used to indicate the number of modes to include in the response. However, there are a number of difficulties in the approach that make it impractical or invalid, so the best approach would still appear to be to obtain specific information about the frequency band for which floor vibrations is a concern instead using arbitrary broad-band frequency ranges.

DISCUSSION

The paper was motivated by experiences in vibration control of micro-electronic component manufacturing facilities which showed the lack of a rational approach for predicting response of high frequency floors to footfall excitation. Traditional methods such as the AISC 'kf' approach were found wanting, and with access to a database of recorded continuous ground reaction forces (GRFs) the most appropriate procedure appeared to be to model as much of the floor structure as possible and apply GRFs representative of the operational conditions i.e. slow walking in clean-room suit. This is a rather elaborate relying on extremely rare access to a GRF dataset. Hence spot checks were made using the newly published effective impulse approach now found in Concrete Society Technical Report 43 [9].

Surprisingly, the application of the recorded continuous GRF time series showed the significant possibility of a degree of resonant amplification due to energy at or around harmonics of the pacing rate well beyond the fourth multiple. Simultaneously, questions arose as to the level of detail and upper limit of frequencies to include in the modal summation, and this paper has attempted to address these issues and demonstrate the limitations of the effective impulse approach.

The effective impulse approach, with all its provisos, appears to be the most rational available, and is independent of structural material or structural usage. It is also based on first principles and real walking force data rather than outdated idealizations. Nevertheless there is work to be done to improve the method by accounting for resonant affects at the low end of the frequency range, accounting for modes beyond the present upper limit and obtaining more robust empirical functions.

It is clear that a much larger data set of GRFs is required and that more experimental data for floors with frequencies in the range 12 to at least 20Hz are required to provide experimental validation.

CONCLUSIONS

Significant vertical response of a floor can be generated by higher order harmonics of the walking pace rate, even up to 20Hz, and this can lead the existing effective impulse approach to underestimate response of floors at least in the range up to 15Hz, possibly higher.

The upper frequency bound for modal superposition needs to needs to be extended to include all modes that are excited by GRFs, contribute to response, and are relevant to the occupants, rather than perhaps being specified by a perhaps arbitrary limit.

Determining coefficients of an effective impulse formula is an ill-conditioned problem, but there are strong indications that significantly different coefficients should be employed from those in the current model, and this requires a very extensive data set of real continuous walking forces.

For regular structures, as few bays need to be simulated as is necessary to represent boundary conditions. This is because more bays generate more modes that contribute but these have correspondingly higher modal masses. There is apparently no need to develop extensive floor models if only the local behaviour of a single panel needs to be studied.

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Mode	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Frequency /Hz	18.1	18.1	18.16	22.44	22.45	22.5	37.6	38.8	40.8	44.2	45.4	45.4	46.2	46.4	46.9
Mode order in long dir	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Nodes in transverse dir	0	0	0	1	1	1	2	2	2	3	3	3	4	4	4

Table 1 - Mode characteristics of the hypothetical 3 bay structure

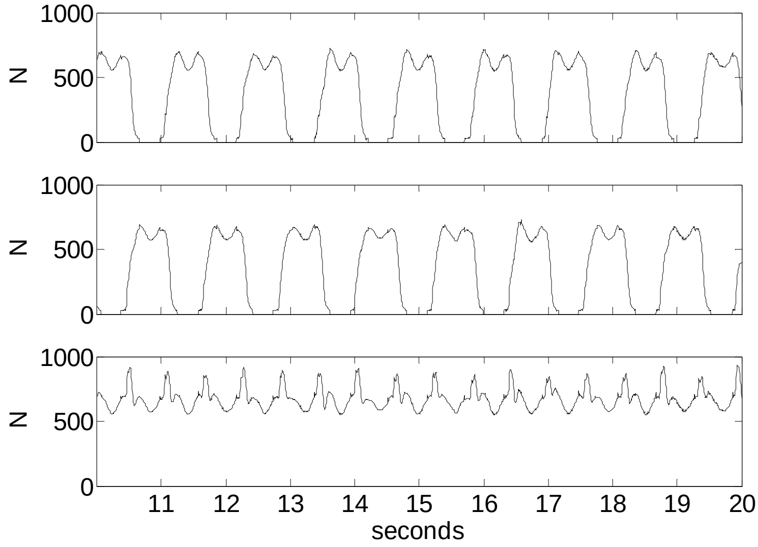


Figure 1a - Footfall GRF of a 640N pedestrian pacing at 1.71Hz

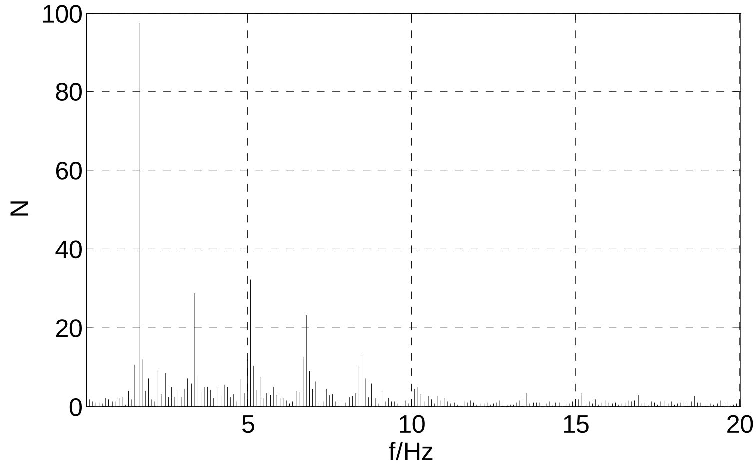


Figure 1b – Fourier amplitudes of a 640N pedestrian pacing at 1.71Hz

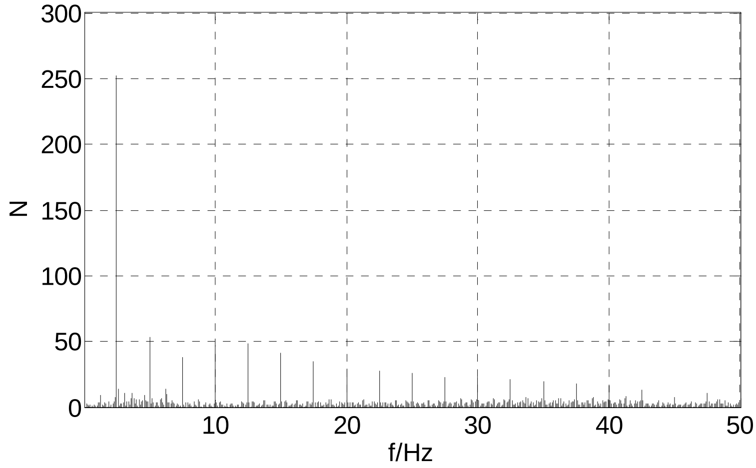


Figure 1c – Fourier amplitudes of a 870N pedestrian pacing at 2.4Hz

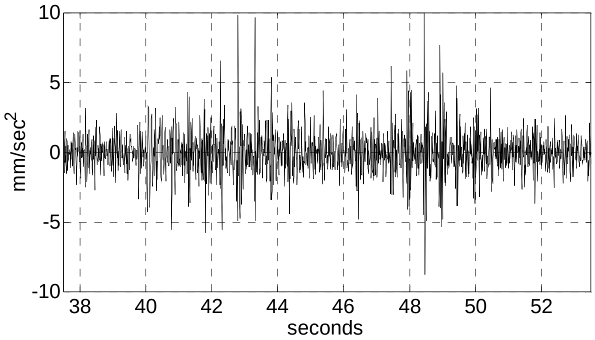


Figure 2a - High frequency floor impulsive response from walking

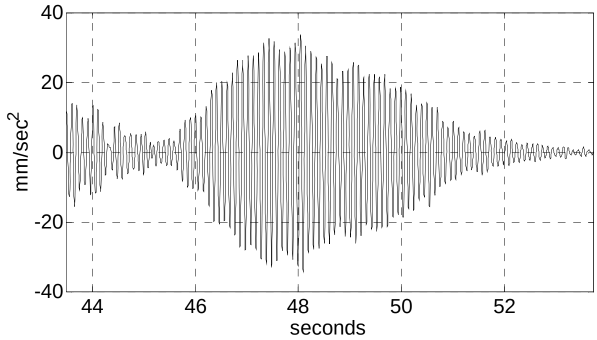


Figure 2b - Low frequency floor resonance response from walking

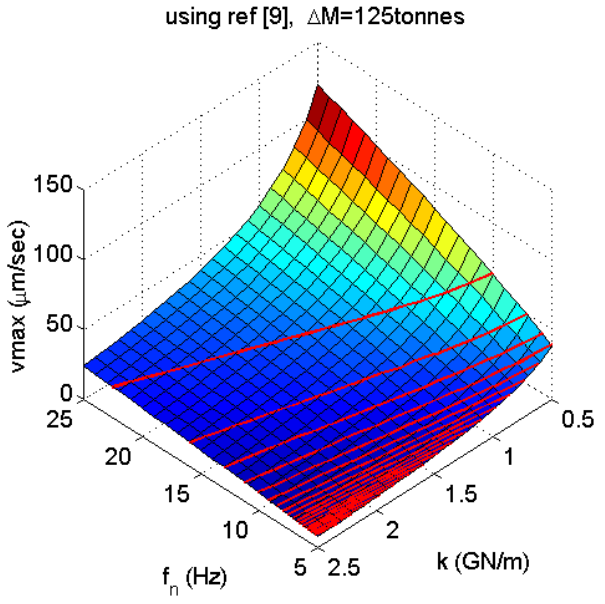


Figure 3a - Maximum velocity using equations (1) and (2) with t=0.

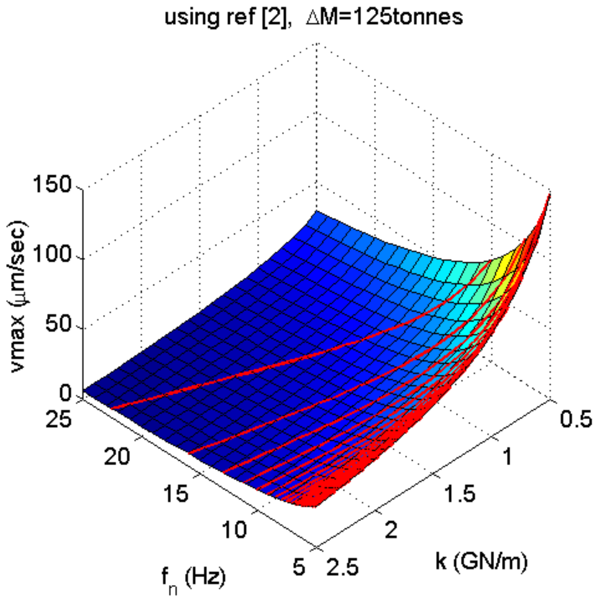


Figure 3b - Maximum velocity using equation (3)

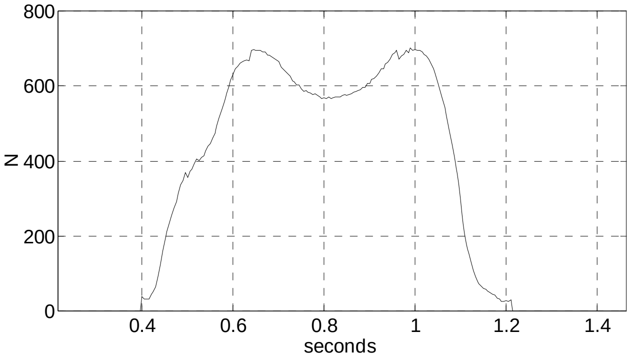


Figure 4a - Footfall force of a 640N male at 1.71Hz

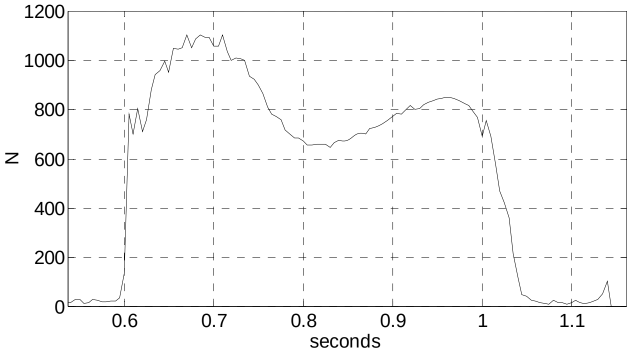


Figure 4b - Footfall force of a 870N male at 2.4Hz

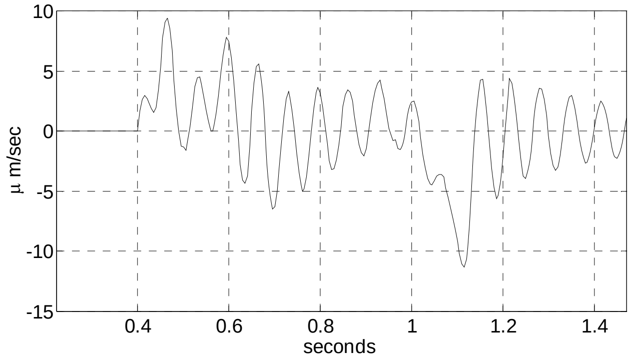


Figure 4c – Velocity response for 640N male on a 15Hz oscillator, 3% damping

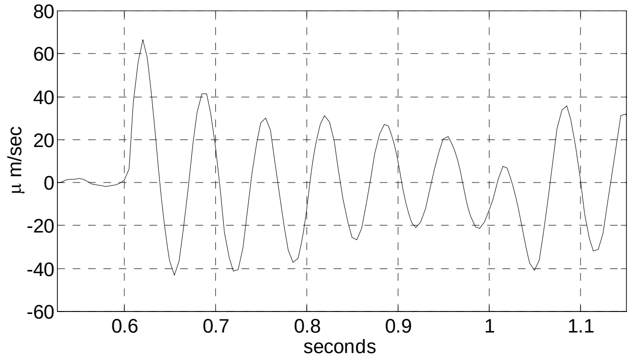


Figure 4d – Velocity response for 870N male on a 15Hz oscillator, 3% damping

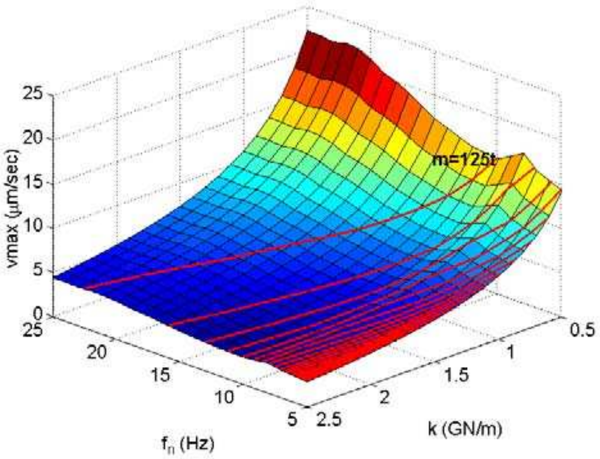


Figure 4e – Peak velocity responses for 640N male

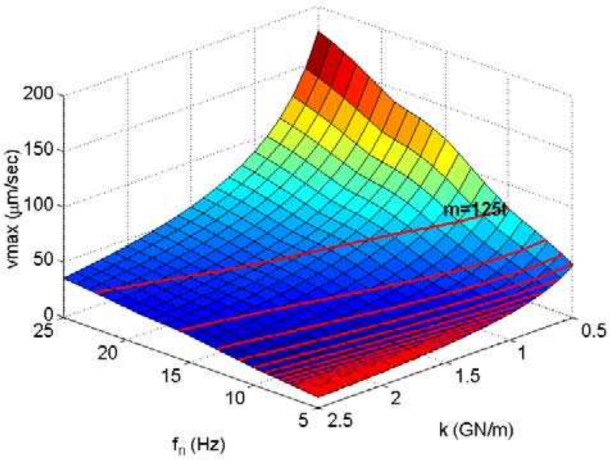
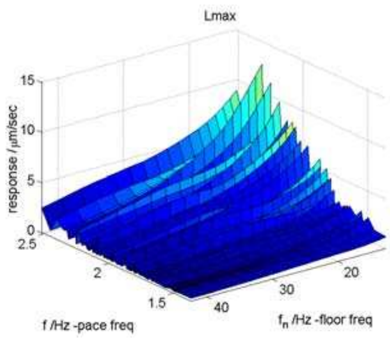
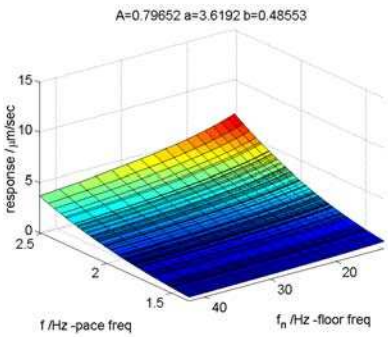


Figure 4f – Peak velocity responses for 870N male





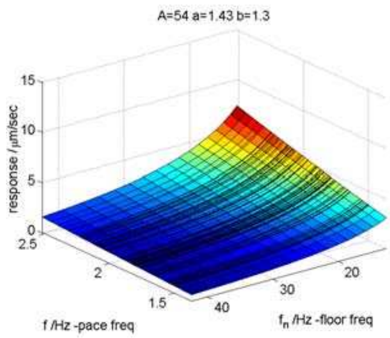


Figure 5a - Peak velocity responses from real individual footfalls

Figure 5b – Best fit of the peak velocity response from real individual footfalls

Figure 5c – Peak velocity responses from the CSTR43 [9] effective impulse

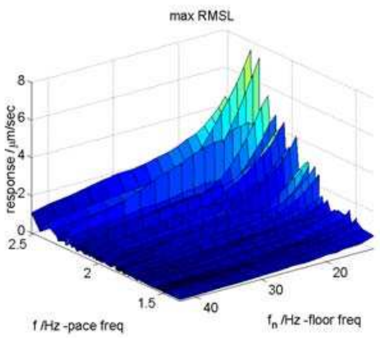


Figure 6a - RMS velocity responses from real individual footfalls

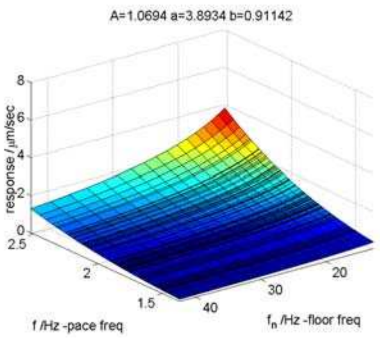


Figure 6b - Best fit of the RMS velocity response from real individual footfalls

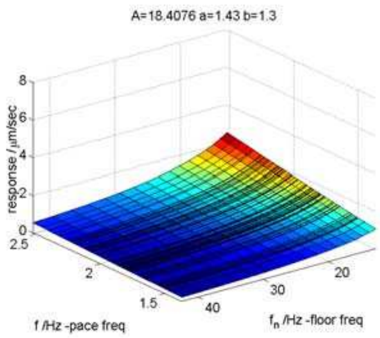


Figure 6c - RMS velocity responses from the CSTR43 [9] effective impulse

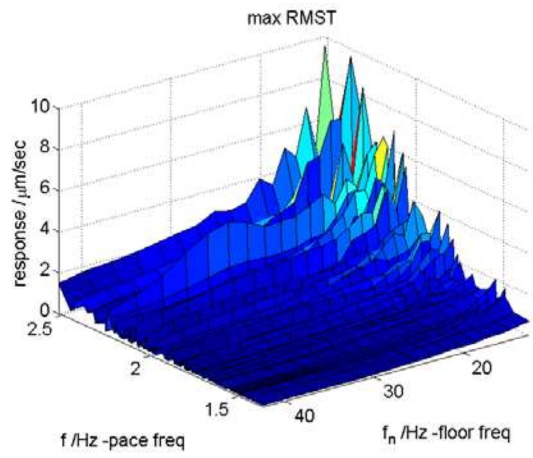


Figure 7a - RMS velocity responses from real continuous footfall time histories

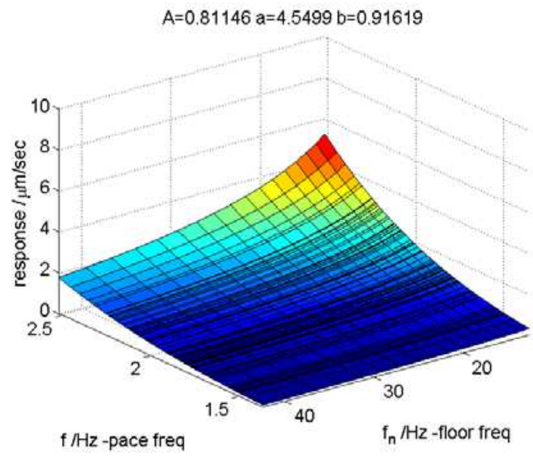


Figure 7b - Best fit of the RMS velocity response from real continuous footfall time histories

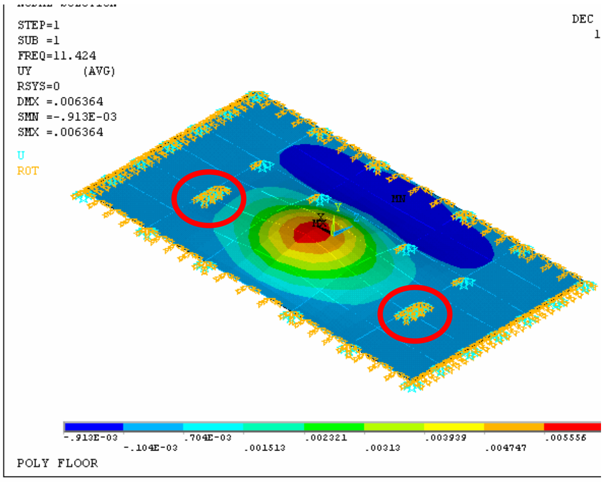


Figure 8a - Singapore Polytechnic floor viewed from below, with near and far columns highlighted

Figure 8b – Fundamental mode shape of Singapore Polytechnic floor, also indicating columns

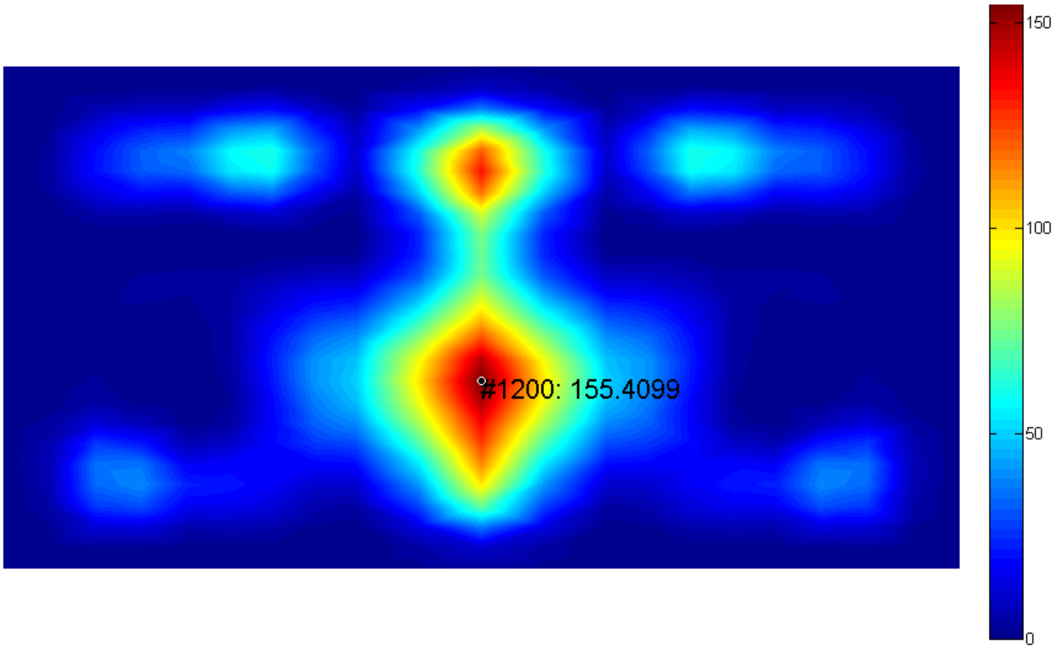
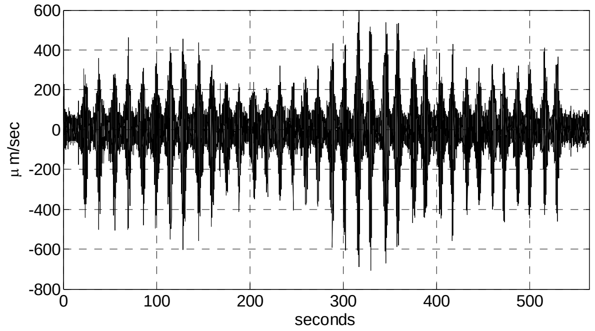


Figure 9 – Maximum vibrations levels using the CSTR43 [9] effective impulse for 2Hz pace rate



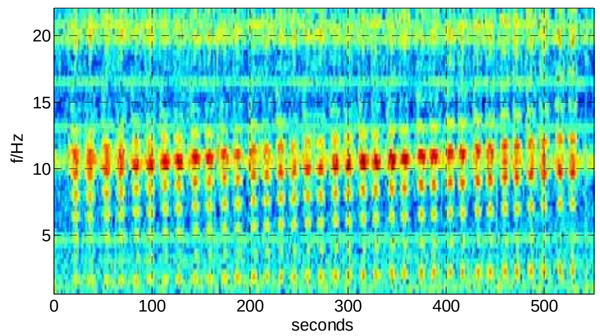


Figure 10a – Velocity response of the first author walking on the floor

Figure 10b – Spectrogram of the velocity response of the first author walking on the floor

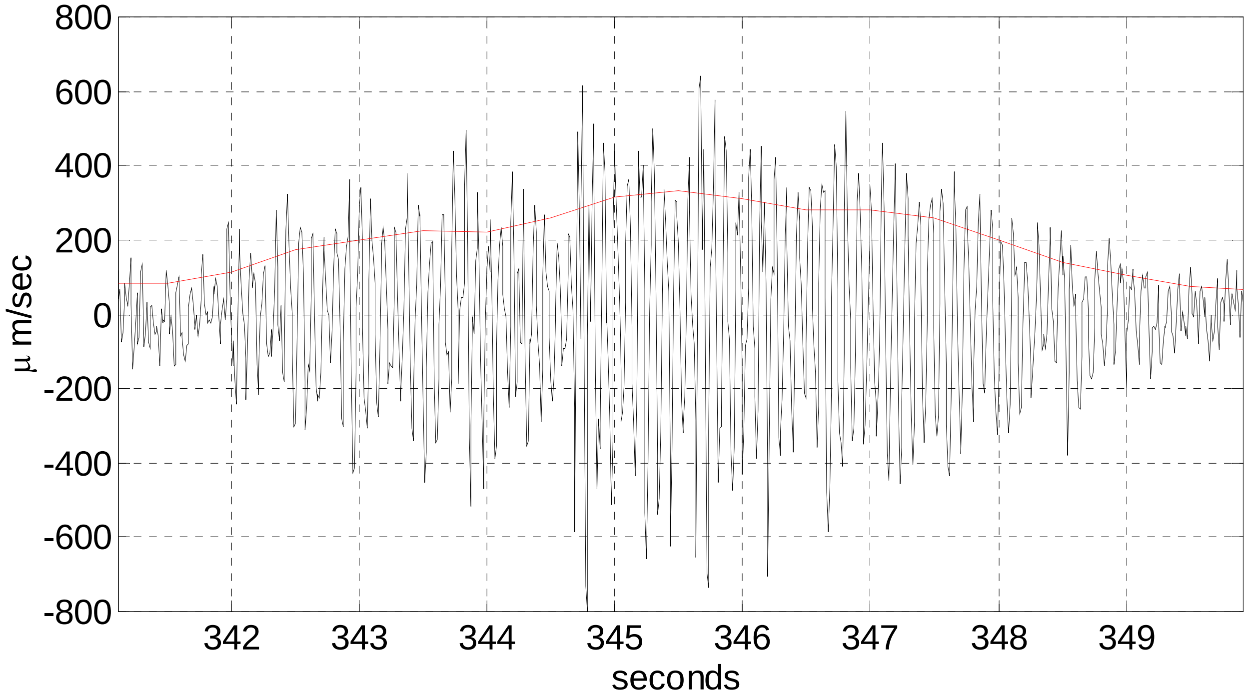


Figure 10c – Zoomed section of the velocity response with an RMS trend line

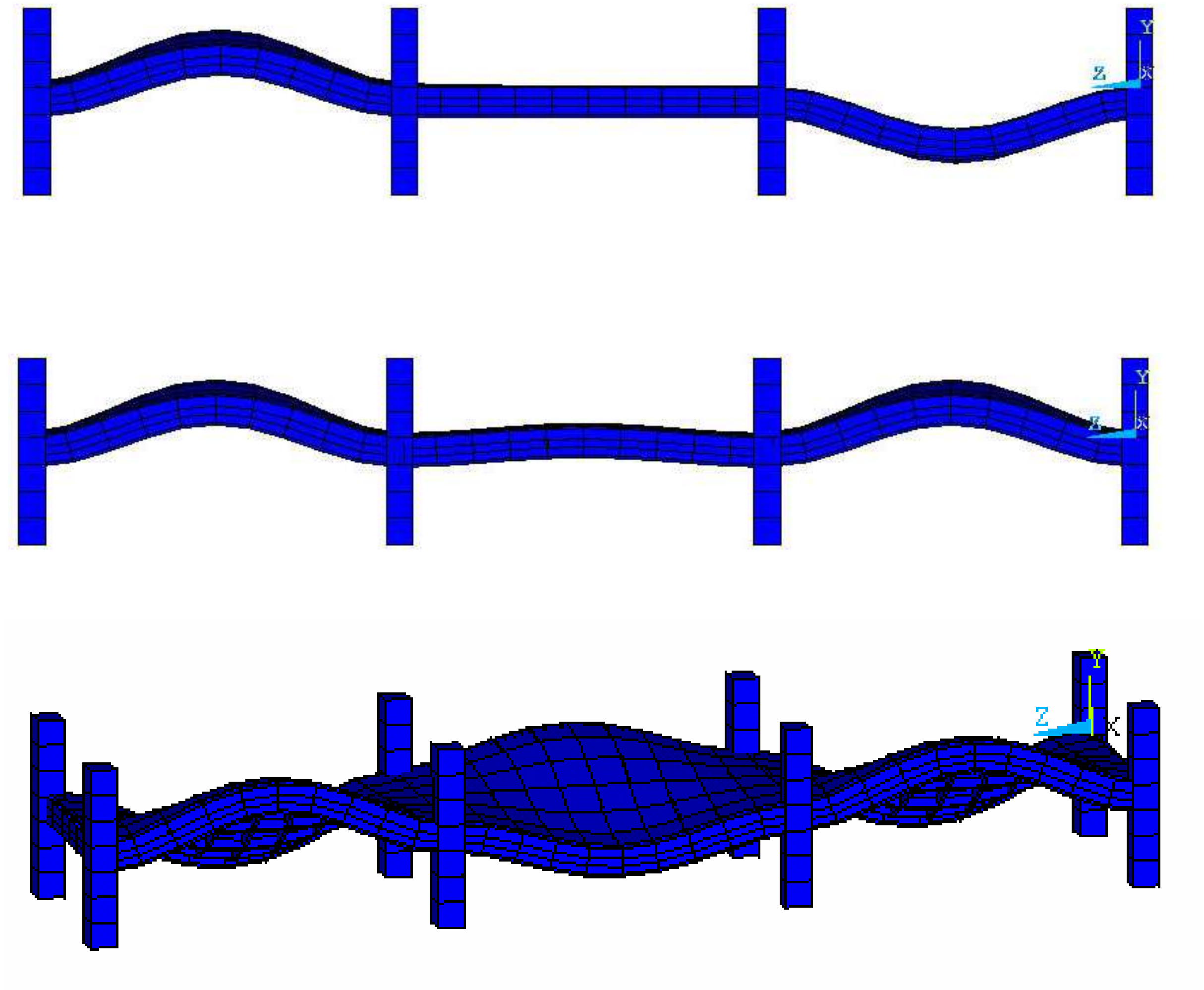


Figure 11 – Hypothetical 3 bay structure and vibration modes 1, 2 and 6

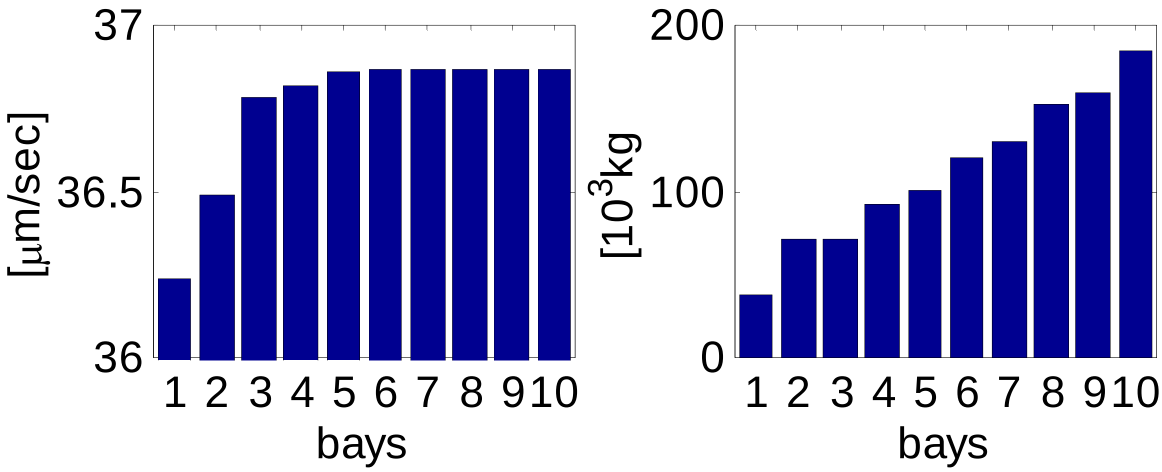


Figure 12 - Change of the maximum velocity response and average modal mass vs . number of bays

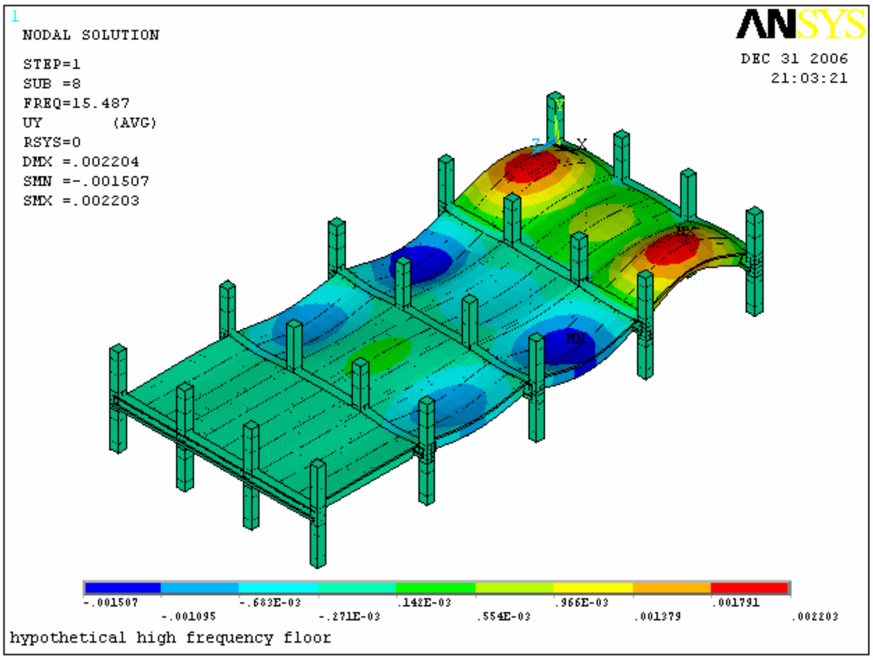


Figure 13a – Hypothetical multi-bay ribbed slab floor

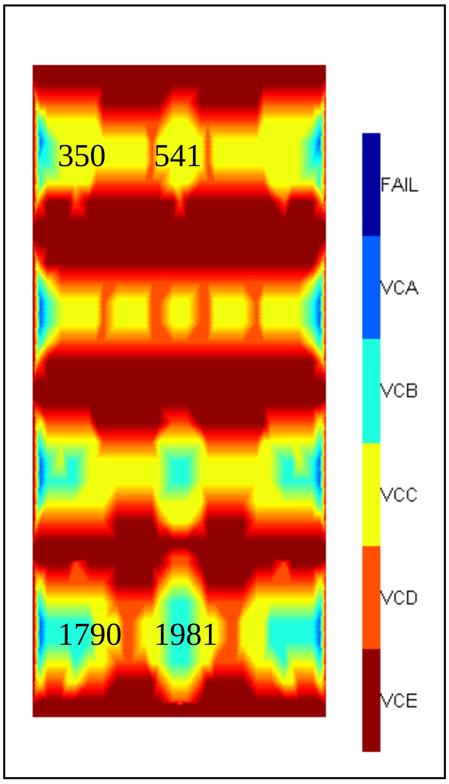


Figure 13b – Maximum velocity response of the multi-bay floor.

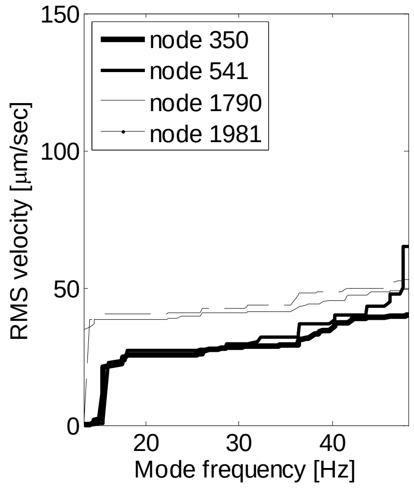


Figure 14a – Response to CSTR43[9] effective impulse

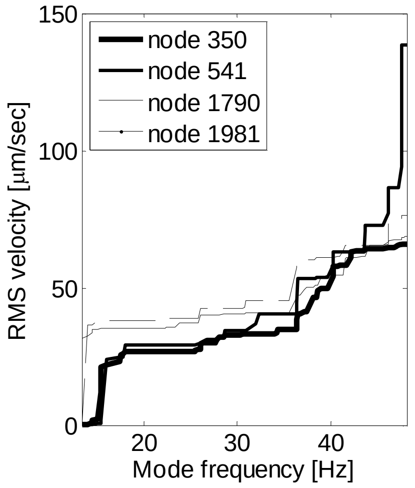


Figure 14b – Response to effective impulse based on parameters of Figure 5b

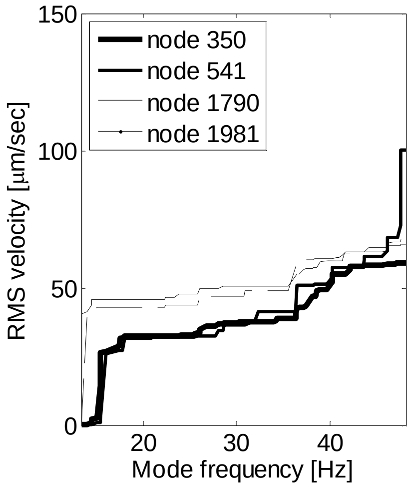


Figure 14c – Response to single footfall impulse time series