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# On the delivery robustness of train timetables with respect to production replanning possibilities

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## Abstract

Measuring timetable robustness is a complex task. Previous efforts have mainly been focused on simulation studies or measurements of time supplements. However, these measurements don't capture the production flexibility of a timetable, which is essential for measuring the robustness with regard to the trains' commercial activity commitments, and also for merging the goals of robustness and efficiency. In this article we differentiate between production timetables and delivery timetables. A production timetable contains all stops, meetings and switch crossings, while a delivery timetable only contains stops for commercial activities. If a production timetable is constructed such that it can easily be replanned to cope with delays without breaking any commercial activity commitments it provides delivery robustness without compromising travel efficiency. Changing meeting locations is one of the replanning tools available during operation, and this paper presents a new framework for heuristically optimising a given production timetable with regard to the number of alternative meeting locations. Mixed integer programming is used to find two delivery feasible production solutions, one early and one late. The area between the two solutions represents alternative meeting locations and therefore also the replanning enabled robustness. A case study from Sweden demonstrates how the method can be used to develop better production timetables.

*Keywords:* Robustness, replanning, train timetable.

## 1 Introduction

In Sweden the rail traffic is highly inhomogeneous, and there are inconsistencies between the traffic originally planned for, and the traffic eventually operated. This makes robustness an important issue for the Swedish Transportation Administration. In railway planning, robustness is generally realised by introducing

time supplements to absorb delays. As this causes longer travel times and calls for more capacity, various additional recovery strategies have also been explored. In this paper we focus on how the train agent can replan train meetings to minimise the effects of delays, and we present methods for visualising and measuring such replanning robustness.

An important concept in this paper is the difference between production timetables and delivery timetables. *Production timetables* contain all stops, meetings and switch crossings, while *delivery timetables* only include stops for commercial activities such as passenger exchanges or associations. We argue that the main goal during planning and operation should be to meet all the commercial activity commitments, and that production timetables should be constructed and later adapted in order to reach this goal. Adaptation is particularly important in Sweden as currently the train diagram which is taken from the yearly train plan will be for a generic day rather than the actual day of operation. We propose that the production timetables should be optimised for each individual day shortly before operation, and envision the methods presented in this paper to be useful during such an optimisation. Note that the delivery timetable can be constructed from the train plan by extracting the times and locations of all commercial activities.

The paper is outlined as follows: Section 2 presents the state of the art and Section 3 the problem characteristics. Section 4 shortly introduces the modelling tool while Section 5 focuses on the optimisation. We conclude with examples and some final remarks.

## 2 State of the art

There is a consensus in the railway timetabling research field on the characteristics of a robust timetable: A timetable that remains valid despite the small, stochastic disturbances in everyday operations. Likewise, the relationship between time supplements (slack) and robustness as such is undisputed, as well as the implication that constructing timetables involves a trade-off between robustness and short travel times.

What is not agreed upon, is exactly the role of slack sizes in relation to other parameters that affect robustness, and how the slack should be distributed in the timetable to maximise its robustness. Research that explicitly explores how time supplements should be distributed in the timetable to maximise robustness is [1,8,10]. In [12], the generally accepted relationship between homogeneity and robustness in timetables is investigated, and the authors establish a relation between robustness and the time gap (headway) between the arrivals and departures of different trains to and from stations.

The basic approach in [2,7,10,11], although the proposed measures and methods differ, is first finding a good (possibly optimal) timetable with respect to a set of relevant parameters but excluding the robustness aspect, and then in the next step maximising its robustness (according to some definition) while preserving most of the desired characteristics of the nominal timetable. Another recent

approach is constructing the timetable while simultaneously evaluating its robustness [8,10].

Entire frameworks like Light Robustness (see e.g. [3]), and Recoverable Robustness [9], have evolved quite recently. [5] empirically compare robustness concepts as well as define their own measure as part of their strategy to get the best trade-off between robustness and other relevant qualities. While different from [5] in many aspects, the measure in [7] is also based on the idea of measuring a distance between a nominal timetable and a proposed (more robust) solution. In [5,7] as well as in the rest of the literature, proposed measures facilitate comparisons of different solutions to the same problem rather than claiming to say something about the robustness of solutions in a broader sense.

### 3 Problem characteristics

During timetable construction, time supplements are added to regulate meetings and to reduce the risk of small disturbances rendering the production timetable infeasible. This means that most trains will have some slack. In a train graph, we define the traversal space as the area between the earliest and latest train paths that fulfil the delivery commitments for a train while ignoring all other trains (see Fig. 1 a). A train thus fulfils its delivery commitments as long as it runs within its traversal space. However, parts of this space will in general not be available due to conflicts with other trains. The time a train can be late (or early) on each link before breaking the schedule, assuming all other trains run as planned, is called link slack.

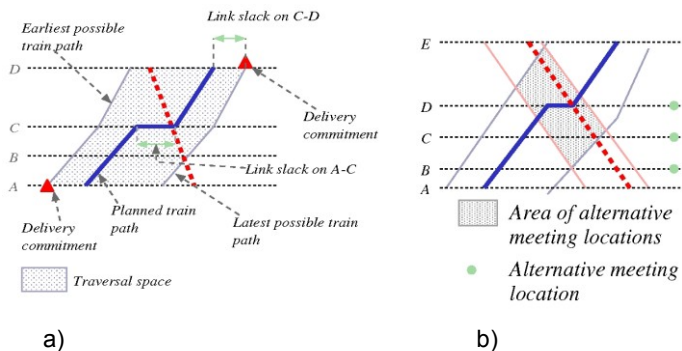


Figure 1. a) Slack, commitments and traversal space for the solid line train. b) Possible meeting locations for a conflict.

When two trains meet, their traversal spaces overlap. If the overlap only contains one meeting location the meeting is *critical* and can not be moved without breaking delivery commitments. If on the other hand there are multiple meeting locations within the overlap there are alternative meeting locations (see Fig 1 b). However, some or all of these meeting locations may be infeasible due to other scheduled trains.

In this paper we focus on how to optimise replanning robustness between two delivery commitments. That is, the times of the delivery commitments are fixed, and the aim is to find a schedule that is robust with regard to fulfilling the next commitments for the trains given that the earlier ones were met. The propagation of lateness over consecutive delivery commitments is not covered.

### 3.1 Similarities with the CPM/Pert

Our approach for visualizing and measuring replanning robustness bears many similarities to CPM (Critical Path Method) and PERT (Project Evaluation and Review Technique) [6]. In CPM/PERT, project events are represented as nodes in a dependency graph. The edges in the graph are activities, and their weights represent the time it takes for the activities to finish. The graph is used to calculate the earliest and latest possible schedules to determine the time-span, or float, within which each event must take place for the project plan to remain feasible.

If all meetings in the train plan are fixed, our situation is analogous to the one above. A meeting is an event, and the delivery commitments are the start and end points. Edges define travel times between meetings. By pushing the meetings as early and late as possible, a float for each link traversal can be obtained. As long as the trains remain within these floats, the delivery commitments can be fulfilled.

If the requirement of fixed train meetings is relaxed, the original weight of an edge in the dependency graph can be changed if one of the edges' meetings has at least one feasible alternative meeting location. This is because the travel time changes if the meeting location is swapped. Changing the edge weight is equivalent to redistributing the slack. This constitutes the core of replanning robustness as it allows for more efficient use of slack. By maximising the number of possible meeting changes, and hence maximising the flexibility of the slack, a more robust timetable can be constructed. An example is shown in Fig. 2.

## 4 Modeling tool

Mixed Integer Programming was used to model and investigate how to adapt the production timetable to obtain replanning robustness. The modelling tool *Maraca* was used for the optimisation [4]. The general model in [4] was adapted to allow for meeting locations and times to change, while all arrivals and departures specified by the delivery timetable were fixed. In this paper we assume that all timetable locations can cater for all conflict meetings, which may not be true in reality. However, trains are only allowed to stop where they had a planned stop in the original train plan.

## 5 Optimising replanning robustness

Ultimately the goal of replanning robustness is to optimise the useful flexibility of slack. To this aim maximising the number of feasible alternative meeting locations is interesting. Some heuristics are presented below.

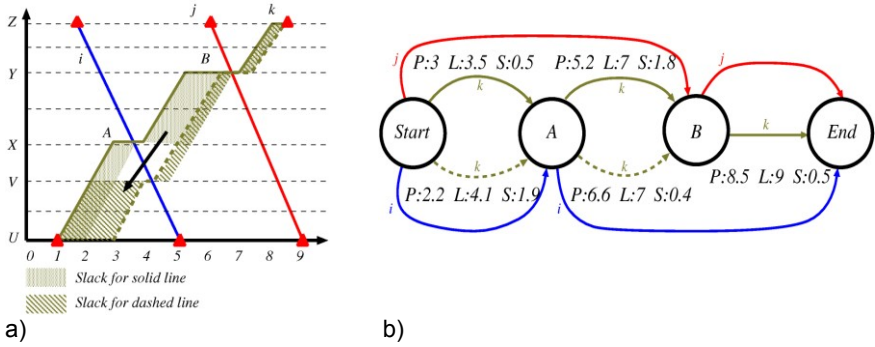


Figure 2. a) The arrow indicates how changing the meeting location from X (solid line) to V (dashed line) redistributes the link slack.

b) The dependency graph for the situation in a). P = planned arrival time to meeting, L = latest possible arrival time, S = slack.

### 5.1 The two-solutions approach

The method used in this paper is based on constructing two feasible solutions (*twin solutions*), and measure how they differ given some objective function. For example, we can maximise the difference in time between the solutions, or the geographical distance in terms of potential alternative meeting locations.

Variables  $x_{ab}^k$  are introduced for trains  $a$  and  $b$  on link  $k$ .  $x_{ab}^k = 0$  if train  $a$  traverses link  $k$  before train  $b$ , else  $x_{ab}^k = 1$ . Since the method requires two solutions, two sets of trains are used,  $\hat{a} \in E$  and  $\tilde{a} \in L$ . A train  $\hat{a}$  belonging to set  $E$  is required to be earlier than its counterpart  $\tilde{a}$  in set  $L$ . To this aim we introduce a variable that defines when a train  $a$  enters a link  $k$ ,  $d_a^k$ .

Constructing one early and one late solution allows the early solution to be adopted as the production timetable, while the late solution serves as a safety net. Both these solutions are schedules that fulfil all the delivery commitments. The original train paths, which also fulfil the commitments, are found somewhere between the early and late solutions.

There will be a number of alternative meeting locations between the early and late solutions, providing a possibility to redistribute slack (see Fig. 3). Although a single meeting location swap may result in a new feasible timetable, there is no guarantee. Sometimes limited further adaptation may suffice to regain feasibility, but in the worst case the late safety solution may have to be put into operation.

Rather than maximising the number of potential alternative meeting locations, we maximise the number of links that have a start and an endpoint where meeting swaps are possible. In Fig. 3 the links fulfilling this criterion are the ones in the blue area. It is clear that for a link to be in the blue area,

$$x_{\hat{i}\hat{j}} \neq x_{\tilde{i}\tilde{j}} \quad (1)$$

or

$$x_{\hat{i}\hat{j}} \neq x_{\tilde{i}\tilde{j}} \quad (2)$$

Binary variables  $C_1^k$  and  $C_2^k$  are introduced to signal whenever Eqns. (1) or (2) are true respectively.  $C$  is zero when its condition is false.

In order to only have links that are either completely in the shaded area, or completely outside it, we force all meetings of early and late trains to be on a timetable location. However, time supplements have not been added to regulate these meetings, so they may not be operable in real life. As a consequence, the uppermost and lowest links may not be feasible for meeting swaps. The model used is the following,

$$\begin{aligned}
x_{i\hat{j}}^k + x_{i\tilde{j}}^k - C_1^k &\geq 0 & x_{i\hat{j}}^k + x_{i\tilde{j}}^k - C_2^k &\geq 0 & x_{i\hat{j}}^k - x_{i\tilde{j}}^k + C_1^k &\geq 0 \\
x_{i\hat{j}}^k + x_{i\tilde{j}}^k + C_1^k &\leq 2 & x_{i\hat{j}}^k + x_{i\tilde{j}}^k + C_2^k &\leq 2 & x_{i\hat{j}}^k - x_{i\tilde{j}}^k - C_1^k &\leq 0 \\
x_{i\hat{j}}^k - x_{i\tilde{j}}^k + C_2^k &\geq 0 & d_i^k &\leq d_{\tilde{i}}^k & & \\
x_{i\hat{j}}^k - x_{i\tilde{j}}^k - C_2^k &\leq 0 & x_{i\hat{j}}^k, x_{i\tilde{j}}^k, x_{i\tilde{j}}^k, x_{i\tilde{j}}^k, C_1^k &\text{ and } C_2^k &\text{ binary}
\end{aligned}$$

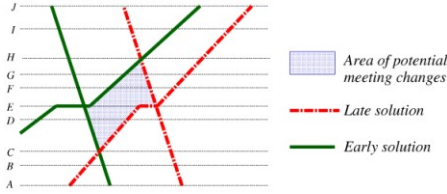


Figure 3. The alternative meeting locations are the ones in the shaded area.

## 5.2 Maximising the time difference

To maximise the time difference between the two solutions define  $df_i = d_{i\tilde{i}}^k - d_{i\hat{i}}^k$ , and use the objective function

$$\max \sum_{ik} df_i^k \quad (3)$$

The model with the objective function defined in Eqn. (3) is called problem A. Although this model allows for different meeting patterns it does not reward it. The objective function results in the safety net being far away in time, but ignores the number of steps between the two solutions (as defined by the possible meeting location swaps for pairs of trains). This will limit the robustness gains.

## 5.3 Maximising the number of alternative meeting locations

Rather than maximising the time difference we may maximise the geographical distance in terms of potential alternative meeting locations between the two solutions. The objective function is,

$$\max \sum_{k \in \text{Links}} C_1^k + C_2^k \quad (4)$$

The model with the objective function defined in Eqn. (4) is called problem B. The objective function maximises the potential alternative meeting locations, but it does not take into account how much slack a potential meeting swap might redistribute. For the slack redistribution to be as efficient as possible this needs to be considered as well.

## 6 Examples

The methods described in Section 5 were tested on a part of the Swedish infrastructure consisting of a single track line with 62 geographic timetable locations. Two days were chosen at random from the 2011 train plan, namely day 98 and 101. The problems included 1488 and 2021 link traversals respectively, which provided for a total of 811 and 1734 potential realisations of meetings if only delivery commitments were considered. The results are presented in Fig. 4 and Table 1. Fig. 4 is the train diagram for day 98 where the twin solutions from problem A are plotted. Table 1 shows the results after optimisation. As expected there is a trade-off between time and potential meeting points.

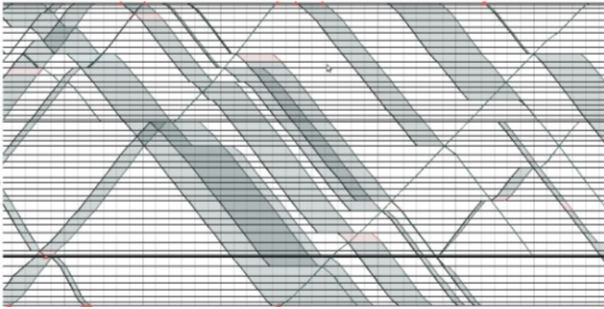


Figure 4. Areas between the two solutions for the distance Boden-Vännäs in the 2011 train plan. The early solution is plotted in black and the late in grey, and the area between the two solutions is shaded.

**Table 1.** The time difference and number of potential alternative meeting locations for the two objective functions.

Objective function	A		B	
Day	98	101	98	101
Sum of time difference over all geographical locations (seconds)	1142986	1277343	641138	920985
Potential alternative meeting locations	118	224	144	239

## 7 Final Remarks and Future Research

This paper presents the concept and models of replanning robustness. The fundamental idea is that by changing train orders on links during operation, slack can be geographically redistributed to absorb delays where they are occurring. We introduce heuristics for investigating and constructing timetables that allow for such flexible use of slack, and test the methods on a case from Sweden.

The concepts and models presented in this article are still being developed, and this paper should be considered a first step towards a full methodology. We plan to further investigate and experiment with these ideas and models.

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