Finite Domain Constraint Programming Systems

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What Is This Tutorial About?

- Focus is on
  - *Services systems provide*
  - *Implementation of these systems*
- No detailed description of one system
  - *Common techniques, approaches, challenges*
  - *No intention to do complete survey*

Finite Domain Constraint Programming Systems

- Offer *reusable* software components for
  - constraint propagation
  - combining constraints (combinators)
  - search
    - branching (labeling)
    - exploration (for example: depth-first, LDS, ...)
  - user extensions
- Services provided
  - environment for integrating components
  - libraries of commonly used components

[Henz & Müller 00]

Systems Discussed

- Prolog-based systems
  - SICStus Prolog, Eclipse Prolog, GNU Prolog, CHIP,...
- Libraries
  - ILOG Solver (C++) and JSolver (Java), Choco (Claire), Figaro (C++), Facile (Ocaml), CHIP Library (C++)
- Specialized languages
  - Claire, Oz

Outline of Tutorial

- Constraint propagation
  - example [Christian]
  - model [Christian]
  - implementation [Mats]
  - optimizations [Mats]
- Search [Christian]
- Combinators [Christian]
- Trends & Challenges [Mats]
- References

Constraint Propagation

Example

Example

$x + y = 9$

$2x + 4y = 24$

x	0	1	2	3	4	5	6	7	8	9
y	0	1	2	3	4	5	6	7	8	9

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Constraint Propagation

Model

Constraint Propagation

- Variables
  - feature variable domain (finite set of integers)
- Propagators
  - implement constraints
- Propagation loop
  - execute propagators until simultaneous fixpoint

Propagator

- Propagator p is procedure
  - implements constraint con(p)
    its semantics (set of tuples)
  - computes on set of variables var(p)
- Execution of propagator p
  - narrows domains of variables in var(p)
  - signals failure
  - signals entailment [discussed later]

Classes of Constraints

- Basic constraints
  - Constraints for which the solver is complete
    - $x \in D$, $x = v$, $x = y$ (variable aliasing)
- Primitive constraints (need propagators)
  - Non-decomposable constraints
    - $x<y$, $x \neq y$, $x+y = z$, $x*y = z$, …
- Global constraints (need propagators)
  - Subsume a set of basic or primitive constraints, usually providing stronger consistency

Reification

- Control constraint by control variable $b \in \{0,1\}$

  $c \quad \Leftrightarrow \quad b=1$
  - also require propagator
- A.k.a.: metaconstraints

Propagators Are Intensional

- Propagators implement narrowing
  - also: filtering, propagation, domain reduction
- No extensional representation of con(p)
  - impractical in most cases (space)
- Extensional representation of constraint
  - can be provided by special propagator
  - often: “element” constraint, “relation” constraint, …

Implementing Propagators

- Implementation uses operations on variables
  - reading domain information
  - narrowing domains
- Variables are the only communication channels between propagators
- More detail later

Propagator Properties

- Propagator *p* is
  - correct: no solution of con(*p*) is removed
  - assignment complete: failure at latest for assignments
    - compatibility with search
- Propagator *p* is
  - contracting: variable domains are narrowed
  - monotonic: application to smaller domains will result in smaller domains than application to larger domains
  - may be idempotent: [discussed later]

Propagation Loop

- Largest simultaneous fixpoint of propagators
  - fixpoint: propagators cannot narrow any further
  - largest: no solutions lost
- Guaranteed
  - termination: domains finite
propagators contracting
  - largest fixpoint: propagators monotonic

Detailed study with proofs: [Apt 00]

Fix and Runnable Propagators

- Propagator is either
  - fix: has reached fixpoint
  - runnable: not known to have reached fixpoint
- Propagation loop maintains propagator sets
  - all propagators *Prop*
  - runnable propagators *Run*
  - initially *Run := Prop*

Sketch of Propagation Loop

```
while (Run ≠ ∅) {
    pick and remove p from Run;
    execute p;
    ModVar := { x | x modified by p };
    DepProp := { q | x∈var(q), x∈ModVar };
    Run := join(DepProp, Run);
}
```

Sketch of Propagation Loop

while (*Run* ≠ ∅) {
    pick and remove *p* from *Run*;
    execute *p*;
    ModVar := { *x* | *x* modified by *p* };
    DepProp := { *q* | x∈var(*q*), *x*∈*ModVar* };
    Run := join(*DepProp*, *Run*);
}

Loop invariant: *p* is fix ⇔ *p*∈(*Prop*-*Run*)

Sketch of Propagation Loop

while (*Run* ≠ ∅) {
    pick and remove *p* from *Run*;
    execute *p*;
    ModVar := { *x* | *x* modified by *p* };
    DepProp := { *q* | x∈var(*q*), *x*∈*ModVar* };
    Run := join(*DepProp*, *Run*);
}

Termination (*Run*=∅): *p* is fix ⇔ *p*∈*Prop*

Sketch of Propagation Loop

while (*Run* ≠ ∅) {
    pick and remove *p* from *Run*;
    execute *p*;
    ModVar := { *x* | *x* modified by *p* };
    DepProp := { *q* | x∈var(*q*), *x*∈*ModVar* };
    Run := join(*DepProp*, *Run*);
}

Ignored: failure (signaled by *p*)

Consistency Level Computed

- Model is generic
- Consistency level defined by each individual propagator
  - accurate way of characterization [Maher 02]
- Supports many different consistency levels
  - propagator for domain-consistent alldifferent
  - propagator for bound-consistent alldifferent
  - propagator for value-consistent alldifferent

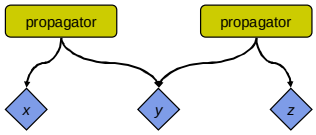
Major Design Decisions

- Implementing *Run* (that is, pick and join)
  - queue: first in – first out
  - stack: last in – first out
  - priority queue
- Implementing *ModVar* and *DepProp*
  - variable-centered representation

Implementing *ModVar* and *DepProp*

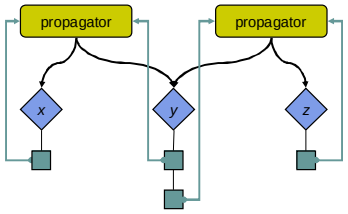
- Variable-centered approach
  - each variable *x* knows dependent propagators
  - typically organized as list (*suspension list*)
  - propagator *p* included in list of *x* ⇔ $x \in \text{var}(p)$
- Upon propagator creation
  - propagator subscribes to its variables
  - becomes runnable

Propagators ⇨ Variables



- Propagators know their variables
  - to perform domain modifications
  - passed as parameters to propagator creation

Variables ⇨ Propagators



- Variables know dependent propagators
  - to perform efficient computation of dependent propagators
  - implemented by suspension lists

Modifying a Variable

- Traverse suspension list
  - add propagators to *Run*
- Optimization
  - mark runnable propagators
  - that is: propagators already in *Run*
- Multiple variable modification by propagator
  - explicitly maintain *ModVar* (as in model)
  - only after propagator execution: process *ModVar*
  - suspension list traversed only once per variable

Propagation Events

- Use knowledge on propagator to avoid its redundant execution
  - redundant: application to fixpoint
- Example: bound-consistent linear equality
  - need to execute, if bound of variable changes
  - no need to execute, if inner value of variable removed
- Suspension list: ⟨propagator,event⟩
  - event describes relevant domain modifications
  - implementation: lists per event, single list of pairs
  - events: VALUE, BOUND, DOMAIN

Idempotent Propagators

- Idempotent propagator
  - always computes fixpoint
- Propagation loop perspective
  - no need to include in *Run*
  - more efficient: saves one invocation of propagator
- Propagator perspective
  - must compute fixpoint itself
  - more efficient: specific method for computing fixpoint
  - might be more challenging

Propagator Entailment

- Propagator will never contribute anything
  - fixpoint property preserved by narrowing
- Delete propagator, if entailment detected
  - remove from suspension-list, or
  - mark as dead, delegate removal to garbage collection
- Similar to consistency, different entailment levels
  - semantically relevant in concurrent constraint programming

Summary of Model

- Variables
  - domain
  - suspension list: ⟨event, propagator⟩
- Propagators
  - intensional, correct, contracting, monotone, define consistency level, …
  - know variables for narrowing
- Propagation loop
  - computes largest simultaneous fixpoint

Constraint Propagation

Implementation

Propagation Queues

- Contents
  - Events, variables, or propagators
- Scheduling Policy
  - LIFO makes sense for “important” events
  - FIFO – fair scheduling, no starvation
  - Compare LIFO and FIFO for:
    - $x>y$, $y>x$, $x \geq 100t$, $y \leq t$, $\{x,y,t\} \in 1..1000$
- Structure
  - Flat or layered

Layered Propagator Queue

- CHOCO’s 8 priority levels
  - VALUE event queue: LIFO
  - BOUND event queue: FIFO
  - DOMAIN event queue: FIFO
  - P’s for extensionally defined constraints (AC-4)
  - O(N) propagators
  - O(< N^2) propagators
  - O(N^2) propagators
  - O(> N^2) propagators

[Laburthe 00]

Stateful Propagators

- Prerequisite for incremental algorithms
  - Can bring down complexity by an “order”
- Arguments checked initially only
- Prolog level state not enough
- State can be used for:
  - Gradually ignoring ground variables
  - Data structures for the filtering algorithm
  - Memory of variables’ min/max/domain
  - Local trailing for backtracking

What Does a Propagator Know?

- Course-grained information
  - Something has changed (SICStus, Mozart)
- Medium-grained information
  - Variables v_3, v_7, v_{11} have changed (CHIP)
- Fine-grained information (ILOG)

Variable	Old domain	New domain	Delta
V_1	1..5	2..4	{1,5}

- Stateful propagators can figure out what changed

What Does a Propagator Need?

- A good filtering algorithm
  - An algorithm library might come in handy
    - Shortest-path, bipartite matching, max flow, min-cost flow, profiles, strongly connected components, …
- ADT: finite domain
- ADT: domain variable
- Host language services
- Solver kernel interface
  - True/false/suspend, replace_by, I_am_not_idempotent, …
- State

ADT: Finite Domain

- Representation
  - List of intervals (ECLiPSe, SICStus)
  - Bounds + bit array (CHIP, GNU Prolog, Mozart)
  - Bounds + list of holes
  - Interval trees
  - Multiple, adaptive (CHOCO, Mozart, …)
- Operations
  - Set operations
  - Constructors, iterators
  - Complexity depends on representation

ADT: Domain Variable

- Representation
  - Logic variable + attributes for domain and suspensions, or
  - Class instance
- Operations
  - Access min, max, domain
  - Adjust min, max, domain; remove values
  - Raise events
  - Attach/detach suspensions

Variable Aliasing ($x = y$)

- Only an issue in logic-based languages
- Normalization:
  - Merge suspension lists
  - Intersect domains
  - Raise events
  - IF con(p) mentions both x and y THEN
    - p may no longer be idempotent
    - p can make more inferences, e.g.:

      xor(x,y,z), x=y $\Rightarrow z=0$

Host Language Services (Generic)

- Memory management
  - Allocation: objects, states
  - Garbage collections: term refs in states and queues
  - Copying
- Trailing
  - Coarse or fine
  - Semantic trailing for self-destruct on backtracking
- Resume/suspend mechanism
  - Full coroutining, multithreading etc. not needed

Host Language Services (Prolog-Based)

- Predicate/function type extensions
  - Constraints must be callable like predicates
  - WAM support for indexicals (coming slides)
- Attributed variables [Holzbaur 92] [Le Huitouze 90]
  - Domains
  - Suspensions
  - Unification hook
- Mutable terms
  - Coarse trailing

Implementation Choices

- In host language
  - Prolog (ECLiPSe, SICStus)
  - C++ (ILOG Solver, Figaro, CHIP Library)
  - Claire (CHOCO)
  - Java (ILOG JSolver)
- C/C++
  - For predefined constraints (ECLiPSe, SICStus)
  - For predefined + user-defined constraints (Mozart)
- Indexicals
  - For “pseudo primitives” (GNU Prolog, SICStus)

Indexicals

- Given $C(X_1,\ldots,X_n)$, for each i, provide a rule
  X_i in R_i
  which computes the feasible values of X_i
  - Example: X = Y + C, arc-consistent version

```
eqcd(X,Y,C) +:
      X in dom(Y)+C,
      Y in dom(X)-C.
```

  - Example: X = Y + C, bound-consistent version

```
eqcd(X,Y,C) +:
      X in min(Y)+C..max(Y)+C,
      Y in min(X)-C..max(X)-C.
```

[Van Hentenryck & Deville & Saraswat 92]

Indexicals: Syntax of *X* in *R*

Range expressions

R ::= T..T | *R*/*R* | *R*\/*R* | *R* | *R*+*T* | *R*-*T* | *R* mod *T* | *{T,…,T}* | dom*(X)*

Term expressions

T ::= T+*T* | *T*-*T* | *T*/>*T* | *T*</*T* | *T* mod *T* | min*(X)* | max*(X)* | *X* | *integer* | inf | sup

Monotone indexicals for propagation
Anti-monotone indexicals for entailment check

Indexicals for Reification

Example: X = Y + C, SICStus syntax

```
?- eqcd(X,Y,5) #⇔ B.

eqcd(X,Y,C) +:    % propagation
  X in dom(Y)+C, Y in dom(X)-C.
eqcd(X,Y,C) -:    % converse propagation
  X in \{Y+C}, Y in \{X-C}.
eqcd(X,Y,C) +?    % entailment check
  X in {Y+C}.
eqcd(X,Y,C) -?    % disentailment check
  X in \dom(Y)+C.
```

[Carlsson & Ottosson & Carlson 97]

Indexicals: Implementation

- Compiled to (bytecode,symbol table)
- Syntax intercepted by *term expansion*
- Executed by a simple stack-based VM
- `eqcd/3` gets defined as a Prolog predicate
  - The WAM escapes to a solver entrypoint

[Carlsson & Ottosson & Carlson 97]

Indexicals: Pros and Cons

- Efficiency: witness GNU Prolog
- A RISC approach to constraint solving
- A VM for propagators
- A language for fine-tuned propagation in a general framework
- Can detect entailment as well as propagate
- Drawbacks
  - Pseudo-primitives only (no global constraints)
  - N propagators needed for 1 constraint

An Entity-Relationship Model (CHOCO, Figaro, ILOG)

- Objects
  - Problems
  - Variables
  - Domains
  - Constraints
- Relationships
  - Links between constraints and variables

(constraint, variable, event)

[Puget 94][Puget & Leconte 95] [Laburthe 00]

Constraint Objects

- State in private data
- Virtual methods for (some of):
  - Posting
  - Propagation
  - Entailment/disentailment test
  - Reification
  - Profiler and visualizer services
  - Memory manager services

The ILOG Global Constraint API (Simplified)

```
class ClassName : public IlcConstraint {
 public:
  ClassName(IloSolver solver, Args);
  ~ClassName(void);
  virtual void post(void);
  virtual void propagate(void);
  virtual IlcBool isViolated(void) const;
};
```


The Mozart Global Constraint API (Simplified)

```
class ClassName : public OZ_Propagator {
public:
  ClassName(OZ_Term Args);
  virtual OZ_Return propagate(void);
  virtual size_t sizeOf(void);
  virtual void gCollect(void);
  virtual void sClone(void);
  virtual OZ_Term getParameters(void) const;
};
```


A Global Constraint API for Prolog (SICStus)

- `fd_global(+C, +S, +V)`
  - Posts a global constraint *C* with initial state *S*, suspended according to *V*, which is a list of `dom(X), min(X), max(X), minmax(X), val(X)`
- `dispatch_global(+C, +S0, -S, -A)`
  - user defined
  - Entrypoint to the propagator of constraint *C* with state *S0*, producing a new state *S* and kernel requests *A* (*true/false/suspend*, events)
- ADTs for domains and domain variables
- Control is implicit

Constraint Propagation

Optimizations

Rewriting

- Generic → special

  3x + y – z = 0, x = 0

  →

  y = z (variable aliasing)

- Gradual decomposition

  alldiff*([T,U,V,X,Y,Z])*, *[T,U,V]* in *1..3*, *[X,Y,Z]* in *4..6*

  →

  alldiff*([T,U,V])*, alldiff*([X,Y,Z])*

Avoiding Useless Work

- Idempotent *p* should be immune to events raised by *p*
  - Kernel may or may not assume idempotence
- An entailed *p* should never be resumed
  - It can even be detached (undoably) from var(*p*)
- IF time of latest event < time of latest resumption THEN don't resume *p*
  - Event queues require timestamps
- Indexicals linked to the same constraint should (sometimes) be immune to each other's events

More Optimizations

- Different event types
  - VALUE > BOUND > DOMAIN
  - *p* is suspended on a set of (*v*,*event*)
  - Demons vs. propagators
- Scheduling policies
  - Poorly understood
  - Complexity-based priority queues make sense
  - Always bear the worst case in mind

Search

Branching and Exploration

- Branching: defines shape of search tree
  - labeling, branching, distribution, …
  - often based on heuristics
- Exploration: explore nodes of search tree
  - often fixed to be depth-first
  - many aspects
    - optimization (branch-and-bound)
    - development tools (Oz Explorer)
    - parallelism (ILOG Solver, Oz)

Branching

- Requires synchronization on fixpoint
  - for implementing dynamic variable orderings
  - by construction: Prolog, ILOG Solver, …
  - explicit synchronization in concurrent setup: Oz
- Programmed
  - from builtin-search: Prolog-based
  - special (language) constructs: ILOG Solver, Oz
- Typically, rich library available

Exploration

- All systems support
  - search for first solution
  - search for some/all solutions
  - search for best solution
- Most systems support
  - LDS and some variants

Exploration Strategy

- Often fixed to be depth-first
- Sometimes can be programmed
  - Oz: spaces (“nodes”) as ADT for exploration
    - exploration programmed from operations
    - for example: copy node in search tree
      access solution
  - ILOG Solver: control exploration by limits and priorities
    - limit cut-off branches
    - priorities which node to explore next

[Schulte 97] [Perron 99]

Infrastructure for Exploration

- State restauration
  - backtrack to a *previous* state
- Approaches
  - trailing: recording and undoing changes
  - copying: put complete state aside
  - recomputation: recompute state on need
- By far dominating approach: trailing

Trailing

- Trailing stores undo and redo information
  - interleaved with constraint propagation
  - uses trail data structure
  - update: put ⟨location,content⟩
  - undo: write location ← content
  - every choice point: put mark or record top of trail
- Requires
  - all updates trail-aware
  - for example: domain change, change of suspension list, …

Time Stamping

- Problem: multiple change of same location
  - for example: multiple narrowing of domain
  - only original value needs restauration
  - intermediate values not needed
- Solution: local time stamp on modified entity
  - new choice point: increase global time stamp
  - upon modification: trail, if local stamp earlier; update local stamp

[Aggoun & Beldiceanu 90] [Aggoun & Beldiceanu 91]

Multiple Value Trail

- Modifying n successive locations
  - record start, number (n) and n locations on trail
  - instead of $2n$ individual entries

[Aggoun & Beldiceanu 90] [Aggoun & Beldiceanu 91]

Copying And Recomputation

- Copying
  - operations ignorant of state restauration
  - support for concurrency and parallelism
  - alone infeasible: excessive memory requirements
- Hybrid strategies: copying and recomputation
  - adaptive: create copy on demand to speed up future recomputation
  - batch: speed up recomputation by avoiding repeated fixpoint computation
  - competitive with trailing

[Schulte 99] [Choi & Henz & Ng 01] [Schulte 02]

Combinators

Combinators

- Reification-based combination
  - reified constraints
  - propositional combination
- Propagation-preserving approaches
- Constructive disjunction

Reification

- Use control variable $b \in \{0,1\}$

  $c \quad \Leftrightarrow \quad b=1$
- Propagate
  - c entailed $\Rightarrow$ propagate $b=1$
  - $\neg c$ entailed $\Rightarrow$ propagate $b=0$
  - $b=1$ entailed $\Rightarrow$ propagate c
  - $b=0$ entailed $\Rightarrow$ propagate $\neg c$ (might be difficult)

Propositional Combination

- Constraints for combining reified constraints
  - constraints as connectives over 0/1 variables
- Combine $(c_1 \wedge c_2) \vee c_3$
  - reify each c_i to 0/1 variable
  - use constraints on 0/1 variables
- Problem: not propagation-preserving
  - no propagation between c_1 and c_2
  - in $c_1 \wedge c_2$, both c_1 and c_2 propagate individually

Compositional Approaches

- Use language-based primitives for implementing combinators
  - encapsulated propagation
  - generalization of ccp-paradigm
  - pioneered by AKL [Haridi & Janson 90]
  - generalized to programming abstraction [Schulte 02]
- Advantages and disadvantages
  - expressive and propagation preserving
  - implementation complex and less efficient than reification
  - unclear how to provide in language-independent setting

Constructive Disjunction

- Idea: make assumptions and generalize
  - propagate locally in each branch of disjunction
  - lift out common information on domains from branches
- Well researched/published idea
  - cc(FD) [Van Hentenryck & Saraswat & Deville 95]
  - many other papers, for example [Codognet & Codognet 95] [Würtz & Müller 96]
  - not of strategic importance
  - technique useful to know about

Trends and Challenges

Explanations

- Definition

  A (minimal) set of constraints and choices made during search justifying a propagation event

- Uses
  - Understanding dead ends
  - Nogoods
  - Conflict-directed backtrack search
  - Debuggers and visualizers
- Challenges
  - Sharp explanations for global constraints
  - Bridging semantic gap between application and CP model

[Jussien & Barichard 00]

Meta-Programming

- Applications:
  - Debuggers
  - Visualizers
  - Static analysers
  - Search strategy synthesizers
  - Test case generators
  - Parser generators for propagators
- Requirement:
  - Exact and formal description of all constructs

Challenges

- Communication between constraints

  Constraints communicate via domain variables only, so constraints are independent of each other

  - Good news: constraints can be posted regardless of already posted ones
  - Bad news:
    - Loss of global view
    - Obvious propagation missing
    - Thrashing
    - Creates artificial global constraints

Challenges

- Exact and formal description of all constructs
  - Syntax and options
  - Declarative semantics
  - Events
  - Level of consistency
  - Complexity
- No information should appear only in the manual

[Beldiceanu 00]

Challenges

- Hybridization
  - with local search
    - Let CP explore the neighborhood
  - with linear programming
    - Benders decomposition

[Eremin & Wallace 01]

- Modelling languages and global constraints
- Optimization
  - Cost-based filtering algorithms
- Over-constrained problems
  - Replace *C(X)* by *C'(X,cost)* where *cost* is the degree to which *X* violates *C(X)*

[Petit & Régin & Bessière 01]

Challenges

- Beyond finite domains

[Jaffar & Maher & Stuckey & Yap 94]

  - Richer set of basic constraints, e.g. TVPI

$X \bmod 11 \in \{1,5\}$, $x \geq 2y+3$, …

- Classification and standardization
- Parametric constraints
  - One constraint family – one filtering algorithm

[Beldiceanu 2000]

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