Information Technology

as and aid to

Teaching Algebra

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Thesis submitted for the degree of

Doctor of Philosophy

Faculty of Education Studies

Dublin City University

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August 1990

Declaration

I hereby declare that the contents of this thesis are based entirely on my own work which was carried out at Dublin City University.

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For

Bridin, Alan and Eavan

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Abstract

This project was concerned with teaching algebra novices, all girls aged 13 or 14 years, to solve algebra word problems using an electronic spreadsheet. It was based on the realisation that a spreadsheet cell provides a suitable cognitive model for an algebraic variable and that the manipulation of a spreadsheet is essentially based on the construction of algebraic expressions. The main objectives were to test the effectiveness of spreadsheet use on the ability to construct algebraic expressions and to examine the effect of manipulating problem contexts (abstract vs. concrete) on this ability. Other objectives were to determine the relationship between general numerical ability, attitude to mathematics, attitude to computers and the experimental treatments.

The particular skill taught was the construction of algebraic expressions to represent relational propositions from verbally stated problems. Problems from current textbooks and examination papers (Intermediate Certificate Syllabus B) were used in the instruction. A pretest - posttest control group design was used. Seventy three volunteers were recruited and received approximately eight hours of instruction in a reasonably natural school setting. There were two treatment groups. One group worked on abstract (numerical) problems and the other group worked on mathematically identical problems set in concrete contexts which were familiar and relevant.

Both treatment groups made considerable gains between pretest and posttest. The abstract group performed significantly better than the concrete group on the total posttest ($p < .01$), on its abstract subsection ($p < .01$) and on its concrete subsection ($p < .05$). Attitude to mathematics was also found to have a significant interaction with the treatment ($p <. 05$). Those with a positive attitude to mathematics learned more from abstract problems, but the difference was much less for those with a negative attitude. Neither numerical ability or attitude to computers had any significant effect.

Acknowledgments

I would like to express my gratitude to Dr. Peter McKenna for his unfailing support and encouragement for this research. Thanks also to Dr. David Owens for his help during the initial stages.

I wish to thank the library staff at DCU, especially Anne Rafter for inter-library loans, and Susan Cuffe for online searches and moral support.

I acknowledge gratefully the cooperation of the students, staffs and principals of the five schools involved. I am especially grateful to Sr. Rosaleen Cunniffe and Sr. Anne Marie Clancey who provided access to computer hardware.

I also wish to thank my own students; Geraldine, Lorraine, Jennifer and Audrey who acted as tutors. Special thanks to my colleague Marita McGrath who also acted as a tutor and who helped prepare the instructional materials.

I would also like to acknowledge my debt to both of my parents who understood the value of education and who made real sacrifices to educate their children.

Finally, I would like to thank my wife Bridin for her constant encouragement and support.

Chapter 1: Introduction

1.1 Background to the Study

In our rapidly developing technological society, there is an increasing need for mathematical skills as more professions move towards quantification, e.g. economics, management science, medicine and the biological sciences (Cockroft 1982). Apart from professional and career considerations, knowledge of quantity and space has a pervasive effect on how well we function in society. For example, 93% of articles on the front page of The New York Times cannot be comprehended without fundamental mathematical competence (Czepiel and Esty, 1980). There is, however, widespread concern at the level of mathematical attainment of school leavers (e.g. Cockcroft, 1982; Howe, 1986; Silver et al., 1988; Brown et al., 1988). For example, the National Assessment of Educational Progress shows a significant decline in mathematical competence of eleven to sixteen-year-olds over the period 1963 to 1983 (Gagne, 1985). In the area of school algebra, Silver et al. (1988) reported that over 50% of seventeen-year-olds had not enrolled in second year algebra. Brown et al. (1988) reported that only 40% of students complete second-year algebra.

There is also concern about the abilities of those students who do complete school algebra successfully. A series of studies, the 'disaster studies', (Rosnick and Clement, 1980; Rosnick, 1981; Clement, 1982) have shown that many students, who seem to be successful when judged by conventional tests, are seriously confused about basic algebraic concepts. These studies revealed that many students develop special-purpose translation algorithms which work for many textbook problems, but which do not involve a semantic understanding of algebra. Many students have profound misunderstandings of algebraic notation (Sleeman 1984; Booth 1984a, 1984b) and very few can use algebra as an "autonomous mode of expression" (Burkhardt 1986). Students who were thought to have successfully learned elaborate formal structures have been found to have serious misconceptions at the most fundamental levels. Rosnick and Clement (1980) found

> "disturbing difficulties in students' conceptualization of the basic ideas of equation and variable" (Rosnick and Clement, 1980, p.5).

among students who had successfully completed five or more years of algebra in high school and college. This is alarming, as these same students had passed tests which involved linear equations, quadratic equations, cubic equations, graphs of conic sections etc. This indicates that many successful mathematics students work by imitation without any real understanding of what they are doing. In the light of these studies, and the increasing requirement for mathematical competence, there is a need to investigate how mathematics curricula can be developed and improved.

Any discussion of mathematical education must take account of the spread of information technology:

> "It is important that the rapid changes in our society in technology, in methods of communication and in knowledge, are reflected in changes in mathematics education, both in what is taught and in the methods by which children learn" (Ball et al. 1987, p.7)

The recent arrival of microcomputers in schools presents new opportunities in mathematics education. There are many ways in which computers can be used in mathematics teaching, ranging from drill on basic facts to simulations and modelling. Computers can facilitate a more experimental, investigative approach. They can relieve the burden of calculation, provide graphical representations and allow learners to explore more realistic problems. They are also very motivating for many children and can facilitate non-traditional problem-solving techniques, such as guess-and-check.

Computers are also changing the ways in which people do mathematics. The recent availability of microcomputer programs which can carry out symbolic algebraic manipulations (Maurer 1984; Kunkle & Burch 1984; Higgo 1985; Howe 1986; Freese et al. 1986) poses serious questions concerning the appropriateness of present algebra syllabi and the content of future syllabi.

> "Students in mathematics classes will soon have access to computer programs that perform all the routine numerical and algebraic manipulations that have traditionally required months to learn. It is only a matter of time until mathematics instruction starts shifting away from the memorization of standard algorithms towards a more conceptual emphasis" (Kilpatrick 1987, p. 123).

> "Much of the arithmetic syllabus, many algebraic techniques and, at a more advanced level, many techniques connected with the calculus are now, or soon will be, no longer required" (Ball 1987, p. 156).

> "It is scary to think that most of what we have been teaching - manipulation drill - may soon be irrelevant" (Maurer 1984, p.423).

Similar questions arose when calculators first became widely available. It would be difficult to sustain the argument that students should be taught pencil-and-paper algorithms for calculating square roots when these can be found at the touch of a button on a calculator.

> "What is important is not that 'I know' in any pencil-and-paper algorithm sense, how to compute a square root. Knowing how to do something is always relative to a technology, and pencil-and-paper technology is sufficiently ill-suited to finding square roots that specialised 'know-how' is required on the part of the person .... The educational issue is not to know how to compute a square root, but when to compute one." (Balzano 1987, p.87)

Similarly, it must be asked whether students should spend the bulk of their mathematics instruction time learning to carry out algebraic manipulations which can be performed with readily available software on computers? Could such an expenditure of time and effort be justified if and when this software becomes available on hand-held calculators?

Ernest (1987) suggests that current school mathematics is essentially the study of algebraic manipulations. Ernest (1987) further suggests that:

> "mathematical expressions are commonly presented not for purposes of comprehension in the psycholinguistic sense of contributing to the construction of a larger meaning context, but as the

initial state of a mathematical task which will be transformed in the performance of the task" (Ernest 1987, p.350).

This may be more a reflection of curricular history than of current need. Balzano (1987) argues that we have been sidetracked into thinking that mathematics is more concerned with the 'how' than the 'when'. This is never the case. The 'how' is always relative to a technology, but the 'when' is much more enduring. There is now a need to re-examine what we teach and to re-evaluate it in the light of new technology. In many cases we may find that what we thought was basic knowledge is merely an artifact of a technology that is now out of date.

This does not mean that merely substituting computer methods for pencil-and-paper manipulation will necessarily improve matters:

> "Being able to run an application program whose result he does not understand is no more useful to a child than executing a pencil-and-paper algorithm whose result he does not understand" (Howe 1986, p.24)

The reduction in the need for manipulative skills in mathematics will not diminish the need for mathematical ability but should alter the focus of curriculum development. If the emphasis in algebra courses is to be shifted from routine manipulation, what should take its place? The abilities that will always be required concern insight and understanding of underlying mathematical principles and, in particular, the ability to apply mathematics to a broad range of problems (Higgo 1985). A recent set of standards developed by the National Council of Teachers of Mathematics (Thompson & Rathmell 1988) stresses conceptual understanding, reasoning and problem-solving while de-emphasising computational and manipulative proficiency. According to the National Council of Teachers of Mathematics:

> "The study of algebra will shift from a focus on manipulative facility to include an increased emphasis on conceptual understanding and using algebra as a means of representing mathematical situations and relationships" (Thompson & Rathmell 1988, p.350).

Gagne (1983) has stressed that one of the areas in need of attention is teaching to translate concretely stated problems into mathematical form. Maurer (1984) lists

> "the ability to express ideas precisely and to translate ideas into symbols" (Maurer 1984, p.425).

among the skills that remain essential. Emphasis should therefore be given to developing the ability to construct mathematical representations from problem statements. This is a vitally important ability if mathematics is to be applied to real problems. Lack of skill in translating problems into symbolic mathematical expressions and equations is the cause of some concern:

> "Many of the problems which arise in mathematics classrooms can be traced to a dislocation in the pupil's thinking between the symbolic expressions being manipulated and the original situations from which they represent an abstraction. This dislocation is particularly common in the case of algebra" (Greer 1984, p.109).

> "These difficulties seem not to be rooted in their quantitative understanding of events or relationships in the natural world, but rather in the translation between those content-based representations and the formal systems of mathematics" (Sims-Knight & Kaput 1983, p.561).

The concept of a variable is central to the construction of expressions and equations from verbally stated problems. According to the National Council of Teachers of Mathematics, it is central to mathematics learning in second level schools and is one of the key ideas to be emphasized as students begin algebra (Thompson & Rathmell, 1988). It provides the basis for the transition from arithmetic to algebra, and is necessary for the meaningful use of all advanced mathematics (Schoenfeld & Arcavi, 1988). It allows generalisation of mathematical knowledge into expressions and equations, thus permiting more comprehensive statements than is possible with specific numbers. For example, without variables it can be said that the area of a particular rectangle is 6cm X 4cm. Using variables, it can be said that the area of any rectangle is L X B, where L is the length and B is the breadth.

Most contemporary curricula treat variables as primitive terms that will be understood relatively easily and used in a straightforward way by most students (Schoenfeld & Arcavi 1988). There is a lot of evidence that this is not so. Quilter and Harper (1988), carried out a study to identify the reasons offered by adults (all of whom were graduates) for their difficulties, anxieties, fears and inability to cope with mathematics at more than a rudimentary level. Of the 15 interviewees, 11 had

> "clearly encountered a sharp discontinuity in their ability to understand and learn mathematics when faced with algebra for the first time. This appeared to be associated with the conceptual difficulty associated with symbolic variables" (Quilter & Harper 1988, p.125).

Many "successful" students do not have a proper understanding of variables, even though they may be able to carry out symbolic manipulations (Rosnick & Clement 1980; Rosnick 1981; Clement 1982; Fisher 1988; Brown et al. 1988).

It is standard practice in second level textbooks (e.g. Morris 1987) to simply introduce the idea of a variable as "something which stands for a number". This is typically followed by showing how to substitute values into an expression, solve equations, find factors etc. There is usually only a short final section on creating equations for story (word) problems. The emphasis, therefore, is firmly placed on manipulative routines and there is usually no reference to earlier work in arithmetic. Most introductory exercises are totally abstract and there is rarely any attempt to set problems in areas that may be of interest to the student. It could be said that this is algebra for its own sake! It is not surprising that many students acquire only a verbal understanding of variables (i.e. "something which stands for a number") without developing a firmer conceptual framework.

This emphasis on symbolic manipulation may be traced to the influence of philosophies of mathematics on education. The two dominant philosophies that have shaped mathematics education today are Platonism and

Formalism. Platonism is the view that the objects (triangles, circles etc.) of mathematics exist in some real realm and that their existence is an objective fact, independent of our knowledge of them. These objects are outside of space and time and are never-changing. It is the job of mathematicians to discern the nature of this realm. This was the dominant view of mathematics up to the 19th Century. For example, Euclidean geometry was considered to be the firmest, most reliable branch of knowledge. It was thought to be exact, final and knowable with certainty. The subsequent invention of non-Euclidean geometries and the development of analysis exposed the vulnerability of this solid foundation upon which mathematics had been based. The influence of Platonism on education can be seen in the idea of mathematics as a fixed body of knowledge, divorced from its applications. This results in a subject-centred approach to teaching, in which topics are built up in a strict, linear step-by-step manner.

In direct opposition, the Formalist view is that the relationships between mathematical objects are important, rather than the objects themselves. Mathematics consists solely of axioms, definitions and theorems (i.e. Formulas). It is the science of rigorous proof. Experience and intuition are irrelevant. Proof is everything. Theorems are not correct in any absolute sense because they are derived from axioms which are purely arbitrary. All we can say is that the theorem follows logically from the axioms. Theorems have no content at all; they are not about anything. Davis & Hersh (1981) have remarked that the typical mathematician is a "Platonist on weekdays but a Formalist on Sundays", i.e. acts as if dealing with an objective reality when doing mathematics, but pretends not to believe in this reality when asked for a philosophical account.

Formalism abandons meaning in favour of formal rules and this gives rise to a cyclical form of teaching built around axioms, definitions, theorems, corollaries, lemmas and more axioms. This is the predominant model of mathematics teaching in third level education and was at the heart of the "New Mathematics" curriculum at second level. It is an approach which requires an appreciation of the overall structure of the topic (e.g. geometry) and this is rarely achieved by second-level students. The Formalist approach is also reflected in current introductory algebra courses which concentrate almost exclusively on manipulative techniques at the expense of meaning.

Teaching practices influenced by Formalism present topics in the opposite order to the historical development of the topics. Historically, the development of topics in mathematics follows from the relatively concrete to the more abstract:

1. Exploration of the topic.
2. Proofs of results.
3. Isolation of principles.
4. Reduction of principles to a minimal set of axioms.

This is a sequence which may be close to the development of an individual's understanding. It is, however, the exact opposite of the standard classroom development of many topics, for which the general pattern is:

1. Teacher explanation of axioms and principles.
2. Demonstration of proof.
3. Imitative exercises.

This can lead to rapid apparent progress in the sense of 'covering topics' but the skills acquired often evaporate when non-routine problems are encountered.

There is a more recent development in the philosophy of mathematics, Fallibilism, which mirrors more closely an individual's understanding and which is beginning to influence educational thought. Fallibilism is the view that mathematics is what mathematicians do, and have done, with all the imperfections inherent in any human activity or creation (Ernest 1985). Mathematics 'in the making' is a tangle of guesswork, analogy, approximation and frustration. Proof is very far from being at the core of discovery but is rather a way of ensuring that our minds are not playing tricks on us. Mathematics, like science is fallible. It grows by the criticism and correction of theories which are never entirely free of ambiguity or the possibility of error. Statements of proof lead to searches for counterexamples. New, refined, proofs explain old counterexamples; new counterexamples undermine old proofs, and so on. This is akin to the cognitive view of learning, in which internal schemata are constantly revised to take account of new evidence. Learners are active builders of their own conceptions and competencies, and mathematics instruction is a context for stimulating and guiding these builders in their own constructive processes.

Fallibilism argues that the formal-logical account of mathematics is a fiction, divorced from the way mathematics is created and understood. Mathematics is a human activity and mathematics itself is to be found in the practice of mathematics. The traditional presentation of mathematics in textbooks and elsewhere is difficult to follow because the presentation is usually backward, with the discovery process eliminated from the description. After the theorem and the proof have been discovered, the whole verbal and symbolic presentation is re-arranged, polished and reorganised by teachers and authors to fit into a logical-deductive structure. This approach was first criticised by George Polya (Polya 1945) who proposed an investigative, intuitive method in its place.

Fallibilism does not provide a basis for mathematical objectivity and truth. However, this may be a bonus in terms of mathematical education. The denial of objectivity places the emphasis on individual experience. There is less stress on rigour, with a corresponding emphasis on approximation and common sense. If it is accepted that proof is relative, then it follows that, at different stages of the educational process, different modes of justification are required. These can range from the very intuitive to the very rigorous. A further implication of this view is that mathematics derives its importance from its usefulness rather than from the facility it provides for conducting arguments at the highest possible level of abstraction. Mathematics should therefore be taught through its application to relevant and suitable problems.

The view that the strictly 'logical' approach to mathematics is an educational impediment has gained increasing support in recent years. There have been many suggestions for a more investigative, exploratory approach focussing on applications, problem-solving and modelling (e.g. Papert, 1980; Cockcroft, 1982; Higgo, 1985). The reduced emphasis on formal geometry in the new Junior Cycle Mathematics syllabi (Department of Education, 1987) is an example of this trend. Computer use may be a very important factor in promoting such an open, experiential approach to teaching mathematics.

1.2 Outline of the study

The purpose of this study was to develop an alternative approach to introductory algebra for children aged 13 or 14. The approach was based on the premise that algebra is not simply a set of techniques for manipulating symbols; it is a set of techniques for solving problems. The ability to construct equations to represent problem situations is one of these techniques. The present study concentrated on story problems, as these are generally the only opportunity that students of this age have to apply algebra to real situations. Mayer (1982) maintains that nearly all story problems consist of combinations of assignment propositions, relational propositions, question propositions and relevant facts. Mayer (1982) found that students had particular difficulty in understanding relational propositions and in formulating expressions to represent such propositions. The present study dealt exclusively with the formulation of algebraic expressions to represent relational propositions in word problems.

In teaching algebra through word problems, consideration must be given to the type of problems to be used. It is possible to vary the context of word problems while maintaining mathematical content. Context variables are important as they indicate the development of the ability to extract essential mathematical information from non-mathematical information. Contexts may be varied across a number of dimensions including abstract/concrete, factual/hypothetical, fantasy/reality etc. While the purpose of using word problems is to develop the ability to apply algebra to concrete problems, current textbooks use more abstract (numerical) problems than concrete ones. In addition, the concrete contexts used are often quite technical and not of immediate interest to students in this age group, e.g. distance/rate/time. A review of all current textbooks for Intermediate Certificate Syllabus B (an Irish public examination taken at age 15/16) revealed that 50% of algebra word problems are set in abstract contexts. For example, "find two consecutive numbers whose sum is 27". A further 40% are set in the contexts of age, money, area, volume and speed. For example "Mary is one year older than Sue. The sum of their ages is 27. How old is each?" The present study examined the effect of manipulating the abstract/concrete variable of word problems. The concrete contexts were designed to be of interest to the students involved; girls aged 13 or 14 years.

Algebra is directly related to arithmetic. Algebraic expressions and procedures are a generalisation and symbolisation of familiar arithmetical ideas and operations. However, many students, while proficient with arithmetic techniques and concepts, have great difficulty in thinking about number in general terms (i.e. thinking algebraically). The present study attempted to build on the students' arithmetic ability by relating algebraic expressions to their existing knowledge of arithmetic. This was achieved by using an electronic spreadsheet. This allowed problems to be represented arithmetically, but required the construction of algebraic expressions for their solution.

Computer use can be very motivating for students in this age group and there are many software packages which provide interesting mathematical environments. These include graphics packages, statistics packages, programming languages, spreadsheets, algebraic symbol manipulating packages etc. At present, the use of computers in mathematics teaching is limited and consists mainly of drill in basic skills. In algebra learning, computers can provide an environment which is rich in variables, interactive, and engaging for children. However, there is no readily available software written specifically for the purpose of teaching students to generate algebraic expressions for verbally stated problems. The present study shows how computer aided instruction for this purpose can be developed around a readily available piece of software, a spreadsheet.

All of the subjects initially received the same brief instruction on spreadsheet concepts and use. They were then split into two groups. The treatment for each group was identical in every way except for the contexts in which the exercises were set. The experimental environment was kept as close as possible to a natural school setting. For one experimental group, problems were set in concrete contexts with which students of this age and background could identify. These included sport, clothes etc. The other experimental group studied problems of identical mathematical structure, set in abstract (i.e. purely numerical) contexts. The dependent variable was a test which required students to translate problem statements into algebraic expressions. The problems used in each treatment were all taken directly from, or adapted from, textbooks and previous examination papers at Intermediate Certificate (Syllabus B) level.

The principal objectives were a) to determine the effectiveness of spreadsheet use on the ability to construct formal algebraic expressions for relational propositions in word problems and b) to examine the effect of manipulating problem contexts on the achievement of this ability. Further objectives were to determine the relationship, if any, between the experimental treatments and a) attitude to mathematics, b) attitude to computers and c) general numerical ability. The investigation of these questions will contribute towards the effective use of computers in whole-class instruction and towards the development of a mathematics curriculum which is relevant to current and future technological environments.

Chapter 2: Review of the Literature

2.1. The Difficulty of Teaching Algebra Word Problems

Algebra word problems are regarded by many authors as an extremely difficult area of the mathematics curriculum (e.g. Ewing, 1984; Feurzeig, 1986; Thaeler, 1986; Thomas, 1987).

> "Algebra word problems have been a source of consternation to generations of students" (Berger & Wilde 1987, p. 123).

> "The solution of word problems is one of the most difficult subjects in school algebra" (Feurzeig 1986, p. 245).

Brown et al. (1988), reporting on the results of the fourth National Assessment of Educational Progress administered in 1986, noted that items requiring eleventh grade students to translate situations into algebraic expressions or equations were very badly answered. Only half of the students with two years of algebra chose the correct equation to describe the following situation;

> "The number of chairs (C) is twice the number of students (S)"

They also had serious difficulties when required to construct expressions for problems of the following type:

> "Jim has 5 fewer marbles than Karen. If Jim has M marbles, how many has Karen?"

The general finding of this report was that, although students were often able to use routine procedures in school mathematics, they appeared not to have gained an understanding of those procedures. Students were generally not able to apply knowledge in problem-solving situations and did not appear to understand many of the structures underlying basic mathematical concepts and skills.

Booth (1984a), in a major study of 13 to 15 year-old children's difficulties with elementary algebra, focussed on algebra as 'generalised arithmetic'. This is the ability to use letters for numbers and to write general statements representing arithmetical rules and operations. Booth's (1984a) study did not concern itself with solving equations, factorising or other 'manipulative' skills normally emphasised in introductory algebra courses.

Booth's research (1984a, 1984b) revealed that very many children had difficulty in grasping the notion of a letter as a generalised number, in the formalisation and symbolisation of method and in their understanding of convention and notation. Some particular difficulties in the understanding of letters were a) assigning a numerical value to each letter from the outset, b) ignoring letters, c) interpreting letters as shorthand for objects or as objects in their own right and d) regarding letters as specific but unknown numbers, i.e. failure to realise that a letter can represent a range of unspecified values. Booth (1984a) also found that children often see no need for brackets in expressions and consequently do not use them. This is because they consider the context of the problem to define the order of operations. It was also noted that children were unwilling to accept an algebraic expression as an 'answer'.

Booth (1984a) considered that introductory algebra instruction required two main problems to be addressed, namely its conceptual difficulty and the justification of its use. It was considered that if its use were not justified, children would make little effort to try to come to terms with its conceptual difficulty. With these considerations in mind, instruction was designed around a notional mathematics machine. The 'machine' performed all calculations, but had to be 'programmed' by the children. The use of this model allowed attention to be focussed on the need to make problem-solving procedures explicit. It also justified the introduction of letters as 'number locations' in the 'machine'. The teaching programme was designed to address the problem of indeterminate answers by specifically considering the kind of answer which the mathematics machine would require. This provided a rationale for algebraic expressions as 'answers'. The students performed the actual calculations themselves with calculators. This approach, which was very similar to using a paper spreadsheet (as opposed to an electronic one):

> "was effective in improving childrens' general level of understanding in elementary algebra, as measured by a sustained improvement in performance on test items" (Booth 1984a, p. 84).

In particular, a gain was noted in the interpretation of letters in expressions (particularly with second-year groups). This suggests that the use of a real mathematics machine, i.e. an electronic spreadsheet, may help novice algebra students to deal with variables and to formulate algebraic expressions.

Rosnick and Clement (1980) examined the interface between mathematical symbols and verbal descriptions of real world problems. They asked groups of engineering and social science majors to translate the following English sentence into an equation:

> "There are six times as many students as professors at a university. Use S for the number of students and P for the number of Professors". (Rosnick and Clement 1980, p.4)

They found that 37% of engineering students and 57% of social science students were unable to answer this problem correctly. Two thirds of the errors in each case took the form of a reversed equation (i.e. 6S=P rather than 6P=S). For slightly harder problems the results were even worse. On the basis of subsequent interviews, they concluded that these errors were not due to careless misinterpretations of the problems but revealed:

> "disturbing difficulties in the students' conceptualization of the basic ideas of equation and variable" (Rosnick and Clement 1980, p.5).

Follow-up tutoring and taped interviews demonstrated that these misconceptions were very resilient. The conclusion reached was that:

> "Fundamental concepts of variable and equation should not be treated lightly in high schools and colleges, nor should we assume that our students

will develop the appropriate concepts by osmosis" (Rosnick & Clement 1980, p. 23).

Rosnick and Clement (1980) conclude that the concept of a letter standing for a number is a fairly abstract one and, for that reason, a very difficult one to teach. They noted the need for the development of specific teaching strategies to address this problem. They stress the need for conceptual development rather than the development of manipulative skills.

Rosnick and Clement (1980) noted that students who make the "reversed equation" error describe the situation in terms of "students", while those who write the correct equation usually say "number of students". Fisher (1988), in an experiment with college students, attempted to exploit this finding by using a more explicit notational system. The notational system involved the use of subscripts, i.e. N_S rather than S for 'number of students'. This was intended to indicate that the variables represented 'numbers of objects' rather than the objects themselves. Although the number of students involved in this study was small (58), the results were very discouraging. Far more errors were made by the group using the explicit notation than by the control group.

In a subsequent study, Clement (1982), conducted clinical interviews in which subjects were asked to think aloud as they worked on the 'Students & Professors' and similar problems. Semantic content was also manipulated to give less guidance (everybody knows that colleges have more students than professors). Analysis of the thinking-aloud protocols revealed two main errors. In the first of these, the subjects assumed that the order of key words in the problem simply mapped directly on to the order of symbols as they appeared in the equation. In the second, the subjects appeared to use the letter S as a label for the word 'student', rather than as a variable representing an unspecified quantity.

Rosnick (1982) discovered that, for many students, an 'algebraic' letter is identified with an entire, complex, overly-generalised referent rather that with a particular quantitative attribute of that referent. For example, in a problem which calls for the creation of a variable, X, to represent the number of books bought, many students will think of X as standing for 'bookness'. Thus, at different stages of working on the problem they may regard X as the name of a book, a physical book, and other qualitative aspects of books. Rosnick (1982) concluded that it was at the interface between the semantic content of a problem and its algebraic representation that students' skills were most lacking.

Sims-Knight & Kaput (1983) examined how students' vulnerable understanding of algebraic syntax can be overridden by natural language syntax and rules of reference. They hypothesised that students have difficulty with algebraic word problems because they map their language-based and image-based representations of quantitative relationships inappropriately onto the algebraic symbol system. It was expected that familiar quantitative relationships (e.g. 5 finger pieces to every palm piece in a glove) would produce more errors. This was tested by manipulating the degree to which various quantitative relationships could be embedded in a natural representation system. Students were more successful when conflict between natural representation and the rules and syntax of algebra were minimised, as in the case of unfamiliar quantitative relationships. Students' difficulties arose from the application of natural language to the formalisms of algebra, based on the fact that they share several symbols (=, numerals etc.) but do not share underlying rules of reference or syntax. They concluded that skill in translating from a natural representation system to an abstract symbol system is differentiable from quantitative understanding of numerical concepts and understanding of algebraic symbols. The current practice of simply telling students that letters stand for numbers, and then providing them with plenty of context-free manipulative exercises followed by some word problems, does not adequately develop this translation capability.

Davis (1984) has applied a special kind of knowledge representation system called a frame (originally developed in studies of reading comprehension) to the problem of equation reversal errors. There are several steps involved in using frames:

1. Input: Input generally offers many cues, only some of which need to be attended to. A major difference between novices and experts is their choice of cues to attend to, versus cues to ignore.

2. Retrieval: The input cues trigger the retrieval of a seemingly appropriate frame from the huge number stored in long term memory. A student can fail at this stage if a) he/she does not possess the required frame, b) the relevant frame in memory is incomplete or c) the retrieval mechanism does not locate the correct frame. If a frame is not retrieved the search may be terminated.

3. Mapping input data into frame variables: The specific data from the input must be used to instantiate the frame variables. If there is no appropriate input for some of the frame variables, a frame will typically make a default evaluation by drawing on past typical situations.

4. Once an instantiated frame has been judged acceptable, nearly all subsequent processing uses this instantiated frame as a data base. The original 'primitive' data is thereafter ignored.

5. Frame modification: When modifications are made, the new (modified) frame is added to memory but the previous version is not deleted.

The "reversal" errors described by Clement and colleagues are explained in terms of retrieval of a "Labels" frame rather than the appropriate numerical-variables frame. The labels frame is used for dealing with situations such as stating the relationship between feet and inches:

$12I = F$ (12 Inches = 1 Foot)

but this is inappropriate if 'i' is the <u>number</u> of inches and 'f' is the <u>number</u> of feet. In this case the correct equation is:

$i = 12f$ (No. of inches = No. of feet X 12)

Both frames have legitimate uses but they are often confused and used inappropriately. Davis (1984) concludes that, in addition to the formal level of algorithms, definitions, notations etc., instructional programmes are needed at the experiential level. Such programmes should connect with the students' existing representation structures and help to build, revise and extend these structures by the process of 'assembly'.

There have been a number of studies which highlight the fact that many students err on mathematical problems because of difficulties in the comprehension stage of problem-solving rather than the solution phase (e.g. Lewis & Mayer, 1987; De Corte, Verschaffel & De Win, 1985). It has been found that students have particular difficulty understanding and representing statements which specify a relationship between two variables, e.g. "a man is three times as old as his son" (Mayer 1982; Reed, Dempster & Ettinger 1985). This is not simply due to reading difficulties. It has been found that variations in readability of a few grades have no significant effect (Paul, Nibbelink & Hoover, 1986). Mayer (1981) developed a system for classifying algebra story problems by compiling and categorising 1097 such problems from 10 algebra textbooks used in public schools in California. Mayer (1981) found eight major families of formulas, e.g. distance/rate/time, cost/unit cost etc. Within each family there were several categories of problems. Some of these categories were "simple" as they directly involved a well known source formula. Others which required the source formula to be used in a larger equation, were "complex". Most of the story problems in the textbooks were complex (82%). The final part of the analysis explored a more detailed level of classification within each category - classification by template. A template refers to the specific propositional structure and story line of a problem. There were three major types of propositions:

1. Assignment of a value to a variable (e.g. Length=2)

2. Relation between two variables (e.g. Length=2xBreadth)

3. Assignment of a variable to an unknown value (e.g. Length=X)

Problems belong to the same template if they share the same story line and the same list of propositions, regardless of the values assigned to each variable, the actual relation assigned to a pair of variables, or which variable is the unknown. Approximately 90 templates were identified in the sample, with about half of these occurring at least 10 times. In a follow-up study, Mayer (1982) found that recall was poorer for relational propositions than for assignments. When the subjects (college freshmen) were asked to construct their own word problems, they rarely made use of relational propositions. This suggests that students have difficulty in interpreting relational propositions. Mayer (1982) found that relational statements were often changed to assignments. For example, "has 3 marbles more than" might become "has 3 marbles". A dynamic computer representation may provide a better model for such a relationship than a static format such as an algebraic equation or expression.

In a later study, Lewis and Mayer (1987) examined college students' difficulties in comprehending relational statements in arithmetic word problems. They found that errors were more likely when the required operation was inconsistent with the statement's relational term, e.g. having to subtract when the relational term was "more than". They suggest that many errors are due to difficulties in the comprehension stage of problem-solving rather than the solution phase. They recommend that students need more training in skills of problem representation, particularly representation of relational statements.

Reed, Dempster and Ettinger (1985), using Mayer's (1981) classification, also found that students had most difficulty in specifying the relations between variables. They suggested that:

> "practice in constructing tables might increase the likelihood that students would express the correct relations among the variables before they attempt to formulate an equation" (Reed, Dempster & Ettinger 1985, p. 123).

The purpose of these tables is to assist the students to represent the problem and to focus on the algebraic expressions which form the equation. The kind of tables suggested are exactly the same as the tables constructed in a spreadsheet. A sample problem and table are shown below (Reed, Dempster & Ettinger 1985, p. 124):

Problem

"A car travelling at a speed of 30 mph left a certain place at 10.00 a.m. At 11.30 a.m. another car departed from the same place at 40 mph and travelled the same route. In how many hours will the second car overtake the first car?"

Table

Car	Distance (miles)	Rate (mph)	Time (hr)
First	30(t + 1.5)	30	t + 1.5
Second	40 x t	40	t

This table is exactly equivalent to a spreadsheet representation of the problem. The "cells" for the distance travelled by each car are connected by formulas to the "cell" for the time (t) of the second car. In a spreadsheet, a value would be substituted for the "cell" containing t, and the distance "cells" would then be automatically calculated. The value of t could be varied until the two distances were equal. This is precisely how spreadsheets were used in the present study.

2.2 The Effect of Varying Problem Contexts

A great deal of research has been done on the effect of varying the contexts of mathematical word problems. However, a lot of studies examine either student preferences for different contexts, or the relative difficulty of problems in different contextual settings. Comparatively few studies examine the suitability of abstract and concrete contexts for promoting learning. With verbal problems, contexts are only "concrete" in the sense that they refer to real objects and real situations. This is not quite the same as the meaning usually associated with the term "concrete problem" (i.e. a manipulative task).

Performance is usually better for verbal problems set in concrete contexts and in contexts for which the participants express a preference. Differences have been found on several measures including learning, recall, recognition and comprehension (Schwanenflugel & Shoben, 1983). However, this is not always the case. There are some studies in which no great difference was found (e.g. Travers, 1967; Cohen and Carry, 1978; Ross and Bush, 1980) and others in which abstract contexts have been found to be more favourable. For example, Ewing (1984) found that numeric problems were easier than coin and distance problems.

Holtan (1964) prepared material on mathematical word problems for ninth grade students in four different contexts. Students who used material which was related to their expressed interests did significantly better on posttest and on a retention test three weeks later. Travers (1967), in a similar study with ninth grade students, found no significant difference in problem-solving success between preferred and non-preferred problem contexts. Higher achievers, however, had fewer preferences overall than lower achievers. Similarly, Cohen and Carry (1978) did not find any relationship between problem-solving success and the interest preference of the eighth grade students in their study.

Ross and Bush (1980) studied the effects of problem context in a self-instructional probability module for preservice teachers. In one treatment, examples and explanations were abstract, while in the other they were related directly to teaching. It was expected that students would perform better on problems that were similar to those used during instruction. Students taught with abstract examples were expected to be superior on abstract test items. Those taught with education examples were expected to be superior on education-related items. It was found that students in the education treatment obtained higher scores on the education items than they did on the other types (medical and abstract contexts). Those in the abstract treatment did not demonstrate a comparable superiority on abstract items. However, those in the education treatment did not perform better on the overall posttest. In addition, the education treatment did not produce a more positive reaction from students. Ross and Bush (1980) have suggested that these slightly disappointing results may be due to the very short treatments and the limited number of practice items presented. The study involved only 38 subjects and there was pressure on the participants to work rapidly in order to complete the material in one session.

In a follow-up study (Ross 1983) examined the effect of context on five different types of learning, ranging from memory of formulas to far transfer. The subjects were preservice teachers and nursing students, with both groups studying statistical probability. It was hypothesised that contexts adapted to the interests of each group would facilitate assimilation and integration of the rules in memory and that, as a result, subjects would excel on far transfer tasks. Non-adaptive contexts were expected to produce more rigid, specific encoding. This was expected to lead to relatively better performance in remembering specific formulas and procedures than in solving transfer problems. It was found that preservice teachers benefitted from education-related material but not from medical-related material. The nursing students showed the opposite tendency. These effects could not simply be accounted for by increased motivation, as an attitude survey failed to find differences favouring the adaptive contexts. It was found that adaptation effects were relatively strong on transfer items. This was explained by Ross (1983) in terms of meaningful contexts activating relevant past experiences as conceptual anchoring for information.

A further study was conducted by Ross, McCormick and Krisak (1986) with groups of nursing majors and education majors, again studying statistics. In this case individuals opted for problems in one of four different contexts; education, medicine, sport or abstract. Some subjects were given problems in the context of their first choice while others were given their last choice. It was found that achievement was better for first-choice contexts. Those receiving problems in their least favoured context did particularly poorly. It was further noted that the context variation did not have quite as much effect with the nursing group who were older and more advanced academically. This might indicate that context is particularly important for younger, less mature, students such as those in the present study.

The interaction of problem context with subjects' age was studied by Caldwell and Goldin (1987). They examined the effect of the context of algebraic word problems with both junior and senior high school students. Concrete problems were significantly less difficult than abstract problems at both junior and senior levels but the differences became smaller with increasing grade level. This suggests that there may be a developmental component involved and supports the use of concrete contexts in introductory courses for junior pupils.

A slightly different approach was used by Ross and Anand (1987). They tailored the context of the problems to the interests of individual students. They used a computer-based strategy to personalise verbal problems for fifth and sixth grade students. This was done by asking students to provide biographical information about themselves and using this information to formulate problems using both print media and computer aided instruction. In two control treatments, concrete (non adaptive) and abstract contexts were used. Results indicated that the personalised contexts were advantageous for solving both conventional problems and transfer problems, for recognising rule procedures and for developing favourable attitudes.

Ross, Anand and Morrison (1988) extended this personalised approach to students of nursing and education. They again found that personalised materials were beneficial across a variety of learning outcomes, as well as for attitudes. They found that individualised contexts had particular benefits for weaker students. High achievers performed relatively well with either personalised or concrete contexts. The lower and middle groups benefited most from personalised contexts.

Other studies have examined the effect of varying word problem contexts along assumed dimensions of interest. Graf & Riddell (1972) and McCarthy (1976) varied problem contexts in terms of male/female activities. For traditional context problems, boys performed better than girls. When the contexts were made relevant to both sexes, however, this advantage was not nearly as great. Christensen (1980) explored the fantasy/reality dimension of context with grade nine students. Students were slightly more successful on fantasy problems. This study also found that weaker students had stronger preferences.

There is some disagreement concerning the relative importance of reading ability in the solution of mathematical word problems. Some studies report that reading ability is a major factor in solving arithmetic word problems (e.g. Aiken, 1972; Ballew & Cunningham, 1982). This may be due to poor language skills impeding understanding of the problem rather than interfering with the solution process. Others contend that increased reading ability does not result in a significant improvement (e.g. Paul, Nibbelink & Hoover, 1986). Being able to explain what one reads will not necessarily help to determine a course of action leading to a solution. In addition, fluent reading does not guarantee awareness of the precise meaning of the question to be addressed. Muth (1984) found that the syntactic complexity of arithmetic word problems had no effect but that the presence of extraneous information in the problem statement had a negative effect. Moyer and colleagues (Moyer, Moyer et al., 1984; Moyer, Sowder et al., 1984) compared the effect of both telegraphic and drawn formats with conventional verbal formats for simple arithmetic problems. No difference was found between the verbal and telegraphic formats. The drawn format did produce a significant effect, particularly for those with lower reading abilities. This may have been because the drawn format facilitated semantic processing and reduced the burden on working memory for poorer readers. It seems that factors other than reading far outweigh the importance of reading ability.

Developmental theory and modern information theories would indicate that concrete problems should be more easily assimilated and solved by children such as those in the present study. Developmental theory suggests that competence with abstract problems, theories and hypotheses is limited to those who have reached the stage of formal operations. A developmental model would therefore suggest that abstract problems would be more difficult for young children (Goldin & Caldwell 1979).

Information processing theories suggest that a crucial first stage in problem-solving is the construction of an internal representation of the problem (Briars & Larkin, 1984). The difficulty of a problem may be directly related to the complexity of constructing such an internal representation. Better performance on concrete problems in familiar contexts can be explained in terms of a dual representation model; a verbal system and an imaginal system. Differences occur because of the greater availability of the imaginal system for concrete problems. The generation of internal representations should be less difficult for concrete problems in familiar contexts than for abstract problems (Caldwell & Goldin 1987).

An alternative model for solving word problems is to use a 'direct translation' strategy. Darch et al. (1984) used such an approach with arithmetic word problems on skill-deficient fourth graders. They used an explicit word-matching strategy to translate concrete problems into mathematical forms with students from grade 4 through grade 6. The strategy consisted of a number of rules: if you use the same number again and again, you multiply; if you are given the big number you divide; if you do not know the big number you multiply etc. It was found that this method was effective compared to a method compounded from four basal texts adopted by the state of Oregon. They remarked that the basal texts seemed to view mathematical word problems as a vehicle to teach the more generalised ability of problem-solving, encouraging the students to offer their own solutions and to discuss each others' proposed solutions. On the other hand, the subjects in the explicit treatment were taught clearly articulated strategies for translating word problems and, in the initial stages, the teacher modelled each step of the process. They concluded that

> "a program constructed to teach prerequisite skills in a sequential manner and, more importantly, explicitly model each step in the translation process is significantly more effective than approaches advocated in teachers' guides to currently used basal texts"
> (Darch et al. 1984, p.358).

A 'direct translation' approach, embodied in a computer program, was also used by Paige and Simon (1966). They used the computer program as a model of human behaviour in solving algebra word problems. The program, STUDENT, was capable of solving algebra word problems by directly translating the problem text into a series of equations. Direct translation models suggest that concrete problems are more difficult than abstract ones. This is because concrete problems must undergo extra substitutions to reference the number of objects described in the question, rather than the objects themselves (Goldin & Caldwell 1979).

However, re'ying on direct translation processes is not recommended by modern information processing models of learning. Human subjects differ in the extent to which they depend on direct processes, and generally only use this approach for non-standard or unfamiliar problems. Human subjects usually make extensive use of auxilliary cues, physical representations etc.

2.3 The Effectiveness of Computer-Based Instruction

There has been an enormous amount of discussion of the role of computers in the educational system. In 1986 there were over 7000 titles containing the word computer in the ERIC educational database (Diem 1986). By 1985 there were over 7700 educational software packages available in the USA and Canada, with up to 2000 packages being added per year (Dudley-Marling & Owston, 1987). Becker (1987) estimated that there were more than one million computers in K-12 schools in the USA and that the typical high school had twenty computers. Many claims have been made that computers will radically change both the curriculum and methods of instruction but this has not happened yet. Such claims have been made in the past for other educational technologies, but computers do have certain attributes which are unique, notably their interactive nature and their flexibility.

There have been many studies which have attempted to evaluate the effectiveness of CAI (computer aided instruction) compared to traditional forms of instruction. Burns and Bozeman (1981) carried out a meta-analysis of mathematics teaching using CAI in secondary and elementary schools and found a "significant enhancement of learning". Kulik, Kulik and Cohen (1980), in a meta-analysis of 59 CAI studies, found an overall positive effect in both achievement and attitude. They also found a significant reduction in instruction time, compared to traditional teaching methods. In a later meta-analysis of 51 CAI studies at second level (Kulik, Bangert and Williams 1983) similar results were reported. Hasselbring (1986), in a review of research on the effectiveness of CAI conducted over the past twenty years, concludes that students receiving CAI achieve more in less time and have improved attitudes, regardless of the type of CAI used.

These reports seem to indicate that claims for the effectiveness of CAI are justified. Clark (1983) has, however, cast some doubt on the validity of the analyses described above and claims to have found evidence that

> "there are no learning benefits to be gained from employing any specific medium to deliver instruction" (Clark 1983, p.445).

Clark (1983) suggests that the effects found in these meta-analyses are due to instructional method differences between treatments and/or the novelty effect of newer media. These hypotheses are supported by evidence that the positive effect is reduced when the same instructor produces all treatments and that the gains tend to diminish as students become familiar with the new medium. Clark further suggests that the reduction in learning time may be due to the extra effort that students (particularly younger students) are prepared to invest in newer media programmes.

In a later study, Clark (1985) examined a 30% random sample of the studies utilized by Kulik and his colleagues. He found that 75% of these contained serious design flaws. These flaws included failure to control content, amount of instruction and instructional method. Clark then carried out a re-analysis of sub-groups of these studies and found that

> "when adequate method and content controls were applied, significantly fewer outcomes favoured CAI". (Clark 1985, p. 257)

There have also been reports of positive effects on attitude and motivation and of reductions in instruction time for computer based methods (Kulik, Kulik & Cohen 1980; Burns & Bozeman 1981; Swenson & Anderson 1982; Kulik, Bangert & Williams 1983; Roblyer 1985; Hasselbring 1986; Levin et al., 1986; Seymour et al. 1987). Seymour et al. (1987) had two groups of children complete identical tasks using either a computer or pencil-and-paper. The computer group expressed an extremely strong preference for further practice and they rated the learning as more interesting. They also thought that they had achieved more, although this impression was not substantiated by performance. They concluded that

> "factors related to a feeling of self-control over the medium, such as operating the machine itself, automatic self pacing......and immediate responsiveness..... contributed greatly to the computer's appeal. There may have been some novelty effect but this should have been minimized by the subjects' considerable past experience with computers." (Seymour et al. 1987, p.22)

Such reports give grounds for assuming that computer use may be a factor in motivating students to invest more interest and effort in mathematical work.

Th bulk of the evidence suggests that computer use, per se, will not lead to increased learning but that it is extremely motivating for many students. Clark (1985) suggests that CAI may also be more efficient and cost effective than traditional methods. One such study of cost-effectiveness (Levin et al., 1986) analysed eight, adequately designed, CAI interventions. These were all drill-and-practice programs in the areas of mathematics and reading. They found that CAI was "superior in cost-effectiveness in imparting mathematics and reading achievement to extending the school day or reducing class size" (Levin et al. 1986, p.31)

The bulk of the evidence in the evaluation studies described above comes from drill-and-practice and tutorial programs. These are generally used by the learner in isolation, with little or no teacher intervention. Hasselbring (1986) has noted that CAI with no teacher interaction is much less effective than CAI in which teacher interaction is an integral part of the instruction.

There is a case for designing computer-based instruction in which the learners use computers under the direction of a teacher. The choice of software will depend on what type of skills are to be taught but, from the point of view of learner control and flexibilty of use, it can be argued that software with the least embedded content offers the best prospect. A group of leading British mathematics educators (Ball et al., 1987) has recently identified the following list of characteristics of good software:

1. It is powerful: learners encounter important ideas and are encouraged to use these ideas to solve problems.

2. It is accessible: Even beginners can make it work and find aspects of it they wish to develop

3. It is controlled by the learner: Pupils can decide what problems to undertake and what constitutes a solution to these problems. (Ball et al. 1987, p.25)

Software which meets these requirements includes programming languages, data-handling packages, graph plotters and spreadsheets. Of these, spreadsheets are the most promising in mathematics education. They embody a mathematical microworld and are flexible, easy to use and readily available.

2.4 Modes of Computer Use in Education

There have been numerous suggestions for classifying modes of classroom computer use (Manion 1985, Whiting 1985, Howe 1986, Mills 1987, Adams 1988). Mills (1987) used a categorisation based roughly on the amount of control exercised by the learner. Increasing learner control corresponds to decreasing software content. Demonstration and drill-and-practice programs are very content-specific while, at the other end of the scale, word-processors, information handling packages and spreadsheets are content-free. The following classification is based on that of Mills (1987):

A. Instructional Mode

1. Demonstration/Textbook mode
2. Drill and Practice
3. Programmed learning - linear Mode
4. Programmed learning - branching Mode

B. Revelatory Mode

5. Application programs
6. Case studies and Simulations
7. Educational games

C. Exploratory Mode

8. Problem-solving
9. Mathematical Environments (e.g. LOGO)
10. Content-Free Packages (e.g. Spreadsheets)

These are not rigid categories, and some packages may employ more than one mode of interaction (i.e. a tutorial followed by a simulation). Others (e.g. spreadsheets) may be adapted for use in more than one mode (e.g. revelatory and exploratory).

2.4.1 Instructional Mode

Using a computer to deliver a demonstration program in which the user's only interaction is to start, branch or stop the program might be justified if it demonstrates something that cannot be done in a textbook. These programs require no overt interaction and the user's role is as a passive observer.

The majority of mathematics drill-and-practice programs are concerned with basic skills at primary school level (Howe 1986). These are based on a reinforcement model of learning and require more user interaction than demonstration programs. They are generally used to teach verbal knowledge and their usual purpose is to supplement previous instruction. They do this by allowing students to rehearse and automatize basic skills. They assume that the user has the relevant procedural knowledge for the particular task and simply provide an opportunity for self-paced practice. This is by far the most common mode of CAI in mathematics teaching (Dickey & Kherlopian, 1987).
Drill-and-practice programs are often quite sophisticated in their use of graphics, colour and sound. They often employ a game format and this can be motivating. They usually have no diagnostic capability and only tell the learner that he/she is wrong when an incorrect response is given, without attempting to explain why, or providing any remediation. This is of no benefit to a learner who has a 'bug' in his or her procedural knowledge.

In programmed learning (tutorial) mode, the computer acts as teacher by presenting new information and providing opportunities for the learner to practice. Linear programming is based on the principles of operant conditioning. It is characterised by systematic presentation of very small 'frames' of material which elicit a student reponse, followed by immediate reinforcement. The sequence of 'frames' is predetermined by the program author and it is assumed that the learner's response will be correct. In branching programs, the learner's reponse is used to decide on the subsequent choice of material. They may also provide more sophisticated forms of feedback than the typical "correct/incorrect" of drill-and-practice programs. Conventional tutorial programs contain a range of hints and suggestions for the more predictable learner responses. Newer, 'intelligent', tutors maintain a representation of the learner's current state of knowledge and use this to influence the nature of the learner's interaction with the materials. They contain

> "an explicit representation of the knowledge that the student should acquire, an extensive catalogue of error types and their origin, with related hints and explanations, and a teaching model which decides when to interrupt, what kind of help to give, what type of problem to give next and when to move on" (Howe 1986, p.27).

Intelligent tutors have been used successfully in knowledge domains which are formally organised and dependent on logical analysis, such as mathematics and the physical sciences (Allinson & Hammond, 1990). Intelligent mathematics tutors are a very recent development and existed only in prototype form as recently as 1986 (Howe, 1986). It may be some time before they become available to schools.

The principal advantages of the instructional modes of CAI described above are that the learning is self-paced and that feedback, however rudimentary, is provided. With the exception of 'intelligent' tutors, they are quite easy to program and a number of authoring systems are available to assist non-specialist programmers. This relative ease of production may account for their predominance in the educational software market.

Kozma (1987), however, argues that tutorial instruction performs certain cognitive functions for the learner. These include specifying objectives, giving rules, providing examples, asking questions etc. He suggests that this short-circuits the learning process by replacing cognitive strategies such as determining goals, posing and testing hypotheses and inferring rules. The categories of software discussed below place more control in the hands of the learner and are less susceptible to such criticisms.

2.4.2 Revelatory Mode

A mathematical applications program carries out some operation for the user, such as integrating, plotting a graph etc. It embodies an algorithm or algorithms that solve particular types of problems. MuMath, which contains a range of symbolic mathematical functions is an example. The purpose is not to teach the user how to carry out the embodied algorithm but to relieve the burden of calculation and symbolic manipulation.

Held (1988) tested the effect of using graphical and symbol-manipulating software to perform routine manipulations in a fifteen week introductory calculus course. This reversed the normal pattern of instruction in algebra, which begins with the development of algorithmic skills (Berger & Wilde, 1987). The experimental group used a computer to perform the necessary algorithms and spent only the last three weeks on skill development. This group showed better understanding of course concepts and performed almost as well on a test of routine skills as did the comparison group who had practised skills for the full fifteen weeks. While this experiment was limited in its scope, it shows how an applications program can be used to facilitate a "concepts first" teaching strategy. This challenges the traditional belief that concepts cannot be adequately understood without prior mastery of basic skills.

In a simulation, the computer is used to model some real situation and the learner interacts with the model. This is generally done by varying some specified factors and observing the consequent changes in other factors. This implies more activity by the user than any of the modes discussed above, giving the learner a large measure of control over the activity. Simulations concentrate on certain key elements of the real situation, thus allowing the whole package to be programmed on the computer. Case studies typically involve additional non-computer material. The assumption is that interacting with the simulation will give the learner some insight into the underlying rules encoded in the model. Simulations are generally designed for use after skills, concepts and rules have been learned. Their usual purpose is to integrate these newly learned ideas into a meaningful context.

Educational games are often simulations and their distinguishing feature is their highly motivating format. This type of software has been most common in the physical and biological sciences in higher education and has had little impact so far at second level (Howe 1986).

2.4.3 Exploratory mode

Content-free general-purpose software does not overtly engage in instruction. This type of software is designed to assist the learner in tasks such as writing (word processors), analysing data (information handling packages), calculating (spreadsheets) and problem-solving (e.g. programming languages). For example, information handling packages allow learners to compile their own databases or to interrogate other databases in a flexible way. The important learning taking place may be the process of the interrogation rather than the product. A learner interrogating a database may discover useful factual information but will also learn to form and test hypotheses, make generalisations etc. Underwood (1986) investigated the use of computers in developing children's classificatory abilities using an information handling package. The results showed gains in both the computer group and the control group, but with much larger gains in the computer group. The gains were explained by increased motivation for the computer group.

This type of software has not been designed for education but its lack of specific content allows it to be adapted and used in a number of ways.

"It is not finished, complete and targeted to a particular niche in the school curriculum, but instead it is open to curriculum developers, teachers and students to find their own ways of using it according to their own goals and local needs" (DiSessa 1987, p. 359).

2.4.3.1 Programming Languages

The exploratory approach, using computer programming languages (e.g. LOGO) has been advocated by numerous authorities in mathematics education (Wiechers, 1974; Papert, 1980; Higgo, 1985; Howe, 1986; National Mathematics and Statistics CBL Review Panel, 1986; Ball et al., 1987). Papert (1980) has argued that learning computer programming can increase mathematical problem-solving skills. Papert (1980) advocates natural discovery with a minimum of teacher guidance and provides anecdotes of children spontaneously discovering mathematical principles. Papert (1980) claims that children using LOGO will learn mathematics as easily and as naturally as they learn their native language.

Howe (1986) argues that Papert's case is based on a rational analysis and is not supported by empirical evidence. He suggests that while students in higher education should be capable of exploring a system in an organised way, younger students are unlikely to be able to do this unaided. An alternative (Howe et al. 1982) is to provide worksheets to help the learner interact with the system in a structured way. This however, reduces somewhat the degree of learner control. O'Shea and Self (1983) suggest that while Papert's approach is superior when the teacher pupil ratio is very favourable, economic constraints will normally dictate some sort of systematic support such as worksheets.

Programming is often assumed to promote exploration, to encourage students to reflect upon their own thought processes, and to foster general problem-solving skills which will be transferred across problem domains. These skills include planning, problem decomposition, hypothesis generation, hypothesis testing and debugging. However, much of the evidence presented in favour of such benefits is in the form of rational analysis and anecdotal accounts.

Attempts to show that learning programming affects general, domain-independent, problem-solving skills have not been successful. Pea and Kurland (1987) caution against the 'technoromantic' notion that experience with a powerful symbolic system will automatically benefit higher order cognitive skills. They are sceptical about the possibility of spontaneous transfer, given that even adult thinkers often do not recognise connections between isomorphic problems. In a substantial review of the literature on programming they conclude that:

> "there are no substantial studies to support the claim that programming promotes mathematical rigour .... and there are no substantial reports that it aids children's mathematical exploration" (Pea and Kurland 1987, p.172).

Pea & Kurland (1983, 1984a, 1984b) found no measurable influence on the planning ability of students who had studied LOGO for one year.

> "Learning thinking skills and how to plan is not intrinsically guaranteed by the LOGO programming environment: it must be supported by teachers who, tacitly or explicitly, know how to foster such skills through a judicious use of examples, student projects and direct instruction" (Pea, Kurland & Hawkins, 1985, p.212)

Further studies by Howe et al. (1979) and Clements (1985) failed to find significant differences in either logical thinking or cognitive development among children who learned LOGO.

There has been some success, however, in using programming as a vehicle for learning more modest, domain-specific, skills. Noss (1986) examined the extent to which experience with LOGO assisted children to understand elementary algebraic concepts. The study involved 118 children, aged from 8 to 11 years, who learned LOGO for 18 months (approx. 50 hours). It was designed to test whether this experience would assist children to construct formalised algebraic rules and meaningful symbolisations for the concept of a variable. No overt attempt was made to link LOGO with algebra. Nevertheless, the results suggested that extensive exposure to LOGO programming did enable some of the subjects to perceive variables as generalised numbers. Butler and Close (1989) have also identified mathematical skills which benefit directly from LOGO programming. In a project designed to provide enrichment for mathematically more-able children, aged 6 to 12 years, they found that LOGO was particularly beneficial in the areas of geometry and measurement.

Soloway, Lochhead and Clement (1982) examined the effect of learning programming on solving algebra word problems with third level students. An identical word problem was given to two groups of subjects. One group was required to write an equation to represent the problem. The other group was asked to write a BASIC program to do the same. Significantly more students were able to write a program. Another experiment required students to explain an equation embedded in a computer program and a similar equation standing alone. Again, it was found that significantly more students could explain the equation embedded in the program. It was suggested that these effects are due to the procedural, dynamic, nature of equations in programs, which is not obvious in ordinary algebraic equations. Soloway, Lochhead and Clement (1982) have gone as far as to suggest that it would be worthwhile to:

> "redefine much of the early mathematics curricula to be programming-based. That is, teach algebra as an integral part of programming" (Soloway, Lochhead & Clement 1982, p.182).

In summary, there is some justification for thinking that programming may be of benefit in developing understanding in certain restricted mathematical areas. However, learning programming will not automatically develop powerful domain-independent problem-solving skills. Most previous work in this area has concentrated on such broad skills. It may be more fruitful to lower expectations and replace the ideal of attaining mastery of wide-ranging skills with attempts to help children acquire a set of domain specific skills. The ability to construct algebraic expressions is one such domain.

2.4.3.2 Spreadsheets

An important issue that arises in relation to computer use in exploratory mode, is the amount of effort that needs to be invested in learning how to handle the system itself. For example, learning the syntax and semantics of a programming language may require very considerable effort. Boulay (1978) reported that, while LOGO programming promoted understanding of mathematics in trainee teachers, the difficulty of the programming activity often distracted attention from mathematical issues. It has been suggested (Hsiao 1985) that this problem can be alleviated by using more 'friendly' software, such as spreadsheets. These require considerably less effort to learn and use.

Spreadsheet programs are of particular interest to mathematics educators and many have advocated their use (Goodyear 1984, Waddington & Wigley 1985, Howe 1986, Brown 1987, Arad 1987, Luehrmann 1986, Ball et al. 1987; Keeling and Whiteman, 1990). There has, however, been very little work published in this area. Spreadsheets constitute a very rich mathematical environment and have great potential for teaching algebra in second-level schools. In particular, spreadsheets may provide better support for the acquisition of the concept of a variable than is provided by programming languages. Variables have many connotations and uses in computer programming, not all of which are relevant to school mathematics. There are situations in programming in which a variable is like a variable in mathematics:

e.g X:=3 X:=3+5 X:=3*Y X:=Y etc.

There are also situations in which they are unlike variables in mathematics:

e.g X:=X+Y X:=X*Y

These "non-mathematical" uses of variables and the equality sign could be confusing for algebra novices. Variables in programming may also be parameters, a use which is not relevant to elementary algebra and which could also cause confusion. In contrast, the use of spreadsheet cells as variables is so close to traditional algebraic usage that there is little danger of confusion.

In addition a spreadsheet continuously displays the values of variables and expressions whereas a program only displays these at run time. Even then, a program will only display those variables for which output statements have been written. Thus a spreadsheet provides better support for an operative view of variables by highlighting the dynamic nature of their values. There is also the problem, noted by DuBoulay (1980) that using a programming language can tend to shift the initial mathematical problem to a programming problem. This is less likely to occur with spreadsheets as they do not contain input/output or control statements (loops and branches), which are not relevant to elementary algebra and which are difficult to teach to young children. Spreadsheets also constitute a less complex computer environment than programming languages. They have fewer syntax regulations and less non-relevant structures and should require less initial learning. It is expected that favourable results from learning programming would therefore be replicated, and possibly improved, in a spreadsheet-based learning environment.

Dubitsky (1986) explored the use of spreadsheets to enable children with no previous knowledge of algebra to solve algebra word problems. Subjects were academically able sixth graders (aged 11 to 13 years). The purpose of the study was to investigate, through the use of spreadsheets, if students could gain skills in solving algebra word problems without first learning to manipulate algebraic formulas. The children used pre-prepared spreadsheets and were not required to construct their own formulas. They used a 'guess and check' technique and were required to guess the value of all variables. It was found that some children put formulas into the spreadsheets so that they would only have to guess one variable! This was surprising, as it was not expected that the children would be capable of understanding the underlying structure of the spreadsheets. The children were able to manipulate the spreadsheets much better than was expected and were highly motivated by it. However, the population was quite limited and the children were selected from a class of the brightest and most stable population in the school. There was also a very favourable pupil teacher ratio of 2:1.

In another study of spreadsheet use, with third level economics students, Orzech and Shelton (1986) reported that students were able to see how procedures work more clearly when they were freed from manual arithmetic computations.

2.4.4 Recent Developments in CAI

Hypertext and hypermedia are recent innovations in exploratory computer environments. In these environments, windows on a screen are associated with records in a knowledge base and links are provided to these records. This permits exploration in a non-linear and highly interactive manner. Links may be to text, graphical displays, sounds, video etc. These are extremely flexible computer environments but one problem that has been noted is that users can become disorientated because of this very flexibility (Allinson & Hammond, 1990). These developments present interesting new possibilities for education but are not widely available at present.

The recent availability of powerful computer systems has also prompted the development of sophisticated, special purpose, mathematics learning environments. Feurzeig (1986) has described work-in-progress on a computer system which attempts to separate the difficulties in performing the manipulative aspects of algebra from the conceptual and strategic content. The system consists of a) LOGO projects in algebraically rich contexts whose content is compelling for students, b) use of algebra microworlds (bags of marbles) with concrete iconic representations of formal objects and operations, and c) an expert instructional system to aid in performing algebraic operations.

Kaput (1986) has proposed a similar computer system containing simultaneous and interconnected iconic, tabular and graphical representations of word problems. The system is designed to help students overcome "a most pervasive difficulty" of failure to appreciate the idea of a ratio. It is suggested that such failure may be tied to

> "a lack of cognitive models for the critically important idea of variable" (Kaput 1986, p.197).

The current project is primarily concerned with providing a cognitive model (a spreadsheet cell) for the idea of a variable in a much less costly computer environment which is currently available to the majority of second-level students in Ireland.

2.5 Current Usage of Computers in Schools

While there is little dispute concerning the potential of computers to influence education, they have not yet had a major impact in schools (e.g. Alty, 1987). Dickey and Kherlopian (1987), in a survey of computer use in grade 5-9 classrooms, questioned 558 mathematics teachers. They found that 45% had access to computers and used them, 30% had access but did not use them and the rest did not have access. Usage was weighted towards drill-and-practice (83%), games (58%), and tutorials (50%). Content-free software was very poorly represented, with programming being used by a mere 13% and simulations by just 4%. A survey of 52 British secondary schools (Green & Jones, 1986) on computer use in mathematics, revealed that computers were used "very rarely" by 39% and "never" by a further 33%. Drill-and-practice and demonstrations were the most popular modes of use. Spreadsheets were practically never used.

The large-scale use of computers in schools is hampered by, among other things, a shortage of good quality software. The availability of good software had not emerged as an issue in the initial drive to provide computers for schools. Early enthusiasm for computer use in education was largely based on the idea of computers as an object of study in their own right. In the early stages, debate centered on the issues of hardware choice and the appropriateness of various programming languages. More recently, educators have become aware of the potential for using computers to enhance learning in every area of the curriculum. It has also been realised that it is extremely difficult and expensive to produce good quality educational software. Unlike other software, the enormous devlopment costs of educational software are not likely to be offset by a large consumer market. Educational software also has a very short "shelf life", in most cases about 18 months (Walker, 1987). For these reasons, production of educational software is not a very attractive commercial proposition. Most of the important initiatives in this area have come from government funded projects. These have provided a number of models for the production of educational software (e.g. Schoenmaker et al., 1987; Watson, 1987; Walker, 1987). Most models involve the cooperation of various groups of experts including teachers, curriculum developers, educational psychologists, professional programmers, graphic artists etc. These models are very complex and encompass "national policy, curriculum development, teacher training, distribution and dissemination, staff development, technical and educational operations, research and development, financial management and the raising of capital" (Walker 1987, p.317)

Other issues that must be addressed include support services, evaluation, documentation, specification formats, field testing and report mechanisms.

To date, no such models have been implemented in second level education in Ireland. There have been some initiatives by individual teachers, but with very mixed results. This is not surprising as teachers, no matter how enthusiastic, do not have the resources or the training to produce professional quality software. It is also questionable whether teachers should be the producers of their own software. Other professional groups do not write their own software, although they may be involved in its design. There is a similar place for teacher involvement in the production of educational software, particularly in the areas of specification and evaluation.

An alternative model is the adaptation, by teachers, of general-purpose software to meet specific needs. This approach to computer aided learning is within the resources and expertise of many teachers.

Chapter 3: The Hypotheses

This study will examine the following hypotheses:

1. All students who complete a spreadsheet mathematics course will be better able to formulate algebraic expressions for relational propositions in word problems.
2. Those who study problems in concrete contexts will perform better than those who study problems in abstract contexts.
3. The context effect will be greater for those with lower general numerical ability.
4. The context effect will be greater for those with less positive attitudes to mathematics.
5. The context effect will be greater for those with less positive attitudes to computers.

These hypotheses can be justified by reference to theories of cognitive psychology and the empirical research literature.

3.1 Theoretical Background

Cognitive psychology is the science of human information processing (Wessels, 1982) and is currently one of the most rapidly growing areas of psychology. Information processing describes mental endeavour in terms of conversions from inputs to outputs. Cognitive models of learning assume that instruction is mediated by students' use of previously acquired knowledge (Clark & Voogel 1985). It conceives of learning as an active process in which the learner is responsible for attending to, organising, elaborating and encoding material into long term memory. The basic assumption underlying the information processing theories is that human memory is an active organiser and processor of information.

In the area of children's mathematics learning, information processing theory has largely replaced the Piagetian framework as a broad explanatory model (Groen & Kieran 1983). Within the cognitive approach, the teacher's role is to organise activities that facilitate the learner's active construction of his or her own knowledge (Wittrock, 1981). New information is learned by relating it to existing knowledge, so teachers can enhance learning by stimulating learners to connect new information to previously learned material. Hence, it is important to base initial instruction in algebra on a foundation of previous arithmetic knowledge.

While there are many models of learning within the information processing approach, that of Robert Gagne is typical and well accepted (Aronson & Briggs 1983; Sachs 1984; Case & Bereiter 1984). According to Gagne, learning takes place when "a stimulus situation together with the contents of memory affect the learner in such a way that his performance changes from a time before being in that situation to a time after being in it. The change in performance is what leads to the conclusion that learning has occurred" (Gagne 1977, p.5).

Gagne's model postulates a number of internal mental structures through which the flow of information is purposeful and organised. Stimuli from the environment are received by receptors that are sensitive to different forms of energy (light, sound etc.). These receptors send nerve impulses to a sensory register in the central nervous system. From this complete representation of sensory information, a small fraction is kept for use in short term memory and the remainder is lost from the system. Information in short term memory may be coded and stored in long term memory. Once information is in long term memory it may be retrieved to short term memory and thence to the response generator. Alternatively, it may be sent directly to the response generator. The response generator organises the response sequence and guides the effectors. The effectors include all muscles and glands.

Each of these internal structures carries out certain processes during learning. Each process can also be influenced by external events (Gagne 1977). The processes are:

1. Attending

   This is a process whereby the learner concentrates on certain features of a stimulus recorded in the sensory register and ignores others. This process transforms input from the environment and sends the transformed information to short term memory. External events that influence attending include sudden changes in the intensity of stimulation. Selective perception can be activated by verbal instructions, underlining in text, arrows in diagrams etc.

2. Storage in Short-Term Memory

   Information in short-term memory can persist for a limited period only, generally thought to be up to twenty seconds. The amount of information that can be held in short term memory is very limited and is thought to vary from five to nine individual items. While the number of items is very limited, the capacity of short term memory can be enhanced by "chunking". This is where a number of related items are joined together to form one "chunk" of information. For example, the single item "A Minor" would represent a wealth of information about a piece of music to a musician. Storage can be affected externally by assisting the learner to "chunk" information.

3. Encoding

   Encoding is the transfer of information from short term memory to long term memory. In this process the information is transformed from a perceptual mode to a conceptual or meaningful mode. The principal feature of encoded material is that it is semantically organised, that is, it is inherently meaningful. Schemes for encoding may be provided by teachers. These might include the organisation of data into tables, provision of visual images etc.

4. Storage in Long-Term Memory

   There is evidence that material stored in long term memory is permanent and that failure to recall is due to ineffective search and retrieval mechanisms. The storage of learned items in long term memory can be influenced by external events, especially the learning of other items. There is some evidence that interference occurs when specific items directly contradict each other. However, retention of old learning can be enhanced by new learning which is of a complementary nature.

5. Retrieval

This process requires that cues be provided, either by the external conditions or from other memory sources. What is retrieved may be relayed to short term memory or directly to the response generator. If relayed to short term memory it is readily accessible to the learner and may be combined with new inputs to form new entities which may, in turn, be encoded in long term memory. Alternatively it may be transformed to activate the response generator. Externally, the provision of cues is the principal event which affects retrieval.

6. Response generation

This process determines both the form of the reponse (speech, movement etc.) and its sequence and timing. Its overall function is to ensure that an organised performance will occur. Externally, this is usually established by informing the learner of the objective of the learning.

7. Performance

This process results in patterns of external behaviour that can be observed. These may be movements, statements etc.

8. Feedback

This is provided by the learner's observation of the effect of his or her performance. Its main effect is internal, serving to fix the learning, making it permanently available.

The ways in which a learner engages in an act of learning may vary considerably from one individual to another. Such differences are accounted for by two further processes in Gagne's theory; executive control and expectancies. Executive control affects each step in the process of learning by influencing attention, selective perception, choice of encoding scheme etc. This may be influenced externally by instructions and questions. Expectancies represent the specific motivation of learners. What a learner intends to accomplish will influence what is attended to, how it is encoded etc. Verbal communications that inform the learner of the objectives influence the learner's expectancies, as well as all the other internal processes of learning.

Learning and remembering, therefore, are brought about by internal processes that are affected by the external organisation of stimuli, by executive control processes brought to bear by the learner, and by the contents of the learner's memory.

To make sense of the enormous number of outcomes produced by learning, Gagne has classified all kinds of learning into five distinct categories (Gagne 1977). While the processes of learning are the same for all categories, different internal and external conditions are required for different categories. The categories are

1. Verbal information: This is the capability of being able to state or tell information. It may be accomplished orally, in writing, in a drawing etc. Examples include the names and locations of cities and the names of the days of the week. Learning verbal information depends on the recall of internally stored complexes of ideas which constitute "meaningfully organised" structures. Externally, conditions should relate the new information to these existing structures.

2. Intellectual skills: These are learned capabilities through which the individual interacts with the environment by using symbols. Intellectual skills include reading, writing, using numbers, combining, tabulating, classifying, analysing and quantifying. The most important internal condition is the recall of prerequisite skills. External conditions should guide the combining of these simpler skills.

3. Cognitive strategies: These are learned capabilities through which the individual manages his or her own learning, remembering and thinking. They include ways of analysing problems and approaches to the solving of problems. For example, mnemonics are a well known example of a cognitive strategy for remembering and retrieving information from long term memory. Cognitive strategies require, as internal conditions, the recall of intellectual skills and information relevant to the specific learning tasks being undertaken. Externally, frequent practice is required.

4. Attitudes: These are learned capabilities which influence the individual's choice of personal actions. The most reliable way of learning attitudes is by "human modelling". Internally, previously learned respect for a real or imagined person is required. Externally, a display of the required behaviour by the model, followed by the observation of a successful outcome, is required.

5. Motor skills: These are learned capabilities of executing movements in organised motor acts such as throwing a ball, pouring a liquid etc. Usually these individual acts form a part of a more comprehensive activity such as playing basketball, mixing cement etc. Internally, the recall of part-skills and an

executive subroutine to provide the correct sequence are required. External conditions are provided by practice.

Within the category of intellectual skills, Gagne has described a hierarchy of five types of skill which vary in complexity. At the top of this hierarchy are higher-order rules, followed in level of complexity by rules, defined concepts, concrete concepts and discriminations. To acquire a skill in any of these subcategories requires skills in the lower levels of the hierarchy as prerequisites.

The most typical form of intellectual skill is the rule. The learning of rules is of great educational importance, making up the bulk of what is learned in school. A rule is an inferred capability that enables the individual to respond to a class of stimuli with a class of performances. The use of rules allows human beings to respond to an enormous variety of situations and to function effectively, despite the almost infinite variety of stimulation received. Rules are the major factor in intellectual functioning and have largely replaced the stimulus/response connection in the theoretical formulations of many psychologists (Gagne 1977). The learner is not necessarily able to state the rule but, if the learner's behaviour is rule-governed, it is possible to infer that the rule has been learned. To learn a rule it is necessary that the component skills are already available in the learner's memory. A complex rule may be composed of simpler rules, which may in turn be analysed into even simpler components. As this process is continued, the nature of the components changes from rules to concepts. For example, the rule "swallows fly south" requires the concepts of swallow, flight and south.

Concrete concepts are those which can be denoted by pointing out, for example, "red", "circular" and "chair". Defined concepts are abstract and may be thought of as rules that classify objects or events, for example, "square root", "sport" and "jungle". Concepts require discriminations, which are even simpler skills. Thus, in order to acquire the concept "swallow" it is necessary to be able to discriminate the features of swallows from those of other types of birds.

Learning intellectual skills is mainly a matter of "snapping into place" a combination of simpler skills that have been previously learned. The role of the teacher in intellectual skill acquisition is to help the learner to retrieve the simpler skills to working memory and to provide cues for the sequencing of these simpler skills. Problem-solving can be viewed as a process by which the learner discovers a combination of previously learned rules which can be applied to achieve a solution for a novel problem. In this case the learner has not only solved the problem, but has learned something new. One newly learned entity is a higher-order rule which can be used in the solution of similar problems. Another aspect of the new learning may be a cognitive strategy for dealing with problems in general.

Regardless of the category involved, learning and retention are positively influenced when material is presented in a way that has meaning for the learner and when opportunities are provided for using new learning in various different contexts. The job of instruction is to organise learning experiences so that, in the course of learning, students are confronted with new ideas and have opportunities to build them into their own understanding.

3.2 Rationale for Hypothesis 1

In the context of introductory algebra, spreadsheets may be used to mediate between the familiar idea of a constant and the new concept of a variable. This is possible because a cell in a spreadsheet is very similar to a mathematical variable. The ability of a spreadsheet to display a set of numerical values while suggesting underlying relationships is also a major advantage. This can help pupils to view a problem in general (algebraically), as well as in specific terms (arithmetically). Specific numerical examples can provide a vehicle for thinking about more general relationships. This may provide a link between the learner's knowledge of arithmetic and his or her ability to form algebraic abstractions.

The skill of using a spreadsheet is essentially the skill of constructing algebraic expressions. The user deals with variables and their inter-relationships, but it is the values of the variables that are displayed rather than their mathematical names. Explanatory labels are usually displayed alongside the values as in Figure 3.1.

Spreadsheet			Algebraic Equation
Labels	Values	Variable Names	
Length	5	B1	
Width	6	B2	A = L X W
Area	30	B3 (= B1*B2)	

Figure 3.1

Students who are starting algebra should have little difficulty in manipulating constants but often have severe problems both in manipulating letters and in understanding what the letters mean. To a novice, the spreadsheet (on the left in Figure 3.1) may be related to previous work in arithmetic, whereas the algebraic equation (on the right) provides no such link. The notion of multiplying the values of Length (cell B1) and Width (cell B2) to find the value of Area is embodied just as well in the spreadsheet as in the equation. The student must use the variables B1 and B2 to generate the Area formula, but is able to think in terms of the numbers 5 and 6. In this way the new information (the algebraic expression B1*B2) is connected to the old knowledge (multiplication of integers, knowledge of how to find the area of a rectangle).

The spreadsheet explicitly embodies the concept of a name (e.g. B1) standing for a number, whereas this is not obvious with the equation. In the solution of algebra word problems Arad (1987) has remarked that the variable "X" is difficult for students to relate to, is not dynamic, does not encourage experimentation and is unlikely to be motivating. When a word problem is translated into a dynamic table in a spreadsheet, rather than into an algebraic equation, it should become easier to experiment with and relate to. Using the spreadsheet allows the student to see the relationship at work by varying the values of Length and Width and examining their effect on Area. This is equivalent to seeing the equation as an "operation", as recommended by Clement (1982).

Gagne has identified essential events that occur in every act of learning (Gagne, 1975). Spreadsheets can provide external support for these events as outlined below:

1. Attending

   Computer use can be a motivating factor for many students. The novelty and interactive nature of computers should promote alertness on the part of the learners. Access to more relevant and interesting problems may also be a factor in promoting attention. This may be contrasted with textbooks and chalk-and-talk with which students are very familiar.

2. Expectancy

   Many students are very keen to acquire computer-using skills. This should orient the learners towards the learning goal which is the construction of algebraic expressions, albeit disguised as computer problem-solving. It is very unlikely that such an expectancy would exist for learning algebra in the traditional manner.

3. Retrieval

   The relevant information and skills that need to be recalled to working memory are those previously learned in mathematics, notably number facts and formulas. Cues for their retrieval can be provided by supplying spreadsheet templates which students are required to complete.

4. Selective Perception

The neat layout of the rows and columns in a spreadsheet should aid perception and facilitate retention in short term memory so that semantic encoding may take place.

5. Semantic Encoding

What appears on the screen (usually a table of some sort) can be easily related to previous knowledge of arithmetic. The underlying manipulation of algebraic symbols can be related to the manipulation of numbers. This should influence coding of the algebraic ideas. In conventional algebra classes at this level, the manipulation of algebraic symbols is very difficult to relate to any previous learning. The use of a spreadsheet also facilitates the setting of exercises in a variety of familiar contexts. This may help the spread of activation in long term memory and enhance retention.

6. Responding

In this event, the learner retrieves the newly stored information from long term memory and executes a reponse. This phase is supported by external cues, usually given when informing the learner of the objectives. The absence of a numeric answer to the given problem in a spreadsheet will act as a further cue. "Pure" algebraic problems at this level generally do not have any numeric answers and novices find this very difficult to accept (Booth 1984a). It should be noted that, in a spreadsheet, the numeric answer will be provided by responding with the correct algebraic expression.

7. Reinforcement

This confirms the acquisition of the new capability by informing the learner about the achievement of the learning objective. In spreadsheet use it is provided automatically by the computer. The student has immediate information concerning the values of mathematical relationships that have been established. This is much more difficult to arrange in traditional algebra learning and usually requires that the students check an answer book or consult the teacher. Neatly formatted hard copy can also provide material for further reflection.

8. Cueing Retrieval for Transfer

Recall is improved by the use of spaced reviews and practice. Students may be more prepared to practise with a computer than with routine pencil-and-paper problems. Students will also be practising the skills to be learned (i.e. constructing expressions) without the distraction of having to carry out their own calculations or perform symbolic algebraic manipulations.

9. Generalisation

Problems can easily be set in many different contexts if spreadsheets are used. This contrasts with the predominantly abstract contexts of traditional approaches and may help students to generalise their knowledge. Specific instruction may be designed to encourage transfer of spreadsheet skills to more formal algebra.

Therefore, it is expected that experience with a spreadsheet will have a positive effect on students' ability to use variables and expressions.

3.3 Rationale for Hypothesis 2

To solve a problem in any domain, one needs knowledge of the particular domain, but also general perceptual, linguistic and semantic knowledge. A popular view of the problem-solving process holds that solvers construct an initial mental representation while reading the problem, or shortly afterwards. The initial representation changes as it interacts with further information from the task environment or with knowledge retrieved from long term memory. This results in the construction of a more elaborated representation of the problem.

Long term memory contains mathematical knowledge such as basic facts, generalised problem types, heuristics and algorithms. It also contains beliefs and opinions about mathematics, metacognitive knowledge and knowledge about the real world that may be related to the problem setting. Translating a problem into an internal representation requires the possession of linguistic, factual and schematic knowledge about the objects or events in the problem; for example, knowing that "tile" and "floor tile" mean the same thing (linguistic), that there are 100cm in 1m (factual) and that area = length X width (schematic).

Information from long term memory may be accessed and used in working memory. Information in long term memory is organised, i.e. it is not a 'written tablet' of what has been learned. The importance of this organisation is that it reflects functions for long term memory other than simply serving as a store. These functions are 1) providing a format into which new data must fit, 2) serving as a guide for directing attention and 3) filling gaps in information received from the outside world. Access and recall depend to a large extent on the form in which information is stored.

> "Students do not absorb knowledge onto a blank slate, but instead interpret what they read and what is said to them in terms of an already functioning knowledge system" (DiSessa 1987, p.346).

> "Differences in mathematical understanding are caused partly by the differences in the way people organise knowledge. The organisation influences what people attend to in problems, what they recall about problems, and therefore how they solve problems" (Gagne 1985, p. 230).

Transformation of the initial representation may be accomplished in a number of possible ways. It may be a) modified so that it relates to information already stored, b) replaced by another symbol or c) supplemented by additional information to aid in recall (i.e. mnemonics etc.). The resulting representation is the problem-solver's internalised version of the problem task. The nature of these representations is crucial to one's success in creating a solution. It is known that different people can represent the same task in different ways. Hinsley, Hayes and Simon (1977) showed that subjects can very often categorise problems without even reading the full text of the problem. Further investigations (e.g. Reif 1987; Sweller 1988) have distinguished between the ways in which experts and novices differ in their approach to problems. Experts generally categorise problems by their 'deep' structure and invoke an appropriate schema for solution. Novices, on the other hand, categorise problems by their surface features and tend to proceed by means-ends analysis.

Ausubel (1968, 1977) suggests that existing knowledge is used as an assimilative context, or schema, for new information. The activation of this prior knowledge, as new material is presented, is therefore an important aspect of instruction. The less specific the knowledge at the disposal of the problem-solver, the more he or she will have to rely on heuristic, generally applicable strategies. To solve problems in unfamiliar or abstract contexts, pupils may have to translate unfamiliar words and determine the meaning of problem situations that have little relation to their life experiences. This suggests that performance should be improved by adapting contexts to the students' own interests and concerns.

Greeno and colleagues (Greeno 1980, Kintsch & Greeno 1985) have developed a model of arithmetic problem-solving which has been tested by De Corte, Verschaffel and Verschueren (1982). According to this model, good problem-solvers are distinguished by their ability to construct a semantic representation of the problem situation. In constructing a problem representation, the problem-solver infers information that is needed but is not included in the problem statement. It may also be necessary to exclude information that is given but not required. The provision of rich contextual cues related to the pupils' interests should aid in the construction of such representations.

Schwanenflugel & Shoben (1983) maintain that, during comprehension, concrete materials provide easy access to both imaginal and verbal representations. Abstract materials are restricted to verbal representation alone. Comprehension processes are aided by the addition of contextual information to the materials that are to be understood. This contextual information may come from either the stimulus environment or from the problem-solver's existing knowledge. This again indicates that concrete, familiar contexts should be more favourable than abstract contexts.

Gagne & White (1978) have also predicted that:

> "when knowledge is stored as a proposition or as an intellectual skill, its outcome effect in retention and transfer will be greater the more extensive are its associations with interlinked sets of propositions, intellectual skills, images and episodes" (Gagne & White 1978, p.209).

These considerations suggest that concrete contexts should prove a better learning vehicle than abstract ones. However, there is an alternative argument which suggests that the limitations of short term memory might restrict the capacity of young children to learn from problems set in concrete contexts.

Working memory's growing capacity to process information is a fundamental characteristic of cognitive development in a number of theories (Bruner, 1966; Case, 1978a; Flavell, 1971). Young children are quite limited in their ability to deal with all the information demands of complex tasks. Their limited capacity seems to be a critical developmental factor which can constrain learning (Case 1975, 1978a, 1978b). As problem representation takes place in short term memory, there may be a trade-off between the capacity of short term memory and the other resource demands on the system. It is here that information obtained from the task environment interacts with knowledge retrieved from long term memory. It is also here that the processes of planning, monitoring and evaluating take place. If the task to be performed is difficult, there is a danger that short term memory will be overloaded and important aspects of the problem may not be properly represented.

Sweller (1988), found that conventional problem-solving was not effective in schema acquisition. Conventional problem-solving, such as means-ends analysis, was found to require a large amount of cognitive processing capacity. In problem-solving, the solver must consider the current problem state, the goal state and the relation between these two. If subgoals have been used, a goal stack must also be maintained. Novices, who lack domain specific knowledge, and whose knowledge is not sufficiently 'chunked', may find the cognitive load too great. Sweller (1988) found that, even in cases where the problem was solved, subjects failed to acquire the appropriate schema.

Task factors (abstract/concrete) are known to affect the complexity of the procedure for representing problems. Additional substitutions are necessary for concrete problems so that the variables will refer to the numbers of objects rather than to the objects themselves. This is not the case with abstract (numerical) problems where the objects of the problem are themselves numbers. For example, Meijer & Riemersma (1986) studied 13 year-old childrens' interpretations of the following problem:

> "A father and his son together are 50 years old; the father is 36 years older than the son; how old is each of them?"

Most subjects had considerable difficulty with this problem. A common mistake was to subtract 36 from 50 and conclude that the son was 14. Subjects generally only realised that this was inadequate when asked to check on the 'difference in ages' restriction. Some subjects did not recognise the correct answer even when they provided it themselves. This suggests that the two restrictions mentioned in the problem (the sum of the ages is 50, the difference in ages is 36) were not properly integrated in the representation of the problem. Subjects found it hard to concentrate on both restrictions at the same time, possibly because of the limitations of short term memory.

However, the bulk of the research evidence on the effect of concrete and abstract contexts (Holtan, 1964; Ross & Bush, 1980; Ross, 1983; Schwanenflugel & Shoben, 1983; Ross, McCormick & Krisak, 1986; Caldwell & Goldin, 1987; Ross & Anand, 1987; Ross, Anand & Morrison, 1988) supports the hypothesis that concrete contexts and familiar contexts are better vehicles for learning than abstract ones.

3.4 Rationale for Hypotheses 3, 4 and 5

Hypotheses 3, that concrete contexts will especially favour those with lower general numerical ability is suggested by the consideration that such students will be more engaged by problems which are relevant to their own concerns. It is also implied by other research studies. Travers (1967) and Christensen (1980) found that lower achievers had stronger preferences for particular problem contexts. Ross, Anand and Morrison (1988) found that individualised contexts were of special benefit to weaker students. High achievers performed well, regardless of context. The lower and middle groups benefitted most from personalised contexts. Caldwell and Goldin (1987) found that the effect of context was particularly significant for junior high school students when compared to older subjects. Ross, McCormick and Krisak (1986) also noted that context variation had more effect with younger and less academically advanced subjects. This suggests that there is a developmental aspect involved and that the context effect might be greater for less able, less mature subjects, such as those in the present study.

Hypotheses 4 an 5, that concrete contexts will favour those with poorer attitudes to mathematics and computers, are based on the consideration that subjects with poor attitudes are more likely to be motivated by contexts which are of some relevance to themselves.

3.5 Operational definition of the variables

Independent Variables

1. Concrete Group
   Experimental group who pursued an 8 hour course on spreadsheet mathematics with exercise problems set in concrete contexts. The contexts were designed to be relevant and interesting to the participants.

2. Abstract Group

Experimental group who pursued an identical course of instruction with exactly equivalent problems in abstract (numerical) contexts.

Dependent Variable

Achievement

Achievement was measured by a 21-item test based on algebra word problems at the level of Intermediate Certificate Mathematics (Syllabus B). Each item required the construction of an algebraic expression to represent a relational proposition in a word problem.

Moderator Variables

1. Numerical ability

The measure of numerical ability was the raw score attained on the numerical section of AH2 (Heim, Watts & Simmonds 1975). This is a norm-referenced test containing 40 numerical questions designed for a cross-section of the adult population and for children aged 10+. These scores were then converted to nominal categories using a median split. The categories were labelled "High Numerical Ability" and "Low Numerical Ability".

2. Attitude to mathematics

   A mathematics attitude questionnaire, consisting of 10 items, was constructed. The scores were converted to nominal categories using a median split. The categories were labelled "Positive Mathematics Attitude" and "Negative Mathematics Attitude".

3. Attitude to computers

   A computer attitude questionnaire, consisting of 6 items, was constructed. The scores were converted to nominal categories using a median split. The categories were labelled "Positive Computer Attitude" and "Negative Computer Attitude".

3.6 Operational restatement of the hypotheses

The hypotheses investigated were as follows:

1. There will be a significant difference in the ability to construct algebraic expressions representing relational propositions between pretest and posttest for all students who undertake an 8 hour course in spreadsheet mathematics.

2. The concrete group will perform better on the dependent measure than the abstract group.

3. The gains of the concrete group over the abstract group on the dependent measure will be greater for subjects with lower numerical ability than for subjects with higher numerical ability.

4. The gains of the concrete group over the abstract group on the dependent measure will be greater for subjects with negative attitudes to mathematics than for subjects with positive attitudes.

5. The gains of the concrete group over the abstract group on the dependent measure will be greater for subjects with negative attitudes to computers than for subjects with positive attitudes.

3.7 Significance of the study

With the recent availability of microcomputer software for symbolic algebraic manipulations, there is a need to reexamine the content of second-level algebra syllabi. At present, such syllabi are weighted very much towards learning manipulative procedures, with relatively little emphasis on problems and applications (e.g. Department of Education 1987). As symbol-manipulating algebraic software is now becoming widely available, it is unlikely that this emphasis can be justified and maintained. Therefore, it is inevitable that there will be greater emphasis on problems and applications in the future.

The first step in the application of algebra to real problems is the solution of word (story) problems. Despite their importance, it is widely accepted that many students in second-level schools have severe difficulty with word problems (e.g. Brown et al., 1988). The concept of a variable, and the construction of expressions to represent relational propositions, have been identified as major stumbling blocks in previous investigations in this area. This study develops a new approach, applicable in a natural classroom setting, to this problem area. Thus it makes a contribution to classroom practice in an important area of mathematical education.

There is evidence to show that the use of relevant and interesting contexts is of benefit to students in mathematical problem-solving. However, most of this empirical evidence comes from the study of older subjects, mostly in third level education. Some of this evidence suggests that there is a developmental component and that the benefits may be more marked for younger, less mature students (Caldwell & Goldin, 1987). There is a shortage of empirical evidence concerning the importance of problem contexts for students as young as those in the present study. It may be that familiar contexts provide more internal connections which facilitate learning or, alternatively, that the added complexity of examples set in a variety of contexts will have the opposite effect. This research examines the effect of problem contexts on learning algebra and presents evidence which will be of benefit to developers of instructional materials and to teachers.

Computer availability is an important new factor in both mathematics and in mathematical education. This study develops a model for the use of computers in mathematics education in second-level schools. It is based on the adaptation and use of general-purpose software rather than the production of specially written courseware. There are many readily available software packages which might be adapted for use in mathematics instruction. These include spreadsheets, graphics packages, information handling packages, programming languages etc. A major advantage of this approach is that these packages are generally cheap and widely available, often 'bundled' with hardware. Producers of educational material based around such packages are freed from all of the technical, non-educational, issues which must be tackled in the production of specially written courseware. A further advantage is that general-purpose packages are usually not hardware-specific. For example, there are spreadsheets for every type of computer and they all work in essentially the same way. Thus, using a spreadsheet in mathematics education would not involve any software development costs, nor would it involve the purchase of any further hardware or software. The production, by teachers, of computer aided instruction in this manner is therefore a realistic option.

It is widely accepted that new knowledge is not absorbed onto a 'blank slate', but must be related to and integrated with existing knowledge. In practice, mastery of procedures, which have very little meaning for most students, is generally taught before an understanding of concepts is achieved. This research attempts to show that if instruction is initially concentrated on concepts, then stable cognitive structures can be formed as a basis for subsequent skill development. In particular, it shows that computer environments can be used to refocus the curriculum on concepts by allowing the computer to process laborious, but necessary, calculations in the initial stages of instruction.

Chapter 4: Methods

4.1 Subjects

A total of 73 second-year students, all girls aged 13 or 14, from five schools volunteered as subjects. The subjects were recruited by sending a letter to the principals of each school requesting their cooperation (Appendix H). The experimenter then visited each school to speak to the second-year classes. During these visits an explanatory letter and application form were distributed to interested students (Appendix I).

The experiment took place during a week-long mid-term break for which all five schools were closed. Three of the schools were in inner-city areas and the other two drew most of their students from large, working-class housing estates. A large proportion of students in this population come from deprived backgrounds, with a high incidence of family and social problems. These problems often result in poor attendance at school and poor academic achievement.

The subjects were assigned to the two treatments at random, 37 to the concrete group and 36 to the abstract group. To make the best use of the available hardware and personnel, each treatment group was then split into a morning group and an afternoon group. One concrete and one abstract group attended in the mornings and the other two groups attended in the afternoons. Each group shared 10 networked Apple IIe computers. To save time, and to guard against system failure, all files were loaded by the tutors before each session began. Two teams of tutors were involved, each consisting of one experienced teacher and two senior students. Each team instructed one abstract group and one concrete group. The teaching teams were assigned to these groups at random. The experimenter was one of the teachers involved.

4.2 Design

A pretest-posttest control group design (Tuckman 1978) was used. This design controls for all the sources of internal invalidity with the exception of a testing effect. However, as the degree of change in both treatments was of major interest, it was necessary to include a pretest. Such testing is a regular part of school life, and as the testing procedure was not in any way unusual, it is unlikely that the subjects would have been sensitised by the pretest.

The proposed level of significance was $p = 0.05$. The analysis was intended to indicate:

1. The effect of the independent variable on the dependent variable.

2. The effect of the moderator variables on the dependent variable.
3. The interaction between the moderator variables and the independent variable.

4.3 Pilot testing

Pilot testing of the pretest, the posttest and the attitude questionnaire was carried out prior to the experiment. None of the subjects involved [illegible] took part in the final study.

4.3.1 Pretest and Posttest

An examination of Intermediate Certificate textbooks and examination papers revealed that eleven distinct types of algebraic expressions were required. These were:

x+a	x-a	a-x	x+y+z
ax	ax+b	ax-b	ax+by+cz
a(x+b)	a(x-b)	a(b-x)	

where x, y, z are variables and a, b, c are constants.

The initial versions of the pretest and posttest contained eleven questions, each of which required the construction of one of these types of expression. They also contained three further questions which required the construction of equations rather than expressions. Construction of equations was not part of the target capability but it was thought that this would correlate closely with the construction of algebraic expressions. None of the questions were abstract and an attempt was made to make the contexts relevant to the target population. All questions were multiple-choice. The two tests were designed to be equivalent, consisting of isomorphic items. Problems are isomorphic if they are essentially the same problem presented in different contexts. The tests were initially piloted with 126 subjects from two different schools. Each subject took both tests in one forty-minute period.

The results for both tests were almost identical and the results for equivalent questions were essentially the same in most cases. However, the mean score in each test was 63% and this seemed to be remarkably high. It was suspected that the multiple-choice format may have been a factor in producing such high scores. As the target capability was the construction, rather than the recognition, of algebraic expressions, it was decided to change the format. One of the tests was given again to 54 of the original subjects, without the multiple-choice format. None of the questions were changed in any way, except for the removal of the multiple-choice element. This produced a mean score of only 39%. The number of correct answers for every question was substantially reduced and the number of correct answers for the three questions which required the construction of equations was negligible. It was decided to discard the multiple-choice format.

Questions which had been particularly badly answered were then refined by changing their contexts or by adjusting the wording. Both tests were then given to a different group of 59 subjects. The results for these two tests were almost identical. Their means were 33% and 32.5% respectively and the standard deviation was 22% in each case. However, it was noticed that six pairs of mathematically identical questions produced remarkably different results from the same subjects. For example, the following two questions were answered correctly by 58% and 25% of subjects respectively:

1. Jane had £X. She got a present of another £X. She then spent £3. The number of pounds she had left was:

2. Sharon worked for two months during the holidays and earned £X each month. She spent a total of £100 during that time. The number of pounds she had left was:

Such large differences between almost identical questions were unexpected. This illustrates how important context can be in determining the difficulty of a problem. The six pairs of questions which had produced unexpected results were then refined for the final pilot test. In addition, seven abstract questions (i.e. purely relating to numbers), which were mathematically equivalent to the first seven concrete questions, were added. There was no need to test two versions of the abstract questions. There was no possible confounding effect of context, and equivalent questions could be produced by simply changing the numbers involved. The questions requiring the creation of equations were removed. These were rarely answered correctly and were not part of the target capability but rather an attempt to test for transfer. The final pilot test contained only seven abstract questions and revised versions of the six pairs of questions which had produced unexpected results previously. This was tested with a different group of 26 pupils. It was found that the refined pairs of questions were answered equally well and that the new abstract questions produced similar scores to their concrete isomorphs. Thus, sufficient pairs of equivalent questions were available to construct the pretest and posttest.

The final tests contained twenty one questions each (see Appendices A and B). There were seven abstract questions followed by seven equivalent questions set in concrete contexts. The remaining seven questions were also set in concrete contexts and were slightly more difficult. A coin was tossed to decide which would be the pretest and which would be the posttest. The final versions are in Appendices A and B.

During pilot testing it was noticed that, apart from the initial multiple-choice format, a substantial minority of subjects (15% to 20% approximately) scored zero in every test. In most cases this was because the subjects consistently wrote numerical answers rather than algebraic expressions. For example,

> Question: Mary is 6 years older than Kate. Kate is X years. How old is Mary?
>
> Answer : 12 years

This type of answer would suggest that these subjects had never learned algebra. This was not the case as their classmates were able to answer correctly. It was more likely that their need for a 'real' answer prompted them to substitute a number for the variable in the question. In many cases the number substituted was one of the numbers appearing in the problem statement.

4.3.2 Attitude Questionnaire

An instrument was designed to measure the subjects' attitudes to mathematics and computers. The target attitudes were considered to consist of the following eight components:

1. Difficulty of mathematics
2. Usefulness of mathematics
3. Suitability of girls for mathematics
4. Enjoyability of mathematics
5. Confidence in doing mathematics
6. Difficulty of computer use
7. Suitability of girls for computers
8. Enjoyability of using computer

A pool of sixty items was assembled, consisting of both positively and negatively worded statements under each of the above headings. These items were piloted with a group of 78 students from the sample population. Using a Likert scale, items expressing favourable attitudes were scored from 4 for "strongly agree", down to 0 for "strongly disagree". Items expressing negative attitudes were scored from 0 for "strongly agree", up to 4 for "strongly disagree". The range of possible scores was from 0 to 240 inclusive. The preliminary results were analysed by correlating the score for each item with the total score. Items which correlated highly and which gave a good spread of scores were refined and tested again with a different group of 58 students. This test consisted of 16 items, with each of the eight components of the target attitudes represented twice, once in a positively worded statement and once in a negatively worded statement. The possible range of scores varied from 0 to 64. The actual scores ranged from 25 to 53 with a mean of 37 and a standard deviation of 7.5. This was considered satisfactory and so this test was used without further refinement (see Appendix C).

4.4 Preparation of the Instruction

Before word problems can be attempted with a spreadsheet, it is essential to be able to edit a spreadsheet. The hierarchy of tasks involved in basic spreadsheet editing is shown in Figure 4.1 (P.124)

Based on Figure 4.1, performance objectives for spreadsheet use were drawn up:

1. The student can state the following definitions in relation to spreadsheet use:

Cell :	Any 'box' in a spreadsheet
Column:	A vertical line of cells (A,B,C...)
Row:	A horizontal line of cells (1,2,3...)
Cell Name:	Each cell is named by its column and its row, e.g. A1, D12 etc.
Label cell:	A cell containing a word or words
Value cell:	A cell containing only a number

Formula cell: A cell containing a formula. The current value of the formula will be displayed on the screen.

2. Given a completed electronic spreadsheet, the student can execute cursor movements to position the cursor on any named row, column or cell.

3. Given a completed electronic spreadsheet, the student can classify named cells as either Labels, Values or Formulas.

4. Given a blank electronic spreadsheet, the student can enter a) Labels b) Values c) Formulas into named cells.

5. Given a completed electronic spreadsheet, the student can change a) Labels b) Values c) Formulas in named cells.

6. Given a completed electronic spreadsheet, the student can discriminate between the current value and the formula contained in a given cell.

7. Given a paper spreadsheet on a familiar theme (e.g. shopping list) containing labels, values and formulas (only the values of formulas are shown), the student can classify each cell as either a label, a value or a formula.

To achieve these objectives, a 15 minute lecture on spreadsheet concepts and operations was prepared. In addition, a two-page handout summarising the main points (Appendix E) was given to each participant on the first day of instruction. These, combined with exercises on spreadsheet manipulation, Concrete 1 and Abstract 1, (see Appendices F & G), were sufficient to achieve objectives 1 to 7 above.

The target objective was the construction of algebraic expressions to represent relational propositions in word problems. This is a vital step in the solution of word problems and one that has been identified (Mayer 1982) as presenting major difficulties for many students. Figure 4.2 (p.135) is an information processing analysis which details the various steps in solving a word problem. This shows how the target capability ("Express other unknowns in terms of X") fits into the overall process.

A similar flow chart, detailing the various steps in using a spreadsheet to solve a word problem, is shown in Figure 4.3 (p.136). The top part of this flow chart is essentially the same as the top part of Figure 4.2. The target skill ("Write expressions for other unknowns") is in the top part and occupies roughly the same position in both diagrams. This indicates that the processes involved in producing the target expression are roughly the same in both conventional algebra and in spreadsheet algebra. The bottom parts of the diagrams differ considerably. The spreadsheet answer is found by substituting trial values rather than solving an equation, but this is not relevant to the present study.

The key tasks identified in Figure 4.2 and Figure 4.3 may be classified as follows:

1. Identify unknowns:
   Intellectual skill (Defined concept)

   The key prerequisite for this skill is the cognitive strategy of dividing a verbally described situation into parts. A further prerequisite is the ability to identify a question proposition (What is...?, How much...?, In how many years...?, Find.... etc.)

2. Identify statements of relationships between unknowns:
   Intellectual skill (Defined concept)

   The key prerequisite for this skill is also the cognitive strategy of dividing a verbally described situation into parts. A further prerequisite is the ability to identify a relational proposition. (Is twice as old as...., Is 3 more than...., Costs £3 less than... etc.)

3. Let X = (any) unknown .... Figure 4.2

Write label/choose trial value for unknown .... Figure 4.3

Step 1 above is an essential prerequisite.

4. Express other unknowns in terms of X ..... Figure 4.2

Write labels/expressions for other unknowns .... Figure 4.3

Intellectual skill (Higher order rule)

Steps 1, 2, and 3 above are essential prerequisites for this skill. It involves the ability to generate an expression which precisely describes the relationship between X and each of the other unknowns. A further prerequisite is knowledge of algebraic terminology, i.e. 3X means "3 times X" etc.

5. Identify equality statement:

Intellectual skill (Defined concept)

The key prerequisite for this skill is also the cognitive strategy of dividing a verbally described situation into parts. Further prerequisites are the concept of equality and the recognition of statements of equality (The answer is..., Result is the same as..., Total amount is..., Adds up to.. etc.)

6. Write L.H.S. and R.H.S. expressions ...... Figure 4.2

Write label/expression for answer ...... Figure 4.3

Intellectual skill (Higher order rule)

Steps 1, 2, 3, 4, and 5 above are essential prerequisites for this skill. It involves the ability to combine the expressions generated in 4 in accordance with the equality statement.

7. Solve Equation .... Figure 4.2

   Intellectual skill (Higher order rule)

8. Check result .... Figure 4.2

   Answer correct?/Choose different trial value .... Figure 4.3

   Intellectual skill (Rule)

9. Other essential prerequisites:

   Concept of a variable 'standing for' a number.

   Numerical skills (arithmetic)

Performance objectives were then drawn up for solving word problems using a spreadsheet:

1. Given a word problem, the student can identify the unknowns (including the 'answer') by writing labels for them.

2. Given a word problem, the student can generate a trial value for one of the unknowns.

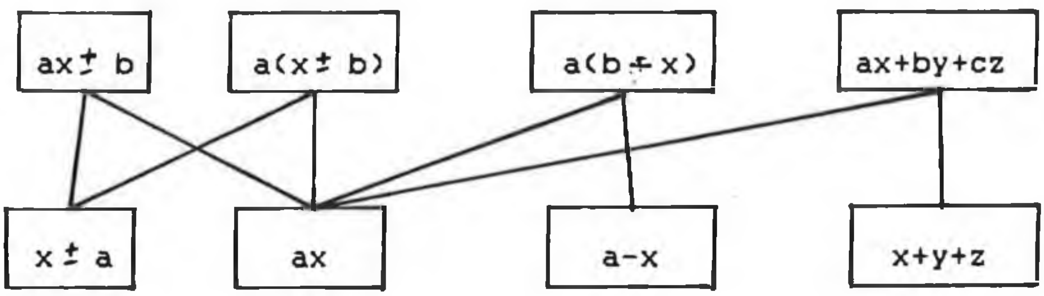
3. Given a word problem and a spreadsheet template which contains labels for all unknowns and a value for one unknown, the student can generate an expression for each of the remaining unknowns.

An examination of word problems from current Irish Intermediate Certificate textbooks and from recent examination papers showed that over 90% of the algebraic expression generated in solving these problems fall into the following categories:

x+a	x-a	a-x	x+y+z
ax	ax+b	ax-b	ax+by+cz
a(x+b)	a(x-b)	a(b-x)	

(x, y and z are variables. a, b and c are constants.)

A learning task analysis of these expressions revealed the following hierarchical structure:



The tasks on the bottom line are essential prerequisites for learning those on the top line. The practice problems (Appendices F & G) were sequenced to take account of this hierarchy.

4. Given a word problem, the student can identify the equality statement.

5. Given simple algebraic word problems, the student can generate complete spreadsheet solutions.

As objective 3 was the target objective, assistance was given with objectives 1 and 2 by providing templates containing labels for each problem. It was expected that this would help in the construction of an internal representation for each problem by breaking down the problem statement into its constituent parts. In addition, this device was used to force the subjects into creating complex algebraic expressions in the later exercises, e.g. Concrete 5 and Abstract 5 (Appendices F & G). In some of these later problems, it is possible to generate a correct spreadsheet solution by breaking the problem down into very simple steps, thus avoiding the necessity to write more complex expressions such as a(b-x) etc. The use of the templates ensured that the subjects constructed the expressions in the desired manner. Providing the templates had the added advantage of saving time, as typing skills were very poor. The use of the same problem layout by each subject also made it easier for the tutors to check solutions.

4.5 Activities

At their first session, each group completed the pretest (15 minutes), the attitude questionnaire (5 minutes) and the test of numerical ability (15 minutes). The subjects in each group then received the same initial introductory lecture on spreadsheet use, which lasted for 15 minutes. This consisted of definitions and examples of the following key concepts; cursor, cell, row, column, label, value, formula. Brief instructions were given concerning basic editing techniques and then sample spreadsheets were shown on an overhead projector. These sample spreadsheets were used to help discriminate between labels, values and formulas. A two-page summary of this introductory presentation was given to each subject (Appendix E). This was the only formal, whole-class instruction in the course. For the remainder of the time, the students worked at the computers at their own pace in a very relaxed atmosphere. They were allowed to talk, eat sweets, take short breaks etc. as they pleased. No external pressure was put on them to complete all of the tasks.

Each group worked on spreadsheet word problems for approximately 8 hours, spread over 4 days. Subjects in both treatment conditions worked in pairs and were allowed to choose their own partners. There were, however, a few subjects in each group who chose to work alone. Each group, or single student, had access to one computer. The spreadsheet used was Appleworks (Apple Computer Inc., 1983). Subjects were given spreadsheet templates and printed worksheets for all of the problems. The spreadsheet templates were organised in files (Abstract 1, Abstract 2 etc.) and the tutors loaded these onto the computers before each session. Each of these files had a printed worksheet associated with it. For each problem, the worksheet contained the problem statement and a copy of the spreadsheet template. Each spreadsheet template contained all the necessary labels for each problem.

The initial exercises (see Abstract 1 in Appendix F and Concrete 1 in Appendix G) were concerned with editing and with spreadsheet concepts. These required approximately two hours to complete. The remainder of the problems were sequenced in accordance with the hierarchy identified in section 4.4 and each group adhered to this sequence. Each group worked at its own pace and the teaching teams were available to check solutions and to provide hints and guidance as required. A few groups completed all of the problems in the four days. These were then shown more advanced editing techniques and were allowed to experiment freely with the spreadsheet. Most of the groups did not complete all of the exercises, but all groups completed at least five sets of exercises.

The concrete group were given problems which were designed to appeal to their own interests e.g. pocketmoney, sports, clothes etc (Appendix G). The abstract group were given isomorphic problems, in the same sequence, all of which concerned only numbers (Appendix F). All of the problems used were adapted from Intermediate Certificate examination papers or from current textbooks. For each concrete problem, an isomorphic abstract one was created and vice-versa.

For each problem, subjects were told to enter a guess for the value of one particular variable (identified on the worksheet by an X) and to construct expressions for all the other variables. When this was done, the original guess was to be adjusted until the expressions produced the correct answer. For example:

> Three suomo wrestlers were weighed at the same time. The second was 143 kg heavier than the first. The third was twice as heavy as the first. Their total weight was 935 kg. How heavy was each?

Supplied Worksheet/Template

	A	B
1	First	X
2	Second	
3	Third	

4	Total	

Computer Solution

	A	B	
1	First	100	(Guess)
2	Second	B1+143	(Expression)
3	Third	2*B1	(Expression)

4	Total	B1+B2+B3	(Expression)

The computer template provided only the labels "First", "Second" etc. The subjects were required to enter a guess for the weight of the first wrestler into cell B1 (e.g. 100). Then expressions for the weights of the others, and the total weight of all three, were constructed and typed into the appropriate cells. If the initial guess was not correct, the value produced for the total weight would not be 935. The initial guess then had to be refined until the correct total was achieved. Subjects were then required to go back to their printed worksheet and write the correct (numerical) answers. In addition, they were required to write the expressions that they had used to produce the correct answers, but using X rather than specific cell numbers. This was motivated by pointing out that the solution did not depend on the use of a particular set of cells. This requirement was intended to aid transfer to 'ordinary' algebra as there was a danger that the students might become absorbed in the process of computing and skip over the mathematical implications of what they were doing. Thus, for the above problem the following written answer was required on the worksheet:

First	198	X	
Second	341	X+143	
Third	396	2X	
	---	----------	
Total	935	X+X+143+2X	(or 4X+143)

At the fourth session, the final half hour was used to administer the posttest (15 minutes) and a debriefing questionnaire (10 minutes).

Editing a spreadsheet: Hierarchy of tasks

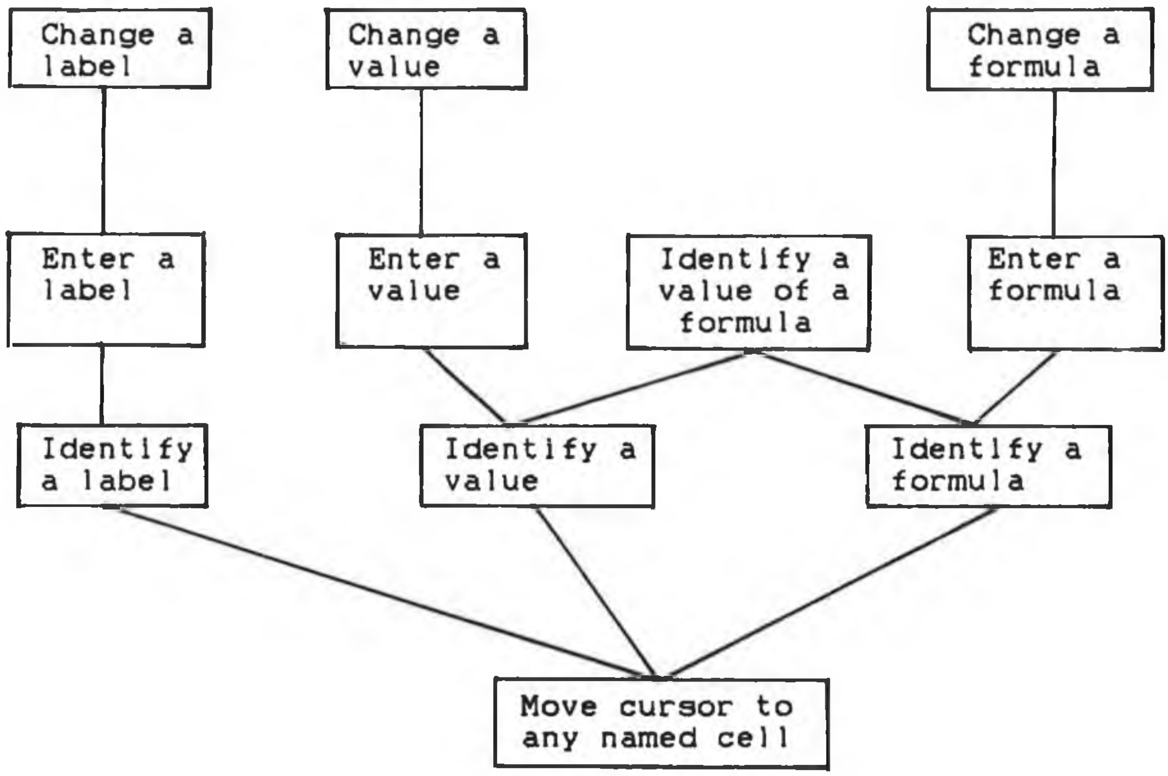


Figure 4.1

Information Processing Analysis: Solution of Word Problems

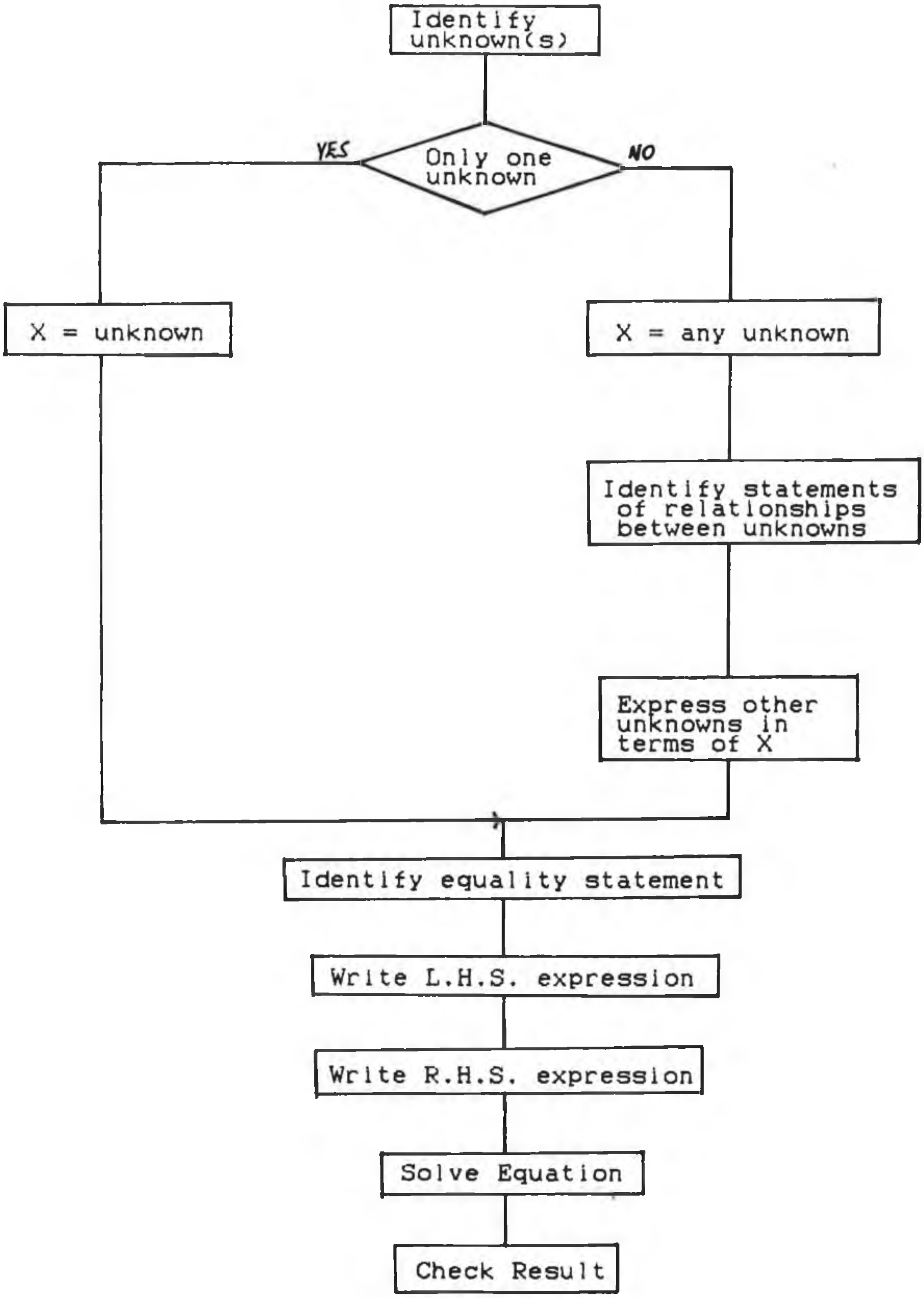


Figure 4.2

Flow chart: Solving a Word Problem Using a Spreadsheet

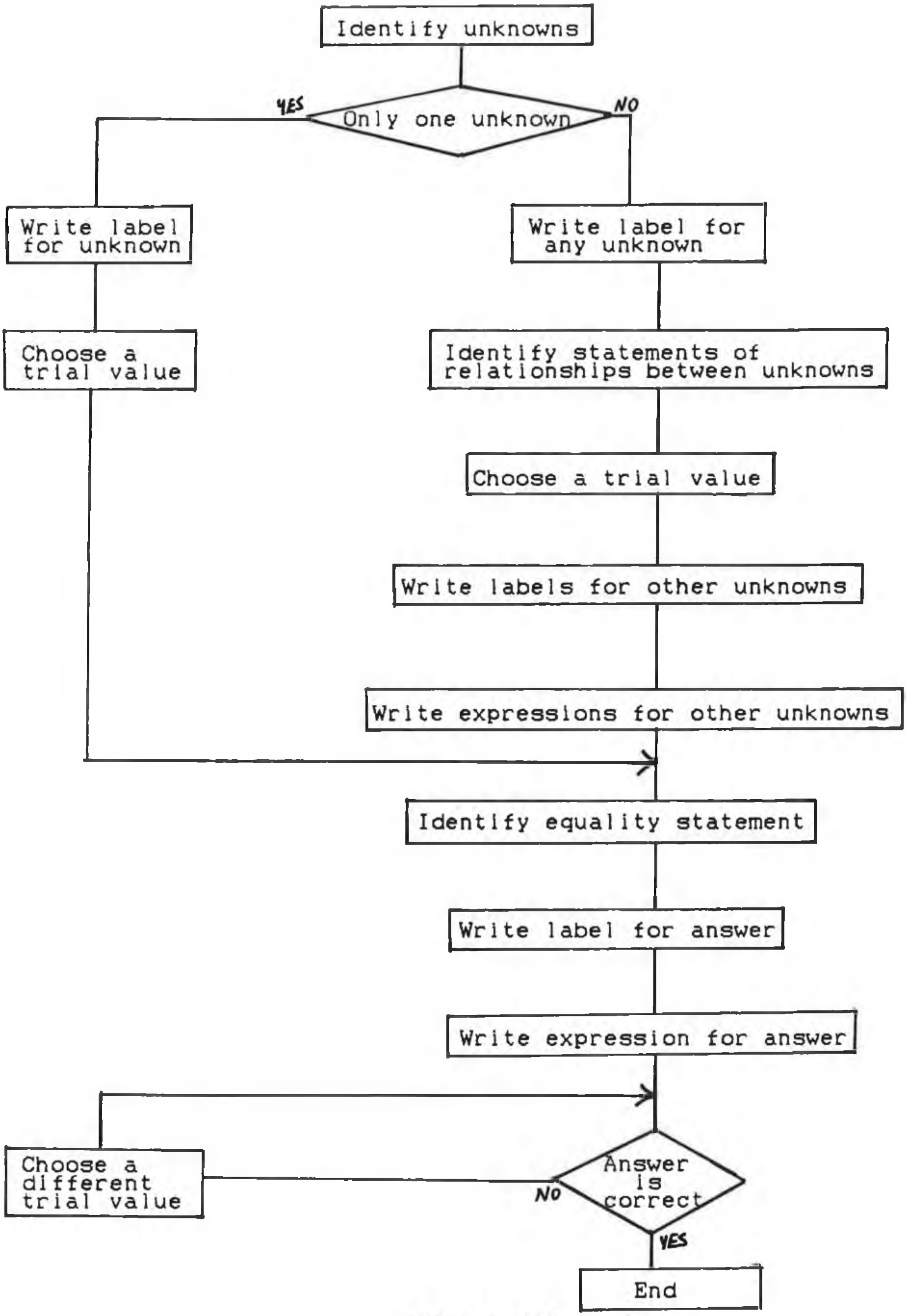


Figure 4.3

Chapter 5: Results

The following measurements were made on each of the 73 subjects:

1. An algebra pretest consisting of 21 questions (Appendix A). The first 7 questions were abstract. The next 7 were isomorphic with the first 7 problems, but were set in concrete contexts. The remaining 7 questions were more difficult and were also set in concrete contexts. There was no time limit on this test. All subjects finished within 15 minutes. Each answer was scored as either right or wrong, giving a possible range of scores from 0 to 21. An answer was considered to be correct if it was mathematically valid, regardless of notation. For example "£X times 3", "X multiplied by 3" or "3X=Y" would be acceptable for "3X".

2. An algebra posttest (Appendix B) which was equivalent to the pretest. Each question was isomorphic with one in the pretest. No time limit was set, but all subjects finished within 15 minutes. This was scored in exactly the same way as the pretest.

3. A norm-referenced measure of numerical ability, AH2 (Heim, Watts & Simmonds 1975). This consisted of 40

multiple-choice items. The time allowed for this was 15 minutes exactly.

4. A questionnaire to determine attitude towards mathematics and computers (Appendix C). This consisted of 16 statements with 5 levels of agreement for each (strongly agree, agree, undecided, disagree, strongly disagree). Ten of these questions concerned attitude to mathematics and the other six concerned attitude to computers. This gave a mathematics attitude scale from 0 to 40 and a computer attitude scale from 0 to 24.

5.1 Pretest and Posttest Results

The pretest scores were remarkably poor in comparison with those recorded in pilot work. It was expected that the mean score on the pretest would be about 6 out of 21. In fact, the mean was only 2.47. 64% of the subjects scored zero. This was much worse than the 20% of zero scores observed in pilot testing. The posttest scores were substantially better, with a mean of 12.96. Five subjects achieved full marks on the posttest. A gain score was calculated for each subject by subtracting her pretest score from her posttest score. The details are shown in Figure 5.1 p.136. T-tests (Figure 5.2) p.137 showed that there was a very significant difference between pretest and posttest for the total group, the abstract group and the concrete group.

To test if ability at pretest was a significant factor in determining the amount of improvement, the gain scores were partitioned into those who scored zero on the pretest (N=47) and those who scored more than zero on the pretest (N=26). Analysis of variance (Figure 5.3) p.138 revealed no significant difference. This indicates that gain scores were not dependent on pretest scores.

Figure 5.4 p.139 shows the difference in gain scores between the two treatment groups. In addition to the total gain scores, gains on subsections of the tests were examined. These were a) gains on abstract questions (7 items), b) gains on concrete questions which were equivalent to these abstract questions (7 items) and c) gains on concrete questions which were more difficult (5 questions only, as 2 of the remaining questions were, in retrospect, judged to be not difficult). The mean scores for the subjects in the abstract treatment were higher in every case. Analysis of variance (Figure 5.5) (p.140.) indicated that the abstract treatment group scored significantly better on the overall test, on problems of an abstract nature and on the equivalent concrete problems. The greatest difference was on the abstract questions. The higher score of the abstract group on the more difficult concrete problems was not found to be statistically significant.

5.2 The effect of numerical ability

The test of numerical ability contained 40 multiple-choice items. The range of possible scores was from 0 to 40. The actual scores ranged from 3 to 34. The mean score was 17.37 and the standard deviation was 6.07. The median was 17.

The gain scores (posttest-pretest) are shown in Figure 5.6 p.141 for those with lower numerical ability (numerical ability score of 17 or less) and those with higher numerical ability (numerical ability score of more than 17). Analysis of variance (Figure 5.7) p.142 indicated that numerical ability did not affect the gain scores within the total group, the abstract group or the concrete group.

The gain scores were then partitioned into four groups (Figure 5.8) p.143. These were

1. Abstract group with high numerical ability.
2. Abstract group with low numerical ability.
3. Concrete group with high numerical ability.
4. Concrete group with low numerical ability.

The graph in Figure 5.8 shows a moderate interaction between numerical ability and treatment, but with the abstract treatment favouring those with low numerical ability. This is the opposite of what was hypothesised (Hypothesis 3). A two-way analysis of variance (2 X 2) was then carried out on these scores (Figure 5.9) p.144. This indicates that the difference in scores between the groups was due to the treatments rather than to numerical ability. The interaction between treatment and numerical ability was not statistically significant.

5.3 The effect of attitude to mathematics

The possible range of scores on the mathematics attitude scale ranged from 0 to 40. The actual scores ranged from 14 to 38. The mean was 27.81 and the standard deviation was 5.70. The median was 28. Gain scores (posttest-pretest) are shown in Figure 5.10 p.145 for the negative attitude group (mathematics attitude score less than 28) and the positive attitude group (mathematics attitude score 28 or more). A one-way analysis of variance (Figure 5.11) p.146 indicated that the attitude of the subjects did not significantly affect gain scores within the total group, the abstract group or the concrete group.

The gain scores were then partitioned into four groups (Figure 5.12) p.147. These were

1. Abstract group with positive mathematics attitude.
2. Abstract group with negative mathematics attitude.
3. Concrete group with positive mathematics attitude.
4. Concrete group with negative mathematics attitude.

The graph in Figure 5.12 p.147 indicates quite a substantial interaction between attitude and treatment. It indicates that abstract contexts favour those with a positive attitude to mathematics more than those with a negative attitude. This is the direction predicted in hypothesis 4. A two-way analysis of variance (2 X 2) was then carried out (Figure 5.13) p.148. This confirmed that the interaction between mathematics attitude and treatment was statistically significant at the .05 level.

5.4 The effect of attitude to computers

The possible range of scores on the computer attitude scale ranged from 0 to 24. The actual scores ranged from 14 to 24. The mean was 18.49 and the standard deviation was 2.57. The median was 18. Gain scores (posttest-pretest) are shown in Figure 5.14 p.149 for the negative attitude group (computer attitude score 18 or less) and the positive attitude group (computer attitude score more than 18). A one-way analysis of variance (Figure 5.15) p.150 indicated that the attitude of the subjects did not significantly affect gain scores within the total group, the abstract group or the concrete group.

The gain scores were then partitioned into four groups (Figure 5.16) p.151. These were

1. Abstract group with positive computer attitude.
2. Abstract group with negative computer attitude.
3. Concrete group with positive computer attitude.
4. Concrete group with negative computer attitude.

The graph in Figure 5.16 shows the interaction between attitude and treatment. It indicates that abstract contexts favour those with a positive attitude to computers more than those with a negative attitude. This is the direction predicted in hypothesis 5. A two-way analysis of variance (2 X 2) was then carried out (Figure 5.17) p.152. This indicated that the interaction between computer attitude and treatment was not statistically significant.

5.5 Some further results

Figure 5.18 p.153 shows the gain scores of each separate school group in the experiment. Of the five schools involved, three were located in inner-city areas and accounted for 21 of the subjects (Schools A, B and C). The fourth and fifth schools were situated in working-class areas a little further from the city centre. One of these (School D) supplied 28 subjects and the other (School E), which had a more upmarket image, supplied the remaining 24. It was noticed that gain scores for school E were lower than for the other schools. Even though the numbers in these groups were quite small, this was interesting and some further analysis was undertaken. Analysis of variance (Figure 5.18) p.153 showed that the school E students were not significantly lower than the other groups. However, when the other four schools were pooled together, school E appeared to perform significantly more poorly than this combined group (Figure 5.19) p.154. This cannot be explained by a ceiling effect caused by the slightly higher pretest scores of school E. All pretest scores were very low and Figure 5.3 p.138 indicates that performance on the pretest did not affect gain scores.

A debriefing questionnaire (Appendix D) was administered after the experiment. This showed that the subjects found the course interesting and enjoyable. There was very little difference between the two treatment groups in their opinions of the course. The only area in which their views diverged was in their perception of the difficulty of the practice problems (Figure 5.20) p.155. The concrete group considered the problems to be more difficult than the abstract group.

PRETEST AND POSTTEST SCORES

(Possible range of scores: 0 to 21)

	All Subjects (N=73)		Abstract Group (N=36)		Concrete Group (N=37)	
	Mean	SD	Mean	SD	Mean	SD
Pretest	2.47	4.42	2.00	4.06	2.92	4.76
Posttest	12.96	5.95	14.31	4.76	11.65	6.73
Gain	10.49	5.12	12.31	4.48	8.73	5.15

(Gain Score = Posttest - Pretest)

Figure 5.1

T-TESTS OF GAIN SCORES

	All Subjects (N=73)	Abstract Group (N=36)	Concrete Group (N=37)
Std.Error of Mean	.60	.75	.85
95% CI for Mean	9.30,11.69	10.80,13.82	7.01,10.45
T-Stat for Zero Mean	17.50	16.49	10.30
Df	72	35	36
P value	.000**	.000**	.000**

**p < .01

Figure 5.2

GAIN SCORES

Zero Pretest Subjects vs. Non-Zero Pretest Subjects

		Mean Gain	SD
Zero Pretest	(N=47)	10.91	5.79
Non-Zero Pretest	(N=26)	9.73	3.62

Analysis of Variance

Source	df	SS	MS	F	Prob. of F
Groups	1	23.47	23.47	.89	.348
Resid	71	1866.77	26.29		
Total	72	1890.25			

Figure 5.3

GAIN SCORES FOR ABSTRACT VS. CONCRETE TREATMENTS

	Abstract Treatment (N=36)		Concrete Treatment (N=37)	
Gains on:	Mean	SD	Mean	SD
Overall test (21 questions)	12.31	4.48	8.73	5.15
Abstract items (7 questions)	4.72	1.78	3.19	2.05
Equivalent Concrete items (7 questions)	4.56	1.99	3.32	2.54
More difficult concrete items (5 questions)	1.81	1.56	1.43	1.64

Figure 5.4

ANALYSIS OF VARIANCE OF GAIN SCORES

Abstract vs. Concrete Treatments

Source	df	SS	MS	F	Prob. of F
Total Gains					
Groups	1	233.31	233.31	10.00	.002**
Resid	71	1656.94	23.34		
Total	72	1890.25			
Gains on Abstract Questions					
Groups	1	42.88	42.88	11.58	.001**
Resid	71	262.90	3.70		
Total	72	305.78			
Gains on Equivalent Concrete Questions					
Groups	1	27.66	27.66	5.29	.024*
Resid	71	371.00	5.23		
Total	72	398.66			
Gains on more difficult Concrete Questions					
Groups	1	2.54	2.54	0.99	.324
Resid	71	182.72	2.57		
Total	72	185.26			

*p < .05
**p < .01

Figure 5.5

GAIN SCORES

Low Numerical Ability vs High Numerical Ability

	All Subjects (N=73)		Abstract Group (N=36)		Concrete Group (N=37)	
	Mean	SD	Mean	SD	Mean	SD
Low Num. Ability	10.05	5.18	12.50	4.26	7.74	5.24
	(N=39)		(N=18)		(N=19)	
High Num. Ability	11.00	5.09	12.11	4.80	9.78	4.99
	(N=34)		(N=18)		(N=18)	

Figure 5.6

ANALYSIS OF VARIANCE IN GAIN SCORES

Low Numerical Ability vs High Numerical Ability

Source	df	SS	MS	F	Prob. of F
All Subjects (N=73)					
Groups	1	16.35	16.35	.62	.434
Resid	71	1873.90	26.39		
Total	72	1890.25			
Abstract group (N=36)					
Groups	1	1.36	1.36	.07	.799
Resid	34	700.28	20.60		
Total	35	701.64			
Concrete group (N=37)					
Groups	1	38.50	38.50	1.47	.233
Resid	35	916.80	26.19		
Total	36	955.30			

Figure 5.7

GAIN SCORES: COMPARISON OF NUMERICAL ABILITY GROUPINGS

	Abstract	Concrete
High Numerical Ability	Mean = 12.11 S.D. = 4.80	Mean = 9.78 S.D. = 4.99
Low Numerical Ability	Mean = 12.50 S.D. = 4.26	Mean = 7.74 S.D. = 5.24

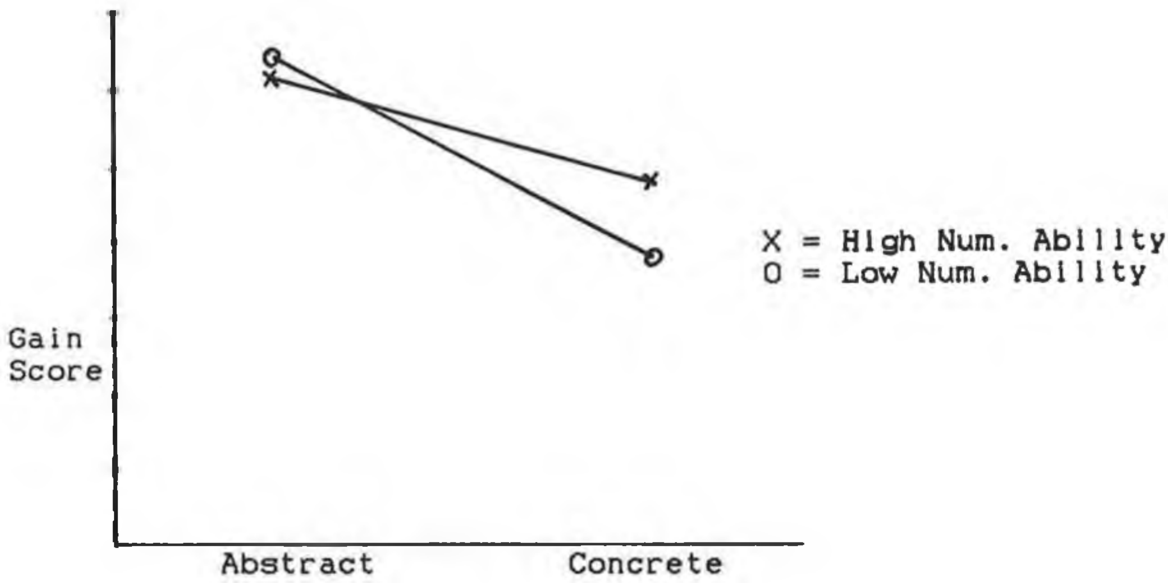


Figure 5.8

TWO-WAY ANALYSIS OF VARIANCE OF GAIN SCORES

Source	df	SS	MS	F	Prob. of F
Numerical Ability	1	12.5	12.5	.53	.471
Treatment	1	227.56	227.56	9.57	.003 **
Interaction	1	26.89	26.89	1.13	.291
Resid	68	1617	23.78		
Total	71	1883.94	5.52		

**p < .01

Figure 5.9

GAIN SCORES

Negative Attitude to Mathematics vs Positive Attitude

	All Subjects (N=73)		Abstract Group (N=36)		Concrete Group (N=37)	
	Mean	SD	Mean	SD	Mean	SD
Negative Attitude	10.45 (N=33)	4.29	11.24 (N=17)	4.97	9.63 (N=16)	3.40
Positve Attitude	10.53 (N=40)	5.77	13.26 (N=19)	3.87	8.05 (N=21)	6.16

Figure 5.10

ANALYSIS OF VARIANCE OF GAIN SCORES

Negative Attitude to Mathematics vs Positive Attitude

Source	df	SS	MS	F	Prob of F
All Subjects					
Groups	1	.09	.09	.003	.954
Resid	71	1890.16	26.62		
Total	72	1890.25			
Abstract Group					
Groups	1	36.90	36.90	1.89	.178
Resid	34	664.74	19.55		
Total	35	701.64			
Concrete Group					
Groups	1	22.59	22.59	.85	.363
Resid	35	932.70	26.65		
Total	36	955.30			

Figure 5.11

GAIN SCORES: COMPARISON OF MATHEMATICS ATTITUDE GROUPINGS

	Abstract	Concrete
Positive Attitude	Mean = 13.26 S.D. = 3.87	Mean = 8.05 S.D. = 6.16
Negative Attitude	Mean = 11.24 S.D. = 4.97	Mean = 9.63 S.D. = 3.40

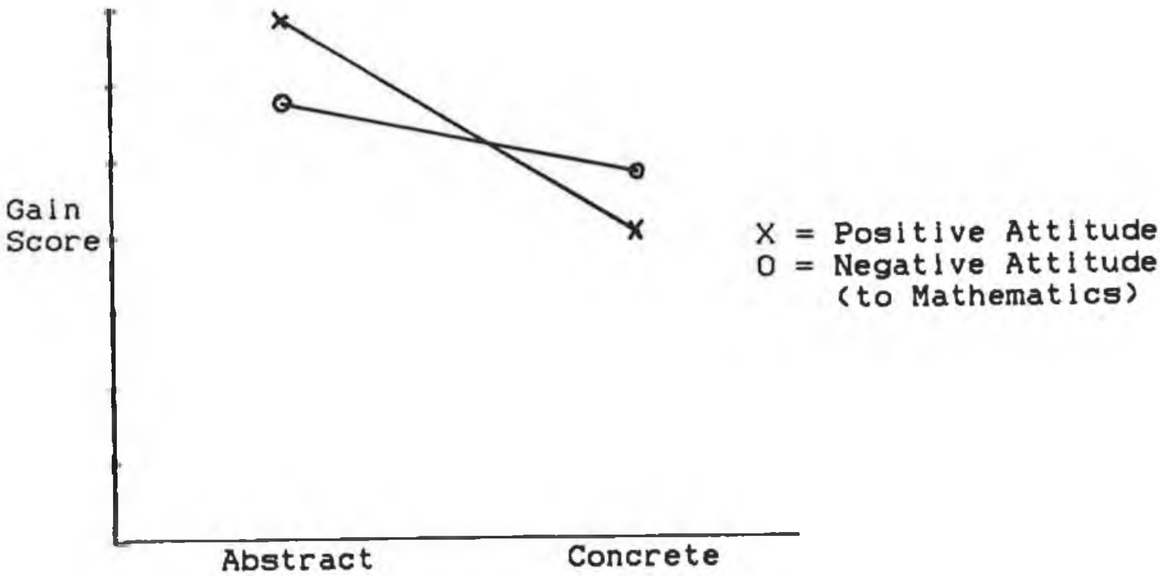


Figure 5.12

TWO-WAY ANALYSIS OF VARIANCE OF GAIN SCORES

Attitude to Mathematics vs. Treatment

Source	df	SS	MS	F	Prob of F
Attitude to Mathematics	1	8.68	8.68	.38	.538
Treatment	1	224.01	224.01	9.89	.002**
Interaction	1	105.13	105.13	4.64	.035*
Resid	68	1540.06	22.65		
Total	71	1877.88	3.75		

**p < .01
*p < .05

Figure 5.13

GAIN SCORES

Negative Attitude to Computers vs. Positive Attitude

	All Subjects		Abstract Group		Concrete Group	
	(N=73)		(N=36)		(N=37)	
	Mean	SD	Mean	SD	Mean	SD
Negative Attitude	10.10 (N=39)	5.26	11.40 (N=18)	4.85	8.74 (N=19)	5.46
Positive Attitude	10.94 (N=34)	5.00	13.44 (N=18)	3.81	8.72 (N=18)	4.97

Figure 5.14

ANALYSIS OF VARIANCE OF GAIN SCORES

Negative Attitude to Computers vs Positive Attitude

Source	df	SS	MS	F	Prob of F
All Subjects					
Groups	1	12.77	12.77	.48	.489
Resid	71	1877.47	26.44		
Total	72	1890.25			
Abstract Group					
Groups	1	36.90	36.90	1.89	.178
Resid	34	664.74	19.55		
Total	35	701.64			
Concrete Group					
Groups	1	.002	.002	.000	.993
Resid	35	955.30	27.29		
Total	36	955.30			

Figure 5.15

GAIN SCORES: COMPARISON OF COMPUTER ATTITUDE GROUPINGS

	Abstract	Concrete
Positive Attitude	Mean = 13.44 S.D. = 3.81	Mean = 8.72 S.D. = 4.97
Negative Attitude	Mean = 11.40 S.D. = 4.85	Mean = 8.74 S.D. = 5.46

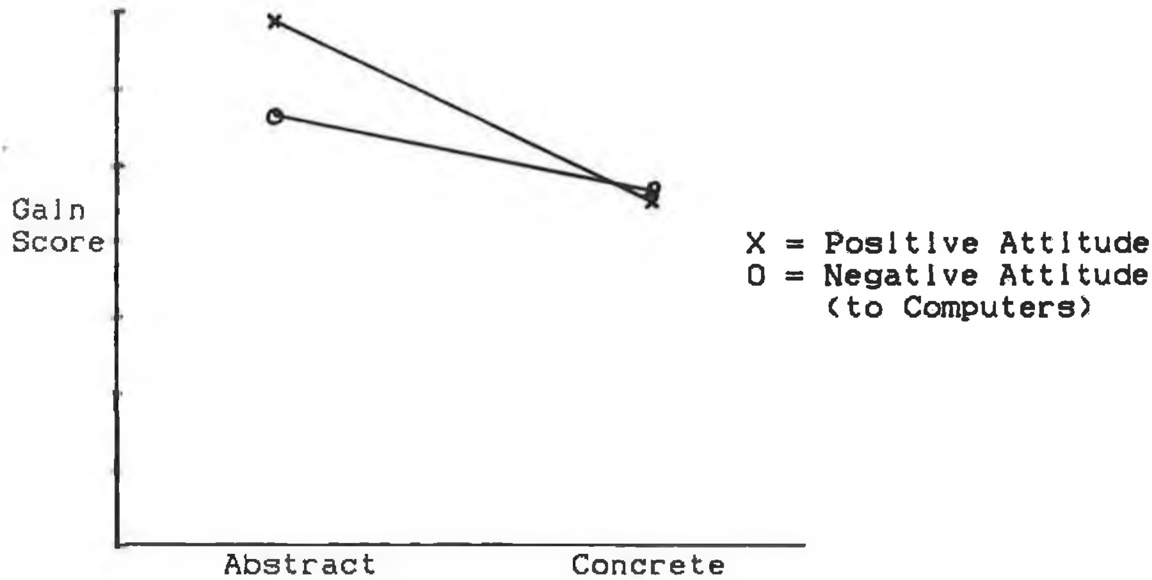


Figure 5.16

TWO-WAY ANALYSIS OF VARIANCE OF GAIN SCORES

Attitude to Computers vs. Treatment

Source	df	SS	MS	F	Prob of F
Attitude to Computers	1	5.56	5.56	.23	.633
Treatment	1	227.56	227.56	9.41	.003**
Interaction	1	6.72	6.72	.28	.600
Resid	68	1644.11	24.18		
Total	71	1883.94			

**p < .01

Figure 5.17

Analysis of Gain Scores by School Groups

	Mean	Std. Dev
School A (N=6)	10.17	6.62
School B (N=9)	10.56	4.19
School C (N=6)	11.40	5.37
School D (N=28)	11.96	4.37
School E (N=24)	8.72	5.62

Analysis of Variance

Source	df	SS.	MS	F	Prob of F
Groups	4	143.99	36.00	1.40	.242
Resid	68	1746.26	25.68		
Total	72	1890.25			

Figure 5.18

Analysis of Gain Scores: School E compared with others

	Mean	Std. Dev
Other Schools (N=49)	11.42	4.64
School E (N=24)	8.72	5.62

Analysis of Variance

Source	df	SS.	MS	F	Prob of F
Groups	1	119.54	119.54	4.79	.032*
Resid	71	1770.71	24.94		
Total	72	1890.25			

* p<.05

Figure 5.19

Student Debriefing Questionnaire

	Abstract Group	Concrete Group
Length of course		
Too long	2	1
Too short	16	17
Just right	18	19
Length of lessons		
Too long	6	9
Too short	30	28
Diffcult to concentrate?		
Yes	21	17
No	15	20
Enough instruction on problems?		
Yes	33	30
No	3	7
Some problems too easy?		
Yes	16 (44%)	6 (16%)
No	20 (56%)	30 (81%)
Some problems too hard?		
Yes	10 (28%)	15 (41%)
No	26 (72%)	21 (57%)
Could be done more easily without computer?		
Yes	11	11
No	25	26
Improved mathematical ability?		
Yes	33	37
No	3	0

Figure 5.20

Chapter 6: Conclusions

6.1 Discussion of results

There are many studies in the mathematics education literature which indicate that introductory algebra, and algebra word-problems in particular, are difficult areas for many students (e.g. Rosnick, 1982; Thomas, 1987; Silver et al., 1988). The extremely poor results, including substantial numbers of zero scores, found in both pilot testing and pretesting confirm that the construction of algebraic expressions is a particular area of difficulty. It was noted in pilot testing that scores were much higher when a multiple-choice format was used. Why is it that students can identify the correct answer but cannot construct it for themselves? One possibility is that they 'work back' from the given answers to decide which one is correct. This is unlikely as students of this age would lack the sophistication for this approach, and it was not possible to confirm any of the answers by simple numerical substitutions. An alternative explanation is that the prompting provided by the multiple-choice answers triggered retrieval of previously learned knowledge from long term memory. The subjects were able to discriminate the fine detail between the correct answers and the carefully constructed 'distractors'.

This suggests that they had substantial experience of algebraic expressions including the use of brackets, order of operations, conjoining etc. However, without the prompts provided by the multiple-choice format, many were totally unable to retrieve this knowledge.

The results of this study support the principal hypothesis that use of a spreadsheet can enhance the ability of algebra novices to construct algebraic expressions to represent relational propositions. The mean score on the 21-item pretest was 2.47 (12%) and the mean score on the equivalent posttest was 12.96 (62%). Thus, the mean gain for all subjects was 50%. On the pretest, the most common error was to write a numerical answer, based on whatever numbers appeared in the problem, rather than an algebraic expression. This unwillingness to write an algebraic expression as an answer was also noted by Booth (1984a). In the initial stages of the course, there was further evidence of this reluctance. A large number of subjects consistently worked out the correct numerical answers themselves and then typed these numbers into the appropriate cells of the spreadsheet. A similar problem was noted by Dubitsky (1986), whose subjects were unwilling to test any value in a spreadsheet before they were sure it was correct. This need for numerical answers has been explained by Booth (1984b) in terms of frameworks of knowledge. Prior to learning algebra, most children's mathematical work is conducted within an arithmetic framework. In this framework, numerical answers are always required and the mathematical structure of problems is generally subordinated to obtaining the correct answer. Algebraic answers conflict with this established framework as they do not supply a 'closed' solution. Furthermore, an algebraic expresssion is a formalisation and symbolisation of arithmetic method and many children are not aware of the precise methods by which they solve problems in arithmetic.

According to information processing theories, students are not empty vessels, but build up knowledge representation structures for themselves. When starting algebra, they already possess conceptualisations that conflict with the new ideas to be learned. These include conjoining and the use of letters. In arithmetic, conjoining indicates addition (25 means "two tens plus 5 units"). In algebra, conjoining indicates multiplication (2X means "two times X"). In arithmetic, letters are labels (2p means "2 pence") but in algebra, letters are variables (2p means "two times the number represented by p"). When new ideas are learned in algebra, older, conflicting ideas are not replaced but retained along with the new knowledge. When confronted with a problem, either conceptualisation may be retrieved. It appears that many students were working within an arithmetic framework during pilot testing, pretesting and the early parts of the instruction. This was possibly because it was better established and more stable than their recently acquired algebraic framework.

The greatly improved performances on the posttest may be explained in terms of the consolidation of the algebraic framework. This was achieved by building on their arithmetic knowledge. A spreadsheet cell which can accept any value, and which in turn affects the value of connected cells, provides a cognitive model for the effect of a variable in an expression. Typing various trial values into a cell supports the conceptualisation of a range of numbers, while only requiring the consideration of one value at a time. Students could see the value of an expression changing as they changed the value of any of its constituent variables. The problems could be thought about, and answers could be checked, in purely arithmetic terms. However, to achieve the correct answer, it was also necessary to symbolise the method of the arithmetic solution in the form of algebraic expressions.

The use of a computer also allowed attention to be focussed on the need for precision when constructing expressions. Booth (1984a) found that children often assign 'intent' to algebraic expressions. That is, they assume that the particular context in which the expression appears defines the required order of its operations. Using the computer, children were able to see the need for accuracy as their expressions needed to be 'understood' correctly by the computer.

Booth's (1984a) teaching approach was similar to that of the present study. A "mathematics machine" was used to motivate the need for precision in the construction of algebraic expressions. Booth (1984a) found that children readily accepted non-numerical answers, and the need for precision in the use of brackets, while working in the "machine" context. However, a delayed posttest showed that this acceptance did not transfer readily to "mathematics", or "algebra", in general. In the present study it was feared that learning to construct algebraic expressions in a spreadsheet context would not guarantee that this skill would be transferred automatically to normal algebra. To aid transfer, subjects were required to generalise all answers to refer to some unspecified cell (i.e. X) rather than to the particular cell they had used in the spreadsheet. That is, subjects were required to rewrite their expressions in conventional algebraic form after they had solved each problem. While some subjects were reluctant to do this, it appears to have been successful as transfer from the specific spreadsheet context to "algebra" was quite smooth.

The method of instruction and the practice problems were obviously motivating as the students were prepared to work for long sessions during their own free time. The post-instruction debriefing questionnaire (Appendix D) indicated that the participants found the course very interesting and enjoyable. Only one student out of 74 failed to complete all four sessions. All of this confirms hypothesis 1, that spreadsheet use can make a valuable contribution to teaching word problems, a difficult but fundamentally important area of school mathematics.

The other principal outcome was that learning through abstract problems was more effective than learning through problems set in concrete contexts. This was so, even though the contexts used were designed to be relevant to the interests of the participants. This was the opposite of what was hypothesised (hypothesis 2). The abstract treatment group performed better, not only on the overall posttest, but also on its abstract subsection and on its concrete subsection. This was unexpected, as previously reported research suggested that performance should be consistently better for problems set in concrete contexts (e.g. Schwanenflugel and Shoben, 1983; Anand and Ross, 1987). It was also unexpected in the light of well-accepted theory, (e.g. Gagne and White, 1978; Glaser, 1984; Ausubel, 1986) which suggests that learning will be enhanced when new material is connected to as many existing reference points as possible. However, there are factors which may explain this anomaly.

Students at this level are more accustomed to abstract mathematics exercises than to 'real' problems. The vast majority of arithmetic exercises in Primary school textbooks are abstract and there is evidence (e.g. Muth 1984) that concrete arithmetic story problems are an area of major difficulty for younger students. So, even before beginning algebra, students will be more familiar with abstract problems and many students will have experienced difficulties in dealing with simple concrete arithmetic story problems. It is well known that these difficulties do not arise from a lack of computational ability. Students do know how to carry out arithmetic procedures, but they do not know when to apply them in solving story problems (e.g. Mayer, 1982; Muth, 1984).

The difference in outcome between the two treatments may be explained in terms of the limitations of short term memory. The limited capacity of short term memory is an important aspect of cognitive development in a number of theories (Bruner, 1966; Flavell, 1971; Case, 1978a). Young children are very limited in their capacity to process all the information in difficult tasks. For example, Muth (1984) studied the effect of extraneous information in arithmetic word problems with sixth graders. It was found that the superfluous information imposed extra demands which reduced the accuracy of solutions. Sweller (1988) found that conventional means-ends problem-solving interfered with schema acquisition due to the excessive demands it placed on cognitive processing capacity. Thus, the limited processing capacity of novices can constrain learning in instructional situations if the demands on the system are too great.

In short term memory, the information obtained from the external problem environment interacts with knowledge retrieved from long term memory. It is here also that the meta-level processes of planning, monitoring and evaluating take place. Relevant knowledge retrieved from long term memory includes linguistic and factual knowledge, processes, schemata, information about problem types, heuristics and algorithms, knowledge about the quantities involved in the problem and other knowledge that may be related to the problem setting. It is from a combination of all this knowledge that the problem-solver's mental representation of the problem is constructed in short term memory (Silver 1987). The construction of this internal representation of the problem is a crucial stage in problem-solving. The difficulty of a problem may depend on the complexity of generating such a representation. Some children may be poor problem-solvers because most of their short term memory is devoted to interpreting and representing the problem. This may leave insufficient capacity available to solve the problem. An abstract word problem may be easier to interpret than a similar one set in a concrete context, thus placing less demands on the information processing system.

The concrete problems in this study generally contained more information than isomorphic abstract problems. Additional 'substitutions' are necessary for concrete problems so that the variables will refer to the numbers of objects rather than to the objects themselves. In abstract numerical problems, no such 'substitutions' are necessary.

For example, consider the following problem which was used during the experiment:

Abstract Version: One number is 3 bigger than another. When the smaller one is multiplied by 5 and the bigger one by 10, the answers add up to 135. Find the numbers.

Concrete Version: Judy had £135 made up of a mixture of £5 and £10 notes. She had 3 more £10 notes than £5 notes. How many of each had she got?

With this problem, the abstract version presented no particular difficulty but nobody solved the concrete version without assistance. With the concrete version, there was no difficulty in setting up the expression for the number of £5 notes (X) and the number of £10 notes (X+3). The difficulty arose in constructing the expression for the total value. Everybody in the concrete group wrote this incorrectly as X+X+3 rather than 5X+10(X+3). This incorrect expression gives the total number of notes rather than the total value of the notes. Every subject then adjusted the value of X to achieve a 'total' of 135. None of the subjects in the concrete group were aware that their answer was incorrect until it was pointed out to them by the tutors.

An important difference between the two versions is that the abstract version explicitly states what must be done (multiply by 5, multiply by 10) whereas, in the concrete version, it is necessary to keep the meaning of the problem in short term memory along with the newly learned, and possibly unstable, capability of constructing an algebraic expression. It is extremely unlikely that the children did not possess the correct factual information in long term memory (i.e. that the value of £5 and £10 notes is got by multiplying the number of notes by 5 and 10 respectively). This information was not integrated into the problem representation by any of the subjects. This suggests that short term memory had reached its capacity in dealing with all the other aspects of the problem.

The contents of a novice's short term memory, when attempting to represent this problem, may be divided into those items which relate to spreadsheet manipulation and those which relate to the mathematical problem. Items relating to manipulating the spreadsheet include recalling that a formula requires a leading + sign, the symbol for multiplication is *, that it is necessary to use 'shift' for certain keys and that the cursor must be correctly positioned before typing the expression. In addition it is necessary to recall keyboarding skills including the technique for using a 'pointer'. Items relating to the mathematics of the problem include recalling which cell contains the smaller number, which cell contains the larger number, which cell should be multiplied by 5, which cell should be multiplied by 10, the total is got by adding, the order in which the computer evaluates expressions, whether brackets are required etc.

In addition to all of this, the concrete version requires the recall of factual information concerning the computation of the value of sums of money. It is generally agreed that short term memory can contain no more than about 7 items at any one time (e.g. Wessels, 1982) and that this limit does not change from person to person or between experts and novices. The difference between experts and novices in any domain can be related to the way in which experts possess domain-specific knowledge (schemata) and can therefore 'chunk' information. As the learners in this case were all novices, it is reasonable to suppose that many of the items listed above existed as discreet entities, thus occupying a large amount of the available memory. The additional information necessary to solve the concrete version, though certainly available in long term memory, was not retrieved. It is likely that this was due to the inability of short term memory to accomodate any further information.

Additional evidence for this explanation may be found from the students' debriefing questionnaire (Figure 5.20). 41% of those in the concrete group thought that some of the problems were 'too hard', compared to 28% in the abstract group. Also, only 16% of the concrete group thought that some of the problems were 'too easy', compared to 44% of the abstract group. The problems in both treatments were mathematically identical, indicating that the contextual settings were the cause of the perceived extra difficulty. However, none of the contexts were unfamiliar to the subjects. This suggests that the difficulty arose from the extra processing required to represent the problems internally, even though all the knowledge required was available in long term memory.

It is suggested that the subjects dealing with abstract problems had less burden placed on their processing capacity in a number of the exercise problems. This would have left them in a better position to concentrate on the target skill, i.e. the construction of algebraic expressions. The results indicate that they were more successful than those in the concrete group, not just on abstract problems but also on concrete problems. Bassok and Holyoak (1989) have suggested that if problem-solving knowledge is encoded along with its applicability conditions, then access to that knowledge may depend on contextual cues relating to the situation in which the initial information was encoded. On the other hand, if knowledge is encoded in a context-free setting then it may be applied more readily to novel settings.

Bassok and Holyoak (1989) examined transfer between two isomorphic subdomains of algebra and physics. Students who had learned arithmetic progressions were very likely to Recognise spontaneously ; that physics problems involving velocity and distance could be addressed using the same equations. In contrast, the students who had learned the physics topic almost never transferred their knowledge to isomorphic algebra problems. Transfer from mathematics to physics occured even when all the example problems were set in a single context (e.g. money). This confirms that the provision of disparate examples is not crucial to transfer. Bassok and Holyoak (1989) did not compare single context mathematics instruction with multiple context. However, they concluded that the degree of transfer observed for the single context instruction was sufficiently high that ceiling effects would make it difficult to observe any added benefit from multiple context instruction.

As the whole purpose of instruction in the domain of word problems is to enable students to solve real problems, some experience with concrete problems is necessary. Perhaps the best teaching strategy would be to use abstract problems initially and then, when some skill has been gained and students are thoroughly familiar with the computer environment, gradually introduce concrete problems. However, as was observed in pretesting, the specific context of a problem can be a major factor in determining its difficulty for subjects in this population. Current textbooks frequently favour technically-oriented contexts such as velocity, currency transactions, weights & measures etc. (Mayer, 1981). When concrete problems are introduced, care should be taken to ensure that they are relevant and understandable to those for whom they are intended.

The third hypothesis, that concrete contexts would favour those with lower numerical ability, was not upheld. Numerical ability had no significant effect on achieving the target capability. While the target capability was not directly related to numerical ability, it was expected that numerical skill, as an index of overall mathematical ability, might have had some effect. The lack of any effect may be explained by the fact that all of the participants were volunteers. Those who volunteer to undertake school work during vacation time may be different from the general population, in terms of both ability and motivation. The distribution of participants in the numerical ability test, compared to the population norm, is shown below:

Population Norm	Participants	
Percentile	Number of participants	
90% - 100%	12	(16%)
70% - 89%	22	(30%)
30% - 69%	34	(47%)
10% - 29%	4	(5%)
0% - 9%	1	(1%)
Total	73	

This shows that the participants were above the population average on this measure. In particular, there were very few participants from the weakest section of the population, i.e. only 6% of the participants were from the lowest 30% of the population. It may be that there just were not enough very weak students involved to show an effect. However, in the light of the other results of this research, very weak students would probably benefit more from working on abstract problems.

In relation to the fourth hypothesis, it was found that attitude to mathematics interacted significantly with the treatments (Figures 5.12 and 5.13). Hypothesis 4 predicted that the gains of the concrete group over the abstract group would be greater for those with negative attitudes to mathematics. The concrete group did not, of course, make any gains over the abstract group at all. However, the 'losses' to the abstract treatment were not nearly as pronounced for the negative attitude group. Therefore, the direction of the effect was as predicted in hypothesis 4. This indicates that students with negative attitudes to mathematics are more motivated by concrete contexts, even though they are likely to learn more from abstract ones.

Hypothesis 5 was not upheld. Attitude to computers had no significant effect. As with hypothesis 3, this lack of effect may be explained by the fact that all participants were volunteers. Computer attitude scores for volunteers in a computer course during holiday time, would be expected to be very high. This was the case. It may be that those with very negative attitudes to computers were under-represented in the sample.

It was noted that overall attitude scores, to both mathematics and computers, in the sample were much higher than those observed during pilot testing. The table below compares the combined attitude scores (mathematics attitude + computer attitude) of those who participated in the study and those observed in pilot testing:

	Pilot Test (N=58)	Participants (N=73)
Mean	37.34 (58%)	46.30 (72%)
Standard Deviation	7.55 (12%)	7.11 (11%)

The participants in the study scored 14% higher (on average) than those observed in pilot testing. Despite this, there was a statistically significant interaction between treatment and attitude to mathematics. While the interaction between treatment and attitude to computers was not significant, it was in the predicted direction. If students with more negative attitudes had participated in the study, this interaction might have been more pronounced.

Finally, it was interesting to note that one particular school group did not gain as much from the course as the other four. This was unexpected and it is only possible to speculate as to why this was so. The school involved was more 'middle class' than the others, and this may have been a factor. It may be that these students were more accustomed to using computers at home, and were therefore less motivated by the novelty of computer use. There could also have been some factor in their school environment which left them less able to benefit from the comparatively relaxed teaching style used. There might even have been some factor in the way they were taught algebra previously which confounded their learning in this situation.

6.2 Extensions and Applications

The sample in this study consisted of 73 volunteers. The number of very weak students, and students with very negative attitudes, was limited by the fact that all were volunteers. However, to replicate the study with non-volunteers would require substantial resources. Working with non-volunteers would necessarily imply working within school hours. This would require that intact groups be split up to allow for random assignment to treatments. In addition, to achieve external validity it would be necessary to involve a number of schools. All of this would require re-arranged timetables, the co-operation of many teachers and principals, the provision of substantial amounts of hardware and the coaching of teachers to implement the course. In the present Irish second-level education system, this would not be feasible because of the inflexibility of school timetables, the predominance of streaming and the lack of hardware standardisation between schools. To compare the use of a spreadsheet with other instructional strategies, such as traditional instruction, would have similar resource implications.

There are, however, a number of questions that could be addressed which would not require such resources. It would be interesting to examine the effect of limiting the context of examples to just one domain (e.g. money problems). Would this be just as effective as using all abstract questions? Would a mixture of abstract and concrete problems produce better results than all abstract examples? How could a spreadsheet method be best integrated into an overall approach to word problems? Does learning how to constuct expressions and equations have any effect on the ability to solve equations?

As the experiment took place in a reasonably natural school setting, its results are directly relevant to the mathematics curriculum. The amount of instruction time involved (8 hours approx.) would represent approximately two weeks' normal mathematics classes. Therefore it could easily be incorporated into present curricula, provided sufficient hardware was available. For whole-class instruction, one computer between two pupils would be sufficient, e.g. ten computers for a class of twenty pupils. Many schools have sufficient hardware at present to implement this.

Because spreadsheets provide a general context for experimenting with mathematical relations, they may be applied to many areas of school mathematics. For example, they could be used to advantage in the following Junior Cycle topics:

Number Theory

Relations and Functions

Mass and Measurement

Ratio and Proportion

Percentages, Profit and Loss

Rates, Taxes etc.

Squares, Square Roots and Indices

Simple Interest

Areas and Volumes

Statistics

Binary Operations (investigating commutativity etc.)

Transformations of the Plane

In addition, spreadsheets could be applied in the following Senior Cycle topics:

Sequences and Series

Graphing Functions

Roots of Quadratic Equations

Co-ordinate Geometry of the Line, Circle, Parabola

Complex Numbers

Vectors

Simultaneous Linear Equations

Compound Interest

Matrices

Roots of Cubic Equations

Linear Programming

Limits

Introductory Calculus

Maxima and Minima

To take just one example from the above lists, maximum and minimum problems are an area of major difficulty for many students. Even those who can successfully solve such problems algebraically, often do so by applying a supplied algorithm, and have no real understanding of what they are doing. Maximum and minimum problems can also be solved by numerical methods using successive approximations. Early pilot work for this research showed that maximum and minimum problems from Leaving Certificate Higher Course papers could be solved by second-year pupils using a spreadsheet. The following problem is a typical example:

> "A poster is to contain 50 cm sq of printed text with margins of 4 cm at the top and bottom and 2 cm at each side. Find the overall dimensions, if the total area of the poster is to be as small as possible"

Spreadsheet Solution:

Area of text = 50

Text Length	Text Width	Total Length	Total Width	Total Area
1	50	9	54	486
2	25	10	29	290
3 etc.				

To solve this problem, the value of text length is varied and the effect of this on the total area is observed. The area decreases as text length increases until text length has a value of 10. When text length is further increased, it is found that the area begins to increase again. It is then necessary to investigate values of text length between 9 and 11. In this way it is possible to 'zoom in' on the correct answer and to calculate it to any required level of accuracy.

Problems such as this are normally dealt with in an 'algebraic' way, i.e. equations are set up, differentiated etc. The difficulty of executing the algebraic procedures often obscures the meaning of the solution for students. However, if the problem is solved numerically and if the figures are tabulated neatly, as they always are on a spreadsheet, then it is quite easy to observe the effect that changing text length has on the total area. This should permit a better understanding of how the total area has a lower limit. As the spreadsheet does all of the arithmetic, the burden on the students' processing capacity is reduced. This allows the students to concentrate on the underlying relationships.

The formulas in the poster example are as follows:

	A	B	C	D	E
1	Area of text	50			
2					
3	Text	Text	Total	Total	Total
4	Length	Width	Length	Width	Area
5					
6	1	+B1/A6	+A6+8	+B6+4	+C6*D6
7	+A6+1	+B1/A7	+A7+8	+B7+4	+C7*D7
8	etc.				

None of these formulas are particularly difficult. If this problem were solved with conventional algebraic notation, the formula for the total area would be:

$$(X + 8)(50/X + 4)$$

The use of the spreadsheet allows this difficult expression to be broken down into its component parts. Only one relationship needs to be considered at a time, for example, the relationship beween length, width and area. To construct the formula for the text width, it is only necessary to divide the text area by the text length. The fact that the value of the text width is instantly displayed gives valuable confirmation that the formula is correct. There is no need to think in terms of a large intimidating algebraic expression but rather in terms of the simpler relationships between the components of the problem. Using such a numerical method with a spreadsheet is a meaningful way to introduce students to the concepts of maxima and minima. This could provide the basis for later algebraic methods by allowing students to develop a deeper understanding of the limiting process involved.

Similar spreadsheet-based approaches to the other topics listed above could be developed in the same way. In addition to its use in mathematics, the spin-off effects of being able to use a spreadsheet would be substantial in other subject areas. Spreadsheets are applicable in any area where calculations are required. These include business studies, physics, chemistry, home economics etc.

This experiment demonstrated that children in early secondary school are capable of using sophisticated software with ease. It is likely that they could also learn to use other general-purpose software packages such as word processors, databases, graphics packages etc. Such packages could be similarly adapted for teaching purposes in areas right across the curriculum. This approach has many advantages over the development of specific courseware packages, including speed and cost of production. Ideally, secondary schools should standardise on a few general-purpose packages which could be taught in first and second year. These could then be used for various applications throughout secondary schooling. The most effective strategy would be to standardise on an integrated package containing a word processor, database, spreadsheet etc. If this were done, students would be dealing with a consistent user interface throughout all of the applications. This research has indicated that young students can deal with a 'serious' software package even though they may not use all of its facilities. Thus, there is no need to design simplified packages for students in this age group. There is no reason why 'industry standard' packages should not be chosen if they are available and reasonably priced.

The design of learning materials based on general-purpose software could be undertaken by individual teachers, or by groups of teachers working in the same subject area. It would be vital to keep in mind that the software package is a vehicle for instruction and not the object of instruction in itself. The development of materials would proceed from the identification of a problem through the specification of objectives, the design of instruction, evaluation and testing and the production of materials. In addition to computer-based materials, print materials would also be required for most applications. Work of this nature by teachers would necessarily require support in terms of hardware, software, secretarial help and, possibly, release from teaching duties for limited periods. Distribution of materials could be accomplished electronically through existing communications systems such as NITEC (National Information Technology in Education Centre).

Support would also be required for teachers who would use such materials. This would be best accomplished by school-based inservice. If a school standardised on a few general-purpose software packages, then it would be in the interest of the whole staff to learn to use the software. In addition, support would be required by subject teachers for each instructional package produced. This would be easily managed if teachers were already familiar with the software. Print materials alone might suffice for this purpose.

If such an initiative were to be taken, its success or failure would be very much dependent on the reaction of teachers. Many teachers who were initially enthusiastic and who undertook inservice computer courses have become disillusioned with computers. This is partly due to the nature of many of the early inservice courses, in which the emphasis was firmly placed on learning programming. One of the main outcomes of such courses was the realisation by many teachers that programming was very difficult. They also realised that programming was not easy to teach. The scarcity of suitable software and the lack of in-school support have also contributed to this disillusion. In order to regain the interest and enthusiasm of teachers, it would be necessary to produce complete ready-to-use instructional packages which were immediately relevant to present syllabi. Anything less would be unlikely to have any impact. It is not suggested that such packages should replace normal classroom teaching. They should complement it. This research shows that such instructional packages can be produced reasonably easily and are effective in normal classroom situations.

References

Adams T. (1988), Computers in Learning: A Coat of Many Colours, Computer Education, 12, 1, 1-6.

Aiken L.R. (1972), Language factors in learning mathematics, Review of Educational Research, 42, 359-385.

Allinson L., Hammond N. (1990), Learning Support Environments: Rationale and Evaluation, in Kibby M. (Ed.), Computer Assisted Learning: Selected Proceedings from the CAL '89 Symposium, Oxford, Pergamon Press.

Alty J.L. (1987), Keynote Address: Working Within Limitations: Computer Aided Instruction and Expert Systems, in Moonen J., Plomp T. (Eds.), EURIT 86: Educational Software and Courseware, 3-12, Oxford, Pergamon Press.

Anand P.G., Ross S.M. (1987), Using Computer-Assisted Instruction to Personalize Arithmetic Materials for Elementary School Children, Journal of Educational Psychology, 79, 1, 72-78.

Apple Computer Inc. (1983), Appleworks, Cupertino, Apple Computer Inc.

Arad O.S. (1987), The Spreadsheet: Solving Word Problems, The Computing Teacher, 14, 4, 13-15.

Aronson D.T., Briggs L.J. (1983), Contributions of Gagne and Briggs to a Prescriptive Model of Instruction, in Reigeluth C.M. (Ed.), Instructional Design: Theories and Models, Hillsdale NJ., Lawrence Erlbaum Associates.

Ausubel D.P. (1968), Educational Psychology: A Cognitive Approach, New York, Holt, Rinehart and Winston.

Ausubel D.P. (1977), The facilitation of meaningful learning in the classroom, Educational Psychologist, 12, 162-178.

Ausubel D.P.(1986), Educational Psychology. A Cognitive Approach, New York, Holt, Rinehart and Winston.

Ball D. (1987), PROLOG and Mathematics Teaching, Educational Review, 39, 2, 155-161.

Ball D., Higgo J., Oldknow A., Straker A., Wood J. (1987), Will Mathematics Count? Computers in Mathematics Education, Hatfield, Advisory Unit for Computer Based Education.

Ballew H., Cunningham J.W. (1982), Diagnosing strengths and weaknesses of sixth-grade students in solving word problems, Journal for Research in Mathematics Education, 13, 202-210.

Balzano G.J. (1987), Reconstructing the Curriculum for Design, Machine-Mediated Learning, 2, 1, 83-109.

Bassok M., Holyoak K.J. (1989), Interdomain Transfer Between Isomorphic Topics in Algebra and Physics, Journal of Experimental Psychology: Learning, Memory and Cognition, 15, 1, 153-166.

Becker H.J.(1987), Using Computers for Instruction, Byte, 12, 2, 149-162.

Berger D.E., Wilde J.M. (1987), A Task Analysis of Algebra Word Problems, in Berger D.E., Pezdek K., Banks W.P., Applications of Cognitive Psychology: Problem Solving, Education, and Computing, Hillsdale NJ., Lawrence Erlbaum Associates.

Booth L. (1984a), Algebra: Children's Strategies and Errors, Windsor, NFER-Nelson.

Booth L. (1984b), Misconceptions Leading to Error in Elementary Algebra, Journal of Structural Learning, 8, 125-138.

Boulay J.B.H. (1978), Learning Primary Mathematics Through Computer Programming, Unpublished Doctoral Dissertation, University of Edinburgh.

Briars D., Larkin J. (1984), An integrated model of skill in solving elementary word problems, Cognition and Instruction, 1, 245-296.

Brown C.A., Carpenter T.P., Kouba V.L., Lindquist M.M., Silver E.A., Swafford J.O. (1988), Secondary School Results for the Fourth NAEP Mathematics Assessment, Mathematics Teacher, 81, 337-347.

Brown J.M., (1987), Spreadsheets in the Classroom, The Computing Teacher, 14, 4, 8-12.

Bruner J.S. (1966), Toward a theory of instruction, New York, Norton.

Burkhardt H. (1986), Computer Aware Curricula: Ideas and Realisation, in Howson A.G., Kahane J.P. (Eds.), The Influence of Computers and Informatics on Mathematics and its Teaching, Cambridge, Cambridge University Press.

Burns P., Bozeman W. (1981), Computer Assisted Instruction and Mathematics Achievement: Is There a Relationship?, Educational Technology, 10, 2, 32-39.

Butler D., Close S. (1989), Assessing the Benefits of a LOGO Problem-Solving Course, Irish Educational Studies, 8, 2, 168-190.

Caldwell J.H., Goldin G.A. (1987), Variables Affecting Word Problem Difficulty in Secondary School Mathematics, Journal for Research in Mathematics Education, 18, 3, 187-196.

Case R. (1975), Gearing the demands of instruction to the developmental capacities of the learner, Review of Educational Research, 45, 59-97.

Case R. (1978a), A developmentally based theory and technology for the design of effective instruction, Review of Educational Research, 48, 439-463.

Case R. (1978b), Implications of developmental psychology for the design of effective instruction, in Lesgold A.M., Pellegrino J.W., Fokkema S.D., Glaser R. (Eds.), Cognitive Psychology and Instruction, New York, Plenum.

Case R., Bereiter C. (1984), From Behaviourism to Cognitive Behaviourism to Cognitive Development: Steps in the Evolution of Instructional Design, Instructional Science, 13, 141-158.

Christenen E.B. (1980), Reality Versus Fantasy in Algebra Word Problems: An Investigation of the Context Preferences of Students, Unpublished Doctoral Dissertation, University of Kentucky.

Clark R.E. (1983), Reconsidering Research on Learning from Media, Review of Educational Research, 53, 4, 445-459.

Clark R.E. (1985), Evidence for Confounding in Computer-Based Instruction Studies: Analysing the

Meta-Analyses, Educational Communications and Technology Journal, 33, 4, 249-262.

Clark R.E., Voogel A. (1985), Transfer of Training Principles for Instructional Design, Educational Technology and Communication Journal, 33, 2, 113-123.

Clement J. (1982), Algebra Word Problem Solutions: Thought Processes Underlying A Common Misconception, Journal for Research in Mathematics Education, 13, 1, 16-30.

Clements D.H. (1985), Logo programming: Can it change how children think?, Electronic Learning, 28, 74-75.

Cockcroft W.H. (1982), Mathematics Counts, London, HMSO.

Cohen M., Carry L.R. (1978), Interest and its relationship to verbal problem solving, International Journal of Mathematical Education in Science and Technology, 9, 207-212.

Czepiel J., Esty E. (1980), Mathematics in the Newspaper, Mathematics Teacher, 73, 582-586.

Darch C., Carnine D., Gersten R. (1984), Explicit Instruction in Mathematics Problem Solving, Journal of Educational Research, 77, 6, 351-359.

Davis P.J., Hersh R. (1981), The Mathematical Experience, Boston, Birkhauser.

Davis R.B. (1984), Learning Mathematics: The Cognitive Science Approach to Mathematics Education, London, Croom Helm.

De Corte E., Verschaffel L., De Win L. (1985), Influence of Rewording Verbal Problems on Childrens' Problem Representations and Solutions, Journal of Educational Psychology, 77, 4, 460-470.

De Corte E., Verschaffel L., Verschueren J. (1982), First graders' solution processes in elementary word problems, in Vermandel (Ed.), Proceedings of the Sixth International Conference for the Psychology of Mathematical Education, Antwerp, Universitaire Instelling Antwerpen.

Department of Education (1987), Rules and Programme for Secondary Schools 1987/88, Dublin, Stationery Office.

DiSessa A.A. (1987), The Third Revolution in Computers and Education, Journal of Research in Science Teaching, 24, 4, 343-367.

Dickey E., Kherlopian R. (1987), A Survey of Teachers of Mathematics, Science and Computers on the Uses of Computers in Grades 5-9 Classrooms, Educational Technology, 27, 6, 10-14

Diem R. (1986), Microcomputer Technology in Educational Environments: Three Case Studies, Journal of Educational Research, 80, 2, 93-98.

Dubitsky B. (1986), Algebraic Problem Solving in Grade Six: An Application of the Computer Spreadsheet, Unpublished Doctoral Dissertation, Columbia University.

DuBoulay J.B.H. (1980), Teaching Mathematics Through Programming, International Journal of Mathematical Education in Science and Technology, 11, 3, 347-360.

Dudley-Marling C., Owston R.D. (1987), The State of Educational Software: A Criterion-Based Evaluation, Educational Technology, 27, 3, 25-29.

Ernest P. (1985), The philosophy of mathematics and mathematics education, International Journal of Mathematical Education in Science and Technology, 16, 5, 603-612.

Ernest P. (1987), A Model of the Cognitive Meaning of Mathematical Expressions, Br. J. Educ. Psychol., 57, 343-370.

Ewing D.E. (1984), A Study Of Cognitive Processes With Respect To Selected Algebra 1 Word Problems By Means Of A Constructed Reading Test, Unpublished Doctoral Dissertation, Kansas State University.

Feurzeig W. (1986), Algebra Slaves and Agents in a Logo-Based Mathematics Curriculum, Instructional Science, 14, 229-254.

Fisher K.M. (1988), The Students-and-Professors Problem Revisited, Journal for Research in Mathematics Education, 19, 3, 260-262.

Flavell J.H. (1971), What is memory development the development of?, Human Development, 14, 272-278.

Freese R., Lounesto P., Stegenga D.A. (1986), The Use of MuMath in the Calculus Classroom, Journal of Computers in Mathematics and Science Teaching, 6, 1, 52-55.

Gagne E.D. (1985), The Cognitive Psychology of School Learning, Boston, Little, Brown and Company.

Gagne R.M. (1983), Some Issues in the Psychology of Mathematics Instruction, Journal for Research in Mathematics Education, 14, 1, 7-18.

Gagne R.M. (1977), The Conditions of Learning, New York, Holt Rinehart and Winston.

Gagne R.M. (1975), Essentials of Learning for Instruction, New York, Holt, Rinehart and Winston.

Gagne R.M., White R.T. (1978), Memory Structures and Learning Outcomes, Review of Educational Research, 48, 2, 187-222.

Glaser R. (1984), Education and Thinking: The Role of Knowledge, American Psychologist, 39, 93-104.

Golden G.A., Caldwell J.H. (1979), Syntax, Content and Context Variables in a Research Study, in Goldin G.A., McClintock C.E. (Eds.), Mathematics Education Reports: Task Variables in Mathematical Problem Solving, Columbus, ERIC.

Goodyear P. (1984), LOGO: A guide to learning through programming, Chichester, Ellis Horwood.

Graf R.G., Riddell J.C. (1972), Sex differences in problem-solving as a function of problem context, Journal of Educational Research, 65, 451-452.

Green D., Jones B. (1986), Using the Computer, Mathematics in Schools, 15, 5, 36-37.

Greeno J.G. (1980), Some examples of cognitive task analysis with instructional implications, in Snow R.E., Federico P.A. and Montague W.E. (Eds.), Aptitude Learning and Instruction: Vol. 2. Cognitive process analysis of learning and problem solving, Hillsdale NJ., Lawrence Erlbaum Associates.

Greer B. (1984), Introduction: Cognitive Psychology and Mathematics Education, Journal of Structured Learning, 8, 103-112.

Groen G., Kieran C. (1983), The Many Facets of Piaget, in Ginsburg H.P. (Ed.), The Development of Mathematical Thinking, New York, Academic Press.

Hasselbring T.S. (1986), Research on the Effectiveness of Computer-Based Instruction: A Review, International Review of Education, 32, 3, 313-324.

Heid M.K. (1988), Resequencing skills and concepts in applied calculus using the computer as a tool, Journal for Research in Mathematics Education, 19, 1, 3-25.

Helm A.W., Watts K.P., Simmonds V. (1975), AH2/AH3 Manual, Windsor, NFER Publishing Co. Ltd.

Higgo J. (Ed.) (1985), Proceedings of the A-Level Conference held at Ware, London, The Mathematical Association.

Hinsley D.A., Hayes J.R., Simon H.A. (1977), From Words to Equations. Meaning and Representation in Algebra Word Problems, in Just M.A., Carpenter P.A. (Eds.), Cognitive Processes in Comprehension, Hillsdale NJ., John Wiley & Sons.

Holtan B. (1964), Motivation and general mathematics students, Mathematics Teacher, 57, 20-25.

Howe J. (1986), Arithmetical and Mathematical Concepts: Report of International Conference on Information

Technology and Basic Learning, Paris, Centre for Educational Research and Innovation (OECD).

Howe J., O'Shea T., Plane F. (1979), Teaching mathematics through LOGO programming: An evaluation study, in Lewis R., Tagg E.D. (Eds.), Computer-assisted Learning: Scope, progress and limits, Amsterdam, North Holland.

Howe J.A.M., Ross P.M., Johnson K.R., Plane F., Inglis R. (1982), Teaching Mathematics Through Programming in the Classroom, Computers and Education, 6, 85-91.

Hsiao F.S.T. (1985), Micros in mathematics education - Uses of spreadsheets in CAL, International Journal of Mathematical Education in Science and Technology, 16, 6, 705-713.

Kaput J.J. (1986), Information Technology and Mathematics: Opening New Representational Windows, Journal of Mathematical Behaviour, 5, 187-207.

Keeling R., Whiteman S. (1990), Simply Spreadsheets, Sutton Coldfield, KW Publications.

Kilpatrick J. (1987), Problem Formulating: Where Do Good Problems Come From?, in Schoenfeld A.H. (Ed.) Cognitive Science and Mathematics Education, Hillsdale NJ., Lawrence Erlbaum Associates.

Kintsch W., Greeno J.G. (1985), Understanding and Solving Word Arithmetic Problems, Psychological Review, 92, 1, 109-129.

Kozma R.B. (1987), The Implications of Cognitive Psychology for Computer-Based Teaching on Secondary School Students, Educational Technology, 27, 11, 20-25.

Kulik J., Bangert R., Williams G. (1983), Effects of Computer-Based Teaching on Secondary School Students, Journal of Educational Psychology, 75, 1, 19-26.

Kulik J., Kulik C., Cohen P. (1980), Effectiveness of Computer-Based College Teaching: A Meta-Analysis of Findings, Review of Educational Research, 50, 1, 525-544.

Kunkle D., Burch C.I. (1984), Symbolic Computer Algebra: The Classroom Computer Takes a Quantum Jump, Mathematics Teacher, 77, 3, 209-214.

Levin H.M., Leitner D., Meister G.R. (1986), Cost-Effectiveness of Alternative Approaches to Computer-Assisted Instruction, Institute for Research on Educational Finance and Governance, Stanford University.

Lewis A.B., Mayer R.E. (1987), Students' Miscomprehension of Relational Statements in Arithmetic Word Problems, Journal of Educational Psychology, 79, 4, 363-371.

Luehrmann A. (1986), Spreadsheet: More Than Just Finance, The Computing Teacher, 13, 7, 24-28.

Manion M.H. (1985), CAI Modes of Delivery and Interaction: New Perspectives for Expanding Applications, Educational Technology, 25, 1, 25-28.

Maurer S.B. (1984), College Entrance Mathematics in the Year 2000, Mathematics Teacher, 77, 6, 422-428.

Mayer R.E.(1981), Frequency norms and structural analysis of algebra story problems into families, categories and templates, Instructional Science, 10, 135-175.

Mayer R.E. (1982), Memory for Algebra Story Problems, Journal of Educational Psychology, 74, 2, 199-216.

McCarthy K.A. (1976), Sex bias in tests of mathematical aptitude, Dissertation Abstracts International, 36, 7301A.

Meijer J., Riemersma F. (1986), Analysis of Solving Problems, Instructional Science, 15, 3-19.

Mills G.M. (1987), Categories of Educational Microcomputer Programs: Theories of Learning and Implications for Future Research, in Rushby N. (Ed.), Technology Based Learning: Selected Readings, London, Kogan Page.

Morris O.D. (1987), Text & Tests 1, Dublin, Celtic Press.

Moyer J.C., Moyer M.B., Sowder L., Threadgill-Sowder J. (1984), Story Problem Formats: Verbal versus Telegraphic, Journal for Research in Mathematics Education, 15, 64-68.

Moyer J.C., Sowder L., Threadgill-Sowder J., Moyer M.B. (1984), Story Problem Formats: Drawn versus Verbal versus Telegraphic, Journal for Research in Mathematics Education, 15, 5, 342-351.

Muth K.D. (1984), Solving Arithmetic Word Problems: Role of Reading and Computational Skills, Journal of Educational Psychology, 76, 2, 205-210.

National Mathematics and Statistics CBL Review Panel (1986), Will Mathematics Count?, Hatfield, Advisory Unit for Computer Based Education.

Noss R. (1986), Constructing a Conceptual Framework for Elementary Algebra through Logo Programming, Educational Studies in Mathematics, 17, 335-357.

O'Shea T., Self J. (1983), Learning and Teaching with Computers, Brighton, Harvester Press.

Orzech Z., Shelton F.A. (1986), The Electronic Spreadsheet as a Didactic Learning Enhancement, Computer Education, 10, 4, 429-437.

Paige J.M, Simon H.A. (1966), Cognitive Processes in Solving Algebra Word Problems, in Kleinmuntz B. (Ed.), Problem Solving: Research, Method and Theory, New York, John Wiley and Sons.

Papert S. (1980), Mindstorms, Brighton, Harvester Press.

Paul D.J., Nibbelink W.H., Hoover H.D. (1986), The Effects of Adjusting Readability on the Difficulty of Mathematics Story Problems, Journal for Research in Mathematics Education, 17, 3, 163-171.

Pea R.D., Kurland D.M. (1983), On the cognitive prerequisites of learning computer programming, (Technical Report No.18), New York: Center for Children and Technology, Bank Street College.

Pea R.D., Kurland D.M. (1984a), On the cognitive effects of learning computer programming, (Technical Report No.9), New York: Center for Children and Technology, Bank Street College.

Pea R.D., Kurland D.M. (1984b), Logo programming and the development of planning skills, (Technical Report No.16), New York: Center for Children and Technology, Bank Street College.

Pea R.D., Kurland D.M. (1987), On The Cognitive Effects of Learning Computer Programming, in Pea R.D., Sheingold K. (Eds.), Mirrors of Minds, New Jersey, Ablex Publishing Corporation.

Pea R.D., Kurland D.M., Hawkins J. (1985), LOGO and the Development of Thinking Skills, in Chen M., Paisley W. (Eds.), Children and Microcomputers, Beverly Hills, Sage Publications Ltd.

Polya G. (1945), How To Solve It, Princeton New Jersey, Princeton University Press.

Quilter D., Harper E. (1988), 'Why we didn't like mathematics, and why we can't do it', Educational Research, 30, 2, 121-129.

Reed S.K., Dempster A., Ettinger M. (1985), Usefulness of Analogous Solutions for Solving Algebra Word Problems, Journal of Experimental Psychology: Learning, Memory and Cognition, 11, 1, 106-125.

Reif F. (1987), Interpretation of Scientific or Mathematical Concepts: Cognitive Issues and Instructional Implications. Cognitive Science, 11, 395-416.

Roblyer M.D. (1985), The Greening of Educational Computing: A Proposal for a more Research-Based approach to computers in instruction, Educational Technology, 25, 1, 40-44.

Rosnick P. (1981), Some Misconceptions Concerning the Concept of Variable, Mathematics Teacher, 74, 418-420.

Rosnick P. (1982), Student Conceptions of Semantically Laden Letters in Algebra, National Science Foundation, Washington D.C.

Rosnick P., Clement J. (1980), Learning without Understanding: The Effect of Tutoring Strategies on Algebra Misconceptions, Journal of Mathematical Behaviour, 3, 1, 3-27.

Ross S.M. (1983), Increasing the Meaningfulness of Quantitative Material by Adapting Context to Student Background, Journal of Educational Psychology, 75, 4, 519-529.

Ross S.M., Anand P.G.(1987), A Computer-Based Strategy for Personalising Verbal Problems in Teaching Mathematics, Educational Communications and Technology Journal, 35, 3, 151-162.

Ross S.M., Anand P.G., Morrison G.R. (1988), Putting the Student into the Word Problem: Microcomputer-based Strategies that Personalize Math Instruction, Focus on Learning Problems in Mathematics, 10, 2, 29-42.

Ross S.M., Bush A.J. (1980), Effects of Abstract and Educationally Oriented Learning Contexts on Achievements and Attitudes of Preservice Teachers, Journal of Educational Research, 74, 1, 19-22.

Ross S.M., McCormick D., Krisak N. (1986), Adapting the Thematic Context of Mathematical Problems to Student Interests: Individualised versus Group-Based Strategies, Journal of Educational Research, 79, 4, 245-252.

Sachs S.G. (1984), Citation Patterns in Instructional Development Literature, Journal of Instructional Development, 7, 2, 8-13.

Schoenfeld A.H., Arcavi A. (1988), On the meaning of Variable, Mathematics Teacher, 81, 6, 420-427.

Schoenmaker J., van der Mast C., Moonen J. (1987), A Methodology for the development of Educational Software, in Moonen J., Plomp T. (Eds.), EURIT 86; Developments in Educational Software and Courseware, 235-242, Oxford, Pergamon Press.

Schwanenflugel P.J., Shoben E.J. (1983), Differential Context Effects in the Comprehension of Abstract and Concrete Verbal Materials, Journal of Experimental Psychology: Learning, Memory and Cognition, 9, 1, 82-102.

Seymour S.L., Sullivan H.J., Story N., Mosley M.L. (1987), Microcomputers and Continuing Motivation, Educational Computing and Technology Journal, 35, 1, 18-23.

Silver E.A. (1987), Foundations of Cognitive Theory and Research for Mathematics Problem-Solving Instruction, in Schoenfeld A.H. (Ed.), Cognitive Science and Mathematics Education, Hillsdale NJ, Lawrence Erlbaum Associates.

Silver E.A., Lindquist M.M., Carpenter T.P., Brown C.A., Kouba V.L., Swafford J.O. (1988), The Fourth NAEP Mathematics Assessment: Performance Trends and Results and Trends for Instructional Indicators, Mathematics Teacher, 81, 9, 720-727.

Sims-Knight J.E., Kaput J.J. (1983), Exploring Difficulties in Transforming between Natural Language and Image Based Representations and Abstract Symbol Systems of Mathematics, in Rogers D., Sloboda J.A., The Acquisition of Symbolic Skills, London, Plenum Press.

Sleeman D. (1984), An Attempt to Understand Students' Understanding of Basic Algebra, Cognitive Science, 8, 387-412.

Soloway E., Lochhead J., Clement J. (1982), Does Computer Programming Enhance Problem-Solving Ability? Some Positive Evidence on Algebra Word Problems, in

Seidel R.J., Anderson R.E., Hunter B. (Eds.), Computer Literacy, New York, Academic Press.

Sweller J. (1988), Cognitive Load During Problem Solving: Effects on Learning, Cognitive Science, 12, 257-285.

Swenson R.P., Anderson C. (1982), The Role of Motivation in Computer-Assisted Instruction, Creative Computing, 8, 10, 134-139.

Thaeler J.S. (1986), A New Solution to an Old Problem - Solving Word Problems in Algebra, Mathematics Teacher, 79, 9, 682-689.

Thomas M.O.J. (1987), Algebra with the Aid of a Computer, Mathematics in School, 16, 1, 36-38.

Thompson C.S., Rathmell E.C. (1988), NCTM's Standards for School Mathematics; K-12, Mathematics Teacher, 81, 348-352.

Travers K.J. (1967), A test of pupil preference for problem-solving situations in junior high school mathematics, Journal of Experimental Education, 35, 9-18.

Tuckman B.W. (1978), Conducting Educational Research, New York, Harcourt, Brace, Jovanovich.

Underwood J. (1986), The role of the computer in developing childrens' classification abilities, Computers and Education, 10, 175-180.

Waddington J., Wigley A. (Eds.) (1985), Secondary Maths with Micros, Hatfield, Advisory Unit for Computer Based Education.

Walker D.D. (1987), Keynote Address: Organisation for Educational Software Production in Scotland, in Moonen J., Plomp T. (Eds.), EURIT 86: Educational Software and Courseware, 317-326, Oxford, Pergamon Press.

Watson D.M. (1987), Developing CAL - The Computers in the Curriculum (CIC) Model, in Moonen J., Plomp T. (Eds.), EURIT 86: Developments in Educational Software and Courseware, 225-233, Oxford, Pergamon Press.

Wessels M.G. (1982), Cognitive Psychology, New York, Harper and Row.

Whiting J. (1985), New Directions in Educational Computing: Coming Changes in Software and Teaching Strategies to Optimize Learning, Educational Technology, 25, 9, 18-21.

Wiechers G. (1974), Programming as an Educational Tool, International Journal of Mathematical Education in Science and Technology, 5, 699-703.

Wittrock M.C. (1981), Learning and Memory, in Farley F.H., Gordon N.J. (Eds.), Psychology and Education: The State of the Union, pps. 242-264, Berkely, McCutchan Publishing Co.

APPENDIX A

Pretest

Pretest

1. One number is 6 bigger than another. If the smaller one is X, what is the bigger one?

2. One number is 12 smaller than another. If the bigger one is X, what is the smaller one?

3. One number is five times as big as another. If the smaller one is X, what is the bigger one?

4. Two numbers add up to 33. If one is X, what is the other one?

5. Two numbers add up to X. If one is 10, what is the other one?

6. One number is got by doubling another and then adding 3. If the smaller number is X, what is the bigger?

7. One number is got by adding 5 to another and then multiplying the answer by 3. If X is the smaller number, what is the bigger?

8. Barbara has £5 more than Jane. If Jane has £X then Barbara has:

9. Helen earned three times as much as Louise. If Louise earned £X then the number of pounds that Helen earned was:

10. Lorraine had X pence. She spent 12 pence. The number of pence that she had left was:

11. Paula had £19. She spent £X. The number of pounds she had left was:

12. Sharon earned £X on Saturday and another £X on Sunday. She spent £13. The number of pounds she had left was:

13. During a footbal season, Liverpool won X matches, drew Y matches and lost Z matches. The total number of matches that they played was:

14. Deirdre earns £X per hour for babysitting. She also gets £3 per week pocket money. In a week when she babysat for 4 hours, her total income, in pounds, was:

15. Large ice creams cost A pence, medium ones cost B pence and small ones cost C pence. How much would it cost to buy 12 of each?

16. Five girls went to the pictures. It cost £X each to get in. They also spent £2 each on sweets. How much was spent altogether by the whole five?

17. There were X pupils in a class. 3 of them were absent on the day of the school outing. Each of the rest paid £5 for the outing. The total amount collected was:

18. A shopkeeper bought a box containing 72 apples but X of them were bad and could not be sold. She sold all the rest for 18 pence each. The amount she made was:

19. One side of a rectangle is 6 metres longer than the other. If the shorter side is X, find the length of the perimeter. (The perimeter is the lengths of all the sides added together).

20. A sweatshirt costs £4 more than a T-shirt. If a T-shirt costs £X, how much would five T-shirts and two sweatshirts cost?

21. A jeweller sold 12 chains, some silver at £15 each and some gold at £24 each. If X of the chains were silver, how much was the whole lot sold for:

<u>APPENDIX B</u>

Posttest

Posttest

1. One number is 8 bigger than another. If the smaller one is X, what is the bigger one?

2. One number is 7 smaller than another. If the bigger one is X, what is the smaller one?

3. One number is three times as big as another. If the smaller one is X, what is the bigger one?

4. Two numbers add up to 27. If one is X, what is the other one?

5. Two numbers add up to X. If one is 12, what is the other one?

6. One number is got by doubling another and then adding 7. If the smaller number is X, what is the bigger?

7. One number is got by adding 4 to another and then multiplying the answer by 5. If X is the smaller number, what is the bigger?

8. Kate is 12 years older than Mary. If Mary is X years old then Kate is:

9. Mary has twice as much money as Sue. If Sue has £X then the number of pounds that Mary has is:

10. A lady weighed X kg. She went on a diet and lost 4 kg. Her new weight, in kg, was:

11. There were 25 students in a class. X students left. The number of students remaining was:

12. Jane had £X. She got a present of another £X. She then spent £3. The number of pounds she had left was:

13. A girl bought a meal for £A and spent £B on clothes. She also lost £C on slot machines. The total amount that she spent was:

14. Susan worked for 3 days and earned £X each day. Her boss also gave her a present of £5. How much did she get altogether?

15. A burger costs A pence, a bag of chips costs B pence and a coke costs C pence. How much would it cost to buy 5 of each:

16. A mother sent her 3 children to summer camp for a week. It cost £X each for the week. She also gave each one £5 pocket money. The total cost for the mother was:

17. A shop was selling jeans for £X each. In a sale they were reduced by £2 each. The cost of 3 pairs of jeans in the sale was:

18. A girl was earning £105 per week. It cost her £X per week to live. She saved the rest of her money. In 4 weeks she saved:

19. Adults pay £3 more than children for an excursion. If a child's fare is £X then the total cost for a family of two adults and two children is:

20. A bar of chocolate costs 6 pence more than an orange. If an orange costs X pence, how much would four oranges and 3 bars of chocolate cost?

21. A shop sold 40 jumpers, some made of wool at £15 each and some made of cotton at £12 each. If X of the jumpers were made of wool, how much was paid for the whole 40 jumpers?

APPENDIX C

Attitude Measuring Instrument

SA = Strongly Agree A = Agree U = Undecided D = Disagree SD = Strongly Disagree					<u>Name:</u>
Strongly Agree				Strongly Disagree	
SA	A	U	D	SD	I have less confidence in myself with maths than with other subjects.
SA	A	U	D	SD	I would like to get a job that involved a lot of computer use.
SA	A	U	D	SD	Doing Honours Maths is more important for boys than for girls.
SA	A	U	D	SD	Working with a computer is always very interesting and enjoyable.
SA	A	U	D	SD	Maths is one of the hardest subjects in school.
SA	A	U	D	SD	What we learn in maths is useful in other school subjects
SA	A	U	D	SD	Using a computer is no harder than using most other machines.
SA	A	U	D	SD	I often find maths classes very boring.

SA	A	U	D	SD	The girls I pal around with are just as good at maths as the boys I know.
SA	A	U	D	SD	I usually expect to get good marks in maths tests.
SA	A	U	D	SD	Most people could never learn to use a computer properly.
SA	A	U	D	SD	Less time should be spent on maths and more time spent on useful subjects.
SA	A	U	D	SD	I think that maths is one of my best subjects.
SA	A	U	D	SD	Girls' schools don't need computers as much as boys' schools.
SA	A	U	D	SD	Sometimes I worry that I might make a mistake which would damage the computer
SA	A	U	D	SD	I enjoy mathematical puzzles and problems.

APPENDIX D

Debriefing Questionnaire

Debriefing Questionnaire

Was the course a) too long b) too short c) just right

Did you find that the sessions were too long?

Did you find it hard to concentrate for the full session?

Was the course like anything else you have done either in school or elsewhere? (If 'yes' then give brief details)

Were you given enough instruction on how to use the computer?

Did you find it difficult to use the computer?

Do you now feel more confident in your ability to use a computer?

Were you given enough instruction on the problems?

Were some of the problems too easy? (If 'yes' then say which types)

Were some of the problems too difficult? (If 'yes' then say which types)

Which problems did you find most interesting?

Which problems did you find boring?

Did you think that the problems could have been done more easily without a computer?

What did you like best about the course?

What did you like least?

Do you think that the course helped to improve your mathematical ability?

Can you suggest any way in which the course could be improved?

Any further comments?

APPENDIX E

Handout on Spreadsheet Concepts

SPREADSHEETS

Spreadsheets contain **ROWS** and **COLUMNS**.

Rows go across the screen and Columns go down the screen.

A CELL is where a row meets a column.

Each COLUMN has a name .... A,B,C etc.

Each ROW has a name .... 1,2,3 etc.

Every CELL is named by its Column and its Row ...A1,C12,E36

****

The CURSOR is the bright bar on the screen. It shows you what cell you are at in the spreadsheet. Whenever you type something, it goes into the cell where the cursor is positioned. To move the cursor around the spreadsheet, use the four arrow keys (on the bottom right of the computer).

****

Each cell may contain either a **LABEL**, a **VALUE**, or a **FORMULA**.

LABEL CELLS
These contain words or letters ... Cost, Area, X, Y, etc.

VALUE CELLS
These contain numbers .....23, 5, 26.7 etc.

FORMULA CELLS
These contain a formula. Formulas always involve the names of other cells. For example, if cell D23 contains the formula +D20+C20, then it will show a value equal to the sum of the values in cells D20 and C20. The formula will not appear on the screen. To discover the formula, you have to place the cursor on cell D23. The formula will then be shown at the bottom of the screen.

A formula cell contains a formula linking it to other cells but shows the <u>value</u> of the formula rather than the formula itself.

****

TYPING IN LABELS, VALUES AND FORMULAS

Place the cursor on the chosen cell. Then type in the label, value or formula as required. Formulas always begin with + or -. If you make a mistake, use the <u>DELETE</u> key. If you get stuck, press <u>ESC</u> and start again.

CHANGING LABELS, VALUES AND FORMULAS

Place the cursor on the cell that is to be changed. Then type in the new label, value or formula and press RETURN. You don't have to "rub out" the old one.

****

SPREADSHEETS: HOW THEY WORK

The key idea in a spreadsheet is the formula. A cell which contains a formula will show the value of the formula on the screen:

THIS IS WHAT YOU SEE

	A	B
1	Length	6
2	Width	8
3	Area	48

THIS IS WHAT YOU TYPE

	A	B
1	Length	6
2	Width	8
3	Area	+B1*B2

Cell B3 contains the formula +B1*B2. This means that it will show the value got by multiplying cells B1 and B2. In this case the value is 48 (6*8). If the values of B1 and B2 are changed, then the value of B3 will change automatically. So if the length (B1) was changed to 10, then the area (B3) would <u>automatically</u> change to 80.

APPENDIX F

Abstract Exercise Problems

Abstract Problems

Abstract 1

1a. Using the spreadsheet called Example 1, move the cursor to the cells named below. Fill in the contents of each cell on the chart below.

Cell	Contents
A5	
E7	
A3	
H6	
E5	

1b. Using the same example, make lists of all the cells that contain: a)Labels b) Values c) Formulas

Label Cells	Value Cells	Formula Cells

2. Use the spreadsheet below to type the Labels into the spreadsheet called Example 2 on your computer.

Side 1		Side 2	
A =	3.25	F =	4.50
B =	3.00	G =	5.50
C =	3.50	H =	8.75
D =	1.55		
E =	3.00		
TOTAL 1	14.30	TOTAL 2	18.75
SUBTRACT	4.45		

3. Look at the spreadsheet below. Use it to correct the Labels in the spreadsheet called Example 3 on your computer.

Type	Value	Number	Total
X	4.95	8	39.60
Y	5.53	3	16.59
Z	4.99	5	24.95
		Total	81.14

4. Look at the spreadsheet below. Use it to enter the numbers in the column called Number in Example 4 on your computer. Then take a note of the results that appear in the % column and the overall average.

Type	Number	Total	%
A	100	200	
B	120	300	
C	200	250	
D	180	300	
E	67	100	
F	88	200	
G	165	300	
H	90	150	

Overall Average

5. Enter the values below into the column called Normal Price in Example 5 on your computer. Watch what happens and then take a note of the new Grand Total.

Type	Normal Price
A	415.00
B	375.00
C	585.00
D	342.00
E	94.00
F	59.00
G	1075.00

New Grand Total =

6. Look at Example 6 on your spreadsheet. Name the cell that contains a formula. Say what the formula is and give its value.

Cell with Formula	Formula	Value of Formula

7. Look at Example 7 in your spreadsheet. The formula for the answer is missing. The answer is got by multiplying the value of Num1 by the value of Num2. The correct formula for the answer then is +C94*C95. (C94 contains the value of Num1 and C95 contains the

value of Num2. All formulas begin with +). Type this formula into cell C97. What is the answer?

Answer =

Use the spreadsheet to find the answer for these values of Num1 and Num2:

Num1	Num2	Answer
22	31	
152	9	
398	456	

Abstract 2

1. Use the spreadsheet called Exercise 1 to fill in the missing numbers in the chart below. The column "Add" is got by writing formulas to add the numbers in the first two columns, "Subt" is got by writing formulas to subtract etc.

Num1	Num2	Add	Subt	Mult	Divide
24	6				
60	20				
33	11				

2. Use the spreadsheet called Exercise 2 to fill in the missing numbers in the chart below. The first column is got by writing formulas to multiply Num by 12, the next by writing formulas to multiply Num by 24 etc.

Num	By 12	By 24	By 36
10			
16			
8			
12			

3. Use the spreadsheet called Exercise 3 to calculate the missing numbers in the chart below. The first column is got by writing formulas to find 12% of the numbers in the column called Num. (Divide by 100 and multiply by 12). The next column is got by finding 26% etc.

Num	12%	26%
225		
259		
413		

4. Use the spreadsheet called Exercise 4 to calculate the missing numbers in the chart below. Write formulas in the column called Mult which multiply Num1 by Num2. The formulas in the column called Answer should add up a number in column Mult and the number beside it in column Num3.

Num1	Num2	Mult	Num3	Answer
20	5		30	
35	4		80	

5. Use the spreadsheet called Exercise 5 to fill in the missing numbers in the chart below. To get column Mult, multiply Num1 by Num2 by Num3. To get the Answers, divide a number in column Mult by the one beside it in column Num4.

Num1	Num2	Num3	Mult	Num4	Answer
100	2	3		12	
250	6	3		24	

<u>Abstract 3</u>

1. One number is 17 bigger than another. They both add up to 85. Find the two numbers.

   Smaller number X
   Bigger number

   Total

2. One number is 6 smaller than another. They both add up to 36. Find the two numbers.

   Bigger number X
   Smaller number

   Total

3. One number is 28 bigger than another. They both add up to 54. Find the two numbers.

   Smaller number X
   Bigger number

   Total

4. The sum of two consecutive numbers is 53. Find the numbers. (Consecutive numbers come immediately after each other, like 6, 7, 8, 9 etc.)

Smaller number X
Bigger number

Total

5. Three consecutive numbers add up to 45. Find the numbers.

Smallest number X
Next number
Biggest number

Total

6. One number is twice as big as another. They both add up to 642. Find the two numbers.

Smaller number X
Bigger number

Total

7. A, B and C are numbers. A is twice as big as B, and B is twice as big as C. They all add up to 42. Find the numbers.

A = X
B =
C =

Total

8. A, B and C are three numbers that add up to 2475. B is 143 bigger than A and C is 256 bigger than A. Find the three numbers.

A = X
B =
C =

Total

9. M, N and P are three numbers that add up to 37. N is 6 bigger than M. P is eight smaller than M. Find the three numbers.

M = X
N =
P =

Total

10. One number is 16 bigger than another. They both add up to 352. Find the two numbers.

Smaller number X
Bigger number

Total

11. Three numbers add up to 212. The second is 4 bigger than the first and the third is three times as big as the second. Find the numbers.

First X
Second
Third

Total

12. A, B, C and D are 4 numbers. A is the smallest. B is 2 bigger than A, C is 5 bigger than A and D is 6 bigger than A. Thay all add up to 41. Find the numbers.

A = X
B =
C =
D =

Total

13. Three numbers add up to 58. The second is twice as big as the first. The third is 2 less than the second. Find the numbers.

First X
Second
Third

Total

Abstract 4

1. D, E and F are three numbers. E is twice as big as D. F is three times as big as D. They all add up to 126. Find the three numbers.

D = X
E =
F =

Total

2. One number is 48 bigger than another. They both add up to 902. Find the two numbers.

Smaller number X
Bigger number

Total

3. One number is twice as big as another. They add up to 969. Find the numbers.

Smaller number X
Bigger number

Total

4. A, B and C are three numbers. B is 14 bigger than A. C is 6 times as big as A. The three numbers add up to 46. Find the numbers.

A = X
B =
C =

Total

5. R, S and T are three numbers. S is four times as big as R. T is 7 less than S. They all add up to 704. Find the three numbers.

R = X
S =
T =

Total

6. H, I and J are three numbers. I is twice as big as H. J is 20 bigger than H. They all add up to 216. Find the three numbers.

```
H =        X
I =
J =
        -----
Total
```

7. Three numbers add up to 708. The second is 30 bigger than the first and the third is 4 times as big as the second. Find the three numbers.

```
First          X
Second
Third
            -----
Total
```

8. A, B and C are three numbers. B is half of A and C is twice as big as A. They add up to 273. Find the three numbers.

```
A =            X
B =
C =
             -----
Total
```

9. Three numbers add up to 406. The second is twice as big as the first and the third is twice as big as the second. Find the numbers.

```
First           X
Second
Third
             ------
Total
```

10. Two numbers add up to 273. One is twice as big as the other. Find the two numbers.

```
Smaller number        X
Bigger number
                    -----
Total
```

11. Two numbers add up to 185. One is four times as big as the other. Find the two numbers.

Smaller number	X
Bigger number	

Total	

12. Find three consecutive numbers whose sum is 222.

Smallest number	X
Next number	
Biggest number	

Total	

13. A, B and C are three numbers. B is three times as big as A and C is four times as big as A. They all add up to 16. Find the three numbers.

A =	X
B =	
C =	

Total	

Abstract 5

1. One number is 4 bigger than another. If both numbers are doubled they add up to 40. Find the numbers.

Smaller number	X
Bigger number	

Total	

2. One number is 5 bigger than another. When they are both doubled the answers add up to 54. Find the two numbers.

Smaller number	X
Bigger number	

Total	

3. One number is 3 less than another. If both are doubled they add up to 22. What are the numbers?

Bigger number	X
Smaller number	

Total	

4. One number is twice as big as another. If both are doubled they add up to 270. What are the numbers?

Smaller number X
Bigger number

Total

5. A and B are two numbers. B is four times as big as A. Twice A added to four times B is 594. Find A and B.

A = X
B =

Twice A + Four times B =

6. A, B and C are three numbers. B is three times as big as A and C is 2 less than A. They all add up to 63. Find the three numbers.

A = X
B =
C =

Total

7. One number is 3 bigger than another. When the smaller one is multiplied by 10 and the bigger one by 5, the answers add up to 330. Find the numbers.

Smaller number X
Bigger number

Total

8. A certain number is multiplied by 5. It is also multiplied by 10. The two answers add up to 495. Find the number.

Number = X

Total

9. One number is three bigger than another. Six times the smaller one added to five times the bigger one is 202. Find the numbers.

Smaller number X
Bigger number

Total

10. One number is six smaller than another. Three times the bigger one minus twice the smaller one is 29. Find the numbers.

Bigger number X
Smaller number

Total

11. One number is 12 smaller than another. Five times the bigger one plus three times the smaller one is 137. Find the numbers.

Smaller number = X
Bigger number =

Total

12. One number is 8 bigger than another. If the smaller is multiplied by 2 and the bigger by 4, the answers add up to 134. Find the two numbers.

Smaller number X
Bigger number

Total

13. One number is 9 bigger than another. Three times the smaller one plus four times the bigger one is 505. Find the numbers.

Smaller number X
Bigger number

Total

14. One number is twice as big as another. If the smaller one is multiplied by 120 and the bigger one by 320, the answers add up to 9880. Find the numbers.

Smaller X
Bigger

Total

15. Two numbers add up to 200. If the first one is multiplied by 25 and the other one by 15, the answers add up to 3600. Find the numbers.

First number X
Second number

Total

16. Two numbers add up to 32. If the first one is multiplied by 3 and the other by 4, the answers add up to 108. Find the two numbers.

First number X
Second number

Total

17. Two numbers add up to 100. If the first one is multiplied by 15 and the other by 2, the answers add up to 655. Find the two numbers.

First number X
Second number

Total

18. Two numbers add up to 7. If the first one is multiplied by 6 and the other by 4, the answers add up to 32. Find the two numbers.

First number X
Second number

Total

19. Two numbers add up to 12. If the first one is multiplied by 5 and the other by four, the answers add up to 52. Find the two numbers.

First number X
Second number

Total

20. Two numbers add up to 15. If the first one is multiplied by 6 and the other by 4, the answers add up to 70. Find the two numbers.

First number X
Second number

Total

21. Two numbers add up to 20. If the first one is multiplied by 15 and the other by 25 the answers add up to 380. Find the numbers.

First number X
Second number

Total

22. The difference between two numbers is 12. One third of the smaller added to half the larger number is 21. Find the numbers.

Smaller number X
Bigger number

Total

<u>Abstract 6</u>

1. What number should be subtracted from both 36 and from 78 so that one answer will be four times as big as the other?

Number that is subtracted X

Answer when taken from 36
Answer when taken from 78

2. When 5 times a certain number is reduced by 8, the result is the same as four times the number increased by 14. Find the number.

Guess for what the number is X

5 times number minus 8
4 times number plus 14

3. A is five bigger than B. If 2 were subtracted from each then A would be twice as big as B. Find A and B.

Value of B X
Value of A

B minus 2
A minus 2

4. If 8 is added to a certain number, the result is twice as big as subtracting 2 from the number. What is the number.

Guess for what the number is X

8 added to the number
2 subtracted from the number

5. The same number is added on to both 19 and 47. One answer is twice as big as the other. What number is added?

Guess for what the number is X

Number added to 19
Number added to 47

6. A certain number is subtracted from both 45 and 49. One answer is twice as big as the other. Find the numbers.

Guess for what the number is X

Number subtracted from 45
Number subtracted from 49

7. A is three times bigger than B. If both were increased by 30 then A would be twice as big as B. Find A and B.

B = X
A =

B plus 30
A plus 30

8. A is a number. If 16 is added to it the result is three times as big as subtracting 40 from it. Find A.

Guess for what A is X

16 added to A
40 taken from A

9. One number is 5 times another. If I add 7 to each number, then one is equal to 3 times the other. What are the two numbers?

Smaller number X
Bigger number

7 added to smaller
7 added to bigger

10. What number should be subtracted from 120 and subtracted from 240 so that one answer is three times as big as the other?

Guess for what number should be subtracted X

When subtracted from 120
When subtracted from 240

11. A and B are two numbers. A is three times as big as B. If 5 is subtracted from each then A is 4 times as big as B. Find A and B.

B = X
A =

B minus 5
A minus 5

12. If I subtract a certain number from 21, I get A. If I add the same number to 43 I get an answer which is three times as big as A. What is the certain number?

    Guess for what number is X

    21 minus the number
    43 plus the number

13. Find three consecutive numbers such that 5 times the smallest number is equal to twice the sum of the other two numbers.

    Smallest number X
    Next number
    Biggest number

    5 times the smallest
    Twice sum of others

14. When 24 is taken from 5 times a certain number, the result is the same as when 20 is added to 3 times the same number. Find the number.

    Guess for what the number is X

    24 from 5 times the number
    20 added to 3 times the number

15. Two numbers add up to 315. If one is doubled and the other is multiplied by three, the total is 748. What are the numbers?

    One number X
    Other number

    Total

16. If I subtract 5 from a number, multiply this result by 7 and then add 11, the answer is 67. What is the original number?

    Guess for what the number is X

    Answer

17. If 11 is added to a certain number and the result is divided by 11, the answer is 9. What is the number?

    Guess for what the number is X

    Answer

18. Two numbers add up to 50. If one of the numbers is halved then they add up to 35. Find the numbers.

One number	X
Other number	
Answer	

Abstract 7

1. The bill for a truckload of goods was lost. All that was known was the total price for each type of item. The VAT on all the items was 25%. Enter a guess for the cost of Item 1. Then fill in formulas to calculate the VAT and the total cost for Item 1. If your first guess was wrong then try again until you get the correct total.

Item	Item 1	Item 2	Item 3	Item 4
Cost				
VAT @ 25%				
Number	15	20	12	8
Totals	6750	10500	5100	4600

2. The table below shows part of the wage bill for four workers. Guess the rate per hour for each worker. Then enter formulas to calculate the 5% bonus for each worker, and formulas to calculate the total wage for each worker. Then adjust your guesses until you get the correct totals.

	Worker1	Worker 2	Worker 3	Worker 4
Rate per hour				
Bonus @ 5%				
Number of hours	40	38	46	37
Totals	126	135.66	135.24	139.86

3. A shopkeeper bought boxes of goods as shown below. She could not sell them all because there were some damaged in each box. Complete the spreadsheet by guessing the number in each box and then entering formulas to calculate the total amount for each box. Adjust your guesses until you get the correct totals.

	Box 1	Box 2	Box 3	Box 4
Number in Box				
Number faulty	7	9	12	2
Price of each	1.25	2.34	1.79	3.56
Totals	27.90	27.75	58.28	13.44

4. A school bought a number of schoolbooks as shown below. A discount of 8% was allowed because so many were bought. Complete the spreadsheet by guessing the cost of each book and by entering formulas to find the discount allowed on each book and to find the total cost for each book. Adjust your guesses until you get the correct totals.

	Book 1	Book 2	Book 3	Book 4
Cost				
Discount @ 8%				
Number of books	32	56	43	28
Totals	191.36	437.92	217.58	218.96

5. A band played gigs in four different venues. The prices for each venue were different. They had to pay 40% of each ticket sold to their manager. Complete the spreadsheet by guessing the admission price at each venue and entering formulas to calculate 40% of each ticket and to find the total they made at each venue. Adjust your guesses until you get the correct totals.

	Venue 1	Venue 2	Venue 3	Venue 4
Ticket Price				
Less 40%				
Number at concert	527	240	280	345
Totals	1581	576	756	724.50

6. Three classes were going on different school trips, all of which cost different amounts. A discount of 8% was allowed for each pupil. Complete the table below by guessing the cost per pupil for each class and writing formulas for the discount and the totals. Adjust your guesses until you have the correct totals.

	2A	2B	2C
Cost per pupil			
Discount @8%			
Number in class	23	26	19
Total	84.64	71.76	104.88

7. A travel firm sold holidays to Spain for three years. The cost of the holidays went up each year. 30% tax was paid on each holiday. Complete the spreadsheet by guessing the cost per holiday each year and entering formulas for the tax @ 30% and the totals. Then adjust your guesses until you get the correct totals.

	1986	1987	1988
Cost per holiday			
Plus tax at 30%			
Number sold	66	85	74
Total	30030	43095	40404

8. A shop bought in boxes of three different items, A, B and C. Complete the spreadsheet by guessing the cost of one item A, one item B etc. Then enter formulas to find the tax at 30% and the totals. Adjust your guesses until the totals are correct.

	Item A	Item B	Item C
Cost for one			
Tax at 30%			
Number per box	50	70	120
Total	845	728	1716

APPENDIX G

Concrete Exercise Problems

<u>Concrete Questions</u>

<u>Concrete 1</u>

1A. Using the spreadsheet called Example 1, move the cursor to the cells named below. Fill in the contents of each cell on the chart below:

Cell	Contents
A9	
E8	
A11	
H10	
H6	
E9	

1B. Using the same example, make lists of all the cells that contain a) Labels b) Values c) Formulas

Label Cells	Value Cells	Formula Cells

2. Use the spreadsheet below to type the correct Labels into the spreadsheet called Example 2 on your computer.

EXPENSES		INCOME	
--------		------	
BUS FARES	3.25	POCKET MONEY	4.50
SNACKS	3.00	BABYSITTING	5.50
DISCO	3.50	SATURDAY JOB	8.75
MAGAZINES	1.55		
SCHOOL TRIP	3.00		
	14.30		18.75
SAVINGS......	4.45		

3. Look at the spreadsheet below. Use it to correct the spellings in the spreadsheet called Example 3.

Item	Cost per unit	No.Units	Total
WALLPAPER	4.95	8	39.60
GLOSS PAINT	5.53	3	16.59
MATT PAINT	4.99	5	24.95
		Total	84.14

4. Look at the spreadsheet below. Use it to enter the correct marks in the spreadsheet called Example 4. Then check the % mark for each subject and the overall average and write them in below.

	SUBJECT	MARK	MAXIMUM MARK	% MARK
1	IRISH	100	200	
2	ENGLISH	120	300	
3	MATHS	200	250	
4	HISTORY	180	300	
5	GEOGRAPHY	67	100	
6	COMMERCE	88	200	
7	SCIENCE	165	300	
8	FRENCH	90	150	

Overall Average

5. A music shop ordered a large amount of equipment as shown in the spreadsheet in Example 5. Because they bought so much, a discount of 10% was given as shown in the spreadsheet. Before the equipment was delivered the prices of all the items increased as shown below. Type in the new prices under the heading "Regular Price" and then take a note of the new Grand Total.

Item	Regular Price
Electric Guitars	415.00
Amplifiers	375.00
Drumkits	585.00
Keyboards	342.00
Microphones	94.00
Mike Stands	59.00
P.A. Systems	1075.00

New Grand Total =

6. Look at Example 6 in your spreadsheet. Name the cell that contains a formula. Say what the formula is and give its value.

Cell with Formula	Formula	Value of Formula

7. Look at Example 7 in your spreadsheet. The formula for the area is missing. The area of a rectangle is got by multiplying the value of the length by the value of the width. The correct formula for the area then is +C104*C105. (C104 contains the value of the Length and C105 contains the value of the Width. All formulas begin with +). Type this formula into cell C107. What is the area?

   Area =

   Use the spreadsheet to find the area of these rectangles:

Length	Width	Area
22	31	
152	9	
398	456	

<u>Concrete 2</u>

1. Use the spreadsheet called Exercise 1 to fill in the missing numbers in the chart below. The column "Add" is got by writing formulas to add the numbers in the first two columns, "Subt." is got by writing formulas to subtract them etc.

Number 1	Number 2	Add	Subt.	Mult.	Divide
24	6				
60	20				
33	11				

2. Use the spreadsheet called Exercise 2 to calculate the amounts of each ingredient needed to make 12 scones, 24 scones etc. The first column is got by writing formulas to multiply each of the Amounts by 12, the second is got by writing formulas to multiply them by 24 etc.

	Amounts (grams)	By 12	By 24	By 36
Raisins	10			
Flour	16			
Margarine	8			
Sugar	12			

3. The rates of VAT are different in different countries. Use the table in Exercise 3 to find the amount of VAT due on each type of item at 12%, then at 15% etc. In the first column write formulas to calculate 12% of the price of each item etc. (To get 12%, divide by 100 and multiply by 12) etc.

	Price	12%	15%
Stereo	225.00		
14" TV	259.00		
20" TV	413.00		

4. A company has offices in Ireland and England. In each country there is a different price for a unit of electricity. There is also a different fixed charge for electricity to be paid. Use the spreadsheet in Exercise 4 to calculate the cost of the units used in each country (use a formula that multiplies the number of units by the price of one unit). Then find the total bills by using formulas to add the cost of the units on to the fixed charge.

	Number of units	Price of unit	Cost of units	Fixed charge	Total Bill
Ireland	450	.08		30.50	
England	375	.06		80.75	

5. Use the spreadsheet in Exercise 5 to calculate the Simple Interest for each example. Write formulas to calculate PrincipalxRatexTime in each case. The

interest is got by formulas to divide PxRxT by 100 in each case.

Principal	Rate	Time	PxRxT	Interest
300	6	3		
250	5	8		

Concrete 3

1. Michelle bought a cup of coffee and a cake. The coffee cost 9p more than the cake. If the total bill was 95p, how much did each cost?

   Cost of cake X
   Cost of coffee

   Total

2. Sandra bought a framed picture. The picture cost £16 more than the frame. If the total cost was £54, how much did each part cost?

   Cost of frame X
   Cost of picture

   Total

3. Patricia watched two TV programmes. One was 28 minutes longer than the other. She watched for 54 minutes altogether. How long was each programme?

   Short Programme X
   Longer Programme

   Total Time

4. Class 2A has one pupil more than class 2B. Between them there are 53 pupils. How many are in each class?

   Class 2B X
   Class 2A

   Total

5. Class 5A has one pupil more than class 5B. Class 5C has four pupils less than class 5B. Between them there are 60 pupils. How many in each class?

Class 5B X
Class 5A
Class 5C

Total

6. Sandra spent twice as much as Jenny. They spent £57 altogether. How much did each spend?

Jenny X
Sandra

Total

7. The number of girls in a club is 14 less than the number of boys. There are 120 members. Find the number of girls.

Number of boys X
Number of girls

Total

8. Sandra bought shoes, jeans and a shirt. The jeans cost twice as much as the shirt and the shoes cost twice as much as the jeans. She spent a total of £42. How much did each cost?

Shirt X
Jeans
Shoes

Total

9. Three suomo wrestlers were weighed at the same time. The second was 143 kg heavier than the first. The third was twice as heavy as the first. Their total weight was 935 kg. How heavy was each?

First X
Second
Third

Total

10. Caroline, Christine and Lorraine had a meal. Christine's cost £2 more than Caroline's and Loraine's cost £1 less than Christine's. The total bill was £18. How much did each spend?

Caroline	X	
Christine		
Lorraine		

Total		

11. Linda weighs 6 kg less than Karen. Between them they weigh 64 kg. How much does each weigh?

Karen's weight	X
Linda's weight	

Total	

12. Geraldine, Vicky and Deirdre were the top three students in a test. Geraldine got 4 marks more than Vicky. Deirdre got 14 marks less than Vicky. The marks for the three girls added up to 197. How many marks did each get?

Vicky	X
Geraldine	
Deirdre	

Total	

13. A family consists of 4 girls. Tracy is two years older than Michelle. Louise is three years younger than Michelle and Audrey is twice as old as Michelle. All their ages add up to 39. How old is each girl?

Michelle	X
Tracy	
Louise	
Audrey	

Total	

14. Karen has twice as much as Debbie and Linda has £2 less than Debbie. They have a total of £58 between them. How much has each girl?

Debbie	X
Karen	
Linda	

Total	

Concrete 4

1. Susan bought two tickets for her Debs. Catherine bought three and Jane bought four. The total cost was £189. How much did each ticket cost?

   Price of one ticket X
   Susan's tickets
   Catherine's tickets
   Jane's tickets

   Total

2. A shop sold 960 bags of crisps. The number of cheese & onion was 48 more than the number of salt & vinegar. How many of each type were sold?

   Salt & Vinegar X
   Cheese & Onion

   Total

3. A bowl has 24 pieces of fruit. Some are oranges and some are grapefruit. There are twice as many oranges as grapefruit. How many are oranges?

   No. Grapefruit X
   No. Oranges

   Total

4. A bag contains a mixture of red cubes, white cubes and black cubes. There are 14 more red than white. There are 6 fewer black than white. If there are 44 cubes altogether, how many are there of each colour?

   No. White X
   No. Red
   No. Black

   Total

5. Julie, Jean and Andrea formed a team for a marathon. Julie took twice as long as Jean. Andrea took one hour less than Julie. The total time for the team was 14 hours. How long did each take?

   Jean X
   Julie
   Andrea

   Total

6. I spent 180 minutes on homework over a period of three days. On the second day I spent twice as long as the first day and on the third day I spent 20 minutes less than the first day. How long did I spend each day?

First Day	X
Second Day	
Third Day	

Total	

7. Liz did a test in Irish, English and Maths. Her mark for Maths was 30 higher than for Irish. Her mark for English was 3 lower than for Irish. She got a total of 219 marks. How much did she get for each subject?

Irish	X
Maths	
English	

Total	

8. I have some money to spend. My sister has £5 less than me but my brother has twice as much as me. Altogether we have £27. How much has each of us got?

My Money	X
Sister's Money	
Brother's money	

Total	

9. There are 370 pupils in a school. There are twice as many doing English as French and there are 10 less doing German than French. How many are studying each subject?

French	X
English	
German	

Total	

10. Each girl in a class attended a meeting with both of her parents. The total attendance was 87. How many girls were in the class?

Number of girls	X
Number of parents	

Total	

11. In a school there are four times as many sixth years doing Pass Irish as there are doing Honours Irish. If there are 165 sixth years, how many are doing Honours?

Hons. Irish X
Pass Irish

Total

12. In a relay race each team consisted of 3 runners. On the winning team runner 2 took one second more than runner 1 and runner 3 took two seconds less than runner 1. The winning time was 71 seconds. How long did each runner take?

Runner 1 X
Runner 2
Runner 3

Total

13. Three sisters have 16 Barbie dolls between them. Niamh has three times as many as Emer and Deirdre has four times as many as Emer. How many has each?

Emer X
Niamh
Deirdre

Total

14. Joe had 6 records more than his sister Susan. Susan doubled the number of records she had and Joe bought 10 new records. They then had 46 records between them. How many did each have at the start?

Number that Susan had X
Number that Joe had

Total number that they have now

<u>Concrete 5</u>

1. One set of twins is 4 years older than another set. The ages of all four children add up to 40. How old is each child?

Age of each of younger set X
Age of each of older set

Total of ages

2. A family has two sets of twins. The older ones each get £1 more pocket money than the younger ones. The total amount of money that all four get is £14. How much does each child get?

Amount for each of younger X
Amount for each of older

Total

3. In a sale, some tapes were reduced by £3. Lisa bought two full priced tapes and two tapes at the reduced price. She spent a total of £22. How much were the full priced tapes?

Cost of one full priced tape X
Cost of one cheap tape

Total cost

4. An adult's train fare is twice a child's fare. A family of two adults and two children paid £24 for a trip. How much was the adult fare?

Child Fare X
Adult Fare

Total cost

5. A pear costs 3p more than an apple. 2 Apples and 4 pears cost 96p. How much does each cost?

Price of one apple X
Price of one pear
------

2 Apples & 4 Pears

6. Books cost £1 more than tapes. Mary bought 2 tapes and 3 books. The total bill was £28. How much was it for a book?

Cost of one tape X
Cost of one book
------

3 books & 2 tapes

7. Judy has £135 made up of a mixture of £5 and £10 notes. She has 3 more £10 notes than £5 notes. How many of each has she got?

No. of £5 notes	X
No. of £10 notes	

Total Value	

8. You have three times as many £20 notes as £5 notes in your pocket. If you have a total of £520, how many of each have you got?

No. of £5 notes	X
No. of £20 notes	

Total Value	

9. I have twice as many £20 notes as £10 notes. If I have a total of £300, how many of each have I got?

No. of £10 notes	X
No. of £20 notes	

Total Value	

10. A family has three times as many sisters as brothers. Each brother got five books and each sister got two books. They got 22 books in all. How many brothers are there?

No. of brothers	X
No. of sisters	

Number of books	

11. Adrienne bought twice as many pots as pans. Each pan cost £13 and each pot cost £15. She spent £129. How many of each did she buy?

Number of pans	X
Number of pots	

Total Cost	

12. On a farm there are eight more hens than cows. All together the animals have 118 legs. How many cows are there.

Number of cows	X
Number of hens	

Number of legs	

13. Angela bought twice as many plants as flowerpots. Flowerpots cost £5 and plants cost £3. She spent £44 in all. How many plants did she buy?

No. of flowerpots	X	
No. of plants		

Total cost		

14. Farmer Murphy bought cows for £320 each and sheep for £120 each. She bought twice as many sheep as cows. If the total cost was £1120, how many sheep were bought?

Number of cows	X	
Number of sheep		

Total Cost		

15. A shop sells chocolate cakes for £3 and sponge cakes for £2. They sold a total of 200 cakes and took in £485. How many of each type were sold?

Number of chocolate cakes	X	
Number of sponge cakes		

Amount taken in		

16. 32 childern shared out a sum of £77. The girls got £3 each, and the boys got £2 each. How many girls were there?

Number of Girls	X	
Number of Boys		

Amount of money		

17. A shopkeeper bought some scarves for £3 and others for £4. In all she bought 100 scarves for a total of £336. How many £3 scarves did she buy?

Number at £3 each	X	
Number at £4 each		

Total Cost		

18. A shopkeeper bought 7 packs of tights containing a total of 32 pairs. Some packs had 6 pairs and others had 4 pairs. How many packs had 6 pairs?

Number of 4-packs X
Number of 6-packs

Number of pairs

19. A shop sold 120 Easter eggs. Some were filled with sweets and cost £5 each. Others were not filled and cost £4 each. The total amount taken in was £520. How many of each type were sold?

Number filled X
Number not filled

Total Cost

20. Sharon bought a mixture of 6-packs and 4-packs of Coke for a party. She bought 70 cans altogether. If she bought a total of 15 packs, how many of these were 6-packs?

Number of 4-packs X
Number of 6-packs

Number of cans

21. A bakery makes two kinds of cakes. One type costs £2 and the other type costs £3. A total of 20 cakes were sold for £47. How many of each type were sold?

Number at £2 X
Number at £3

Total Cost

22. One class in a school had 10 pupils more than another. One third of those in the smaller class and half of those in the larger class got honours maths. The total number that got honours maths was 20. How many were in each class?

Smaller Class X
Larger Class

Hon.Maths

<u>Concrete 6</u>

1. Nelly had £40 and Katie had £60. They both spent the same amount. Write formulas to find out how much

each had left. Katie had twice as much left as Nelly. How much did each spend?

Amount that each spent X

Amount Nelly has left
Amount Katie has left

2. In a school there were 5 first year groups and 4 second year groups, all with the same number of pupils. Eight first year pupils left, but fourteen new pupils came into second year. The total number of first years was then the same as the total number of second years. How many were in each group at the start?

Number in each group at start X

Number of first years at end
Number of second years at end

3. In Home Economics, Sharon made 5 cakes more than Jane. They ate two cakes each and Sharon then had twice as many as Jane. How many did each make.

Number that Jane made X
Number that Sharon made

Number Jane had left
Number Sharon had left

4. Siobhan and Denise were playing poker. At the start Siobhan had £8 less than Denise. Siobhan lost £2 and Denise won £8. Denise then had ten times as much as Siobhan. How much did each start with?

Amount Denise had at start X
Amount Siobhan had at start

Amount Denise had at end
Amount Siobhan had at end

5. Mary's bag of sweets had 19 sweets and Susan's had 47. The same number of sweets was added to each bag. Susan then had twice as many sweets as Mary. How many sweets were added to each bag?

Number added to each bag   X

Number in Mary's bag at end
Number in Susan's bag at end

6. One bus had 15 passengers and another had 19. The same number of people got off each bus. One then had twice as many passengers as the other. How many got off each bus?

Number that got off each bus   X

Number left on first bus
Number left on second bus

7. For a wedding, the bride invited three times as many guests as the groom. They then decided to invite 30 more guests each. When they did this the bride had twice as many guests as the groom. How many did each invite?

Groom's guests at start   X
Bride's guests at start

Groom's guests at end
Bride's guests at end

8. 10 more boys than girls were present at the start of a disco. Before the end, 40 girls had left and 16 more boys had arrived. There were then twice as many boys as girls. How many of each were present at the start?

Number of girls at the start   X
Number of boys at the start

Number of girls at the end
Number of boys at the end

9. One bus contains four times as many passengers as another. If six passengers get off each bus, then it will have seven times as many. How many passengers were on each bus at the start?

Number on first bus at the start X
Number on second bus at the start

Number on first bus at the end
Number on second bus at the end

10. Samantha had 120 records and Petulla had 240. Petulla was a bully and swiped some of Samantha's records. She then had three times as many as Samantha. How many records did she swipe?

Number swiped X

Number that Samantha then had
Number that Petulla then had

11. A box contains three times as many red buttons as blue. If 5 of each colour were removed then there would be four times as many red ones as blue ones. How many of each colour are there?

Number of blue at start X
Number of red at start

Number of blue at end
Number of red at end

12. Susan had £21 and John had £43. Susan lost some money and John found it. John then had 3 times as much as Susan. How much did Susan lose?

Amount that Susan lost X

Amount Susan had at end
Amount John had at end

13. Jane has 9 animals, some of which are dogs and some of which are cats. If she had four times as many cats and twice as many dogs, she would have 30 pets in all. How many dogs and cats does she have?

Number of cats that Jane has now X
Number of dogs that Jane has now

What she would have

14. Karen has £1 more than Jenny, and Angela has £1 less than Jenny. 10 Times Angela's money is the same as

four times Jenny's and Karen's put together. How much has each?

Amount that Jenny has X
Amount that Karen has
Amount that Angela has

Ten times Angela's
Four times Jenny's & Karen's

15. Joan bought 5 sheets of stamps and used 50 stamps to post letters. Claire bought 3 sheets of stamps and 10 extra 'loose' stamps. They then had the same number of stamps. How many stamps were on each sheet.

Number on each sheet X

Number Joan had left
Number Claire had left

16. A vocational school and a secondary school both have boys and girls as pupils. The vocational school has 315 pupils altogether. The secondary school has three times as many boys and twice as many girls. It has total of 748 pupils. How many boys are there in the vocational school?

Number of boys in Vocational school X
Number of girls in Vocational school

Total number in Secondary school

17. Half of the passengers on a bus got off at a stop and another 23 people got on. There were then 41 people on the bus. How many passengers were there at the start?

Number at the start X
Number at end

18. A job was expected to take 5 days, working the same number of hours each day. To get it finished, it was necessary to work one extra day for 14 hours. The average time worked for the whole project was 9 hours per day. How many hours were worked on each of the first 5 days?

Number of hours worked per day X

Total number of hours worked
Average hours per day

19. A barrel of honey weighs 50 kg. Honey is twice as heavy as water. The same barrel with water in it weighs 35 kg. How much does the empty barrel weigh?

Weight of barrel alone	X
Weight of honey alone	
Weight of water alone	
Weight of barrel plus water	

Concrete 7

1. The bill for a truckload of goods was lost. All that was known was the total price for the TVs, Videos etc. The VAT on all the items was 25%. Enter a guess for the cost of one T.V. Then fill in the formulas to calculate the VAT for a TV and the total cost for all the TVs. If your first guess was wrong then try again until you get the correct total.

Item	T.V.s	Videos	Washing Machines	Cookers
Cost				
VAT @ 25%				
Number	15	20	12	8
Totals	6750	10500	5100	4600

2. The table below shows part of the wage bill for four workers. Guess the rate per hour for each girl, then enter formulas to calculate the 5% bonus for each girl and the total wage for each girl. Then adjust your guesses to get the correct totals.

	Deirdre	Sharon	Mary	Sarah
Rate per hour				
Bonus @ 5%				
Number of hours	40	38	46	37
Totals	126.00	135.66	135.24	139.86

3. A shopkeeper bought boxes of chocolate bars as shown below. She could not sell them all because there were some damaged in each box. Complete the spreadsheet by guessing the number in each box and then by entering formulas to calculate the total amount for each box. Adjust your guesses until you get the correct totals.

	Mars	Topic	Bounty	Marathon
Number in Box				
Number damaged	7	9	12	2
Price of each	.30	.25	.31	.28
Totals	27.90	27.75	58.28	13.44

4. A school bought a number of schoolbooks as shown below. A discount of 8% was allowed because so many were bought. Complete the spreadsheet by guessing the cost of each type of book and entering formulas to find the discount allowed and the total cost of each type of book. Adjust your guesses until you get the correct totals.

	Maths	English	Irish	History
Cost				
Discount @ 8%				
Number	32	56	43	28
Totals	191.36	437.92	217.58	218.96

5. A band played gigs in four different venues. The prices for each venue were different. They had to pay 40% of each ticket sold to their manager. Complete the spreadsheet by guessing the ticket price for each venue and entering formulas to calculate 40% of each ticket and to find the total for each venue.

	Dublin	Cork	Galway	Limerick
Ticket Price				
Less 40%				
Number at concert	527	240	280	345
Totals	1581.00	576.00	756.00	724.50

6. Three classes were going on different school trips, all of which cost different amounts. A discount of 8% was allowed for each pupil. Complete the table below by guessing the cost per pupil for each class and writing formulas for the discount and the totals. Adjust your guesses until you have the correct totals.

	2A	2B	2C
Cost per pupil			
Discount @8%			
Number in class	23	26	19
Total	84.64	71.76	104.88

7. A travel firm sold holidays to Spain for three years. The cost of the holidays went up each year. 30% tax was paid on each holiday. Complete the spreadsheet by guessing the cost per holiday each year and entering formulas for the tax @ 30% and the totals. Then adjust your guesses until you get the correct totals.

	1986	1987	1988
Cost per holiday			
Plus tax at 30%			
Number sold	66	85	74
Total	30030	43095	40404

8. A shop bought in boxes of bracelets, rings and chains. Complete the spreadsheet by guessing the cost of one bracelet, one ring etc. Then enter formulas to find the tax at 30% and the totals. Adjust your guesses until the totals are correct.

	Bracelets	Rings	Chains
Cost for one			
Tax at 30%			
Number per box	50	70	120
Total	845	728	1716

APPENDIX H

Letter sent to School Principals

28 September 1989

Dear Principal,

I am a full-time teacher of mathematics at the Holy Faith Secondary School, The Coombe. I am also studying, part-time, for a Ph.D. degree at Dublin City University (formerly NIHE). My research project involves the use of computer technology in introductory Algebra courses for girls in second level schools.

For the experimental part of my project I need some second year students to undertake a short programme of study during the October mid-term break and I would like some girls from your school to participate. The programme will be spread over four days from Tuesday 31 October to Friday 3 November and will last for about two and a half hours per day. The course content is from the Inter Cert Maths Syllabus and will be of direct relevance to the participants. The approach, using computers, will be interesting and enjoyable. I will be employing a number of assistants to ensure that there will be one tutor for every seven students. I would like to include pupils from all ability levels and no previous computer experience is required.

I would be very grateful if you would consider this request favourably and I will contact you by phone in a few days. I would also appreciate if, in the interests of experimental validity, you did not mention this course to any potential applicants before I contact you. If you have any queries I can be contacted at The Coombe (542100).

Yours sincerely,

APPENDIX I

Information Sheet/Application Form

Holy Faith Secondary School
The Coombe
Dublin 8

5 October 1989

I am carrying out a research project for Dublin City University which involves giving young girls the opportunity to use computer technology. As part of the project I will be running a short course in computer problem-solving during the October mid-term break. The course will be spread over four days from Tuesday 31 October to Friday 3 November and will last for two and a half hours per session. Students will be offered a choice of attending in the mornings (10 to 12.30) or in the afternoons (2 to 4.30) and every effort will be made to suit all those applying. It is very important for the success of this experiment that all participants attend each of their four sessions. Please do not apply unless you are able to attend <u>all</u> of the four sessions.

The course will be both interesting and enjoyable. There will be one tutor for every seven students to ensure that everybody will get sufficient individual attention. No computer experience is required. Pupils from a few girls' secondary schools in your area are being asked to attend and the course will be held at the Holy Faith Secondary School, The Coombe. If you would like to take part please fill out the application form below, tear it off and return it as soon as possible. There is a strict limit on the numbers taking part and students will be accepted on a 'first come first served' basis. This is a non profit-making course but to cover the substantial costs involved, it is necessary to charge a fee of £8 per pupil.

Name:

School:

Home Phone (if any):

Mornings _____ Afternoons _____ Don't Mind _____
(Tick one only)

I enclose £8 fee (cheques payable to Michael Brady)

I agree to attend all four sessions of the course and to follow all instructions from the course tutors.
