

Empirical Investigation of Nonlinear Asset Pricing Kernel with Human Capital and Housing Wealth

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By

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ABSTRACT

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In a traditional framework, asset returns are captured by simple linear asset pricing models. They include Capital Asset Pricing Model (CAPM) and Fama-French three-factor model. However, the empirical study shows that the asset returns are fat tailed, that cannot be accurately predicted by normal distribution. Kurtosis and skewness should be considered when pricing those non-normal assets. Various literatures can be found focused on this topic. Bansal and Viswanathan (1993) and Chapman (1997) developed nonparametric model. They find that the nonparametric models perform better in explaining expected returns. Most recently, nonlinear asset pricing models developed by Dittmar (2002) shows more significantly improvements in return estimation, compared to the linear single and linear multi-factor models.

In this study, I focus on an asset-pricing model of higher order risk factors and use polynomial pricing kernel to generate the empirical performance of a nonlinear model. This is an extension to both Bansal and Dittmar's work, by extending the definition of the total wealth including human capital and housing wealth. This research work is novel and especially important to understand asset price behavior after year 2007, the credit crisis. Housing price growth rate is a very critical indicator for long-term investment, reflecting consumer confidence on the long-term global economy. It can be used to estimate the turning point for the recent economic downturn. In addition, since the credit crisis 2008 is triggered by liquidity shortage in banking systems, the level of housing price has direct impact on the balance sheet of those banking sectors. The higher the house price, the more willingness banks have to

release the credit to the market. The housing wealth factor can be used to estimate when the credit crunch will disappear and global economy gets fully recovered.

In this study, the risk factors that represent the aggregate wealth in the economic are tested. The best possible proxy of return on the total wealth is discussed. The thesis can be divided into 2 parts. In the first part of my thesis, a higher order moment model to explain the asset price behaviour is developed. Similar to the work presented by Dittmar (2002), pricing kernel is approximated using Taylor Series expansion and Hansen-Jagannathan (1997) weighting matrix. The time-varying coefficients with respected sign of coefficients are estimated. Housing factor is added to extend the model, as we believe that housing plays an important role in the return on aggregate wealth. In the second part of my thesis, I test models in three time periods. They include Dittmar's period from 1963 to 1995, the full sample period from 1963 to 2009 and recent period from 1996 to 2009.

Our results confirm that nonlinear models outperform than linear models in explaining the cross section of returns. The higher order risk factors give the magnitude improvement in model fitting. This is consistent with the result given by Dittmar (2002). Moreover, my results conclude that the models with the housing wealth included performs significantly better than the models with human capital only.

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Chapter 1

INTRODUCTION

The topic of this thesis is to investigate non-normal asset price behaviour. In this research, I focus on nonlinear asset pricing models with higher order moment of risk factors. Especially, I am interested in the impact of housing wealth risk factor on modelling nonlinear asset price. Empirical historical data for returns and stock prices have proved that returns cannot be predicted accurately by normal distribution. In particular, when economic boom or crunch happens, prices and returns are highly changeable in the market. A large number of literatures have carried out investigations on nonlinear asset pricing models, which have more power in explaining large price fluctuations.

Housing wealth is an important element in today's market-oriented economic. In year 2007, following house market collapse in the US, a liquidity crisis has started in the united banking system and expanded all over the world. Since year 2005, US mortgage lenders sell many expensive mortgages to customers even if they are with poor credit, high chance of default. To increase the profitability on mortgage, other financial companies bought mortgage debts as a package. The idea is to spread the risk, but it makes bigger problem as rating agencies gave these risky mortgages a low risk rating and hide these risks in financial system. Many of these housing mortgages had an introductory period of 1-2 years of very low interest rates. In 2007, interest rates increased at the end of this introductory period. In addition, due to inflation in 2007, US had to increase interest rate and mortgage payments were getting more expensive. Many new house owners could not afford mortgage payments and they choose to default. Then US houses price started to fall, the bank couldn't recoup the initial loan. It became a bad loop for US housing market and caused many medium sized mortgage companies to go bankrupt. Not only this, many banks were also facing big losses. To write off large losses, banks tighten their fund and became more and more reluctant to fund enterprises. There was not enough liquidity in the market. It affected many firms who have difficulty in borrowing money. The total volume of

money in the market shrinks and money movement velocity decreased significantly. It has direct impact on the GDP growth and slowed down the global economy. Thus, housing price growth rate becomes a very critical indicator, reflecting consumer confidence on the long-term global economy, which can be used to estimate the turning point for the recent economic down turn. In addition, the level of housing price has direct impact on the balance sheet of the banking sectors. The higher the house price, the more willingness banks have to release the credit to the market. It can be used to estimate when the credit crunch will disappear and global economy gets fully recovered. Therefore, it is very essential to include housing wealth into the model especially to understand asset price behavior after year 2007, the credit crisis.

Previous empirical studies on asset pricing models use a linear single factor model (e.g. consumption based CAPM model) and multifactor models (such as Fama-French model). However, there are some limitations. Bansal and Viswanathan (1993), who observe that the pricing kernel from a linear model cannot price securities whose payoffs are nonlinear functions of the factors. Chapman (1997), using polynomial approximation shows that nonlinear model is more capable of explaining variations in small firm returns. Another example is by Dittmar (2002), that he investigates nonlinear pricing kernels in examining the impact of risk factors including market returns and human capital and comparing the results with Fama-French model and power utility model. Dittmar use Taylor Series approximation with return on aggregate wealth for each polynomial term. One of the advantages for using Taylor series approach is that it represents a link between linear and nonlinear model specifications, as the leading term of polynomial is linear, follows quadratic term and cubic term. Second, the preference theory applied to nonlinear models is better approach in solving the truncation and avoid over fitting problems. Thus, the nonlinear pricing kernel is a suitable basis for studying non-normal asset price behaviour. I follows Dittmar (2002), modifies the model by adding the additional proxies for the return on aggregate wealth. That means, we specify the priced factor as a function of the return on equity, the return on human capital and return on housing wealth. The model helps to extract the housing returns that are related to financial asset returns.

This research can be divided into two steps: one is the modelling, which explained as above. Second is the test. Our test method provided in this study has many important impacts and findings. We test the performance of the model in three sub periods so that our results provide a better view on behaviour of nonlinear asset pricing model with the proxy choice over both the long time horizon and short time horizon, including economic crisis period. We conduct analysis that covers recession period aim to find out that how important is the housing factor in the whole economic return in modelling the asset prices.

In estimation, parameters of the polynomial series expansion are estimated using generalized method of moments (GMM), similar to the work presented by Hansen's (1982). As the linear model and nonlinear model are not nested, we use Hansen and Jagannathan (1992) distance measure to compare the linear single pricing kernel, linear multiple pricing kernel and nonlinear pricing kernels in three different time series framework. The Hansen Jagannathan distance measure is the distance measure between the pricing kernel under study and the class of valid pricing kernels. A proxy that is valid pricing kernel will have a zero HJ distance. Therefore, a proxy with a smaller HJ distance is closer to the class of valid pricing kernels and can be considered a better pricing kernel than one with a larger HJ distance.

The data we use are sampled on a monthly basis from July 1963 to December 2009. The raw data includes the returns on 17 industry-sorted portfolios and the instrument set $Z_t = \{1, r_{m,t}, dy_t, ys_t, tb_t\}$, where one denotes a vector ones, $r_{m,t}$ is the excess return on the CRSP value weighted index at time t, ys_t is the yield on the three-month Treasury bill in excess of the yield on the one month Treasury bill at time t, and tb_t is the return on a Treasury bill closest to one month to maturity at time t. in addition, labor income at time t is computed as the per capita difference between total income and dividend income. The data cover the period July 1963, through December 2009, totalling 558 observations. All the data are taken from Kenneth R. French website. Our results confirm the findings in Dittmar (2002), who investigates nonlinear pricing kernels in describing cross sectional variation in equity returns and test models use aggregate wealth that includes market return and human capital as risk factors.

Dittmar (2002) finds that nonlinear pricing kernel improves upon the pricing kernel's ability to describe the cross section of equity returns. Especially, when human capital is included into the measure of aggregate wealth, both quadratic and cubic pricing kernels are able to fit the cross section of industry sorted portfolio returns, and linear single or multi-factor models cannot. To extend Dittmar's work, I add in a very important risk factor-return on housing wealth to the total of economic returns. Our results show that the nonlinear term of the housing risk factors fit the models very well, that means housing wealth is one of the important factors in economic returns. Second, we examine the nonlinear model specification by testing the total wealth including equity returns, human capital returns and housing wealth returns. We find that housing wealth in cubic and quadratic terms improve the fit of nonlinear asset pricing kernel. Finally, the test of augmented Fama-French four-factor model offers strong support of return relationship in housing. Similar result can be found in Case, Cotter and Gabriel (2010).

For the linear model specification test, we examine and compare a set of linear single (CAPM) pricing kernels and a set of linear multi-factor (Fama-French) pricing kernels with respect to equity returns only, human capital returns only and housing wealth returns only. We find that the linear model specifications are not admissible for the cross section of industry portfolios, whereas a cubic and quadratic pricing kernel are. This result is consistent with Dittmar (2002), for details in comparison see Dittmar's sample period from July 1963 to December 1995 in Chapter Five.

For the nonlinear model specification test, we augmented polynomial model by examining the housing wealth risk factor in quadratic term and cubic term. The figure of distance measure obtained from nonlinear models with housing factor included has further improvement than Dittmar's results. In addition, we examine the impact of other risk factors including size and book to market Fama-French factors. We find that the cubic term in the pricing kernel drives out the significance of both size and book to market factor in the augmented Fama-French model. This result is again consistent with those in Dittmar (2002).

In the empirical applications of these model used in my research, I test the model in the recent period from July 1996 to December 2009 and compare with the results

obtained from sub period July 1963 to December 2009. Our results show that nonlinearity of the data that were observed in the most recent sample period, drive out the importance of the nonlinear model in explaining the cross section of returns. Furthermore, our results strongly support that the housing wealth factor improves the fit of the asset pricing model.

The thesis is organized as follows: In Chapter II, we discuss the methodology. Chapter III develops the asset pricing model. Chapter IV presents the detail regarding the data set used and instrument test. Chapter V, test and discuss the empirical performance of the model and compare it with those results obtained form Dittmar (2002) and Chapter IV concludes.

Chapter 2

THE ASSET PRICING MODEL

2.1 Models

2.1.A The CAPM Model

The static Capital Asset Pricing Model (CAPM) is the first important capital asset pricing model that developed by Sharpe (1964) and Lintner (1965) and generalized by Black (1972). The general idea behind CAPM is that expected return of a security or portfolio equals the rate on a risk-free security plus a risk premium. If the expected return does not meet required return, then the investment should not be undertaken. The CAPM model is widely cited in the asset pricing field. Jagannathan and Wang (1996) examine the conditional version of the CAPM model to explain the cross sectional variation in average returns in a large collection of stock portfolios. Their result shows that when human capital is included in measuring wealth, the unconditional CAPM is able to explain 50 percent of the cross sectional variation in average return and market portfolio in measuring wealth, which can only explain 30 percent. The asset pricing model of portfolio choice problem also includes Flavin and Yamashita (2002). They use mean-variance efficiency framework to examine the household's portfolio choice problem with exogenous returns of an agent who invests in both financial assets and real estate. They assume covariance matrix and expected return vector are time invariant. They estimate the covariance matrix and expected return for housing and financial asset such as stocks, bonds, T-bills for solving the efficient frontiers and optimal portfolios. And their analysis shows that the housing constraint is relaxed over the life cycle, the baby boom generation may have a systematic effect on asset price. Kullmann (2003) generates CAPM framework to test the importance of housing wealth in asset pricing. She applies linear models in her study: conditional CAPM model and assumes that both betas and expected returns are time invariant, it is:

$$E[R_{it}] = c_0 + c_{vw}\beta_i^{vw} + c_{labor}\beta_i^{labor} + c_{RE}\beta_i^{RE} + c_{tbill}\beta_i^{tbill} \quad (1)$$

And linear stochastic discount factor model, the model assume as long as the law of one price hold, there exist at least one random variable m_t that prices all assets in the economy. The formula as:

$$m_{t+1} = b_0 + b_{vw}R_{t+1}^{vw} + b_{labor}R_{t+1}^{labor} + b_{RE}R_{t+1}^{RE} + b_{tbill}R_{t+1}^{tbill} \quad (2)$$

Both test results show that the proxy for the return to real estate improves the performance of the CAPM model. Most recently, Piazzasi, Schneider, Tuzel (2007) use consumption based asset pricing model (CCAPM) with housing factor to forecast excess return on stocks. The model motivates a two-factor model, which is the consumption growth rate and the growth rate of α . The model predicts that the housing share can be used to forecast excess return on stocks. During recessions stocks have low payoffs, when non-housing consumptions is low, payoffs even lower. They use non-housing consumption as the numeraire and start with Euler equation, it is:

$$E \left[M_{t+1}^C R_{t+1}^{Si} \frac{P_t}{P_{t+1}} \right] = 1 \quad (3)$$

The pricing kernel takes the form

$$M_{t+1}^C = M_{t+1} \frac{P_{t+1}}{P_t} \frac{p_t^c}{p_{t+1}^c} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-1/\sigma} \quad (4)$$

where:

C_t - the appropriate deflator for nominal dividends,

p_t^c - the price of non-housing consumption,

$\frac{P_{t+1}}{P_t}$ - the true inflation rate,

In order to explain the cross section returns, they compare CAPM model with standard CCAPM model. They find that the CAPM model does poorly over the sample period (1936-2001) with R^2 of 8%. The CCAPM explains much more than the CAPM in terms of R^2 58%. Moreover, the consumption-housing CAPM, which called CHCAPM, explains 71% of the cross section variation in returns. Their result also consistent with the results stated in Cochrane (1996) that real investment growth helps pricing the cross section of returns and residential real estate matters to consumers. Klinkowska (2008) used stochastic discount factor, which only depend on the current period information for modelling the coefficients of CAPM model.

However, there are two major difficulties in examining the empirical support for the static CAPM. One is the real world is dynamic and not static. The other one is the return on the portfolio of aggregate wealth is not observable. Thus these limitations drive researchers to look at the multifactor models of asset prices.

The most recent paper by Case, Cotter and Gabriel (2010) investigate the risk-return relationship in determination of housing asset pricing. They conduct aggregate US house price series into the Housing asset pricing model (HCAPM) by examining the impact of additional risk factors including aggregate stock market returns, idiosyncratic risk, momentum, and Metropolitan Statistical Area (MSA) size effects. They find that the basic housing CAPM results are robust to the inclusion of other explanatory variables, including standard measure of risk and other housing fundamentals. Moreover, their findings are supportive of the application of a housing investment risk-return framework in explanation of variation in metro-area cross-section and time series US house price returns.

2.1.B The Multifactor Models

In the past study, the multifactor models have been noticed as successful in pricing the cross section of equity than single factor model, which developed by Eugene Fama and Kenneth French is called Fama-French three-factor model. Fama-French three-factor model expands on the Capital asset pricing model (CAPM) by adding size and value factors on addition to the market risk factor in CAPM. Through research they found that value stocks outperform growth stocks, small cap stocks tend to outperform large cap stocks. So this model includes two additional factors, SMB “small market capitalization minus big market capitalization” and HML “high book-to-market ratio minus low book-to-market ratio”. The model is very often compared to other models in many papers. Ross uses multifactor model and argues that CAPM model ignores the fact that human capital is an important component of wealth. Jagannathan and Wang (1996) estimate the cross-sectional of returns by using factors from Fama and French (1993). That is

$$E[R_{it}(\delta_0 + \delta_{vw}R_t^{vw} + \delta_{prem}R_t^{prem} + \delta_{labor}R_t^{labor} + \delta_{SMB}SMB_t + \delta_{HML}HML_t)] = 1 \quad (5)$$

Where R_{it} is the return on portfolio i in month t , R_t^{vw} is the return on the value weighted index of stocks, R_t^{prem} is the yield spread between low and high grade

corporate bonds, R_t^{labor} is the growth rate in per capita labor income, and SMB_t , HML_t denotes the respective Fama and French (1993) factors that are designed to capture the risk related to firm size and book-to-market equity. They also compare this model with the model without human capita factor and bonds returns. The results show that Equation (5) fits the data set at least as well as the model without human capita factor and bonds returns. The results suggest that the two Fama-French factors SMB and HML may proxy for the risk associated with the return on human capital and beta instability. Piazzasi, Schneider, Tuzel (2007) compare the CHCAPM model (the consumption-housing CAPM) with Fama-French three factor model in terms of R^2 . The result shows that F-F three factors model explains 86% of the cross section variation in excess returns, and the CHCAPM model explains 82%. Furthermore, Carhert (1997) constructed 4-factor model using Fama and French (1993) three-factor model plus the momentum factor. He estimates performance relative to the CAPM, three-factor model, and four-factor models. It shows as

$$r_{it} = \alpha_{iT} + \beta_{iT}VWRF_t + e_{it} \quad t = 1, 2, \dots, T \quad (6)$$

$$r_{it} = \alpha_{iT} + b_{iT}RMRF_t + s_{iT}SMB_t + h_{iT}HML_t + e_{it} \quad t = 1, 2, \dots, T \quad (7)$$

$$r_{it} = \alpha_{iT} + b_{iT}RMRF_t + s_{iT}SMB_t + h_{iT}HML_t + p_{iT}PRIYR_t + e_{it} \quad t = 1, 2, \dots, T \quad (8)$$

Where r_{it} is the return on a portfolio in excess of the one-month T- bill return; VWRF is the excess return on the CRSP value weighted portfolio of all NYSE, Amex, and NASDAQ stocks; RMRF is the excess return on a value-weighted aggregate market proxy; and SMB, HML and PRIYR are returns on value weighted, zero-investment, factor-mimicking portfolios for size, book-to-market equity, and one year momentum in stock returns. He finds that the four-factor model substantially improves on the average pricing errors of the CAPM and the three-factor model.

2.1.C The Nonlinear Pricing Model

Dittmar (2002) use the similar approach as Jagannathan and Wang (1996) for testing Fama-French three-factor model. The difference is that he introduces nonlinear pricing kernel in estimating the cross sectional returns and compares it with augmented Fama-French in quadratic form and cubic form. The results turn out that the Fama-French model fares poorly in describing the cross section of industry returns compare with nonlinear pricing kernels. In other words, the nonlinear pricing kernels

outperform the Fama-French model in pricing the cross section of industry returns. Other examples are Bansal and Viswanathan (1993) and Bansal, Hsies and Viswanathan (1993). They use a nonlinear arbitrage-pricing model, a conditional linear model and an unconditional linear model to price international equities, bonds, and forward currency contracts. And their result shows that nonlinear arbitrage-pricing model is the only model does the job in explaining the time series behaviour of cross section of international returns.

Over all, the nonlinear asset pricing kernels become more and more important in explaining the time-varying behaviour of cross section of returns. Especially, during the economic crisis period, the returns of assets are non-normality distributed. The tails of this distribution become an important issue in considering the asset pricing.

2.2 Aggregate Wealth

2.2.A Measurement of the Human capital

The asset pricing models are tested with the respect of the return on the wealth portfolio. The wealth portfolio is the total return of all the assets in the economy. The return on the market portfolio (value-weighted index of common stocks) is a commonly used proxy by financial economists to test the asset pricing model. However, it might not capture the return on human capital. Jagannathan and Wang (1996) note that stocks form only a small part of the aggregate wealth. The monthly per capita income in the United States from dividends during the period 1959:1-1992:12 was less than 3 percent of the monthly personal income from all sources, whereas income from salaries and wages was about 63 percent during the same period. Diaz-Gimenez (1992) points that almost two thirds of nongovernment assets are owned by the household sector and only one-third is owned by the corporate sector. Approximately a third of the corporate assets are financed by equity. This suggests that the human wealth contribute significantly to the total wealth. This is why many researchers have considered the measure of the return on human capital as proxy of aggregate wealth and applied to asset pricing.

Next, I will introduce the definition of human capital and the most commonly used expression. Human capital refers to the stock of competences, knowledge and

personality attributes embodied in the ability to perform labor so as to produce economic value. Many economists believe that there is a strong relation between human capital and labor income and it is not hard to see that an individual worker related to his productive skills, technical knowledge, experience and capabilities. The employee can use these skills to improve their productivity and increase their salary. They all take into account for the human capital. Klinkowska (2008) not agree with Jagannathan and Wang (1996)'s definition of human capital that includes the social benefit from government and pension after retirement and so on. These types of income should not account for the compensation for working. Only these types income that are reward form the work and can reflect the abilities and knowledge of the employees. Economists apply different measurements of the return on human capital in their research. The commonly used definitions are

R_t - growth rate in aggregate labor income

and

R_t - growth rate in per capita labor income

Data can be taken from the National Income and Product Account (NIPA) Table 2.6. Where labor income = wage income + proprietary income + personal interest income.

The first people use the approach that to assess the effect of human capital upon pricing was Fama and Schwert (1977). In the observation, they test two models: Sharp-Lintner-Black model and Mayers model. The difference of these two models was their systematic risk measure. So they estimate the difference between the Mayers and Sharp-Lintner-Black risk measures for various classes of financial securities (such as NYSE common stock, U.S. treasury bills and bonds) by using a measure of the return to aggregate human capital for the entire U.S. labor force. To measure the return on human capital, they assume that maintenance costs are not highly related to the returns on marketable assets so that net income, like gross income, is likely to be more or less unrelated to the returns on marketable assets. So they use the gross income per capita as the measure of the payoff to a unit of human capita when net income, that is:

Net income = gross income – the maintenance cost

They use data computed by the U.S. Department of Commerce and reported in the "Survey of Current Business". The data is monthly frequency during the period of

1953-1972. Their result shows that the difference between the Mayers and Sharp-Lintner-Black risk measures are very small. That means the human capital in aggregate wealth has little to do with capital market pricing. They suspect that the result is robust with respect to different definitions of income.

Liberman (1980) follows Fama and Schwert (1977), but Liberman concentrates on individual human capital not on aggregate human capital. His proxy for the return on individual human capital is the growth rate in per capita earnings. In order to capture the diversity of individually held human capital, he uses three different sets of per capita earnings data:

To examine the effect of industry affiliation, he use the Bureau of Labor Statistics (BLS) data of monthly per capita production worker earnings for all eight industry classifications reported for in BLS's "Employment and Earnings: United states, 1909-72". The data consist primarily of wage disbursements to predominantly blue-collar-type employees and include only workers actually employed. To adjust the data to per capita for the entire industry labor force, he uses monthly industry unemployment rates as reported in the February 1973 issue of the monthly "Employment and earnings Journal".

For the effect of occupation type, he uses a time series of median annual per capita labor earnings data for men classified by occupation collected from the annual issues of the Bureau of the Census's "Current Population Reports: consumer Income" (Series P-60), beginning with 1958. All occupational classifications reported for the used except for those whose content was not consistent over time or was nor available for the entire period.

For the effect of level of educational attainment, he uses the Panel Study of Income Dynamics (PSID). PSID consists of annual longitudinal labor earnings data by individual from a representative sample of white male Americans for the years 1967 through 1974 arranged by years of school; attended. Being ungrouped, the data should be of special interest, for they should allow him to observe more directly the effect of individually held human capital than it is possible with the grouped industry and occupational data.

Liberman's results confirm those of Fama and Schwert (1977). He states that in the fact human capital is weakly related with the financial market (the changes in labor

earnings and the rate of return on financial assets is weakly correlated). But he also points out the future research directions on human capital and financial market: first, introducing the human capital in to a mutiperiod pricing model, as human capital is a mutiperiod commitment. Second, deem human capital as a purely exogenously determined human capital and use it in the pricing and portfolio composition.

Campbell (1993b) derives a measure for the return on human capital, which is current growth rate of labor income, plus a term that depends on expected future growth rates of labor income and the expected future asset returns:

$$R_{m,t+1} = (1 - v_t)R_{a,t+1} + v_t R_{y,t+1} \quad (9)$$

where

v_t is the ration of human wealth to total wealth,

$R_{a,t+1}$ is the gross simple return on financial wealth (a, refers to financial assets),

$R_{y,t+1}$ is the gross simple return on human wealth (y, refers to the stream of labor income)

Campbell assumes that the average log return on financial wealth equals the average log return on human wealth, and then the result is:

$$r_{m,t+1} \approx k_m + (1 - v)r_{a,t+1} + v r_{y,t+1} \quad (10)$$

where

k_m is a constant.

V is the mean of v_t

The model is multifactor asset pricing model and he argues that CAPM model ignores the fact that human capital is an important component of wealth.

Jagannathan and Wang (1996) base on Fama and Schwert (1977) and Campbell (1993b)'s observation, justify their work and follow Mayers' suggestion that Human capital is the important factor in measuring the total capital in the economy and it also forms the aggregate wealth. They make a simple assumption that return on human capital is a linear function of the growth rate in per capita labor income. They make this assumption and construct a human capital wealth assume return on human capital is linear function of the growth rate in per capital labor income and per capita labor income L_t follows an autoregressive process of the form:

$$L_t = (1 + g)L_{t+1} + \varepsilon_t \quad (11)$$

Where:

L_t - per capita labor income at time t,

g- the average growth rate of per capita labor income L_t ,

ε_t - has mean zero and is independently distributed over time.

Moreover, Jagannathan and Wang assume that the capital gain part of return on human capital is growth rate in per capita labor income and the per capital labor income is discounted at the constant rate r. under their assumptions above, the wealth due to human capital is given by:

$$W_t = \frac{L_t}{r - g} \quad (12)$$

So the rate of change in human capital wealth is then given by

$$R_t^{labor} = \frac{W_t - W_{t-1}}{W_{t-1}} \quad (13)$$

$$R_t^{labor} = \frac{L_t - L_{t-1}}{L_{t-1}} \quad (14)$$

However in their empirical work Jagannathan and Wang make a small change on the growth rate in per capita monthly labor income formula:

$$R_t^{labor} = \frac{L_{t-1} + L_{t-2}}{L_{t-2} + L_{t-3}} \quad (15)$$

Where:

R_t^{labor} - growth rate in per capital labor income that becomes known at the end of period t,

L_{t-1} - per capita labor income for period t-1 but which becomes known at the end of period t

Jagannathan and Wang define the return in human capital as a two month moving average of the growth rate in labor income. The return on human capital is a function of lagged labor income since the data are published with a one-month delay. The reason he use this formula as a proxy to the return on human capital is to minimize consequence of measurement errors. The data on personal income and population are taking from Table 2.2 in the National Income and Product Account of the U.S.A. the

labor income used in their work is the difference between the total personal income and the dividend income, which is

$$\text{Labor Income} = \text{Personal Income} - \text{Personal dividend Income}$$

They define the labor income by the total population in U.S. It includes wage compensation, proprietary income, rental income, net interest payments, social benefits and other types of income.

Heaton and Lucas (2000) test the importance of proprietary income for asset returns using aggregate income measures and an extension of frame work developed by Jagannathan and Wang (1996) in which aggregate wealth is stock market wealth plus human capital. The human capital is consisting by the value of future wage income and the value of future proprietary income (and the traditional approach to test asset pricing model is to use a stock market index alone as the proxy for the return to aggregate wealth). The returns of two elements of human capital are constructed using the growth in aggregate wage income R_t^{wage} and the growth in aggregate nonfarm proprietary income R_t^{prop} . The formula is defined below:

$$R_t^{wage} = \frac{W_t + W_{t-1}}{W_{t-1} + W_{t-2}} \quad (16)$$

$$R_t^{prop} = \frac{P_t + P_{t-1}}{P_{t-1} + P_{t-2}} \quad (17)$$

Where:

W_t - wage income at time t,

P_t - aggregate nonfarm proprietary income,

R_t^{wage} - growth rate in aggregate wage income,

R_t^{prop} - growth rate in aggregate nonfarm proprietary income,

In comparing to Jagannathan and Wang's measure of the return to human capital, Heaton and Lucas distinguish two components of human capital in which JW look human capital as one whole (JW use growth rate in per capital labor income). Furthermore, in comparison to the timing used by JW, Heaton and Lucas's equation (see equation (17)) is not lagged relative to asset returns as was done in equation (15). The reason may due to the different models they choose. Under HL timing assumption for the returns to human capital and stock returns, whether R_t^{wage} exists

does not have significant effect in the GMM function. However the proprietary income factor turns to be more robust to the exact timing. The authors explain “the potential reason for the effect of timing on the importance of wage growth is the observed reaction of the stock market to announcement effect and not due to a direct link between wage income growth and the current wealth of stock holders”. Therefore, the results indicate that proprietary income may be a more important wealth factor for individuals holding stocks, thus the equation (18) in the value of this wealth is more important for the determination of asset returns. Heaton and Lucas, similar to Jagannathan and Wang, also use the monthly data from NIPA Table 2.6. Aggregate wage income is taken from line 2 and aggregate nonfarm proprietary income comes from line 9.

Dittmar viewed human capital as endogenously determined human capital. He follows Jagannathan and Wang define the return on human capital as a two-month moving average of the growth rate in labor income:

$$R_{i,t+1} = \frac{L_t + L_{t-1}}{L_{t-1} + L_{t-2}} \quad (18)$$

where

L_t denotes the difference between total personal income and dividend income at t

The data used to compute the labor income is obtained from the NIPA data. Labor income at time t is computed as the per capita difference between total personal income and dividend income. The data is monthly data and covers the period July 31,1963 through December 31, 1995. Dittmar assumes that the cross products in higher order term of the return on wealth portfolio are zero. He defines the nonlinear pricing kernel as follows:

$$m_{t+1} = (Z_t \delta_0)^2 + \sum_{n=1}^3 I_n \left[(Z_t \delta_{n,vw})^2 R_{vw,t+1}^n + (Z_t \delta_{n,lbr})^2 R_{l,t+1}^n \right] \quad (19)$$

where

Z_t - a set of instrument, $Z_t = \{1, r_{m,t}, dy_t, ys_t, tb_t\}$

$R_{vw,t+1}^n$ - the return on the value weighted equity portfolio,

$R_{l,t+1}^n$ - the growth rate in labor income

and

$$d_n = I_n (\delta_n' Z_t)^2 \quad I_n = \begin{cases} -1 & n=1,3 \\ 1 & n=2 \end{cases} \quad (20)$$

Where

d_n is the value of coefficients corresponding to the n^{th} order of the return on the market portfolio.

Dittmar analyzes the linear, quadratic and cubic pricing kernel. And results are compared with and without human capital as a component of the return on aggregate wealth. The result shows that asset returns are affected by higher order moment coskewness and cokurtosis. Nonlinear terms improve the fit of asset pricing model, especially when human capital is added into the measure of aggregate wealth.

2.2.B Measurement of the Housing wealth

A liquidity crisis in the united banking system and overvaluation of assets are the main causes of the financial crisis, which starts from 2007 until now. However, the bubble in the booming house market is the one of the main triggers for this credit crunch. Since year 2005, US mortgage lenders sell many expensive mortgages to customers even if they are with poor credit, high chance of default. To increase the profitability on mortgage, other financial companies bought mortgage debts as a package. The idea is to spread the risk, but it makes bigger problem as rating agencies gave these risky mortgages a low risk rating and hide these risks in financial system. Many of these housing mortgages had an introductory period of 1-2 years of very low interest rates. In 2007, interest rates increased at the end of this introductory period. In addition, due to inflation in 2007, US had to increase interest rate and mortgage payments were getting more expensive. Many new house owners could not afford mortgage payments they choose to default. Then US houses price started to fall, the bank couldn't recoup the initial loan. It became a bad loop for US housing market and caused many medium sized mortgage companies to go bankrupt. Not only this, many banks were also facing big losses. To write off large losses, banks tighten their fund and became more and more reluctant to fund enterprises. There was not enough liquidity in the market. It affected many firms who have difficulty in borrowing money. The total volume of money in the market shrinks and money movement velocity decreased significantly. It has direct impact on the GDP growth and slowed down the global economy.

Housing price growth rate becomes a very critical indicator, reflecting consumer confidence on the long-term global economy, which can be used to estimate the turning point for the recent economic down turn. In addition, the level of housing price has direct impact on the balance sheet of the banking sectors. The higher the house price, the more willingness banks have to release the credit to the market. It can be used to estimate when the credit crunch will disappear and global economy gets fully recovered. Therefore, it is very essential to include housing price into the model below especially to understand asset price behavior after year 2007, the credit crisis.

In the past study, most researchers have focused on the inclusion of human capital as the proxy for the return of the total wealth. Such as Jagannathan and Wang find that human capital factor improves the fit of the CAPM model specification; however, the human capital is not the best proxy of the return for aggregate wealth such as Fama and Schwert (1977) point that the real estate accounts for a substantial portion of the total financial wealth. More recent papers are focus on housing return as an important component of aggregate wealth. Flavin and Yamashita (2002) state the housing plays an important role in both consumption bundle and the asset portfolio of the household. They estimate the risk and return to financial assets and housing, and address the optimise portfolios issue by using mean-variance efficiency framework. Flavin and Yamashita use the panel study of Income Dynamics (PSID) data on house prices to estimate the housing returns over the 1968 to 1992 period. Every year The PSID asks homeowners how much their house would sell for if the house were put on the market on the date of the interview. The responds enable to calculate the return to owner-occupied housing at the household level. The return to housing depends on the appreciation of the value of the house, the value of the housing services expressed by rental value, and costs of ownership and maintenance. However, there are no direct observations of the rental value of the house and the maintenance costs so Flavin and Yamashita model these components as below:

$$D_t = (r + d)P_{t-1} + \tau \text{PropertyTax}_t \quad (21)$$

$$COM_t = dP_{t-1} + (1 - \tau)\text{PropertyTax}_t \quad (22)$$

Where:

r - short-term real interest rate, equals 5%,

d – the depreciation rate,

τ - the marginal income tax rate, equals 33%,

$Pr\text{opertyTax}_t$ – the property tax rate, equals 2.5%.

The imputed annual rental value, denoted D_t , reflects the assumption that property taxes are passed through into rents. Moreover in the absence of expenditures on maintenance and repairs, physical depreciation at the rate d would be reflected in the real value of the house P_t . However, Flavin and Yamashita assume that both landlords and house owners spend on maintenance and repair an amount equal to the annual depreciation of the house so that the physical condition of the house is constant. In addition the cost of ownership and maintenance COM_t includes the net property tax payment (the net of deduction against income taxes). In computing the real return to housing, the nominal house value and the nominal property tax payments as reported by the respondent are converted into real term using the CPI-U deflator to obtain P_t and $Pr\text{opertyTax}_t$. The real return on housing $R_{H,t}$ (this return is on individual level) is then computed in the following way:

$$\begin{aligned}
 R_{H,t} &= \frac{P_t + D_t - COM_t - P_{t-1}}{P_{t-1}} \\
 &= \frac{P_t + rP_{t-1} + \tau Pr\text{opertyTax}_t - P_{t-1}}{P_{t-1}} \\
 &= \frac{P_t}{P_{t-1}} + r + 0.33 \times 0.025
 \end{aligned} \tag{23}$$

Kullmann (2003) follows the Jagannathan and Wang's approach; use the same measurement of return to human capital. The measurement of income that she use is

Labor income = the total personal income- dividend income

Data is monthly frequency, which is taken from NIPA of the USA published by the BEA. Moreover, she uses an aggregate house price index to examine the impact of real estate risk on asset prices. The results indicate that including proxies for the return to real estate improve the performance of different empirical specifications of the CAPM. She uses two proxies for the return on the two types of real estate: residential real estate and commercial real estate.

To proxy for return to residential real estate, she applies the monthly percentage change in the median price of existing homes sold from the

National Association of Realtors (NAR) to proxy for the returns to residential real estate. Residential real estate assets have large impact on the household's net worth and consumption possibilities. The total return to home ownership contains the price appreciation and the value of the consumption of house services. The house price change does not take into account the consumption of housing services or implicit rent on owner occupied housing, as well as other house related costs and benefits. Kullmann assume that the implicit consumption benefit is a constant fraction of the return to home ownership. To examine this assumption, she tests whether the variance of real estate returns over time is driven primarily by house price changes. She also checks that most of the fluctuation in the total return to housing comes from house price changes and not from changes in the value of housing services received.

To proxy for return to commercial real estate, she uses the National Association of Real Estate Investment Trusts' (NAREIT) equity REIT index (which is commonly used portfolio-based measure of commercial real estate returns).

Davis and Heathcote (2005) measure house price changes from the different data source. The data are taken from the Freddie Mac Conventional Mortgage Home Price Index (CM HPI). This is a repeat sales index calculated using mortgage transaction data provided by Freddie Mac. They focus on individual one-family houses. Moreover not all single family residential properties are taken into account-only these that are financed by mortgages purchased by Freddie Mac. The index is published quarterly since 1970. However since 1975 exists the OFHEO Home Price Index which also by Fannie Mae. The OFHEO HPI is then superior to the CM HPI. They conduct a stochastic growth model to explain the dynamics of residential investment. They identify capital and labor as three different technologies: construction, manufactures and service and they index by the subscript b, m and s respectively. They assume that the representative sells household a constant acreage of new land suitable for residential development each period. The stochastic component of productivity shocks follows an autoregressive process:

$$\bar{z}_{t+1} = (\log \bar{z}_{b,t+1}, \log \bar{z}_{m,t+1}, \log \bar{z}_{s,t+1}) = B\bar{z}_t + \varepsilon_{t+1} \quad (24)$$

To derive the equilibrium changes in the price of land is the formula below:

$$E\left[R_{i,t} \left(\delta_0 + \delta_{vw} R_t^{vw} + \delta_{prem} R_t^{prem} + \delta_{labor} R_t^{labor} \right)\right] = 1 \quad (25)$$

where:

p_{dt} - price of new structure purchased, if land's share is zero $p_{dt} = p_{ht}$. When land's share is positive, house prices are increasing both in the price of structure and quantity of structure purchased. Their analysis results indicate that the volatility of residential investment is more than twice bigger as business investment, and non-residential investment co-move positively and the residential investment leads the business cycle. One failure of this model is that it does not show the fact that residential investment leads GDP. Davids and Heathcote (2007) actually use the OFHEO Index to measure the returns on residential properties in US. They also show that the per capital income and interest rates systematically correlate with house prices only though their connection to the price of residential land.

Qi and Wu (2006) create the return on housing by using the 1976 to 1997 waves of the Panel Study of Income Dynamics (PSID) Family Income Files and generate a time series of annual growth rates of housing value and take it as a measure of the return on residential properties. The housing value is based on the home equity and the home equity is defined as the net worth of self-reported market value of a house minus unpaid mortgage balance. Qi and Wu (2006) have only 21 observations on the return on housing, which is not a lot. Moreover, similar to Flavin and Yamashita (2002), the returns are at the household, individual level and not at the aggregate level and the two values may significantly differ from each other.

Piazzasi, Schneider, Tuzel (2007) in their paper define housing both as an asset and consumption good. They use consumption-based asset pricing model (CCAPM) with housing factor to forecast excess return on stock. They define the housing returns by the NIPA-based measurement. The real house return is given by the following formula:

$$\frac{(p_t^h h_t + q_t s_t)/h_{t-1}}{p_{t-1}^h h_{t-1}/h_{t-1}} - \delta - (1 - 0.33) \times 0.025 \quad (26)$$

Where:

$p_t^h h_t$ is the real housing value, is taken from the NIPA Fixed Asset Table 2.1 (line 59), called Current-Cost Net Stock of Private Fixed Assets, Equipment and Software, and Structures by Type. This housing value is calculated using

the current value method, which measure the current market value of the assets (as opposed to the historical value method, which measures the book value of assets).

To include the value of land, Piazzasi, Schneider, Tuzel assume that land prices are perfectly correlated with the price of structures. Using Census Data, they estimate that the value of the land is 36% of the total housing value. Therefore they adjust houses price to $p_t^h / (1 - 0.36)$.

q_t, s_t is dividends on housing which are the rent payments during that year. However, they do not specify which series from NIPA Table they are using.

Piazzasi, Schneider, Tuzel follow Flavin and Yamashita (2002) and assume that maintenance roughly equals depreciation, so that they subtract $\delta p_{t-1}^h h_{t-1}$ from dividends. Here they do not specify the value of δ .

They also subtract net property tax payments: $(1 - 0.33) \times 0.025 \times p_{t-1}^h h_{t-1}$, where the marginal rate is about 33% and the property tax rate is 2.5%

To compare the differences between Piazzasi, Schneider, Tuzel and Flavin and Yamashita: (i) Flavin and Yamashita calculate return on housing for individual level, as they use data from the Panel Study of Income Dynamics (PSID), this housing return is not accompanying rent. Piazzasi, Schneider, Tuzel and Flavin consider the rent data correspond to the house price series. They use aggregate housing returns. (ii) They also find out average returns on individual housing are more than three times as high as those on aggregate housing. The difference in standard deviation is even bigger: returns on individual houses are more than five times as volatile as return on the aggregate housing. (iii) Flavin and Yamashita use data range from 1968 to 1992 period and Piazzasi, Schneider, Tuzel and Flavin and Yamashita use data range from year 1930 to 2000.

Klinkowska (2008) construct the measures the return on human capital and residential properties. He introduces the dynamics into the asset pricing model CAPM and tests the CAPM model with these factors. The result shows that the CAPM augmented with human capital and housing can explain around 80% of the variation in the cross section of excess return. It works well than simple CAPM model. Fama-French three-factor model is slightly less 79%.

Chapter 3

NONLINEAR ASSET PRICING KERNEL WITH HUMAN CAPITAL AND HOUSING WEALTH

Dittmar (2002) investigates the nonlinear pricing kernels in which the risk factor is endogenously determined. His analytical results show that the nonlinear pricing kernels perform significantly better than the linear model and multifactor models for the cross section of returns. This research modifies Dittmar's nonlinear models by introducing owner-occupied housing. While Dittmar use the returns on the 20 industry portfolios and human capital which is represented by labor income to obtain the empirical results of nonlinear pricing kernels. This paper also applies the intertemporal consumption based nonlinear asset pricing model. However, a unique feature of this model is that we consider owner-occupied housing into the analysis as the housing choice reflects household's expectation about future asset returns, thus the housing factor contains information about financial asset returns. Moreover, purchasing a house reflect the householder's income level or consumers' confidence. The householder who has higher income that has potential to pay higher price for better property. At the same time, he also can get tax return benefit depends on how much he pays the income tax. So the housing has high relation with labor income.

3.1 risk factors

The model I in this research that is tested by three types of factors. The first is returns on market portfolio. Market portfolio is a portfolio consisting of a weighted sum of every asset in the market, with weights in the proportions that they exist in the market. The concept of a market portfolio plays an important role in many financial theories and models. It is used to represent the world aggregate return in the world market. After Roll's critique states that these proxies cannot provide an accurate representation of the entire market. Researchers have refined the definition of the total wealth, for instance human capital risk factor.

The second is the returns on human capital. Human capital is the largest asset in any economy. It affects consumption decisions and the riskiness of assets and therefore

their prices. However, it is a non-marketable asset and it's not easy to define. Most economists use wages represents the return on human capital and the most commonly used formula is two month moving average of respective monthly income measures developed by Jagannathan and Wang (1996) (see equation (15)). Heaton and Lucas (2002) and Dittmar (2002) and Kullmann (2003) and Klinkowska (2008) follow Jagannathan and Wang 's measure of return on human capital that growth rates in per capita labor income, where labor income is defined as total personal income minus dividend income. But Klinkowska excludes the personal dividend income in his paper. The wage income accounts for most (more than 60%) of the labor income and proprietary income is the second largest component of labor income. No matter how you define the labor income, the results are similar stated by Fama and Schwert (1977). My approach also follows Jagannathan and Wang (1996).

The third is the returns on housing wealth. Home equity constitutes roughly one fifth of total net wealth and the proportion of people own a home is much higher than the proportion of people own stocks. Heaton and Lucas (2000) use the survey of Consumer Finances to examine the cross-sectional variation in the composition of the household's wealth. Their analysis shows that the real estate is an extremely large component of individuals' financial wealth as well as total wealth. Housing as a component of household wealth that is indirect contributions to economic growth however, in recent years research, model estimates suggest that housing-related effects accounted for at least one quarter of the growth in personal consumption expenditures. For instance, consumers spend quicker in the gains of housing wealth rather than in the gains of stock wealth. That is because the consumers are cautious about making lifestyle changes based on near-term movements in stock prices that could well prove unsustainable. In addition the housing wealth composes a large proportion of national wealth that about six in ten homeowners had more home equity than stock wealth, the share was even larger among low-income homeowners reported by the Survey of Consumer Finances. The reason is that the housing wealth is far less volatile than the stock wealth that can rise or fall rapidly in one-day time, housing primary store of wealth for most households and is an important component of overall household wealth and the broader economy. Finally, housing is also a leveraged investment that even small percentage gains in home values can be large relative to

the down payment invested a home. So, investing on housing might become another resource of retirements saving.

The real return of housing wealth is complicated and made more interesting. Kullmann (2003), that she examines the performance of the factor pricing model by introducing real estate risk as an additional risk factor. She differentiates between residential and commercial real estates. She uses two proxies for housing wealth. Kullmann finds that the inclusion of real estate risk can greatly improve the performance of linear factor pricing models in terms of the explanatory power for cross-sections of stock returns. However, she does not theoretical explain it. Also I think residential real estates are not highly related on home equity return of over all householders. So I don't account for commercial real estates in return of housing wealth.

Davis and Heathcote (2005,2006) and Klinkowska (2008) use the net change in the Office for Federal Housing Enterprise Oversight (OFHEO) House Price Index (HPI). The data measures the movement of single-family house price in the U.S. It reflects the cost of structure and land, and simultaneously controlling for the quality of the house. The structure can be priced as the replacement cost, after accounting for depreciation of the physical building. The land is shorthand for the size and attractiveness of the plot. In addition, the volatility in housing returns may due to the house price movements on the specific cities and regions, also discussed in Caplin et al. (1997). The HIPI is a weighted, repeat-sales index that it measures average price changes in repeat sales. The index is published quarterly since 1970. However, my sample period starts from 1963 and monthly frequency, thus I don't use their measurement in my test.

Another example of real estate risk as a common risk factor in asset pricing models by Flavin and Yamashita (2002), they estimate the risk and return to financial assets and housing, address the optimal portfolios issue by using mean-variance efficiency framework. Flavin and Yamashita calculate the return to owner-occupied housing at the household level. They consider the appreciation of the value of the house and the loss incurred on a home, like investing on a real estate involves maintenance expenditures, debt service and transaction costs to buy and sell. The calculations are under the assumption that the property taxes pass through into rents and they assume that the amount of spending on maintenance and repairs equals to the annual depreciation of the house. (see equation (23)). Piazzesi, Schneider and Yuzel follow

also Flavin and Yamashita (2002) that use the same method to calculate the real return of housing wealth but they consider the value of the land when calculate this real house price. In this paper, I use the same approach as Flavin and Takashi Yamashita (2002). One reason is that this return on housing is on individual level and the volatile is five times higher than the return on aggregate wealth. Second is the factor such as the depreciation rate that can be cancelled out in the calculation and it can reduce the computing error. (the calculation detail is in chapter 4)

In the more recent paper, Case, Cotter and Gabriel (2010), examine the housing factor in CAPM framework. They find the basic housing CAPM results are robust to the inclusion of standard measures of risk and other housing market fundamentals. Their findings supports the application of a housing investment risk return framework in explanation of variation in time-varying US house price returns. They use quarterly house price indices from the Office of Federal Housing Enterprise Oversight (OFHEO) for the 1985-2007 timeframe. Our research goal is very similar. In this paper, as Case, Cotter and Gabriel, I model both market return and housing. I augment the Fama-French model by adding momentum factor. But I use different framework, my asset pricing model is non linear, because I focus on the higher order moment. Also I want to show the kurtosis has higher power to explain cross section of return than linear multifactor models. Dittmar (2002) use consumption based nonlinear asset pricing framework. I follow his model. Although my results are similar to the results in Dittmar (2002), some differences are needed to point out here. In his paper, his data is from July 1963 to Dec1995. While my models are tested by three time periods; first period is as same period as Dittmar's, then I am able to compare the results to see whether our results are similar. The second time period covers the recession period from July 1963 to Dec 2009, as housing return is more volatile than before, it might become suitable data in examining the nonlinearities of the risk factors. The third period is from Jan1996 to Dec 2009. In Dittmar's model, the two pricing factors are market return and human capital. While in my model the pricing factors are market portfolio return, human capital return and housing return. Further more, Dittmar focus on the preferences restricted pricing kernels and their nonlinearities driven out the importance of the factors in the linear multifactor models. I also establish the fitting results in this paper, but I focus more on the housing wealth contribution on cross section of aggregate returns.

3.2 Theoretical model set up

In this section I introduce the nonlinear asset pricing model that used in Dittmar (2002) paper. In my research I also use this nonlinear model to conduct the empirical analysis. I describe the theory behind it and justify why I think this model may provide better results than linear single or multifactor models, such as CAPM and Fama French three factor model.

The asset price equation that I use can understand as a state price weighted average of the payoffs in each state of nature that the ratio of state price multiplies the possibility for each state. The basic equation of asset pricing can be written in the term of conditional moments as follow:

$$p_{it} = E[m_{t+1}x_{i,t+1}], \quad (27)$$

This formula represents the theory that there is the positive random discount factor that prices all the payoffs, if and only if this law of one price hold and no arbitrage exists. p_{it} is the price of an asset i at time t , $x_{i,t+1}$ is the random payoff on asset i at time $t+1$ and m_{t+1} is the stochastic discount factor converts expected payoffs tomorrow into value today and always positive in reality.

We use the utility function to capture what value of payoff the investor wants. The formula defined over current and future values of consumption:

$$u(c_t, c_{t+1}) = u(c_t) + \beta E[u(c_{t+1})], \quad (28)$$

Where c_t denotes consumption at time t . It's random;

We assume the investor is able to freely buy and sell asset i , at a price p_{it} . The volume of the trade can be obtained by solving the problem

$$\max u(c_t) + E[\beta u(c_{t+1})], \quad (29)$$

let $c_t = e_t - p_t \xi$; $c_{t+1} = e_{t+1} + x_{t+1} \xi$; substitute the constraints into the objective, and setting the derivative with respect to $\xi = 0$, then we get the first condition:

$$p_{it} u'(c_t) = E[\beta u'(c_{t+1}) x_{i,t+1}], \quad (30)$$

or

$$p_{it} = E[\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{i,t+1}]. \quad (31)$$

Where $u'(c_{t+1})$ is the marginal utility of consumption at time t ; $p_{it} u'(c_t)$ is the loss in utility if the investor buys another unit of the asset; $E[\beta u'(c_{t+1}) x_{i,t+1}]$ is the increase in

utility the investor obtains from the extra payoff at $t+1$. The investor continues to buy or sell the asset until the marginal loss equals the marginal gain.

If the value of p_{it} is nonzero, we can also write the formula as

$$1 = E[m_{t+1}(1 + R_{i,t+1})], \quad (32)$$

Where $(1 + R_{i,t+1}) \equiv x_{i,t+1}/p_{it}$ is the $n \times 1$ vector of gross return. $m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$

is the stochastic discount factor and generated from the consumption-based or utility-based asset pricing theory. β is called the subjective discount factor. We

could rewrite the marginal rate of substitution of $\frac{u'(c_{t+1})}{u'(c_t)}$, $\frac{u'(W_{t+1})}{u'(W_t)}$ as the

consumption and wealth are equivalent in this case. You will see this from is commonly used in my empirical work later.

This asset pricing theory has been using and proving by many researchers. Leory (1973) and Lucas (1978) are the first one presenting Euler equation for asset pricing. Cox and Ross (1976) and Ross (1978) apply the Arrow-Debreu model of general equilibrium on option pricing. Harrison and Kreps (1979) provide this asset pricing model in continuous time. The first people who did the empirical study in discrete representation are Grossman and Shiller (1981). Hensen and Richard (1987) develop discrete time representation of this theory further, emphasizing the distinction between conditional and unconditional expectations. More recently, Hansen and Jagannathan (1991) use this expression to discuss the solution of an investor's portfolio choice problem. Cochrane (1999) states the whole of asset pricing theory within this framework. Thus I continue to apply this theory on intertemporal consumption and portfolio choice problem in my research.

Next we discuss the pricing kernels. Many literature researches the standard choices for agent's utility function $U(\cdot)$ and information for the investor's risk aversion or the riskless rate is unrealistic (e.g., Dittmar (2002)). Thus, to mitigate this problem Dittmar (2002) express the pricing kernel as a nonlinear function of the return on aggregate wealth. He approximate the nonlinear pricing kernel using a Taylor series expansion:

$$m_{t+1} = h_0 + h_1 \frac{U''}{U'} R_{w,t+1} + h_2 \frac{U'''}{U'} R_{w,t+1}^2 + \dots, \quad (33)$$

Where $R_{w,t+1}$ represents the return on end-of-period aggregate wealth. As show in equation (33), the marginal rate of substitution can be estimated by a polynomial of order n (includes an infinite number of terms). In practice, we can only take a finite number of terms, and there will be truncation error due to the contribution of the terms that are dropped. Dittmar (2002) let the preference theory to determine the order at which the expansion should be truncated. He stated that the preference theory is more powerful than let data determine the polynomial term of truncation in Bansal et al. (1993). Dittmar considers the four moments, which the fourth moment is cubic term. The reason he stated in his paper is that preference theory does not guide us in determining the sign of additional polynomial terms. Thus, under this assumption that the pricing kernel can be described by three terms (linear, quadratic and cubic) in aggregate wealth, it starts with lower order polynomial (linear pricing kernel) in aggregate wealth, imposing standard risk aversion on agents' preferences and expand to nonlinear pricing kernels that quadratic and cubic polynomial in the return on aggregate wealth. The pricing kernel is decreasing in the linear term of the pricing kernel, increasing in the quadratic term and decreasing in the cubic term. Then he modifies this pricing kernel by imposing restrictions on the signs of the coefficients. Consequently, the pricing kernel that we investigate has form

$$m_{t+1} = (\delta'_0 Z_t)^2 - (\delta'_1 Z_t)^2 R_{w,t+1} + (\delta'_2 Z_t)^2 R_{w,t+1}^2 - (\delta'_3 Z_t)^2 R_{w,t+1}^3 \quad (34)$$

Where

$\delta'_n Z_t$ is the value of coefficients corresponding to the n^{th} order of the return on the market portfolio. δ'_n is five elements vector. The number of element in δ'_n consist with a set of instrument Z_t , $Z_t = \{1, r_{mt}, dy_t, ys_t, tb_t\}$, where 1 indicates a vector of ones, r_{mt} is the excess return on the CRSP value-weighted index at time t, dy_t is the dividend yield on the CRSP value-weighted index at time t, ys_t is the yield on the three-month Treasury bill in excess of the yield on the one month Treasury bill at time t, and tb_t is the return on a Treasury bill closest to one month to maturity at time t.

$R_{w,t+1}^n$ is the return on the aggregate wealth with power of n.

3.3 Hansen-Jagannathan Estimator

We follow Dittmar (2002) using the Hansen-Jagannathan estimator to estimate the parameter. There are three advantages to using H-J estimator rather than standard GMM estimator. First, the Hansen-Jagannathan approach provides better statistics on both nested models and nonnested models. Second, it can produce less volatile pricing errors in the calculation. Third, the results may be more robust. The orthogonally conditions are:

$$E[((1 + R_{t+1}) \otimes Z_t)m_{t+1}] = 1 \otimes Z_t \quad (35)$$

Or
$$E[((1 + R_{t+1}) \otimes Z_t)m_{t+1}] = E[1 \otimes Z_t] \quad (36)$$

Where Z_t is a vector of elements in the chose instrument and “ \otimes ”denotes the Kronecker product operator. As Cochrane (1996) notes, equation (4) is an implication of (3), and if (4) holds for all choices of Z_t , it implies equation (3). The vector of sample orthogonality conditions can express as:

$$E[((1 + R_{t+1})m_{t+1} - 1) \otimes Z_t] = 0 \quad (37)$$

Or same as
$$g_T(\delta) = \frac{1}{T} \sum_{t=1}^T V_{t+1} \otimes Z_t' = 0_N \quad (38)$$

Where V_{t+1} is the vector of errors. It generated from the Euler equation (32)

$$V_{t+1} = (1 + R_{t+1})((Z_t \delta_0)^2 - (Z_t \delta_1)^2 R_{m,t+1} + (Z_t \delta_2)^2 R_{m,t+1}^2 - (Z_t \delta_3)^2 R_{m,t+1}^3) - 1_N \quad (39)$$

To solve equation (38), we rewrite it in the quadratic form and minimizing the quadratic form:

$$J(\delta) = g_T(\delta)' W^{HJ}(\delta) g_T(\delta) \quad (40)$$

We obtain the HJ-distance is thus

$$\text{Dist}_T(\delta) = \sqrt{\min g_T(\delta)' W^{HJ}(\delta) g_T(\delta)} \quad (41)$$

Where

$$W^{HJ} = E[((1 + R_{t+1}) \otimes Z_t)((1 + R_{t+1}) \otimes Z_t)'] \quad (42)$$

B Wald Tests

The Wald statistic of the joint significance of the coefficients are calculated from

$$\text{Wald}(n) = \frac{d_n^2}{\text{var}(d_n)}, \quad n = 1, \dots, 5 \quad (43)$$

And $\text{var}(d_n)$ is calculated from

$$\text{var}(d_n) = \left(\frac{\partial d_n}{\partial \delta_n} \right)' \text{var}(\delta_n) \left(\frac{\partial d_n}{\partial \delta_n} \right) \quad (44)$$

We take derivatives to get

$$\text{var}(d_n) = 4 \times |d_n| \times (Z_t' \text{var}(\delta_n) Z_t) \quad (45)$$

Then we use the expression (16) as the covariance matrix for standard errors and apply it to delta method above for the parameters tests.

$$\text{var}(\delta) = \frac{1}{T} (D'WD)^{-1} D'WSWD(D'WD)^{-1} \quad (46)$$

Where D is the jacobian of the average moments with respect to the parameters, W is the Hansen and Jagannathan weighting matrix, S is the variance of the moments, and T is the sample size.

Chapter 4

DATA AND INSTRUMENT STATISTICS

4.1 Data Description

Here I give the details of the datasets used in this study. Our data are monthly return data on 17 industry-sorted portfolios are Fama-French Portfolios, which are widely used in many empirical asset pricing literatures and taken from the Kenneth R. French's webpage. I study and test the models across three time periods: first time period covered is from July 1963 to December 1995, which is the same period that Dittmar uses. This gives me 390 time series observations; Second time period covered from July 1963 to December 2009. This gives me 558 time series observations; Third time period covered from January 1996 to December 2009. This gives me 168 time series observations. They are described in Table I.

In order to apply GMM estimation, here consider a set of instrument $Z_t = \{1, r_{m,t}, dy_t, ys_t, tb_t\}$ ¹, whose components also have predictable power. 1 denotes a vector of ones. $r_{m,t}$ is the excess return on the CRSP (centre for research in Security Prices) value-weighted index at time t, dy_t is the dividend yield on the CRSP (centre for research in Security Prices) value-weighted index at time t, ys_t is the yield on the three-month Treasury bill in excess of the yield on the one-month Treasury bill at time t, and tb_t is the return on a Treasury bill closest to one month to maturity time t. The excess return on the value-weighted CRSP (centre for research in Security Prices) index $r_{m,t}$ and return on a Treasury bill closest to one month to maturity tb_t are taken from Fama factors file. Dividend yield dy_t is obtained from stock market data created by Princeton University Press. Yield on the Three month Treasury bill in excess of the yield on the one-month Treasury bill ys_t are calculated by Three month Treasury bill minus risk free rate. In addition the group of factors in Fama-French four-factor

¹ Instruments $r_{m,t}, dy_t, ys_t, tb_t$ are consistent with those adopted by Dittmar (2002).

model used in testing asset are $R_m - R_f$, R_{SMB} , R_{HML} , R_{Mom} and are taken from Kenneth R. French website. $R_m - R_f$ is the return on market portfolios. R_{SMB} is the return on the portfolio of long small stock and short big stock, while R_{HML} is the return on the portfolio of long value stock and short growth stock. Moreover, R_{Mom} is the average of the returns on two (big and small) high prior return portfolios minus the average of the returns on two low prior return portfolios, which formed using independent sorts on size and prior return of NYSE, AMEX, and NASDAQ stocks. Big means a firm is above the median market cap on the NYSE at the end of the previous month; Small firms are below the median NYSE market cap.

4.2 Other Data

A commonly used measure of the return on market portfolio is the return on a value-weighted index, which includes all the assets traded in NYSE, AMEX, and NASDAQ.

The data used to compute the labor return series is taken from DataStream. Follows Jagannathan and Wang (1996), we define the return on human capital as a two-month moving average of the growth rate in labor income.

$$R_{l,t+1} = \frac{L_t + L_{t-1}}{L_{t-1} + L_{t-2}} - 1$$

Where L_t is the labor income is computed as the per capita difference between total personal income and dividend income. The two month moving average of per capita labor income growth is used to reduce the influence of measurement error.

The data used to measure the house price index is calculated from the median monthly figures from US census. I follow Flavin and Takashi Yamashita (2002) and define the return of house price index:

$$R_{H,t} = \frac{P_t + rP_{t-1} + 0.33 \times 0.025 - P_{t-1}}{P_{t-1}}$$

Here the short interest r , is monthly interest rate. It obtains from a fixed 5 percent annual rate. 0.33×0.025 is the income tax payment, where 0.33 is fixed marginal income tax rate and 0.025 is property tax rate.

4.3 Summary Statistics

Table 1 is the sample statistics for the returns on 17 industry portfolios and the components of the market proxy. Panel A provides mean returns for the 17 industry portfolios and Panel B shows the standard deviation of the portfolios. The average returns over the sample period from July 1963 to December 1995 for the payoffs range from 85 basis points per month for steel industry to 128 basis points per month for the food industry. The average returns over the sample period from July 1963 to December 2009 for the payoffs range from 80 basis points per month for Durable good industry to 125 basis points per month for the mines industry. The average returns over the sample period from January 1996 to December 2009 for the payoffs range from 40 basis points per month for durable industry to 152 basis points per month for the mines industry and the next large return industry is oil, which is 116 basis points per month. As shown in the table, the average return for the row material such as oil, steel and mines are getting bigger in the most recent sample period.

Table 1

Summary Statistics: Industry Portfolio

Table 1 presents statistics for monthly means and standard deviations of the payoffs on 17 industry-sorted portfolios that used in this paper. The returns on 17 industry-sorted portfolios are equally weighted and the data cover three different sample periods: July 1963, through Dec 1995; July 1963, through Dec 2009; January 1996, through Dec 2009. Panel A and B presents statistics for the monthly returns on the 17 sized-sorted portfolios.

Panel A: Mean Returns

| Industry | July 1963-Dec 1995 | July 1963-Dec 2009 | Jan 1996-Dec 2009 |
|---------------------|--------------------|--------------------|-------------------|
| Food | 0.0123 | 0.0108 | 0.0073 |
| Mines | 0.0106 | 0.0117 | 0.0143 |
| Oil | 0.0109 | 0.0109 | 0.0110 |
| Cloths | 0.0114 | 0.0099 | 0.0072 |
| Durbl | 0.0092 | 0.0074 | 0.0033 |
| Chemicals | 0.0098 | 0.0091 | 0.0075 |
| Cnsum | 0.0118 | 0.0107 | 0.0082 |
| Construction | 0.0102 | 0.0092 | 0.0070 |
| Steel | 0.0078 | 0.0081 | 0.0087 |
| Fabricated Metals | 0.0101 | 0.0092 | 0.0071 |
| Machinery | 0.0097 | 0.0098 | 0.0100 |
| Autos | 0.0093 | 0.0082 | 0.0058 |
| Transport Equipment | 0.0106 | 0.0097 | 0.0076 |
| Utilities | 0.0085 | 0.0082 | 0.0075 |
| Retail | 0.0107 | 0.0101 | 0.0086 |
| Finance | 0.0107 | 0.0094 | 0.0065 |
| Other | 0.0098 | 0.0083 | 0.0047 |

Table 1-Continued

| Panel B: Standard Deviations | | | |
|------------------------------|--------------------|--------------------|-------------------|
| Industry | July 1963-Dec 1995 | July 1963-Dec 2009 | Jan 1996-Dec 2009 |
| Food | 0.0451 | 0.0444 | 0.0426 |
| Mines | 0.0657 | 0.0731 | 0.0881 |
| Oil | 0.0518 | 0.0538 | 0.0582 |
| Cloths | 0.0610 | 0.0623 | 0.0655 |
| Durbl | 0.0526 | 0.0559 | 0.0627 |
| Chemicals | 0.0529 | 0.0557 | 0.0618 |
| Cnsum | 0.0479 | 0.0470 | 0.0448 |
| Construction | 0.0589 | 0.0602 | 0.0634 |
| Steel | 0.0614 | 0.0737 | 0.0966 |
| Fabricated Metals | 0.0503 | 0.0535 | 0.0602 |
| Machinery | 0.0540 | 0.0656 | 0.0869 |
| Autos | 0.0583 | 0.0630 | 0.0729 |
| Transport Equipment | 0.0595 | 0.0583 | 0.0557 |
| Utilities | 0.0391 | 0.0412 | 0.0457 |
| Retail | 0.0561 | 0.0545 | 0.0506 |
| Finance | 0.0518 | 0.0548 | 0.0614 |
| Other | 0.0449 | 0.0489 | 0.0572 |

Table 2
Summary Statistics: Instruments

Table II displays a summary of the predictive power of the instrument variables, $Z_t = \{r_{m,t}, dy_t, ys_t, tb_t\}$, Where $r_{m,t}$ is the excess return on the CRSP value weighted index at time t, ys_t is the yield on the three-month Treasury bill in excess of the yield on the one month Treasury bill at time t, and tb_t is the return on a Treasury bill closest to one month to maturity at time t. The predictive power of the instruments is assessed by the linear projection

$$R_{i,t+1} = d_0 + dZ_t + u_{t+1}$$

The column labelled χ_4^2 presents Newey and West (1987a) Wald tests of the hypothesis

$$H_0 : d = 0$$

with p-values in parentheses. The statistics are computed using the Newey and West (1987b) heteroskedasticity and autocorrelation-consistent covariance matrix.

| Industry | July 1963-Dec 1995 | July 1963-Dec 2009 | Jan 1996-Dec 2009 |
|-----------|--------------------|--------------------|-------------------|
| | χ_4^2 | χ_4^2 | χ_4^2 |
| Food | 10.065 (0.039) | 11.201 (0.024) | 8.135 (0.086) |
| Mines | 19.469 (0.000) | 19.897 (0.001) | 5.948 (0.203) |
| Oil | 9.534 (0.049) | 2.793 (0.593) | 1.4175 (0.841) |
| Cloths | 24.287 (0.000) | 26.099 (0.000) | 7.4259 (0.115) |
| Durbl | 12.531 (0.013) | 22.171 (0.000) | 13.504 (0.009) |
| Chemicals | 9.280 (0.054) | 11.563 (0.021) | 5.612 (0.230) |
| Cnsum | 5.883 (0.208) | 7.541 (0.110) | 8.250 (0.082) |

Table 2- Continued

| Industry | July 1963-Dec 1995 | July 1963-Dec 2009 | Jan 1996-Dec 2009 |
|---------------------|--------------------|--------------------|-------------------|
| | χ_4^2 | χ_4^2 | χ_4^2 |
| Construction | 19.808 (0.000) | 15.861 (0.003) | 9.960 (0.041) |
| Steel | 5.455 (0.243) | 6.825 (0.145) | 5.540 (0.236) |
| Fabricated Metals | 21.204 (0.000) | 24.334 (0.000) | 12.796 (0.012) |
| Machinery | 24.410 (0.000) | 16.803 (0.002) | 6.714 (0.151) |
| Cars | 19.349 (0.000) | 35.647 (0.000) | 19.052 (0.000) |
| Transport Equipment | 23.349 (0.000) | 20.447 (0.000) | 10.882 (0.027) |
| Utilities | 13.879 (0.007) | 7.273 (0.122) | 5.292 (0.258) |
| Retail | 12.921 (0.011) | 12.315 (0.015) | 7.875 (0.096) |
| Finance | 11.767 (0.019) | 9.861 (0.043) | 7.440 (0.114) |
| Other | 15.206 (0.004) | 11.958 (0.018) | 12.507 (0.013) |

Table 2 is the summary statistics for the predictive power of the instrument variables in three sample periods. It is

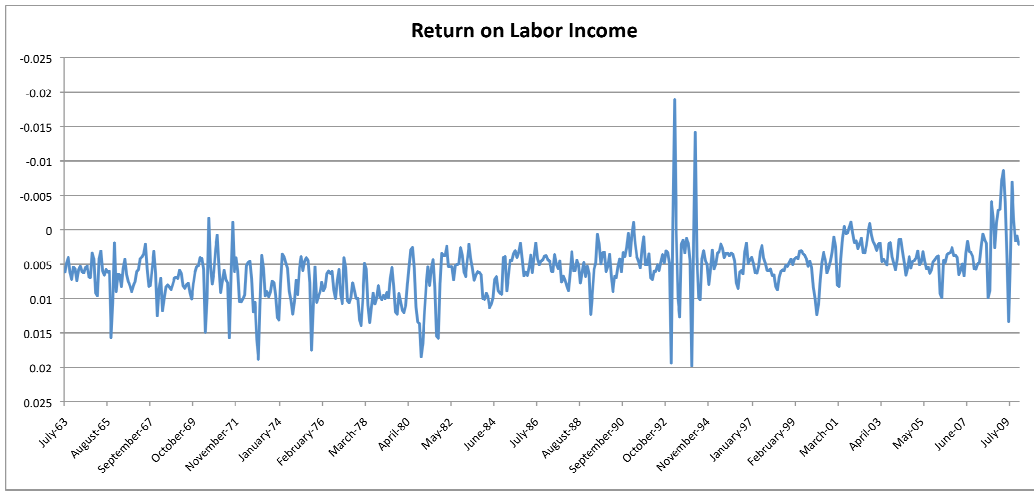
$$R_{i,t+1} = b'Z_t + u_{t+1}$$

where $R_{i,t+1}$ is the return of the 17 industry portfolios.

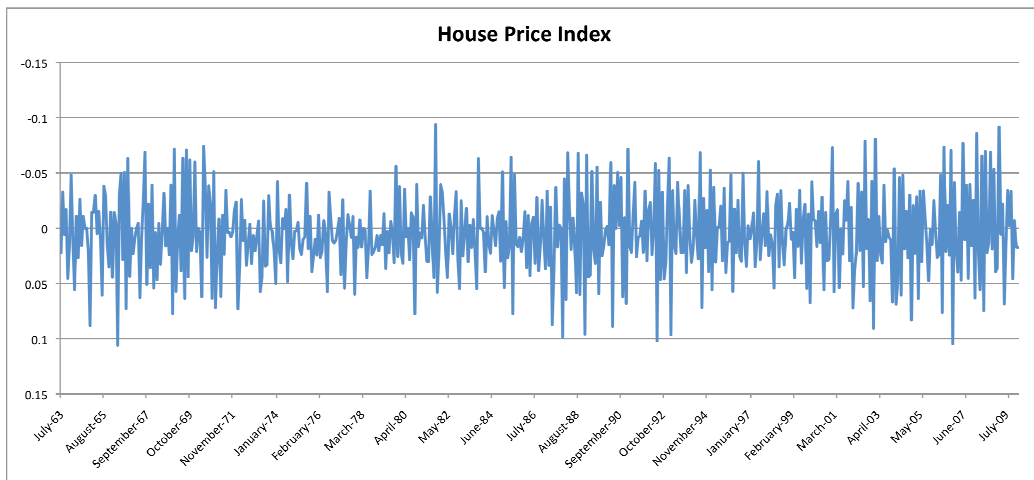
The results obtained from first sample period July 1963:December 1995 and the second sample period July 1963: December 2009 is consistent with those reported by

Dittmar (2002), which the information variables serve as good instruments for the payoffs. The P-value of Chi Square test in sample period January 1996:December 2009 is large, it might due to the small sample size.

Picture 1- represents the time series plot for the return on labor income in full sample period. The plot uses monthly data from July 1963 to Dec 2009.



Picture 2- represents the time series plot for change rates on housing price in full sample period from July 1963 to Dec 2009. The house price variations are roughly around 20%. The recent data from 1996 to 2009 shows after 2005 the house price started to drop until early 2009. This is the time when the US housing market collapse.



Chapter 5

EMPIRICAL ANALYSIS

In this section, I use different wealth proxies across three sample periods in making comparisons among these models and the sample periods. The sample periods are: Dittmar's period (from July 1963 to Dec 1995), the full sample (from July 1963 to Dec 2009), and the period after Dittmar (from Jan 1996 to Dec 2009). I run these sample periods and add in housing wealth risk factor to see whether the housing wealth factor has significant impact on the return to aggregate wealth. Then, I make comparison with the results in Dittmar (2002). I set up the tests on: asset pricing kernel with human capital only; asset pricing kernel with housing wealth only; asset pricing kernel with human capital and housing wealth included. In addition, I test Fama-French four-factor model by adding in momentum factor to Fama-French three-factor model. I will explain more details later. This study not only confirmed a part of Dittmar's report, it also furthered the knowledge on the factor that impact the model, such as us housing wealth contributes to aggregate wealth and momentum factor effects the test result of Fama-French model.

5.1 Comparison of different wealth proxies- sample period from Jul 1963 to Dec 1995

5.1.A Model Specification Tests Using Different Wealth Proxies

5.1.A.1 Specification Tests on Polynomial Pricing Kernels with Human Capital Excluded

Using return data on the 17 industry portfolios described earlier, I first examine the linear model specification, when the measure of aggregate wealth does not include human capital:

$$m_{t+1} = (Z_t \delta_0)^2 - (Z_t \delta_1)^2 R_{m,t+1} \quad (47)$$

Table 3
Specification Tests: Polynomial Pricing Kernels with Human Capital
Excluded

Table 3 presents results of GMM tests of the Euler equation condition,

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

Using the polynomial pricing kernels, m_{t+1} , the coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix $E[(1 + R_{t+1}) \otimes Z_t][(1 + R_{t+1}) \otimes Z_t]'$. The columns present the coefficients of the pricing kernel evaluated at the means of the instruments. The coefficients are modelled as

$$d_n = I_n (\delta'_n Z_t)^2 \quad I_n = \begin{cases} -1, n=1,3 \\ 1, n=2 \end{cases}$$

p-value for Wald tests of the joint significance of the coefficients are presented in parentheses. The final column presents the Hansen-Jagannathan distance measure. The set of returns used in estimation are those 17 industry-sorted portfolios augmented by the return on a one month Treasury bill.

| | $d(\bar{Z})_{0t}$ | $d(\bar{Z})_{1t}$ | $d(\bar{Z})_{2t}$ | $d(\bar{Z})_{3t}$ | Dist |
|--------------------|-------------------|-------------------|-------------------|-------------------|--------|
| Panel A: Linear | | | | | |
| Coefficient | 1.081 | -4.158 | | | 0.6472 |
| P-Value | (0.000) | (0.000) | | | |
| Panel B: Quadratic | | | | | |
| Coefficient | 1.026 | -4.686 | 28.644 | | 0.6406 |
| P-Value | (0.000) | (0.000) | (0.040) | | |
| Panel C: Cubic | | | | | |
| Coefficient | 1.019 | -2.330 | 13.376 | -13.619 | 0.6406 |
| P-Value | (0.000) | (0.002) | (0.044) | (0.284) | |

And I assume the proxy for the return on the wealth portfolio is the return on the value weighted industry portfolio (see Jagannathan and Wang (1996)),

$$R_{w,t+1} = \theta_0 + \theta_1 R_{m,t+1} \quad (48)$$

The results of specification tests are presented in Panel A of Table 3. As show in the table, Panel A: the linear term is statistically significant at the 5% level for this data set, suggesting that $R_{m,t+1}$ play a significant role in constructing a stochastic discount factor in this study. In the GMM test that uses the Hansen-Jagannathan weighting matrix, the estimated distance is 0.6472.

Second, I add in the quadratic term and use the 17-industry portfolio return as a proxy for the market return. This gives the following specification:

$$m_{t+1} = (Z_t \delta_0)^2 - (Z_t \delta_1)^2 R_{m,t+1} + (Z_t \delta_2)^2 R_{m,t+1}^2 \quad (49)$$

The results of specification tests are presented in Panel B of Table 3. As show in the table, Panel B: the quadratic term is statistically significant at the 5% level for this data set, suggesting that $R_{m,t+1}$ play a significant role in constructing a stochastic discount factor in this study. In the GMM test that uses the Hansen-Jagannathan weighting matrix, the estimated distance is 0.6406, indicating that distance has improvement from linear specification to a nonlinear specification. The quadratic pricing kernel reduces the distance measure from 0.6472 to 0.6406. That means the quadratic model specification reduce the pricing errors.

I now add in the cubic term and use the 17-industry portfolio return as a proxy for the market return. This gives the following specification:

$$m_{t+1} = (Z_t \delta_0)^2 - (Z_t \delta_1)^2 R_{m,t+1} + (Z_t \delta_2)^2 R_{m,t+1}^2 - (Z_t \delta_3)^2 R_{m,t+1}^3 \quad (50)$$

The results of specification tests are presented in Panel C of Table III. As show in the table, Panel C: The cubic term does not improve the performance of the pricing kernel. The p-value for the tests of significance of the coefficient of the cubic term is not significant (p-value 0.469). This result indicates that In the GMM test that uses the Hansen-Jagannathan weighting matrix, the estimated distance is 0.6406, indicating that distance does not have improvement from Quadratic to Cubic pricing kernels. These results are consistent with Dittmar (2002).

Table 4
Specification Tests: Polynomial Pricing Kernels with Human Capital
Included Only

Table 4 presents results of GMM tests of the Euler equation condition,

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

Using the polynomial pricing kernels, m_{t+1} , the coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix $E[(1 + R_{t+1}) \otimes Z_t][(1 + R_{t+1}) \otimes Z_t]'$. The columns present the coefficients of the pricing kernel evaluated at the means of the instruments. The coefficients are modelled as

$$d_n = I_n (\delta_n' Z_t)^2 \quad I_n = \begin{cases} -1, n=1,3 \\ 1, n=2 \end{cases}$$

p-value for Wald tests of the joint significance of the coefficients are presented in parentheses. The final column presents the Hansen-Jagannathan distance measure. The set of returns used in estimation are those 17 industry-sorted portfolios augmented by the return on a one month Treasury bill.

| | $d(\bar{Z})_{0t}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | Dist |
|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Panel A: Linear | | | | | | | | |
| Coefficient | 1.426 | -3.775 | -36.419 | | | | | 0.6187 |
| P-Value | (0.000) | (0.001) | (0.004) | | | | | |
| Panel B: Quadratic | | | | | | | | |
| Coefficient | 1.251 | -5.473 | -22.448 | 51.400 | 1396.395 | | | 0.5839 |
| P-Value | (0.000) | (0.001) | (0.038) | (0.022) | (0.051) | | | |
| Panel C: Cubic | | | | | | | | |
| Coefficient | 1.251 | -5.475 | -22.384 | 51.424 | 1393.272 | -0.582 | -1.750 | 0.5839 |
| P-Value | (0.000) | (0.005) | (0.055) | (0.026) | (0.086) | (0.315) | (0.469) | |

5.1.A.2 Specification Tests on Polynomial Pricing Kernels with Human Capital Included Only

Here, I analyze the impact of incorporating a measure of human capital in the return on aggregate wealth. And we assume the proxy for the return on the wealth portfolio is the return on the value weighted industry portfolio and labor income (see Dittmar (2002)),

$$R_{w,t+1} = \theta_0 + \theta_1 R_{m,t+1} + \theta_2 R_{l,t+1} \quad (51)$$

First, I examine the linear model specification, when the measure of aggregate wealth with human capital included:

$$m_{t+1} = (Z_t \delta_0)^2 - (Z_t \delta_1)^2 R_{m,t+1} - (Z_t \delta_1)^2 R_{l,t+1} \quad (52)$$

The results of specification tests are presented in Panel A of Table 4. As show in the table, Panel A: The distance measure of the linear pricing kernel falls to 0.6187 and linear kernel without human capital is 0.6472, approximately dropped 2.85 percent. This result indicates that the human capital improves the performance of the linear pricing kernel (conditional CAPM). These results are consistent with Jagannathan and Wang (1996), who use human capital test conditional CAPM model, and Dittmar's result.

Second, I add in the quadratic term and use the 17-industry portfolio return as a proxy for the market return and use return of labor income as a proxy for human capital. This gives the following specification:

$$m_{t+1} = (Z_t \delta_0)^2 - (Z_t \delta_1)^2 R_{m,t+1} - (Z_t \delta_1)^2 R_{l,t+1} + (Z_t \delta_2)^2 R_{m,t+1}^2 + (Z_t \delta_2)^2 R_{l,t+1}^2 \quad (53)$$

The results of specification tests are presented in Panel B of Table 4. As show in the table, Panel B: In the GMM test that uses the Hansen-Jagannathan weighting matrix, the estimated distance drops sharply to 0.5839, indicating that the quadratic specification of the pricing kernel has further improvement in the distance measure of 3.48 percent relative to the linear kernel with human capital. In addition, the pricing errors of nonlinear model specification are not significantly different from zero. Both quadratic term on $R_{m,t+1}$ and $R_{l,t+1}$ are significant in GMM test. Thus, the proxy of return on human capital and its higher order moment has a dramatic impact on the fit

of the pricing kernel. However, the coefficients are quite large in $d(\bar{Z})_{2l}$ term. It might be driven by the size of the higher orders of the return on labor income.

Last, I add in the cubic term and use the 17-industry portfolio return as a proxy for the market return and use return of labor income as a proxy for human capital. This gives the following specification:

$$\begin{aligned}
m_{t+1} = & (Z_t \delta_0)^2 - (Z_t \delta_1)^2 R_{m,t+1} - (Z_t \delta_1)^2 R_{l,t+1} \\
& + (Z_t \delta_2)^2 R_{m,t+1}^2 + (Z_t \delta_2)^2 R_{l,t+1}^2 \\
& - (Z_t \delta_2)^2 R_{m,t+1}^3 - (Z_t \delta_2)^2 R_{l,t+1}^3
\end{aligned} \tag{54}$$

The results of specification tests are presented in Panel C of Table 4. As show in the table, Panel C: In the GMM test that uses the Hansen-Jagannathan weighting matrix, the estimated distance is 0.5839, indicating that distance does not have improvement from Quadratic to Cubic pricing kernels with human capital. This result is not consistent with those obtained from Dittmar (2002), as his result shows that there is further improvement in distance measure from Quadratic to Cubic.

The results of Table 3 and 4 suggest that human capital is important proxy in estimating the pricing model. Model specifications tests show that the nonlinear model with human capital perform well in pricing the cross section of industry-sorted returns.

Table 5
Specification Tests: Polynomial Pricing Kernels with Housing Wealth
Included Only

Table 5 presents results of GMM tests of the Euler equation condition,

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

Using the polynomial pricing kernels, m_{t+1} , the coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix $E[(1 + R_{t+1}) \otimes Z_t][(1 + R_{t+1}) \otimes Z_t]'$. The columns present the coefficients of the pricing kernel evaluated at the means of the instruments. The coefficients are modelled as

$$d_n = I_n (\delta_n' Z_t)^2 \quad I_n = \begin{cases} -1, n=1,3 \\ 1, n=2 \end{cases}$$

p-value for Wald tests of the joint significance of the coefficients are presented in parentheses. The final column presents the Hansen-Jagannathan distance measure. The set of returns used in estimation are those 17 industry-sorted portfolios augmented by the return on a one month Treasury bill.

| | $d(\bar{Z})_{0t}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1h}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2h}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3h}$ | Dist |
|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Panel A: Linear | | | | | | | | |
| Coefficient | 1.104 | -4.420 | -2.134 | | | | | 0.6446 |
| P-Value | (0.000) | (0.001) | (0.027) | | | | | |
| Panel B: Quadratic | | | | | | | | |
| Coefficient | 0.909 | -5.133 | -1.443 | 31.630 | 94.409 | | | 0.6303 |
| P-Value | (0.000) | (0.000) | (0.041) | (0.036) | (0.014) | | | |
| Panel C: Cubic | | | | | | | | |
| Coefficient | 0.856 | -4.907 | -0.198 | 33.803 | 131.608 | 55.779 | -844.475 | 0.6269 |
| P-Value | (0.000) | (0.001) | (0.120) | (0.055) | (0.012) | (0.116) | (0.044) | |

5.1.A.3 Specification Tests on Polynomial Pricing Kernels with Housing Wealth Included Only

I now analyze the impact of incorporating a measure of housing wealth in the return on aggregate wealth. I assume that the proxy for the return on the wealth portfolio is the return on the value weighted industry portfolio, and housing wealth.

$$R_{W,t+1} = \theta_0 + \theta_1 R_{m,t+1} + \theta_2 R_{h,t+1} \quad (55)$$

First, I examine the linear model specification, when the measure of aggregate wealth with housing factor only:

$$m_{t+1} = (Z_t \delta_0)^2 - (Z_t \delta_1)^2 R_{m,t+1} - (Z_t \delta_1)^2 R_{h,t+1} \quad (56)$$

The results of specification tests are presented in Panel A of Table 5. As show in the table, Panel A: The distance measures of the model specification test has improved from those obtained in Table 3. The coefficient corresponding to the growth rate of housing wealth is significant (p-value is 0.027) in linear model specification. However, the distance measure of linear model specification is larger than the nonlinear model specification tests below.

I add in the quadratic term. This gives the following specification:

$$\begin{aligned} m_{t+1} = & (Z_t \delta_0)^2 - (Z_t \delta_1)^2 R_{m,t+1} - (Z_t \delta_1)^2 R_{h,t+1} \\ & + (Z_t \delta_2)^2 R_{m,t+1}^2 + (Z_t \delta_2)^2 R_{h,t+1}^2 \end{aligned} \quad (57)$$

The results of specification tests are presented in Panel B of Table 5. As show in the table, Panel B: The quadratic pricing kernel reduces the distance measure from 0.6446 to 0.6303 relative to linear pricing model. The quadratic term is statistically significant at the 5% level. Both results indicate that the quadratic return on housing wealth contribute significantly to the fit of the pricing model.

By adding in the cubic term. This gives the following specification:

$$\begin{aligned} m_{t+1} = & (Z_t \delta_0)^2 - (Z_t \delta_1)^2 R_{m,t+1} - (Z_t \delta_1)^2 R_{h,t+1} \\ & + (Z_t \delta_2)^2 R_{m,t+1}^2 + (Z_t \delta_2)^2 R_{h,t+1}^2 \\ & - (Z_t \delta_2)^2 R_{m,t+1}^3 - (Z_t \delta_2)^2 R_{h,t+1}^3 \end{aligned} \quad (58)$$

The results of specification tests are presented in Panel C of Table 5. As show in the table, Panel C: the performance of the pricing kernel has further improvement by the

cubic term, as the distance measure falls to 0.6269. Moreover, the housing wealth does improve the fitting of the pricing model and cubic return on housing wealth contribute significantly to the improvement in distance measure (the p-value of d_{3h} term is 0.044). The coefficients corresponding to the return rate $R_{h,t+1}^2$ and $R_{h,t+1}^3$ are significant in model specification test. These results suggest that the housing wealth factor plays a significant role in this study. It has a significant impact on the fit of the pricing kernel.

In Table 5, all three pricing kernels improve significantly relative to the case which housing wealth is not included in the measure of aggregate wealth. In table 4 and 5, we analyze human capital and housing wealth included in the measure of aggregate wealth individually with different model specification tests. The outcomes of the model specification tests suggest both proxies are important. Next, we analyze the impact of both return on human capital and housing wealth in the measure of aggregate wealth.

Table 6**Specification Tests: Polynomial Pricing Kernels with Human Capital and Housing Wealth Included**

| | $d(\bar{Z})_{0l}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{1h}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{2h}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | $d(\bar{Z})_{3h}$ | Dist |
|--------------------|-------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|--------|
| Panel A: Linear | | | | | | | | | | | |
| Coefficient | 1.460 | -3.989 | -38.819 | -2.081 | | | | | | | 0.6175 |
| P-Value | (0.000) | (0.001) | (0.005) | (0.031) | | | | | | | |
| Panel B: Quadratic | | | | | | | | | | | |
| Coefficient | 1.113 | -5.958 | -21.262 | -2.775 | 45.327 | 1183.006 | 108.094 | | | | 0.5705 |
| P-Value | (0.000) | (0.001) | (0.039) | (0.038) | (0.034) | (0.074) | (0.012) | | | | |
| Panel C: Cubic | | | | | | | | | | | |
| Coefficient | 1.116 | -5.692 | -24.879 | -2.065 | 46.725 | 1006.232 | 118.488 | -43.612 | -48.825 | -187.904 | 0.5685 |
| P-Value | (0.000) | (0.001) | (0.008) | (0.049) | (0.014) | (0.035) | (0.008) | (0.075) | (0.000) | (0.105) | |

5.1.A.4 Specification Tests on Polynomial Pricing Kernels with Human Capital and Housing Wealth Included

Now, I consider the main model developed in this paper: first, I examine linear specification,

$$m_{t+1} = (Z_t \delta_0)^2 - (Z_t \delta_1)^2 R_{m,t+1} - (Z_t \delta_1)^2 R_{l,t+1} - (Z_t \delta_1)^2 R_{h,t+1} \quad (59)$$

And I assume the proxy for the return on the wealth portfolio is sum of the return on the value weighted industry portfolio, return on human capital and return on housing wealth. That is,

$$R_{W,t+1} = \theta_0 + \theta_1 R_{m,t+1} + \theta_2 R_{l,t+1} + \theta_3 R_{h,t+1} \quad (60)$$

The results of specification tests are presented in Table 6. As show in the table, Panel A: The estimated coefficients of $R_{m,t+1}$ (p-value 0.1%), $R_{l,t+1}$ (p-value 0.5%), $R_{h,t+1}$ (p-value 3.1%), is at significantly level for this data set.

Next, I add in the quadratic term. This gives the following specification:

$$m_{t+1} = (Z_t \delta_0)^2 - (Z_t \delta_1)^2 R_{m,t+1} - (Z_t \delta_1)^2 R_{l,t+1} - (Z_t \delta_1)^2 R_{h,t+1} \\ + (Z_t \delta_2)^2 R_{m,t+1}^2 + (Z_t \delta_2)^2 R_{l,t+1}^2 + (Z_t \delta_1)^2 R_{h,t+1}^2 \quad (61)$$

The results of specification tests are presented in Panel B of Table 6. As show in the table, Panel B: The GMM test that uses the Hansen-Jagannathan weighting matrix, the estimated distance is 0.5705. It has further improvement compare to those results that estimated in the previous table III, IV and V. The quadratic term of housing wealth factor perform significantly to the fit of the pricing kernel, with p-value 0.012.

We now add in the cubic term. This gives the following specification:

$$m_{t+1} = (Z_t \delta_0)^2 - (Z_t \delta_1)^2 R_{m,t+1} - (Z_t \delta_1)^2 R_{l,t+1} - (Z_t \delta_1)^2 R_{h,t+1} \\ + (Z_t \delta_2)^2 R_{m,t+1}^2 + (Z_t \delta_2)^2 R_{l,t+1}^2 + (Z_t \delta_1)^2 R_{h,t+1}^2 \\ - (Z_t \delta_2)^2 R_{m,t+1}^3 - (Z_t \delta_2)^2 R_{l,t+1}^3 - (Z_t \delta_1)^2 R_{h,t+1}^3 \quad (62)$$

The results of specification tests are presented in Panel C of Table 6. As show in the table, Panel C: The results suggesting that incorporating the quadratic return on wealth term is statistically significant at 5% level, as indicated by the test of the significance of the d_2 terms (p-values 0.014, 0.035 and 0.008). In the GMM test that uses the Hansen-Jagannathan weighting matrix, the cubic pricing kernel reduces the

distance measure from 0.5705 to 0.5685. These results suggest that nonlinear function of housing wealth risk factor has impact on the fit of the pricing kernel. Furthermore, the performance of the nonlinear asset-pricing kernel is further enhanced by higher order of return on human capital and higher order of return on housing wealth. Compare to Dittmar's model, our model provides a better cross sectional fit for industry-sorted portfolio by adding in housing wealth factor.

5.1.B Multifactor Alternative

In this section I investigate the ability of the Fama-French factors to price the cross section of equity returns. I compare Multifactor Fama-French model and Fama-French four-factor model (includes MRP, SMB, HML and momentum factor) with polynomial pricing kernel incorporating with human capital (see Table 8), housing capital (see Table 9) and human, housing capital (see Table 10).

The momentum factor is the empirically observed tendency for rising asset prices further. This strategy assume that the past performance effect the futures performance such as the stocks with strong performance in the past will continue outperform with poor past performance in the next period. The investors who use this trading strategy will buy the stocks which are perform good in the past and sell the stocks with poor performance in the past. Carhart (1997) includes a momentum factor constructed by the month return difference between the returns on the high and low prior return portfolios, to capture the cross-sectional return patterns. Case, Cotter and Gabriel (2010) examine momentum factor in the housing asset pricing model as momentum trading has been found to have a positive influence on future real estate investment return. Here, I add momentum factor in this multifactor-pricing kernel as this factor has significant explanatory power for equity returns.

Table 7**Specification Tests: Fama-French Pricing Kernel with Human capital Included**

Table 7 presents results of GMM tests of the Euler equation condition,

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

m_{t+1} implied by the Fama and French (1993) three-factor model. The coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix. P-value for Wald tests of the joint significance of the coefficients are presented in parentheses. The final column presents the Hansen-Jagannathan distance measure. The set of returns used in estimation are those 17 industry-sorted portfolios covering the period July 1963, through December 1995, augmented by the return on a one month Treasury bill.

| | $d(\bar{Z})_{or}$ | $d(\bar{Z})_{mrp}$ | $d(\bar{Z})_{smb}$ | $d(\bar{Z})_{hml}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | Dist |
|---|-------------------|--------------------|--------------------|--------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Panel A: Fama-French Factors Only | | | | | | | | | | | |
| Coefficient | 1.185 | -4.455 | -1.902 | -2.027 | | | | | | | 0.5680 |
| P-Value | (0.000) | (0.000) | (0.029) | (0.024) | | | | | | | |
| Panel B: Quadratic Augmented by Fama-French Factors | | | | | | | | | | | |
| Coefficient | 1.397 | | -1.173 | -0.816 | -5.673 | -16.016 | 20.273 | 552.593 | | | 0.5187 |
| P-Value | (0.000) | | (0.055) | (0.065) | (0.000) | (0.019) | (0.038) | (0.054) | | | |
| Panel C: Cubic Augmented by Fama-French Factors | | | | | | | | | | | |
| Coefficient | 1.402 | | 1.254 | -1.075 | -4.817 | -16.745 | 19.292 | 550.187 | -146.687 | -177.846 | 0.5159 |
| P-Value | (0.000) | | (0.038) | (0.043) | (0.005) | (0.065) | (0.041) | (0.010) | (0.046) | (0.000) | |

5.1.B.1 Specification Tests on Fama-French Pricing Kernel with Human capital Included

To do this, I begin with Fama-French three-factor model.

$$m_{t+1}^{FF} = (Z_t \delta_0)^2 + (Z_t \delta_{MRP}) R_{MRP,t+1} + (Z_t \delta_{SMB}) R_{SMB,t+1} + (Z_t \delta_{HML}) R_{HML,t+1} \quad (63)$$

The results for the estimation of the Fama-French model showed in Panel A of Table 7. The results show that three Fama-French factors are statistically significant. However, the distance measure of the Fama-French model is higher than quadratic pricing kernel and cubic pricing kernel. Thus, the results suggest that the nonlinear model perform better than Fama-French three-factor model in explaining the cross section of industry returns.

To further investigate the ability of the Fama-French factors. We consider the following nonlinear models,

$$\begin{aligned} m_{t+1}^{FF} = & (Z_t \delta_0)^2 + (Z_t \delta_{SMB}) R_{SMB,t+1} + (Z_t \delta_{HML}) R_{HML,t+1} \\ & - (Z_t \delta_1)^2 R_{m,t+1} - (Z_t \delta_1)^2 R_{l,t+1} \\ & + (Z_t \delta_2)^2 R_{m,t+1}^2 + (Z_t \delta_2)^2 R_{l,t+1}^2 \end{aligned} \quad (64)$$

$$\begin{aligned} m_{t+1}^{FF} = & (Z_t \delta_0)^2 + (Z_t \delta_{SMB}) R_{SMB,t+1} + (Z_t \delta_{HML}) R_{HML,t+1} \\ & - (Z_t \delta_1)^2 R_{m,t+1} - (Z_t \delta_1)^2 R_{l,t+1} \\ & + (Z_t \delta_2)^2 R_{m,t+1}^2 + (Z_t \delta_2)^2 R_{l,t+1}^2 \\ & - (Z_t \delta_2)^2 R_{m,t+1}^3 - (Z_t \delta_2)^2 R_{l,t+1}^3 \end{aligned} \quad (65)$$

The results for the estimation of the polynomial model augmented SMB, HML and human capital are given in Table 7 Panel B and Panel C. SMB is the return on the portfolio of long small stock and short big stock, while HML is the return on the portfolio of long value stock and short growth stock. The value of coefficients d_n , $n = 1, 2, 3$ corresponds to the n^{th} order of time series regression of asset i on market return, SMB, HML and human capital. In Panel B: Quadratic Augmented by Fama-French Factors, distance measure falls from 0.5680 to 0.5187, indicating the quadratic pricing kernel perform better than Fama-French model. However, neither the quadratic term in $R_{HML,t+1}$ and $R_{SMB,t+1}$ are significantly different than zero. In contrast, when Fama-French factors are included in the cubic pricing kernel (showed

in Panel C) the SMB factor, HML factor and human capital are significant. Cubic pricing kernel reduces the distance measure from 0.5187 to 0.5159. In addition, once this model added in Fama-French HML and SMB factors, the coefficient on return to human capital become significant relative to the models in Table 4.

Table 8**Specification Tests: Fama-French Pricing Kernel with Momentum Factor and Human Capital Included**

Table 8 presents results of GMM tests of the Euler equation condition,

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

m_{t+1} implied by the Fama and French (1993) four-factor model. The coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix. P-value for Wald tests of the joint significance of the coefficients are presented in parentheses. The final column presents the Hansen-Jagannathan distance measure. The set of returns used in estimation are those 17 industry-sorted portfolios covering the period July 1963, through December 1995, augmented by the return on a one month Treasury bill.

| | $d(\bar{Z})_{0t}$ | $d(\bar{Z})_{mrp}$ | $d(\bar{Z})_{smb}$ | $d(\bar{Z})_{hml}$ | $d(\bar{Z})_{mom}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | Dist |
|--|-------------------|--------------------|--------------------|--------------------|--------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Panel A: Fama-French with Momentum factor | | | | | | | | | | | | |
| Coefficient | 1.298 | -5.110 | -2.389 | -5.236 | -10.377 | | | | | | | 0.5299 |
| P-Value | (0.000) | (0.028) | (0.001) | (0.004) | (0.000) | | | | | | | |
| Panel B: Quadratic Augmented by Fama-French Four Factors | | | | | | | | | | | | |
| Coefficient | 1.155 | | -0.481 | -4.658 | -11.296 | -5.640 | -16.491 | 3.750 | 882.118 | | | 0.4720 |
| P-Value | (0.000) | | (0.106) | (0.011) | (0.000) | (0.001) | (0.050) | (0.116) | (0.085) | | | |
| Panel C: Cubic Augmented by Fama-French Four Factors | | | | | | | | | | | | |
| Coefficient | 1.542 | | 0.105 | -5.250 | -12.031 | -5.629 | -17.051 | 3.948 | 1048.964 | -7.534 | 18.269 | 0.4709 |
| P-Value | (0.000) | | (0.166) | (0.004) | (0.000) | (0.001) | (0.019) | (0.107) | (0.029) | (0.181) | (0.000) | |

5.1.B.2 Specification Tests on Fama-French Pricing Kernel with Momentum Factor and Human Capital Included

In table 8 Panel A: Fama-French with Momentum factor. The model as

$$m_{t+1}^{FF} = (Z_t \delta_0)^2 + (Z_t \delta_{MRP}) R_{MRP,t+1} + (Z_t \delta_{SMB}) R_{SMB,t+1} + (Z_t \delta_{HML}) R_{HML,t+1} + (Z_t \delta_{MOM}) R_{Mom,t+1} \quad (66)$$

The results show that the Fama-French factors continuously significant. The p-value for the coefficient of momentum factor is 0.000, indicating $R_{Mom,t+1}$ is significant determinant of the cross section of returns. The distance measure of the Fama-French four-factor model falls to 0.5299 relative to the Fama-French three-factor model. However, it is still higher than that of either quadratic pricing kernel and cubic pricing kernel.

In Panel B: Quadratic Augmented by Fama-French Four Factors. We test the model,

$$m_{t+1}^{FF} = (Z_t \delta_0)^2 + (Z_t \delta_{SMB}) R_{SMB,t+1} + (Z_t \delta_{HML}) R_{HML,t+1} + (Z_t \delta_{MOM}) R_{Mom,t+1} \\ - (Z_t \delta_1)^2 R_{m,t+1} - (Z_t \delta_1)^2 R_{l,t+1} \\ + (Z_t \delta_2)^2 R_{m,t+1}^2 + (Z_t \delta_2)^2 R_{l,t+1}^2 \quad (67)$$

In the GMM test that uses the Hansen-Jagannathan weighting matrix, the estimated distance drops sharply to 0.4720. Once the momentum factor is added, the coefficient on equity return becomes insignificant in either quadratic term or cubic term.

In Panel C: Cubic Augmented by Fama-French Four Factors

$$m_{t+1}^{FF} = (Z_t \delta_0)^2 + (Z_t \delta_{SMB}) R_{SMB,t+1} + (Z_t \delta_{HML}) R_{HML,t+1} + (Z_t \delta_{MOM}) R_{Mom,t+1} \\ - (Z_t \delta_1)^2 R_{m,t+1} - (Z_t \delta_1)^2 R_{l,t+1} \\ + (Z_t \delta_2)^2 R_{m,t+1}^2 + (Z_t \delta_2)^2 R_{l,t+1}^2 \\ - (Z_t \delta_2)^2 R_{m,t+1}^3 - (Z_t \delta_2)^2 R_{l,t+1}^3 \quad (68)$$

The momentum factor is continuously significant with p-value 0.000. However, the coefficient on equity return becomes insignificant in cubic model with Fama-French HML and SMB factors (see Table 4). Overall results confirmed the findings in Carhart (1997) and Case, Cotter and Gabriel (2010).

Table 9**Specification Tests: Fama-French Pricing Kernel with Momentum Factor and Housing Wealth Included**

Table 9 presents results of GMM tests of the Euler equation condition,

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

m_{t+1} implied by the Fama and French (1993) three-factor model. The coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix. P-value for Wald tests of the joint significance of the coefficients are presented in parentheses. The final column presents the Hansen-Jagannathan distance measure. The set of returns used in estimation are those 17 industry-sorted portfolios covering the period July 1963, through December 1995, augmented by the return on a one month Treasury bill.

| | $d(\bar{Z})_{0t}$ | $d(\bar{Z})_{smb}$ | $d(\bar{Z})_{hml}$ | $d(\bar{Z})_{mom}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1h}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2h}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3h}$ | Dist |
|---|-------------------|--------------------|--------------------|--------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Panel A: Quadratic Augmented by Fama-French Factors | | | | | | | | | | | |
| Coefficient | 1.288 | -1.772 | -4.112 | -9.204 | -4.833 | -0.528 | 6.442 | 31.549 | | | 0.4868 |
| P-Value | (0.000) | (0.004) | (0.100) | (0.024) | (0.007) | (0.019) | (0.406) | (0.146) | | | |
| Panel B : Cubic Augmented by Fama-French Factors | | | | | | | | | | | |
| Coefficient | 1.212 | -2.610 | -6.341 | -10.625 | -5.992 | -0.326 | 20.578 | 120.940 | -381.671 | -1127.912 | 0.4142 |
| P-Value | (0.000) | (0.042) | (0.010) | (0.003) | (0.012) | (0.098) | (0.089) | (0.015) | (0.045) | (0.024) | |

5.1.B.3 Specification Tests on Fama-French Pricing Kernel with Momentum Factor and Housing Wealth Included

Here I study the performance of Augmented Fama-French Model including housing as a risky asset class. The cross-sectional implications of the models are:

$$\begin{aligned}
 m_{t+1}^{FF} = & (Z_t \delta_0)^2 + (Z_t \delta_{SMB}) R_{SMB,t+1} + (Z_t \delta_{HML}) R_{HML,t+1} + (Z_t \delta_{MOM}) R_{Mom,t+1} \\
 & - (Z_t \delta_1)^2 R_{m,t+1} - (Z_t \delta_1)^2 R_{h,t+1} \\
 & + (Z_t \delta_2)^2 R_{m,t+1}^2 + (Z_t \delta_2)^2 R_{h,t+1}^2 \quad (69)
 \end{aligned}$$

$$\begin{aligned}
 m_{t+1}^{FF} = & (Z_t \delta_0)^2 + (Z_t \delta_{SMB}) R_{SMB,t+1} + (Z_t \delta_{HML}) R_{HML,t+1} + (Z_t \delta_{MOM}) R_{Mom,t+1} \\
 & - (Z_t \delta_1)^2 R_{m,t+1} - (Z_t \delta_1)^2 R_{h,t+1} \\
 & + (Z_t \delta_2)^2 R_{m,t+1}^2 + (Z_t \delta_2)^2 R_{h,t+1}^2 \\
 & - (Z_t \delta_2)^2 R_{m,t+1}^3 - (Z_t \delta_2)^2 R_{h,t+1}^3 \quad (70)
 \end{aligned}$$

The panel A of Table 9 shows the estimates for model (69). The quadratic term of return to equity and housing wealth are insignificant while the model add in Fama-French SMB and HML factors. In contrast, the cubic terms become significant (see Panel B of Table 9).

Table 10
Specification Tests: Fama-French Pricing Kernel with Momentum Factor and Human Capital and Housing
Wealth Included

Table 10 presents results of GMM tests of the Euler equation condition,

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

m_{t+1} implied by the Fama and French (1993) three-factor model. The coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix. P-value for Wald tests of the joint significance of the coefficients are presented in parentheses. The final column presents the Hansen-Jagannathan distance measure. The set of returns used in estimation are those 17 industry-sorted portfolios covering the period July 1963, through December 1995, augmented by the return on a one month Treasury bill.

| | $d(\bar{Z})_{0r}$ | $d(\bar{Z})_{smb}$ | $d(\bar{Z})_{hml}$ | $d(\bar{Z})_{mom}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{1h}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{2h}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | $d(\bar{Z})_{3h}$ | Dist |
|---|-------------------|--------------------|--------------------|--------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|--------|
| Panel A: Quadratic Augmented by Fama-French Factors | | | | | | | | | | | | | | |
| Coefficient | 1.361 | -1.499 | -5.777 | -14.199 | -7.094 | -6.133 | -0.130 | 0.558 | 494.342 | 89.997 | | | | 0.3998 |
| P-Value | (0.000) | (0.068) | (0.013) | (0.000) | (0.001) | (0.086) | (0.102) | (0.137) | (0.089) | (0.015) | | | | |
| Panel B : Cubic Augmented by Fama-French Factors | | | | | | | | | | | | | | |
| Coefficient | 1.380 | -0.441 | 5.459 | -14.714 | -6.131 | -12.029 | -0.308 | 0.382 | 1197.866 | 146.969 | -590.208 | -30842.032 | -1058.834 | 0.3667 |
| P-Value | (0.000) | (0.117) | (0.010) | (0.000) | (0.002) | (0.034) | (0.071) | (0.127) | (0.020) | (0.007) | (0.010) | (0.000) | (0.012) | |

5.1.B.4 Specification Tests: Fama-French Pricing Kernel with Momentum Factor and Human Capital and Housing Wealth Included

Now I study the performance of Augmented Fama-French Model including human capital and housing as two risky asset classes. The cross-sectional implications of the models are:

$$\begin{aligned}
 m_{t+1}^{FF} = & (Z_t \delta_0)^2 + (Z_t \delta_{SMB}) R_{SMB,t+1} + (Z_t \delta_{HML}) R_{HML,t+1} + (Z_t \delta_{MOM}) R_{Mom,t+1} \\
 & - (Z_t \delta_1)^2 R_{m,t+1} - (Z_t \delta_1)^2 R_{l,t+1} - (Z_t \delta_1)^2 R_{h,t+1} \\
 & + (Z_t \delta_2)^2 R_{m,t+1}^2 + (Z_t \delta_2)^2 R_{l,t+1}^2 + (Z_t \delta_1)^2 R_{h,t+1}^2 \quad (71)
 \end{aligned}$$

$$\begin{aligned}
 m_{t+1}^{FF} = & (Z_t \delta_0)^2 + (Z_t \delta_{SMB}) R_{SMB,t+1} + (Z_t \delta_{HML}) R_{HML,t+1} + (Z_t \delta_{MOM}) R_{Mom,t+1} \\
 & - (Z_t \delta_1)^2 R_{m,t+1} - (Z_t \delta_1)^2 R_{l,t+1} - (Z_t \delta_1)^2 R_{h,t+1} \\
 & + (Z_t \delta_2)^2 R_{m,t+1}^2 + (Z_t \delta_2)^2 R_{l,t+1}^2 + (Z_t \delta_1)^2 R_{h,t+1}^2 \\
 & - (Z_t \delta_2)^2 R_{m,t+1}^3 - (Z_t \delta_2)^2 R_{l,t+1}^3 - (Z_t \delta_1)^2 R_{h,t+1}^3 \quad (72)
 \end{aligned}$$

The empirical results are given in Table 10. The panel A of Table 10: Quadratic Augmented by Fama-French Factors with human capital and housing wealth, gives the estimates for model (71). Comparing with previous model specification tests, the distance falls sharply to 0.3998 in quadratic model and falls to 0.3667 in cubic model. The return on human capital and housing wealth are statistically significant in the cubic term. The coefficients of $R_{m,t+1}$, $R_{l,t+1}$ and $R_{h,t+1}$ in d_3 term are playing a significant role in cross section of equity returns. Thus, the estimation results suggest that there is a strong relation between housing wealth returns and market risk, confirm the results in Case, Cotter and Gabriel (2010). In addition, it is interesting to note that while the model without human capital, the SML factor become significant (see Table 9); while the model with human capital, the SML factor become insignificant.

5.1.C Summary of Distance Measure

In this section I apply the least squares measures. The norm of a random variable can be decomposed into a mean component and a standard deviation component via the formula

$$\|\tilde{p}\| = [E(p)]^2 + [std(p)^2]^{1/2}$$

Where $\|\tilde{p}\|$ is the measurement for distance, it is the specification error. Then, I use optimization problem to solve \tilde{p} (see Hansen and Jagannathan (1997))

$$\delta^2 = \max E[y^2 - (y - \lambda'x)^2 - 2\lambda'q].$$

The first order conditions for this problem are

$$E[x(y - \tilde{\lambda}'x) - q] = 0,$$

Next I need to find the vector $\tilde{\lambda}$ such that $y - \tilde{\lambda}'x$ is an admissible stochastic discount factor. The vector λ we can get from

$$\tilde{\lambda} = (Exx')^{-1}E(xy - q),$$

As the proposition 2.1 stated in Hansen and Jagannathan (1997), under assumption that P is a closed linear subspace of L^2 and the function π is continuous and linear on P , and there exists a payoff $p \in P$ such that $\pi(p) = 1$. The random variable \tilde{p} represents the approximation-error is given by

$$\tilde{p} = \tilde{\lambda}'x$$

Where $\tilde{\lambda}'x$ is the ‘pricing factor’, is the smallest adjustment in a least squares sense required to make $y - \tilde{\lambda}'x$ an admissible stochastic discount factor. The measurement for misspecification model is the norm of this random variable. It is

$$p = [(Exy - Eq)'(Exx')^{-1}(Exy - Eq)]^{1/2}$$

As explained in Dittmar (2002) Hansen and Jagannathan distance captures the average and the variability of a proxy pricing kernel’s pricing errors. And most of the distance measure results from $Std(p)$. The results show in Table VII presents estimates of $Mean(p)$ and $Std(p)$. The cubic pricing kernel with human capital and housing wealth has the lowest value for $Std(p)$, which means this pricing kernel with a small distance measure requires the least adjustment to be admissible. Quadratic

Table 11**Decomposition of Distance Measure**

Table 11 presents a decomposition of the Hansen-Jagannathan distance measure, $\|p\| = \sqrt{E[p]^2 + Var[p]}$. The variable p represents the adjustment to the model the pricing kernel needed to make it admissible. The column labelled “ $Mean(p)$ ” represents the average of the estimated p , the column labelled “ $Std(p)$ ” represents its standard deviation, and the column labelled “Distance” represents the Jagannathan distance. The row labelled “Linear: No HC, HW”, “Quadratic: No HC, HW”, and “Cubic: No HC, HW” represents the decomposition for the polynomial pricing kernels, omitting human capital and housing wealth. The row labelled “Linear: HC Only”, “Quadratic: HC Only”, and “Cubic: HC Only” represents the decomposition for the polynomial pricing kernels, including human capital only and housing wealth excluded. The row labelled “Linear: HW Only”, “Quadratic: HW Only”, and “Cubic: HW Only” represents the decomposition for the polynomial pricing kernels, including housing wealth only. The row labelled “Linear: HC+HW”, “Quadratic: HC+HW”, and “Cubic: HC+HW” represents the decomposition for the polynomial pricing kernels, including both human capital and housing wealth.

| Model | $Mean(p)$ | $Std(p)$ | Distance |
|----------------------|-----------|----------|----------|
| Linear: No HC, HW | 0.0003 | 0.6472 | 0.6472 |
| Quadratic: No HC, HW | 0.0003 | 0.6406 | 0.6406 |
| Cubic: No HC, HW | 0.0003 | 0.6406 | 0.6406 |
| Linear: HC Only | 0.0004 | 0.6187 | 0.6187 |
| Quadratic: HC Only | 0.0005 | 0.5839 | 0.5839 |
| Cubic: HC Only | 0.0005 | 0.5839 | 0.5839 |
| Linear: HW Only | 0.0004 | 0.6449 | 0.6449 |
| Quadratic: HW Only | 0.0005 | 0.6303 | 0.6303 |
| Cubic: HW Only | 0.0006 | 0.6269 | 0.6269 |
| Linear: HC + HW | 0.0005 | 0.6175 | 0.6175 |
| Quadratic: HC + HW | 0.0004 | 0.5705 | 0.5705 |
| Cubic: HC + HW | 0.0004 | 0.5685 | 0.5685 |

Pricing kernel with human capital and housing wealth is the second smallest distance measure.

5.1.D Other Model Specification Tests

In this section I analyse power utility pricing kernel with and without human capital included. Also compare these results with Dittmar's.

The Table 12 panel A: specification tests on power utility pricing kernel without human capital, takes the form

$$m_{t+1} = a_0(1 + R_{w,t+1})^{-a_1} \quad (73)$$

$$R_{w,t+1} = R_{m,t+1} \quad (74)$$

The Table 12 panel B: specification tests on power utility pricing kernel with human capital, takes the form

$$m_{t+1} = a_0(1 + R_{w,t+1})^{-a_1} \quad (75)$$

$$R_{w,t+1} = a_2 R_{m,t+1} + (1 - a_2) R_{l,t+1} \quad (76)$$

Results of this estimation are represented in Table 12. Human capital does not improve the performance of power utility pricing kernel. As show in the table, Hansen and Jagannathan distance measure is 0.6896 with human capital excluded and is 0.6873 with human capital included. And both form of perform worse than linear pricing kernel. It is consistent with Dittmar's results.

Table 12**Specification Tests: Power Utility Pricing Kernel**

Table 12 presents results of GMM estimation of the Euler equation restriction

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

m_{t+1} implied by power utility. The coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix. P-value for Wald tests of the joint significance of the coefficients are presented in parentheses. The final column presents the Hansen-Jagannathan distance measure. The set of returns used in estimation are those 17 industry-sorted portfolios covering the period July 1963, through December 1995, augmented by the return on a one month Treasury bill.

| | a0 | a1 | a2 | Dist |
|---------------------------------|-------|---------|-------|--------|
| Panel A: Human Capital Excluded | | | | |
| Coefficient: | 1.008 | -1.747 | | 0.6896 |
| Panel B: Human Capital Included | | | | |
| Coefficient: | 1.144 | -20.964 | 0.102 | 0.6873 |

Table 13
Specification Tests: Polynomial Pricing Kernels with Global
Restrictions and Human Capital Included

Table 13 presents results of GMM tests of the Euler equation condition,

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

Using the polynomial pricing kernels, m_{t+1} , the coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix $E[(1 + R_{t+1}) \otimes Z_t][(1 + R_{t+1}) \otimes Z_t]'$. The columns present the coefficients of the pricing kernel evaluated at the means of the instruments. The coefficients are modelled as

$$d_n = I_n (\delta_n' Z_t)^2 \quad I_n = \begin{cases} -1, n=1,3 \\ 1, n=2 \end{cases}$$

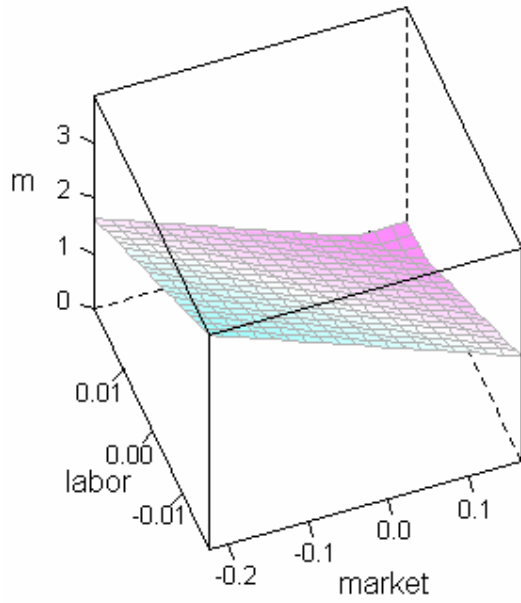
In addition to constraining the signs of the coefficients, the following constraints are placed on the pricing kernel.

$$m_{t+1} \geq 0 \quad m'_{t+1} \leq 0$$

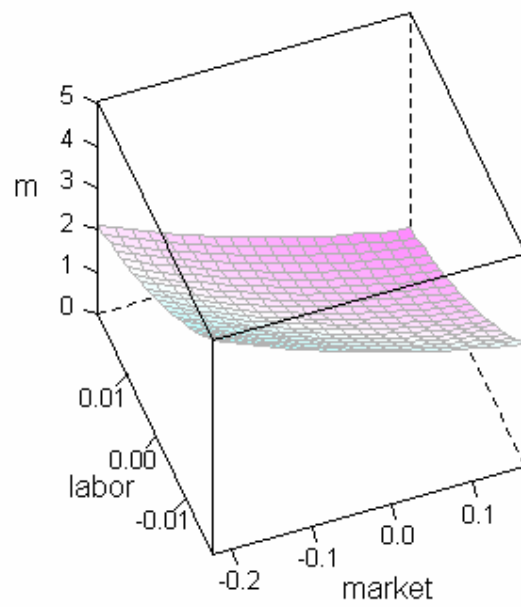
p-value for Wald tests of the joint significance of the coefficients are presented in parentheses. The final column presents the Hansen-Jagannathan distance measure. The set of returns used in estimation are those 17 industry-sorted portfolios covering the period July 1963, through December 1995, augmented by the return on a one month Treasury bill.

| | $d(\bar{Z})_{0r}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | Dist |
|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Panel A: Linear | | | | | | | | |
| Coefficient | 1.641 | -5.004 | -57.361 | | | | | 0.6003 |
| P-Value | (0.000) | (0.000) | (0.005) | | | | | |
| Panel B: Quadratic | | | | | | | | |
| Coefficient | 1.781 | -5.134 | -77.170 | 6.300 | 983.341 | | | 0.6003 |
| P-Value | (0.000) | (0.001) | (0.011) | (0.032) | (0.013) | | | |
| Panel C: Cubic | | | | | | | | |
| Coefficient | 1.702 | -4.067 | -37.140 | 19.037 | 2090.599 | -234.825 | -27977.991 | 0.5846 |
| P-Value | (0.000) | (0.005) | (0.014) | (0.055) | (0.031) | (0.053) | (0.000) | |

(a) Linear Pricing Kernel



(b) Quadratic Pricing Kernel



(c) Cubic Pricing Kernel

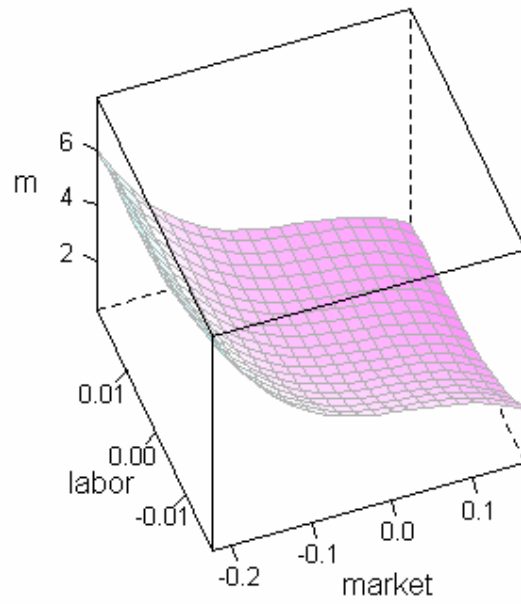


Figure 1. Estimated Pricing Kernels

The point estimates are calculated at the mean of the instrumental variables and with the return on human capital and value-weighted index to support for the graph. The Euler equation is used to generate the coefficient of the pricing kernels:

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

Using the polynomial pricing kernels, m_{t+1} , the coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix $E[(1 + R_{t+1}) \otimes Z_t][(1 + R_{t+1}) \otimes Z_t]'$. The columns present the coefficients of the pricing kernel evaluated at the means of the instruments. The coefficients are modelled as

$$d_n = I_n (\delta_n' Z_t)^2 \quad I_n = \begin{cases} -1, n=1,3 \\ 1, n=2 \end{cases}$$

The set of returns used in estimation are those 17 industry-sorted portfolios augmented by the return on a one month Treasury bill.

Over all, these results are consistent with those obtained from Dittmar 2002. The results indicate the nonlinear model perform well on explaining the cross section of returns. In contrast, linear models perform poorly according to the Hansen-Jagannathan distance measure. In particular, when human capital returns add into the measure of the total wealth, nonlinearity term is important for improving the fit of the pricing kernel. In this thesis, I am not only testing human capital as a proxy of risk asset, also I am testing the impact of the measurement of return on the housing wealth in total wealth. My results show that housing wealth is highly correlated with aggregate return in the economic market. It further improves the fit of the pricing kernel. In addition, momentum factor plays an important role in predicting the asset returns.

5.2 Comparison across different sample Periods

5.2.A Model Specification Tests Across Different Sample Periods

5.2.A.1 Specification Tests on Polynomial Pricing Kernels with Human Capital Excluded

Using return data on the 17 industry portfolios described earlier, I examine the linear, quadratic and cubic model specification in the case of when the measure of aggregate wealth does not include human capital (see equation (47)).

The results of linear specification tests across three different sample periods are presented in Panel A of Table 14. In the GMM test that uses the Hansen-Jagannathan weighting matrix, the linear model specification test during the period July 1963 to December 2009 gives the smaller pricing error 0.5004, compare to Dittmar's sample period (JH-dist 0.6472) and most recent period Jan 1996:Dec 2009 (JH-dist 1.1904). It is interesting to note that the distance measure for the sample period Jan 1996:Dec 2009 is statistically significantly from zero. As the results shown in Table 2, the chi-square test for this sample period is insignificant due to the small sample size. In contrast, the full sample period, which covers the most rent data provide a better fitting of pricing kernel.

The results of nonlinear specification tests across three different sample periods are presented in Panel B and C of Table 14. The full sample period continuously provide small distance measure in quadratic and cubic pricing kernel. However, the estimated coefficients of higher order equity return are insignificantly at 5% level. Those results suggest that return on value-weighted index portfolio does not play a significant role in explaining cross section of expected return.

Table 14
Specification Tests: Polynomial Pricing Kernels with Human Capital
Excluded

Table 14 presents results of GMM tests of the Euler equation condition,

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

Using the polynomial pricing kernels, m_{t+1} , the coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix $E[(1 + R_{t+1}) \otimes Z_t][(1 + R_{t+1}) \otimes Z_t]'$. The columns present the coefficients of the pricing kernel evaluated at the means of the instruments. The coefficients are modelled as

$$d_n = I_n (\delta'_n Z_t)^2 \quad I_n = \begin{cases} -1, n=1,3 \\ 1, n=2 \end{cases}$$

p-value for Wald tests of the joint significance of the coefficients are presented in parentheses. The final column presents the Hansen-Jagannathan distance measure with p-values for the test of model specification in parentheses. The set of returns used in estimation are those 17 industry-sorted portfolios augmented by the return on a one month Treasury bill.

Panel A: Linear Pricing Kernel without Human Capital

| | $d(\bar{Z})_{0t}$ | $d(\bar{Z})_{1t}$ | $d(\bar{Z})_{2t}$ | $d(\bar{Z})_{3t}$ | Dist |
|---|-------------------|-------------------|-------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | |
| Coefficient | 1.081 | -4.158 | | | 0.6472 |
| P-Value | (0.000) | (0.000) | | | |
| Period from July 1963 to December 2009 | | | | | |
| Coefficient | 1.044 | -2.331 | | | 0.5004 |
| P-Value | (0.000) | (0.001) | | | |
| Period from January 1996 to December 2009 | | | | | |
| Coefficient | 1.037 | -2.115 | | | 1.1904 |
| P-Value | (0.000) | (0.012) | | | |

Panel B: Quadratic Pricing Kernel without Human Capital

| | $d(\bar{Z})_{0t}$ | $d(\bar{Z})_{1t}$ | $d(\bar{Z})_{2t}$ | $d(\bar{Z})_{3t}$ | Dist |
|---|-------------------|-------------------|-------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | |
| Coefficient | 1.026 | -4.686 | 28.644 | | 0.6406 |
| P-Value | (0.000) | (0.000) | (0.040) | | |
| Period from July 1963 to December 2009 | | | | | |
| Coefficient | 1.044 | -2.326 | 0.002 | | 0.5004 |
| P-Value | (0.000) | (0.002) | (0.366) | | |
| Period from January 1996 to December 2009 | | | | | |
| Coefficient | 1.044 | -2.326 | 0.002 | | 1.1904 |
| P-Value | (0.000) | (0.014) | (0.411) | | |

Panel C: Cubic Pricing Kernel without Human Capital

| | $d(\bar{Z})_{0t}$ | $d(\bar{Z})_{1t}$ | $d(\bar{Z})_{2t}$ | $d(\bar{Z})_{3t}$ | Dist |
|---|-------------------|-------------------|-------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | |
| Coefficient | 1.019 | -2.330 | 13.376 | -13.619 | 0.6406 |
| P-Value | (0.000) | (0.002) | (0.044) | (0.284) | |
| Period from July 1963 to December 2009 | | | | | |
| Coefficient | 1.044 | -2.357 | 0.000 | -0.502 | 0.5003 |
| P-Value | (0.000) | (0.004) | (0.472) | (0.258) | |
| Period from January 1996 to December 2009 | | | | | |
| Coefficient | 1.037 | -2.103 | 0.000 | -0.008 | 1.1904 |
| P-Value | (0.000) | (0.034) | (0.445) | (0.434) | |

Table 15**Specification Tests: Polynomial Pricing Kernels with Human Capital
Included Only**

Table 15 presents results of GMM tests of the Euler equation condition,

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

m_{t+1} implied by the Fama and French (1993) three-factor model. The coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix $E[(1 + R_{t+1}) \otimes Z_t ((1 + R_{t+1}) \otimes Z_t)']$. The columns present the coefficients of the pricing kernel evaluated at the means of the instruments. The coefficients are modelled as

$$d_n = I_n (\delta_n' Z_t)^2 \quad I_n = \begin{cases} -1, n=1,3 \\ 1, n=2 \end{cases}$$

P-value for Wald tests of the joint significance of the coefficients are presented in parentheses. The final column presents the Hansen-Jagannathan distance measure. The set of returns used in estimation are those 17 industry-sorted portfolios covering the period July 1963, through December 2009, augmented by the return on a one month Treasury bill.

Panel A: Linear Pricing Kernel with Human Capital

| | $d(\bar{Z})_{0t}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | Dist |
|---|-------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | | | | |
| Coefficient | 1.426 | -3.775 | -36.419 | | | | | 0.6187 |
| P-Value | (0.000) | (0.001) | (0.004) | | | | | |
| Period from July 1963 to December 2009 | | | | | | | | |
| Coefficient | 1.212 | -2.370 | -9.145 | | | | | 0.4708 |
| P-Value | (0.000) | (0.000) | (0.029) | | | | | |
| Period from January 1996 to December 2009 | | | | | | | | |
| Coefficient | 1.589 | -2.705 | -69.564 | | | | | 1.1213 |
| P-Value | (0.000) | (0.001) | (0.000) | | | | | |

Panel B: Quadratic Pricing Kernel with Human Capital

| | $d(\bar{Z})_{0r}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | Dist |
|---|-------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | | | | |
| Coefficient | 1.251 | -5.473 | -22.448 | 51.400 | 1396.395 | | | 0.5839 |
| P-Value | (0.000) | (0.001) | (0.038) | (0.022) | (0.051) | | | |
| Period from July 1963 to December 2009 | | | | | | | | |
| Coefficient | 1.173 | -2.783 | -57.515 | 4.766 | 7605.891 | | | 0.4543 |
| P-Value | (0.000) | (0.001) | (0.016) | (0.093) | (0.006) | | | |
| Period from January 1996 to December 2009 | | | | | | | | |
| Coefficient | 1.647 | -2.798 | -127.408 | 0.000 | 13899.038 | | | 1.1060 |
| P-Value | (0.000) | (0.002) | (0.002) | (0.136) | (0.004) | | | |

Panel C: Cubic Pricing Kernel with Human Capital

| | $d(\bar{Z})_{0r}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | Dist |
|---|-------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | | | | |
| Coefficient | 1.251 | -5.475 | -22.384 | 51.424 | 1393.272 | -0.582 | -1.750 | 0.5839 |
| P-Value | (0.000) | (0.005) | (0.055) | (0.026) | (0.086) | (0.315) | (0.469) | |
| Period from July 1963 to December 2009 | | | | | | | | |
| Coefficient | 1.174 | -2.752 | -59.288 | 12.678 | 7490.575 | -55.560 | -124.727 | 0.4484 |
| P-Value | (0.000) | (0.002) | (0.005) | (0.047) | (0.002) | (0.064) | (0.000) | |
| Period from January 1996 to December 2009 | | | | | | | | |
| Coefficient | 1.817 | -2.574 | -218.432 | 0.000 | 24858.388 | -0.002 | -233.023 | 1.0856 |
| P-Value | (0.000) | (0.005) | (0.004) | (0.148) | (0.011) | (0.118) | (0.096) | |

5.2.A.2 Specification Tests on Polynomial Pricing Kernels with Human Capital Included Only

Using return data on the 17 industry portfolios described earlier, I examine the linear, quadratic and cubic model specification across different sample periods in the case of when the measure of aggregate wealth include human capital (see equation (47), also Figure 2,3,4).

The over all results of specification tests across three different sample periods are presented in Table 15, indicating that incorporating human capital improve the performance of all three model specifications and across three sample periods. Especially, in Panel C of Table 15, the cubic term of return on human capital is statistically significant at the 5% level, in which Dittmar's period and most rent period are not. The full sample period has more observed data as it include both Dittmar's period and most recent period. The data is more volatile during the recession time and better fit in nonlinear model. Thus, full sample period in this analysis provide better result.

5.2.A.3 Specification Tests on Polynomial Pricing Kernels with Housing Wealth Included Only

I assume that the proxy for the return on the wealth portfolio is the return on the value weighted industry portfolio, and housing wealth (equation (55), Figures 5, 6, 7).

The results of specification tests across three different sample periods are presented in Table 16: The coefficients of return to housing wealth in different orders are estimated by using the mean return of the housing wealth. As shown in the Table 16, the higher order term of housing wealth is not statistically significant in the full sample period, and the higher order term of housing wealth in the other sub periods are significant. However, the results indicate that incorporating housing wealth improves the performance of three model specifications across three sample periods and the full sample period continuously provide less pricing kernel than that of either Dittmar's period or most recent period. Thus, the housing wealth contributes significantly in explaining the cross section of expected return.

Table 16
Specification Tests: Polynomial Pricing Kernels with Housing Wealth
Included Only

Table 16 presents results of GMM tests of the Euler equation condition,

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

m_{t+1} implied by the Fama and French (1993) three-factor model. The coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix $E[(1 + R_{t+1}) \otimes Z_t ((1 + R_{t+1}) \otimes Z_t)']$. The columns present the coefficients of the pricing kernel evaluated at the means of the instruments. The coefficients are modelled as

$$d_n = I_n (\delta_n' Z_t)^2 \quad I_n = \begin{cases} 1, n=1,3 \\ 1, n=2 \end{cases}$$

P-value for Wald tests of the joint significance of the coefficients are presented in parentheses. The final column presents the Hansen-Jagannathan distance measure. The set of returns used in estimation are those 17 industry-sorted portfolios covering the period July 1963, through December 2009, augmented by the return on a one month Treasury bill.

Panel A: Linear Pricing Kernel with Housing Wealth

| | $d(\bar{Z})_{0r}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1h}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2h}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3h}$ | Dist |
|---|-------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | | | | |
| Coefficient | 1.104 | -4.420 | -2.134 | | | | | 0.6446 |
| P-Value | (0.000) | (0.001) | (0.027) | | | | | |
| Period from July 1963 to December 2009 | | | | | | | | |
| Coefficient | 1.077 | -2.255 | -3.363 | | | | | 0.4845 |
| P-Value | (0.000) | (0.001) | (0.012) | | | | | |
| Period from January 1996 to December 2009 | | | | | | | | |
| Coefficient | 1.087 | -2.2538 | -4.084 | | | | | 1.1724 |
| P-Value | (0.000) | (0.009) | (0.014) | | | | | |

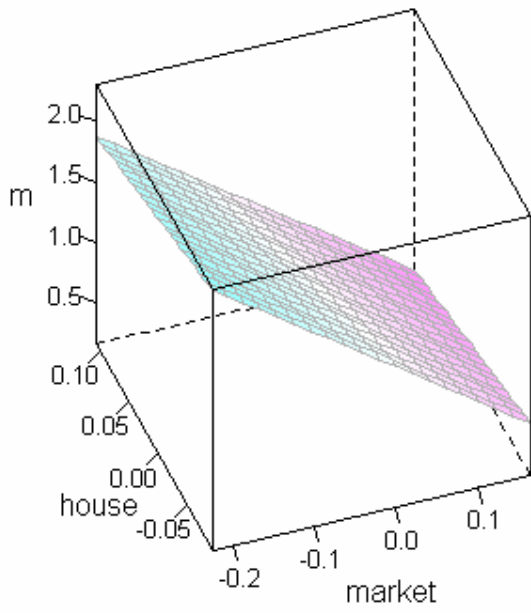
Panel B: Quadratic Pricing Kernel with Housing Wealth

| | $d(\bar{Z})_{0r}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1h}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2h}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3h}$ | Dist |
|---|-------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | | | | |
| Coefficient | 0.909 | -5.133 | -1.443 | 31.630 | 94.409 | | | 0.6303 |
| P-Value | (0.000) | (0.000) | (0.041) | (0.036) | (0.014) | | | |
| Period from July 1963 to December 2009 | | | | | | | | |
| Coefficient | 1.077 | -2.256 | -3.364 | 0.002 | 0.272 | | | 0.4845 |
| P-Value | (0.000) | (0.002) | (0.014) | (0.391) | (0.255) | | | |
| Period from January 1996 to December 2009 | | | | | | | | |
| Coefficient | 0.926 | -2.354 | -5.119 | 0.000 | 105.716 | | | 1.1613 |
| P-Value | (0.000) | (0.011) | (0.012) | (0.448) | (0.011) | | | |

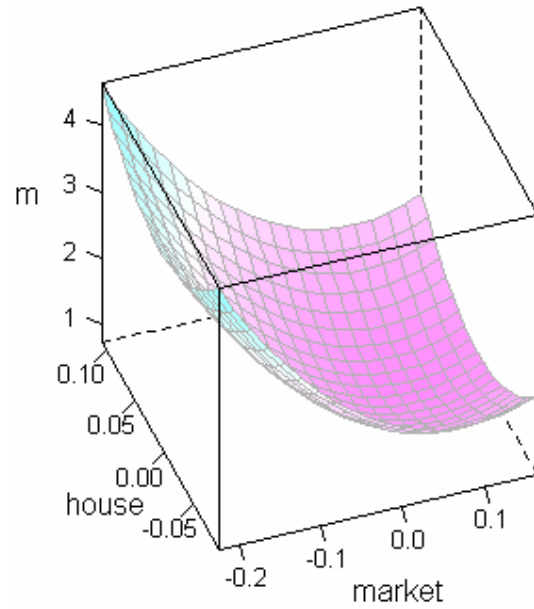
Panel C: Cubic Pricing Kernel with Housing Wealth

| | $d(\bar{Z})_{0r}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | Dist |
|---|-------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | | | | |
| Coefficient | 0.856 | -4.907 | -0.198 | 33.803 | 131.608 | 55.779 | -844.475 | 0.6269 |
| P-Value | (0.000) | (0.001) | (0.120) | (0.055) | (0.012) | (0.116) | (0.044) | |
| Period from July 1963 to December 2009 | | | | | | | | |
| Coefficient | 1.069 | -2.547 | -3.436 | 4.541 | 0.259 | -15.712 | -0.001 | 0.4816 |
| P-Value | (0.000) | (0.006) | (0.021) | (0.108) | (0.259) | (0.128) | (0.468) | |
| Period from January 1996 to December 2009 | | | | | | | | |
| Coefficient | 0.714 | -4.030 | -1.112 | 0.001 | 38.933 | -0.062 | -1411.770 | 1.0647 |
| P-Value | (0.000) | (0.015) | (0.099) | (0.404) | (0.028) | (0.338) | (0.036) | |

(a) Linear Pricing Kernel



(b) Quadratic Pricing Kernel



(c) Cubic Pricing Kernel

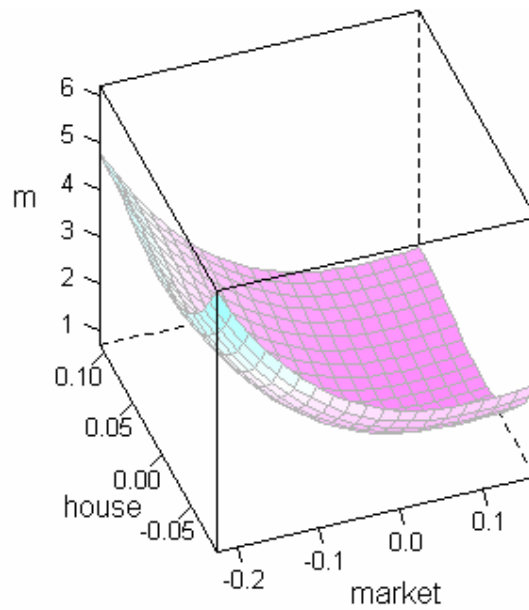


Figure 2. Estimated pricing kernels for Dittmar's period Jul 1963:Dec 1995

The point estimates are calculated at the mean of the instrumental variables and with the return on housing and value-weighted index to support for the graph. The Euler equation is used to generate the coefficient of the pricing kernels:

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

Using the polynomial pricing kernels, m_{t+1} , the coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix $E[(1 + R_{t+1}) \otimes Z_t ((1 + R_{t+1}) \otimes Z_t)']$. The columns present the coefficients of the pricing kernel evaluated at the means of the instruments. The coefficients are modelled as

$$d_n = I_n (\delta_n' Z_t)^2 \quad I_n = \begin{cases} -1, n=1,3 \\ 1, n=2 \end{cases}$$

The set of returns used in estimation are those 17 industry-sorted portfolios augmented by the return on a one month Treasury bill.

5.2.A.4 Specification Tests on Polynomial Pricing Kernels with Human Capital and Housing Wealth Included

Now, I compare the main model developed in this paper (see equation (59), (61) and (62)). I assume the proxy for the return on the wealth portfolio is sum of the return on the value weighted industry portfolio, return on human capital and return on housing wealth (see equation (60)).

The overall results of specification tests across three sample periods are presented in Table 17, indicating that the housing factor has further improved the fit of the pricing kernel. The full sample period continuously provide less pricing kernel than that of either Dittmar's period or most recent period. The distance measure implied by the full sample period falls sharply to 0.4396, a decline of 0.1291 relative to Dittmar's period. Further more, the incorporating the cubic return on housing wealth in most recent sample period also improves the performance of the pricing kernel. And this cubic term is statistically significant at the 5% level (p-value 0.008).

Table 17**Specification Tests: Polynomial Pricing Kernels with Human Capital and Housing Wealth Included**

Table 17 presents results of GMM tests of the Euler equation condition,

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

m_{t+1} implied by the Fama and French (1993) four-factor model. The coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix. P-value for Wald tests of the joint significance of the coefficients are presented in parentheses. The final column presents the Hansen-Jagannathan distance measure. The set of returns used in estimation are those 17 industry-sorted portfolios covering the period July 1963, through December 2009, augmented by the return on a one month Treasury bill.

Panel A: Linear Pricing Kernel with Human Capital and Housing Wealth

| | $d(\bar{Z})_{0r}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{1h}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{2h}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | $d(\bar{Z})_{3h}$ | Dist |
|--|-------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | | | | | | | |
| Coefficient | 1.460 | -3.989 | -38.819 | -2.081 | | | | | | | 0.6175 |
| P-Value | (0.000) | (0.001) | (0.005) | (0.031) | | | | | | | |
| Period from July 1963 to December 2009 | | | | | | | | | | | |
| Coefficient | 1.301 | -2.719 | -19.271 | -3.858 | | | | | | | 0.4596 |
| P-Value | (0.000) | (0.000) | (0.014) | (0.014) | | | | | | | |

Period from January 1996 to December 2009

| | | | | | | | | | | | |
|-------------|---------|---------|---------|---------|--|--|--|--|--|--|--------|
| Coefficient | 1.727 | -2.438 | -85.737 | -5.836 | | | | | | | 1.0918 |
| P-Value | (0.000) | (0.008) | (0.001) | (0.010) | | | | | | | |

Panel B: Quadratic Pricing Kernel with Human Capital and Housing Wealth

| | $d(\bar{Z})_{0r}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{1h}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{2h}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | $d(\bar{Z})_{3h}$ | Dist |
|--|-------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|------|
|--|-------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|------|

Period from July 1963 to December 1995

| | | | | | | | | | | | |
|-------------|---------|---------|---------|---------|---------|----------|---------|--|--|--|--------|
| Coefficient | 1.113 | -5.958 | -21.262 | -2.775 | 45.327 | 1183.006 | 108.094 | | | | 0.5705 |
| P-Value | (0.000) | (0.001) | (0.039) | (0.038) | (0.034) | (0.074) | (0.012) | | | | |

Period from July 1963 to December 2009

| | | | | | | | | | | | |
|-------------|---------|---------|---------|---------|---------|----------|---------|--|--|--|--------|
| Coefficient | 1.176 | -3.432 | -55.705 | -2.912 | 8.744 | 7063.475 | 10.669 | | | | 0.4437 |
| P-Value | (0.000) | (0.001) | (0.019) | (0.025) | (0.078) | (0.008) | (0.102) | | | | |

Period from January 1996 to December 2009

| | | | | | | | | | | | |
|-------------|---------|---------|----------|---------|---------|-----------|---------|--|--|--|--------|
| Coefficient | 1.780 | -3.000 | -194.443 | -5.923 | 0.000 | 18746.690 | 76.053 | | | | 1.0564 |
| P-Value | (0.000) | (0.003) | (0.002) | (0.025) | (0.008) | (0.378) | (0.006) | | | | |

Panel C: Cubic Pricing Kernel with Human Capital and Housing Wealth

| | $d(\bar{Z})_{0r}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{1h}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{2h}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | $d(\bar{Z})_{3h}$ | Dist |
|---|-------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | | | | | | | |
| Coefficient | 1.116 | -5.692 | -24.879 | -2.065 | 46.725 | 1006.232 | 118.488 | -43.612 | -48.825 | -187.904 | 0.5685 |
| P-Value | (0.000) | (0.001) | (0.008) | (0.049) | (0.014) | (0.035) | (0.008) | (0.075) | (0.000) | (0.105) | |
| Period from July 1963 to December 2009 | | | | | | | | | | | |
| Coefficient | 1.200 | -3.054 | -65.044 | -3.143 | -12.712 | -7659.569 | 5.947 | -41.897 | -197.211 | -45.572 | 0.4394 |
| P-Value | (0.000) | (0.002) | (0.005) | (0.027) | (0.049) | (0.002) | (0.121) | (0.069) | (0.000) | (0.152) | |
| Period from January 1996 to December 2009 | | | | | | | | | | | |
| Coefficient | 1.794 | -3.402 | -200.330 | -1.211 | 0.000 | 20878.778 | 132.141 | -0.001 | -16824.993 | -1928.385 | 1.0261 |
| P-Value | (0.000) | (0.002) | (0.002) | (0.057) | (0.350) | (0.004) | (0.011) | (0.244) | (0.000) | (0.008) | |

5.2.B Comparison with Multifactor Model Across Different Sample Period

5.2.B.1 Specification Tests on Fama-French Pricing Kernel with Human capital Included

I begin with Fama-French three-factor model (see equation (63)). The results for the estimation of the Fama-French model in three sample periods showed in Panel A of Table 18. The results of distance measures indicate that the Fama-French factors do not provide significant explanatory power to the pricing kernel.

To further investigate the ability of the Fama-French factors. I consider the nonlinear models (see equation (64) and (65)). The results for the estimation of the polynomial model augmented SMB, HML and human capital are given in Table 18 Panel B and Panel C. not surprisingly, once adding the return to human capital significantly improves the fit of the model. All sample periods in nonlinear model specification reduce the pricing error with respected to linear model. Moreover, the full sample period has small pricing errors in linear, quadratic and cubic model. The SMB and HML factors in cubic model are significant with p-value of 0.038 and 0.043, except most recent period and neither the SMB nor the HML coefficients are significantly different from zero. These results suggest in most recent sample period pricing kernels captures much of the variation in returns.

Table 18

Specification Tests: Fama-French Pricing Kernel with Human capital Included

Table 18 presents results of GMM tests of the Euler equation condition,

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

m_{t+1} implied by the Fama and French (1993) four-factor model. The coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix. P-value for Wald tests of the joint significance of the coefficients are presented in parentheses. The final column presents the Hansen-Jagannathan distance measure. The set of returns used in estimation are those 17 industry-sorted portfolios covering the period July 1963, through December 2009, augmented by the return on a one month Treasury bill.

Panel A: Fama-French Three-Factor Model

| | $d(\bar{Z})_{0t}$ | $d(\bar{Z})_{mfp}$ | $d(\bar{Z})_{smb}$ | $d(\bar{Z})_{hml}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | Dist |
|--|-------------------|--------------------|--------------------|--------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | | | | | | | |
| Coefficient | 1.185 | -4.455 | -1.902 | -2.027 | | | | | | | 0.5680 |
| P-Value | (0.000) | (0.000) | (0.029) | (0.024) | | | | | | | |
| Period from July 1963 to December 2009 | | | | | | | | | | | |
| Coefficient | 1.099 | -3.843 | -2.836 | 0.492 | | | | | | | 0.4601 |
| P-Value | (0.000) | (0.000) | (0.009) | (0.079) | | | | | | | |

| Period from January 1996 to December 2009 | | | | | |
|---|---------|---------|---------|---------|--------|
| Coefficient | 1.065 | -3.246 | 0.773 | 5.018 | 1.1571 |
| P-Value | (0.000) | (0.004) | (0.065) | (0.004) | |

Panel B: Quadratic Pricing Kernel with Fama-French Factors and Human Capital

| | $d(\bar{Z})_{0t}$ | $d(\bar{Z})_{mrp}$ | $d(\bar{Z})_{smb}$ | $d(\bar{Z})_{hml}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{ll}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | Dist |
|---|-------------------|--------------------|--------------------|--------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | | | | | | | |
| Coefficient | 1.397 | | -1.173 | -0.816 | -5.673 | -16.016 | 20.273 | 552.593 | | | 0.5187 |
| P-Value | (0.000) | | (0.055) | (0.065) | (0.000) | (0.019) | (0.038) | (0.054) | | | |
| Period from July 1963 to December 2009 | | | | | | | | | | | |
| Coefficient | 1.215 | | -1.385 | -0.185 | -3.177 | -1.243 | -9.707 | 3760.715 | | | 0.4120 |
| P-Value | (0.000) | | (0.041) | (0.131) | (0.003) | (0.125) | (0.056) | (0.012) | | | |
| Period from January 1996 to December 2009 | | | | | | | | | | | |
| Coefficient | 1.583 | | -0.333 | -0.008 | -0.004 | -115.860 | 0.000 | 12398.722 | | | 1.0796 |
| P-Value | (0.000) | | (0.109) | (0.261) | (0.143) | (0.001) | (0.406) | (0.007) | | | |

Panel C: Cubic Pricing Kernel with Fama-French Factors and Human Capital

| | $d(\bar{Z})_{0r}$ | $d(\bar{Z})_{mrp}$ | $d(\bar{Z})_{smb}$ | $d(\bar{Z})_{hml}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | Dist |
|---|-------------------|--------------------|--------------------|--------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | | | | | | | |
| Coefficient | 1.402 | | 1.254 | -1.075 | -4.817 | -16.745 | 19.292 | 550.187 | -146.687 | -177.846 | 0.5159 |
| P-Value | (0.000) | | (0.038) | (0.043) | (0.005) | (0.065) | (0.041) | (0.010) | (0.046) | (0.000) | |
| Period from July 1963 to December 2009 | | | | | | | | | | | |
| Coefficient | 1.215 | | -1.342 | -1.125 | -2.485 | -1.490 | 11.332 | 3724.93 | -101.87 | -175.98 | 0.4061 |
| P-Value | (0.000) | | (0.038) | (0.043) | (0.005) | (0.065) | (0.041) | (0.010) | (0.046) | (0.000) | |
| Period from January 1996 to December 2009 | | | | | | | | | | | |
| Coefficient | 1.821 | | -0.621 | -2.873 | -0.994 | -198.127 | 0.000 | 23883.703 | -107.725 | -32.602 | 1.0472 |
| P-Value | (0.000) | | (0.090) | (0.021) | (0.019) | (0.004) | (0.293) | (0.010) | (0.070) | (0.000) | |

5.2.B.2 Specification Tests on Fama-French Pricing Kernel with Momentum Factor and Human Capital Included

The results for the estimation of the polynomial model augmented SMB, HML, Momentum and human capital are given in Table 19. Panel A of table 19, estimate include the results for Fama-French four-factor model (see equation (66)). Very obviously, adding momentum factor improves the fit of the linear pricing kernel. However, this distance measure suggests that the Fama-French four-factor model is not better than nonlinear pricing kernel.

In Panel B of Table 19, I compare the results for quadratic pricing kernel augmented by Fama-French Factors, momentum and housing wealth across three sample periods. The comparison results obtained from this table are similar to Table 18. The quadratic model in full sample period provides better fitting of pricing kernel, the estimated distance measure falls to 0.3785.

In Panel C of Table 19, I compare the results for cubic pricing kernel augmented by Fama-French Factors, Momentum and human capital. The cubic model for the full sample period reduces the pricing errors from linear to nonlinear pricing specification. In addition, the SMB, HML factors and cubic term of return to human capital are statistically significant in full sample period of cubic model specification test. These results suggest that Fama-French factors do add more information to our nonlinear pricing kernel.

Table 19**Specification Tests: Fama-French Pricing Kernel with Momentum Factor and Human Capital Included**

Table 19 presents results of GMM tests of the Euler equation condition,

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

m_{t+1} implied by the Fama and French (1993) four-factor model. The coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix. P-value for Wald tests of the joint significance of the coefficients are presented in parentheses. The final column presents the Hansen-Jagannathan distance measure. The set of returns used in estimation are those 17 industry-sorted portfolios covering the period July 1963, through December 2009, augmented by the return on a one month Treasury bill.

Panel A: Fama-French model with Momentum factor

| | $d(\bar{Z})_{0t}$ | $d(\bar{Z})_{mrp}$ | $d(\bar{Z})_{smb}$ | $d(\bar{Z})_{hml}$ | $d(\bar{Z})_{mom}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | Dist |
|--|-------------------|--------------------|--------------------|--------------------|--------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | | | | | | | | |
| Coefficient | 1.298 | -5.110 | -2.389 | -5.236 | -10.377 | | | | | | | 0.5299 |
| P-Value | (0.000) | (0.028) | (0.001) | (0.004) | (0.000) | | | | | | | |
| Period from July 1963 to December 2009 | | | | | | | | | | | | |
| Coefficient | 1.166 | -4.240 | -4.065 | -0.901 | -4.716 | | | | | | | 0.4257 |
| P-Value | (0.000) | (0.000) | (0.003) | (0.055) | (0.002) | | | | | | | |

| | | | | | | | | | | | |
|---|---------|---------|---------|---------|---------|--|--|--|--|--|--------|
| Period from January 1996 to December 2009 | | | | | | | | | | | |
| Coefficient | 1.102 | -4.432 | 1.418 | 4.450 | -2.687 | | | | | | 1.1486 |
| P-Value | (0.000) | (0.006) | (0.046) | (0.015) | (0.018) | | | | | | |

Panel B: Quadratic pricing kernel with Fama-French Factors, Momentum and Human Capital

| | $d(\bar{Z})_{0t}$ | $d(\bar{Z})_{mrp}$ | $d(\bar{Z})_{smb}$ | $d(\bar{Z})_{hml}$ | $d(\bar{Z})_{mom}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | Dist |
|---|-------------------|--------------------|--------------------|--------------------|--------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | | | | | | | | |
| Coefficient | 1.155 | -0.481 | -4.658 | -11.296 | -5.640 | -16.491 | 3.750 | 882.118 | | | | 0.4720 |
| P-Value | (0.000) | (0.106) | (0.011) | (0.000) | (0.001) | (0.050) | (0.116) | (0.085) | | | | |
| Period from July 1963 to December 2009 | | | | | | | | | | | | |
| Coefficient | 1.155 | -1.933 | 0.167 | -0.997 | -3.496 | -3.504 | -8.319 | 4702.514 | | | | 0.3785 |
| P-Value | (0.000) | (0.029) | (0.136) | (0.068) | (0.004) | (0.081) | (0.046) | (0.011) | | | | |
| Period from January 1996 to December 2009 | | | | | | | | | | | | |
| Coefficient | 1.822 | 0.681 | 0.167 | -2.540 | -3.883 | -3.380 | -141.012 | 0.079 | | | | 1.0433 |
| P-Value | (0.000) | (0.098) | (0.047) | (0.012) | (0.008) | (0.012) | (0.247) | (0.019) | | | | |

Panel C: Cubic pricing kernel with Fama-French Factors, Momentum and Human Capital

| | $d(\bar{Z})_{0t}$ | $d(\bar{Z})_{mtp}$ | $d(\bar{Z})_{smb}$ | $d(\bar{Z})_{hml}$ | $d(\bar{Z})_{mom}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | Dist |
|---|-------------------|--------------------|--------------------|--------------------|--------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | | | | | | | | |
| Coefficient | 1.542 | | 0.105 | -5.250 | -12.031 | -5.629 | -17.051 | 3.948 | 1048.964 | -7.534 | 18.269 | 0.4709 |
| P-Value | (0.000) | | (0.166) | (0.004) | (0.000) | (0.001) | (0.019) | (0.107) | (0.029) | (0.181) | (0.000) | |
| Period from July 1963 to December 2009 | | | | | | | | | | | | |
| Coefficient | 1.243 | | -1.710 | -2.179 | -2.917 | -2.844 | -5.551 | 19.202 | 4233.398 | -169.648 | -267.089 | 0.3766 |
| P-Value | (0.000) | | (0.030) | (0.023) | (0.011) | (0.004) | (0.042) | (0.030) | (0.008) | (0.027) | (0.000) | |
| Period from January 1996 to December 2009 | | | | | | | | | | | | |
| Coefficient | 1.821 | | 0.872 | -2.407 | -3.617 | -3.459 | -142.216 | 0.403 | 18706.098 | -0.004 | -5930.291 | 1.0429 |
| P-Value | (0.000) | | (0.070) | (0.030) | (0.006) | (0.001) | (0.002) | (0.139) | (0.008) | (0.255) | (0.000) | |

Table 20

Specification Tests: Fama-French Pricing Kernel with Momentum Factor and Housing Wealth Included

Table 20 presents results of GMM tests of the Euler equation condition,

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

m_{t+1} implied by the Fama and French (1993) three-factor model. The coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix. P-value for Wald tests of the joint significance of the coefficients are presented in parentheses. The final column presents the Hansen-Jagannathan distance measure. The set of returns used in estimation are those 17 industry-sorted portfolios covering the period July 1963, through December 2009, augmented by the return on a one month Treasury bill.

Panel A: Quadratic pricing kernel with Fama-French Factors, Momentum and Housing Wealth

| | $d(\bar{Z})_{0t}$ | $d(\bar{Z})_{smb}$ | $d(\bar{Z})_{hml}$ | $d(\bar{Z})_{mom}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1h}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2h}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3h}$ | Dist |
|--|-------------------|--------------------|--------------------|--------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | | | | | | | |
| Coefficient | 1.288 | -1.772 | -4.112 | -9.204 | -4.833 | -0.528 | 6.442 | 31.549 | | | 0.4868 |
| P-Value | (0.000) | (0.004) | (0.100) | (0.024) | (0.007) | (0.019) | (0.406) | (0.146) | | | |
| Period from July 1963 to December 2009 | | | | | | | | | | | |
| Coefficient | 1.157 | -4.104 | -0.449 | -2.614 | -1.935 | -3.002 | 0.001 | 5.062 | | | 0.4213 |
| P-Value | (0.000) | (0.004) | (0.100) | (0.024) | (0.007) | (0.019) | (0.406) | (0.146) | | | |

| Period from January 1996 to December 2009 | | | | | | | | | | | |
|---|---------|---------|---------|---------|---------|---------|---------|---------|--|--|--------|
| Coefficient | 1.054 | 2.530 | 3.793 | -4838 | -1.071 | -5.010 | 0.000 | 86.190 | | | 1.0915 |
| P-Value | (0.000) | (0.004) | (0.100) | (0.024) | (0.007) | (0.019) | (0.406) | (0.146) | | | |

Panel B: Cubic pricing kernel with Fama-French Factors, Momentum and Housing Wealth

| | $d(\bar{Z})_{0t}$ | $d(\bar{Z})_{smb}$ | $d(\bar{Z})_{hml}$ | $d(\bar{Z})_{mom}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1h}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2h}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3h}$ | Dist |
|---|-------------------|--------------------|--------------------|--------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | | | | | | | |
| Coefficient | 1.212 | -2.610 | -6.341 | -10.625 | -5.992 | -0.326 | 20.578 | 120.940 | -381.671 | -1127.912 | 0.4142 |
| P-Value | (0.000) | (0.042) | (0.010) | (0.003) | (0.012) | (0.098) | (0.089) | (0.015) | (0.045) | (0.024) | |
| Period from July 1963 to December 2009 | | | | | | | | | | | |
| Coefficient | 1.169 | -3.255 | -0.124 | -3.899 | -3.552 | -3.742 | 6.835 | 2.784 | -13.316 | -0.034 | 0.4130 |
| P-Value | (0.000) | (0.008) | (0.150) | (0.005) | (0.002) | (0.010) | (0.078) | (0.136) | (0.115) | (0.185) | |
| Period from January 1996 to December 2009 | | | | | | | | | | | |
| Coefficient | 1.034 | 3.057 | 4.127 | -4.495 | -0.787 | -0.383 | 0.024 | 141.838 | -0.001 | -2169.637 | 1.0765 |
| P-Value | (0.000) | (0.016) | (0.011) | (0.006) | (0.029) | (0.104) | (0.284) | (0.013) | (0.299) | (0.007) | |

5.2.B.3 Specification Tests on Fama-French Pricing Kernel with Momentum Factor and Housing Wealth Included

Now I study the performance of augmented Fama-French Model including housing wealth as a risky asset class. The cross-sectional implications of the models have introduced in section 5.1 (see equation (69) and (70)). The empirical results are given in Table 20.

The panel A and B of Table 20 gives the estimates for model (69) (70) across three sample periods. Not surprisingly, adding housing wealth improve the fit of pricing kernel. The distance measure of quadratic model specification in full sample period reduces to 0.4213; while in Dittmar's period is 0.4868. Due to the small sample size, for the sub period January 1996 to December 2009, the distance measure of model specification is large. The same results apply to cubic model specification test.

5.2.B.4 Specification Tests: Fama-French Pricing Kernel with Momentum Factor and Human Capital and Housing Wealth Included

Now I study the performance of Augmented Fama-French Model including human capital and housing wealth as two risky asset classes. The cross-sectional implications of the models introduces in previous section 5.1 (see equation (71) and (72)). The empirical results are given in Table 21. Both human capital and housing wealth risk factor have further improvement on the performance of the augmented model. By comparing the distance measure across the different sample period, I find that the model specification test for full sample period perform well than other sample periods.

Table 21

Specification Tests: Fama-French Pricing Kernel with Momentum Factor and Human Capital and Housing Wealth Included

Table 21 presents results of GMM tests of the Euler equation condition,

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

m_{t+1} implied by the Fama and French (1993) three-factor model. The coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix. P-value for Wald tests of the joint significance of the coefficients are presented in parentheses. The final column presents the Hansen-Jagannathan distance measure. The set of returns used in estimation are those 17 industry-sorted portfolios covering the period July 1963, through December 2009, augmented by the return on a one month Treasury bill.

Panel A: Quadratic pricing kernel with Fama-French Factors, Momentum, Human Capital and Housing Wealth

| | $d(\bar{Z})_{0t}$ | $d(\bar{Z})_{smb}$ | $d(\bar{Z})_{hml}$ | $d(\bar{Z})_{mom}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{1h}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{2h}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | $d(\bar{Z})_{3h}$ | Dist |
|--|-------------------|--------------------|--------------------|--------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | | | | | | | | | | |
| Coefficient | 1.361 | -1.499 | -5.777 | -14.199 | -7.094 | -6.133 | -0.130 | 0.558 | 494.342 | 89.997 | | | | 0.3998 |
| P-Value | (0.000) | (0.068) | (0.013) | (0.000) | (0.001) | (0.086) | (0.102) | (0.137) | (0.089) | (0.015) | | | | |

Table 21-Continued

| | $d(\bar{Z})_{0t}$ | $d(\bar{Z})_{smb}$ | $d(\bar{Z})_{hml}$ | $d(\bar{Z})_{mom}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{1h}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{2h}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | $d(\bar{Z})_{3h}$ | Dist |
|---|-------------------|--------------------|--------------------|--------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|--------|
| Period from July 1963 to December 2009 | | | | | | | | | | | | | | |
| Coefficient | 1.156 | -2.748 | 0.954 | -1.959 | -4.170 | -4.448 | -3.207 | 14.127 | 1917.188 | 25.726 | | | | 0.3857 |
| P-Value | (0.000) | (0.017) | (0.068) | (0.043) | (0.001) | (0.087) | (0.028) | (0.065) | (0.037) | (0.069) | | | | |
| Period from January 1996 to December 2009 | | | | | | | | | | | | | | |
| Coefficient | 1.700 | -2.559 | 4.215 | -2.540 | -3.443 | -174.685 | -0.010 | 0.000 | 19400.361 | 17.226 | | | | 0.9295 |
| P-Value | (0.000) | (0.017) | (0.068) | (0.043) | (0.001) | (0.087) | (0.028) | (0.065) | (0.037) | (0.069) | | | | |

Table 21-Continued

Panel B: Cubic pricing kernel with Fama-French Factors, Momentum, Human Capital and Housing Wealth

| | $d(\bar{Z})_{0r}$ | $d(\bar{Z})_{smb}$ | $d(\bar{Z})_{hml}$ | $d(\bar{Z})_{mom}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{1h}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{2h}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | $d(\bar{Z})_{3h}$ | Dist |
|---|-------------------|--------------------|--------------------|--------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|--------|
| Period from July 1963 to December 1995 | | | | | | | | | | | | | | |
| Coefficient | 1.380 | -0.441 | 5.459 | -14.714 | -6.131 | -12.029 | -0.308 | 0.382 | 1197.866 | 146.969 | -590.208 | -30842.032 | -1058.834 | 0.3667 |
| P-Value | (0.000) | (0.117) | (0.010) | (0.000) | (0.002) | (0.034) | (0.071) | (0.127) | (0.020) | (0.007) | (0.010) | (0.000) | (0.012) | |
| Period from July 1963 to December 2009 | | | | | | | | | | | | | | |
| Coefficient | 1.177 | -4.262 | 1.216 | -1.585 | -2.323 | -2.935 | -0.433 | 3.636 | 2041.125 | 0.069 | -229.604 | -1.281 | -446.060 | 0.3306 |
| P-Value | (0.000) | (0.005) | (0.058) | (0.044) | (0.008) | (0.057) | (0.067) | (0.069) | (0.028) | (0.168) | (0.017) | (0.000) | (0.048) | |
| Period from January 1996 to December 2009 | | | | | | | | | | | | | | |
| Coefficient | 1.817 | -1.953 | 5.354 | -0.988 | -4.064 | -201.309 | -0.670 | 4.365 | 24243.703 | 16.700 | -0.001 | -90.936 | -1598.621 | 0.8900 |
| P-Value | (0.000) | (0.047) | (0.014) | (0.066) | (0.002) | (0.001) | (0.050) | (0.068) | (0.002) | (0.042) | (0.244) | (0.000) | (0.009) | |

Table 22**Decomposition of Distance Measure**

Table 22 presents a decomposition of the Hansen-Jagannathan distance measure, $\|p\| = \sqrt{E[p]^2 + Var[p]}$. The variable p represents the adjustment to model pricing kernel needed to make it admissible. The column labelled “ $Mean(p)$ ” represents the average of the estimated p , the column labelled “ $Std(p)$ ” represents its standard deviation, and the column labelled “Distance” represents the Jagannathan distance. The row labelled “Linear: No HC, HW”, “Quadratic: No HC, HW”, and “Cubic: No HC, HW” represents the decomposition for the polynomial pricing kernels, omitting human capital and housing wealth. The row labelled “Linear: HC Only”, “Quadratic: HC Only”, and “Cubic: HC Only” represents the decomposition for the polynomial pricing kernels, including human capital only and housing wealth excluded. The row labelled “Linear: HW Only”, “Quadratic: HW Only”, and “Cubic: HW Only” represents the decomposition for the polynomial pricing kernels, including housing wealth only. The row labelled “Linear: HC+HW”, “Quadratic: HC+HW”, and “Cubic: HC+HW” represents the decomposition for the polynomial pricing kernels, including both human capital and housing wealth.

Panel A: Sample Period from July 1963 to December 2009

| Model | $Mean(p)$ | $Std(p)$ | Distance |
|----------------------|-----------|----------|----------|
| Linear: No HC, HW | 0.0002 | 0.5004 | 0.5004 |
| Quadratic: No HC, HW | 0.0002 | 0.5004 | 0.5004 |
| Cubic: No HC, HW | 0.0003 | 0.5003 | 0.5003 |
| Linear: HC Only | 0.0002 | 0.4708 | 0.4708 |
| Quadratic: HC Only | 0.0000 | 0.4543 | 0.4543 |
| Cubic: HC Only | 0.0009 | 0.4496 | 0.4496 |
| Linea: HP Only | 0.0002 | 0.4845 | 0.4845 |
| Quadratic: HP Only | 0.0002 | 0.4845 | 0.4845 |
| Cubic: HP Only | 0.0009 | 0.4496 | 0.4496 |
| Linear: HC+HW | 0.0003 | 0.4596 | 0.4596 |
| Quadratic: HC+HW | 0.0016 | 0.4437 | 0.4437 |
| Cubic: HC+HW | 0.0017 | 0.4394 | 0.4394 |

Table 22-Continued

Panel B: Sample Period from January 1996 to December 2009

| Model | $Mean(p)$ | $Std(p)$ | Distance |
|----------------------|-----------|----------|----------|
| Linear: No HC, HW | 0.0003 | 1.1904 | 1.1904 |
| Quadratic: No HC, HW | 0.0003 | 1.1904 | 1.1904 |
| Cubic: No HC, HW | 0.0003 | 1.1904 | 1.1904 |
| Linear: HC Only | 0.0027 | 1.1213 | 1.1213 |
| Quadratic: HC Only | 0.0048 | 1.1060 | 1.1060 |
| Cubic: HC Only | 0.0080 | 1.0856 | 1.0856 |
| Linear: HW Only | 0.0006 | 1.1724 | 1.1724 |
| Quadratic: HW Only | 0.0022 | 1.1613 | 1.1613 |
| Cubic: HW Only | 0.0030 | 1.0647 | 1.0647 |
| Linear: HC + HW | 0.0050 | 1.0918 | 1.0918 |
| Quadratic: HC + HW | 0.0076 | 1.0563 | 1.0563 |
| Cubic: HC + HW | 0.0088 | 1.0261 | 1.0261 |

5.2.C Summary of Distance Measure

The results show in Table 22 presents estimates of $Mean(p)$ and $Std(p)$. The cubic pricing kernel with human capital and housing wealth again has the lowest value for $Std(p)$, which means this pricing kernel with a small distance measure requires the least adjustment to be admissible. Quadratic pricing kernel with human capital and housing wealth is the second smallest distance measure

Table 24
Specification Tests: Polynomial Pricing Kernels with Global
Restrictions and Human Capital Included

Table 24 presents results of GMM tests of the Euler equation condition,

$$E[(1 + R_{t+1})m_{t+1} | Z_t] - 1 = 0$$

Using the polynomial pricing kernels, m_{t+1} , the coefficients are estimated using the Hansen-Jagannathan (1997) weighting matrix $E[(1 + R_{t+1}) \otimes Z_t][(1 + R_{t+1}) \otimes Z_t]'$. The columns present the coefficients of the pricing kernel evaluated at the means of the instruments. The coefficients are modelled as

$$d_n = I_n (\delta'_n Z_t)^2 \quad I_n = \begin{cases} -1, n=1,3 \\ 1, n=2 \end{cases}$$

In addition to constraining the signs of the coefficients, the following constraints are placed on the pricing kernel.

$$m_{t+1} \geq 0 \quad m'_{t+1} \leq 0$$

p-value for Wald tests of the joint significance of the coefficients are presented in parentheses. The final column presents the Hansen-Jagannathan distance measure. The set of returns used in estimation are those 17 industry-sorted portfolios covering the period July 1963, through December 2009, augmented by the return on a one month Treasury bill.

| | $d(\bar{Z})_{0t}$ | $d(\bar{Z})_{1vw}$ | $d(\bar{Z})_{1l}$ | $d(\bar{Z})_{2vw}$ | $d(\bar{Z})_{2l}$ | $d(\bar{Z})_{3vw}$ | $d(\bar{Z})_{3l}$ | Dist |
|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------|
| Panel A: Linear | | | | | | | | |
| Coefficient | 1.262 | -2.475 | -1.841 | | | | | 0.4598 |
| P-Value | (0.000) | (0.006) | (0.068) | | | | | |
| Panel B: Quadratic | | | | | | | | |
| Coefficient | 1.251 | -2.566 | -1.446 | 0.074 | 247.997 | | | 0.4583 |
| P-Value | (0.000) | (0.007) | (0.102) | (0.274) | (0.121) | | | |
| Panel C: Cubic | | | | | | | | |
| Coefficient | 1.265 | -2.156 | -0.300 | 6.143 | 812.826 | -35.584 | -7702.968 | 0.4482 |
| P-Value | (0.000) | (0.016) | (0.113) | (0.085) | (0.058) | (0.098) | (0.000) | |

Chapter 6

CONCLUSIONS

In the study of asset pricing model of higher order risk factor, I follow an approach similar to Dittmar (2002), using polynomial pricing kernels to generate the empirical performance of a nonlinear model. I extend the polynomial model by examining the impact of additional risk factors including 17 industry portfolio returns, human capital and housing wealth. I also extend the Fama-French model by adding momentum factor.

My initial modelling and test result is consistent with Dittmar (2002)'s findings. When examining the nonlinear asset pricing kernels, the higher order risk factors affect the empirical performance of the models significantly. This conclusion is also valid when considering the housing wealth factor. In addition, the result presented in this thesis provides strong evidence that, including proxies for the return to housing wealth is very beneficial. It significantly improves the different empirical specification performance. This finding is robust for both nonlinear asset pricing models and linear pricing models.

The main difficulty in this research is that, the total wealth is not observable. I extend total wealth and include human capital and housing wealth, as returns on both contribute the significant portion of the aggregate wealth. I follow Jagannathan and Wang (1996) and measure the returns on human capital by calculating two-month moving average of the growth rate in labour income. To estimate the returns on housing wealth, I follow Flavin and Yamashita (2002) and calculate the return to owner occupied household level, which is more volatile than the returns on aggregate wealth. The factors involved in the calculation including the real value of the house, the real interest rate, the marginal income tax rate and the net property tax payment. To simplify the model for housing wealth calculation, the real interest rate, the marginal income tax rate and the net property tax payment are fixed in this thesis. They are 5%, 33% and 25% respectively. The resulting house returns are quite close to risk factor rate, which is around 2% to 3%. The calculated variation of the housing returns is a bit smaller, when considering the recent real estate price trembling in US and Irish market. Even though, the models, including housing wealth, still performs much better as discussed below.

The models are tested in three different time periods. First period is Dittmar's period, from July 1963 to December 1995. Obtained result is consistent with those obtained by Dittmar (2002) that, nonlinearity substantially improves upon the pricing kernel ability to explain the cross section of returns are better than Fama-French model. When the proxy for the return on human capital is included in measuring the return on aggregate wealth, a quadratic and cubic pricing kernels are able to fit the cross section of returns with reduced pricing errors. Moreover, when testing the model, that includes the proxy for the return on housing wealth, the fit of the pricing model are even better as the distance falls to 0.5685, the best distance measure from model specification tests without housing wealth risk factor is 0.5685. And the p-value obtained from higher order risk factors show that they are important for improving the fit of the pricing kernel. Further, I examine the augmented Fama-French three-factor model by adding in momentum factor. The result shows that the model performs substantially better than Fama-French three-factor model. In particular, when including the proxy for the return on housing wealth, the model further reduces the distance measure to 0.3667.

Second period is from July 1963 to December 2009. As explained in the previous chapter, owner-occupied housing plays an important role in economic market. When doing the model specification test on pricing kernel obtained with housing wealth only and pricing kernel with human capital only, the result indicates that housing wealth is a relatively insufficient risk factor as human capital over this sample period. Similar conclusion can be drawn when testing the pricing kernel with both human capital and housing included. However, the results turn out oppositely when examining the size, book to market, human capital and housing wealth in the augmented Fama-French model. Both of the human capital and housing wealth are significant in the model. The performance of the nonlinear pricing kernel significantly improved, the cubic term perform better than quadratic term. Thesis results suggest that nonlinear measures of human capital and housing wealth are able to improve the performance of the pricing kernel. Further, we find that the cubic term in the pricing kernel drives out the significance of both size and book to market factor in the augmented Fama-French model. This finding is similar to the result presented by Dittmar (2002).

Third period is the most recent period from January 1996 to December 2009, which covers the recession period. The results suggest that the housing wealth is sufficient in

this sample period. When estimating the model specification test on housing wealth only, both cubic and quadratic terms are rejected at the significant level. It is not the same case for the previous period from July 1963 to December 2009. The total sample size in this period is much smaller comparing to the previous two. In the future, a similar research can be done with more completed dataset.

In summary, this thesis has important implications for future work in empirical asset pricing. It gives overview of the choice of proxy for the total return examined through the nonlinear asset pricing models. The empirical tests suggest that the measures of aggregate wealth should include housing wealth. It is worth to find a better definition for the housing wealth factor in the future research, to represent better the violate housing price behaviour observed in the recent credit crisis.

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Appendix

R-code for Table 1

```
##clear
rm(list=ls());
graphics.off();
options(warn=1, htmlhelp=TRUE);

## read data from file
##30 industry portfolio returns (monthly %)
p17 = read.table("17_Industry_Portfolios1.txt", header=TRUE);
p17[,-1] = p17[,-1]/100;
##write to file
write.table(p17, "p171.csv", sep=",", row.names=FALSE);

##Calculate mean
y = read.csv("p171.csv");
Mean <- mean(y)
print(Mean)

##calculate standard deviation
SD <- var(y)^0.5
print(SD)
```

R code for Table 2

```
library(lmtest)
library(sandwich)
##clear
rm(list=ls());
graphics.off();
options(warn=1, htmlhelp=TRUE);

##merge and write data file (monthly fractions)
##30 industry portfolio returns (monthly %)
p17 = read.table("17_Industry_Portfolios1.txt", header=TRUE);
p17[,-1] = p17[,-1]/100;

##3 month treasury bill (annualized %)
tb3 = read.table("TB31.txt", header=TRUE);
tb3[,-1] = tb3[,-1]/12;
##fama-french factors (monthly %)
ff = read.table("F-F_Research_Data_Factors1.txt", header=TRUE);
ff[,-1] = ff[,-1]/100;

##dividend yield (monthly fractions)
dy <- read.table("dividend yield1.txt", header=TRUE);
##combine all regressors into one data frame
zt <- data.frame(rmrf = ff$rmrf,
                 divyld = dy$Y,
                 yldspr = tb3[,2] - ff$RF,
                 tb = ff$RF);

##write to file
write.table(p17, "p171.csv", sep=",", row.names=FALSE);
y = read.csv("p171.csv"); #R{t+1}
# get rid of "date" from y
y$Date = NULL;
#regress each column of y on zt
n <- nrow(y)
stopifnot(n==nrow(zt))

for(i in seq.int(ncol(y))){

ols<- lm(y[-1,i]~ rmrf + divyld + yldspr + tb, data = zt[-n,]) #regress y[,i] on zt with
appropriate lead/lag

w<-waldtest(ols,test="Chisq",vcov=NeweyWest) #wald joint test with newey-
west covariance

cat("---", names(y)[i], "---\n") #print portfolio name

print(w) #print test result
```


R-code for Table 3

```
##clear
rm(list=ls());
graphics.off();
options(warn=1, htmlhelp=TRUE);

##merge and write data file (monthly fractions)
##merge and write data file (monthly fractions)
data = function(fnam="dat.csv") {

  ##17 industry portfolio returns (monthly %)
  p17 = read.table("17_Industry_Portfolios1.txt", header=TRUE);
  p17[, -1] = p17[, -1]/100;

  ##3 month treasury bill (annualized %)
  tb3 = read.table("TB31.txt", header=TRUE);
  tb3[, -1] = tb3[, -1]/12;
  ##fama-french factors (monthly %)
  ff = read.table("F-F_Research_Data_Factors1.txt", header=TRUE);
  ff[, -1] = ff[, -1]/100;

  ##Human capital
  hc = read.table("HC1.txt", header=TRUE);
  rl = hc$Rl    # net return

  one <- rep(1,390);
  ##dividend yield (monthly fractions)
  dy <- read.table("dividend yield1.txt", header=TRUE);
  ##combine all regressors into one data frame
  dat <- data.frame(one, ff, rmrf = ff$rmrf, rm = ff$rmrf + ff$RF, divyld = dy$Y,
  yldspr = tb3[,2] - ff$RF, tb = ff$RF, rl);

  ##write to file
  write.table(p17, "p17.csv", sep=",", row.names=FALSE);
  write.table(dat, "zt.csv", sep=",", row.names=FALSE);
}

##column-wise kronecker product
ckron = function(A, B) {
  stopifnot(nrow(A)==nrow(B))
  ca = ncol(A);
  cb = ncol(B);
  AB = matrix(NA, nrow(A), ca*cb);
  jdx = seq.int(cb);
  for (j in seq.int(ca)) {
    AB[,jdx] = A[,j]*B;
    jdx = jdx + cb;
  }
  return(AB)
}
```

```

}

##gmm with hansen-jaganathan fixed weights
gmm3 = function(vp, R, rm1, zt, pow=3L, method=c("optim", "nlminb"),
control=list()) {
  method = match.arg(method);
  ##optimal weighting matrix using E[(R*zt)(R*zt)']
  W <- solve( crossprod( ckron(1+R, zt )/nrow(R) );
  ##check only once
  stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1));
  ##gmm objective function to minimize
  objf <- function(vpar){
    ##pricing kernel (vectorized)
    m1 = (zt %*% vpar[1:5])^2;#constant
    m1 = m1 - (zt %*% vpar[6:10])^2*rm1;
    if (pow>1) m1 = m1 + (zt %*% vpar[11:15])^2*rm1^2
    if (pow>2) m1 = m1 - (zt %*% vpar[16:20])^2*rm1^3
    ##pricing error (matrix)
    v = ckron((1+R)*c(m1) - 1, zt);
    g = colMeans(v);#(12)
    return( crossprod(g, crossprod(W, g) )
  }

  if (method=="optim") optim(vp, objf, control=control) else nlminb(vp, objf,
control=control)
}

##optimization
tab3a = function(pow=3L) {
  ##read data
  y = read.csv("p17.csv"); #R{t+1}
  y$Date = NULL;
  z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
  R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
  zt = as.matrix(z0[-nrow(z0),]); #z{t}
  rm = read.csv("zt.csv")["rm"];
  rm1 = as.matrix(rm[-1,]) #R{m,t+1}
  ##fit gmm with hansen-jaganathan fixed weights
  vp = rep(0.1, ncol(zt)*(1+pow));#starting values
  ##polytope
  gmm = gmm3(vp, R, rm1, zt, pow, method="optim", control=list(maxit=9000));
  cat(sprintf("fmin = %13.9f, info=%i, iter=%i", gmm$value, gmm$convergence,
gmm$counts[1]), "\n");
  ##quasi-newton
  gmm = gmm3(gmm$par, R, rm1, zt, pow, method="nlminb", control=list(trace=0,
eval.max=9000, iter.max=9000));
  cat(sprintf("fmin = %13.9f, info=%i, mesg=%s", gmm$objective, gmm$convergence,
gmm$message), "\n");
  save(gmm, file=paste("tab3", pow, ".Rdata", sep=""));
}

```

```

print(gmm$par);
}

###test parameters at means
tab3b = function(pow=3L) {
  ##read data
  y = read.csv("p17.csv"); #R{t+1}
  y$Date = NULL;
  z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
  R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
  zt = as.matrix(z0[-nrow(z0),]); #z{t}
  rm = read.csv("zt.csv")["rm"];
  rm1 = as.matrix(rm[-1,]) #R{m,t+1}
  ##check only once
  stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1));

  ##read estimated parameter values
  load(paste("tab3", pow, ".Rdata", sep=""));#gmm
  vp = gmm$par;
  ##optimal weigting matrix using E[(R*zt)(R*zt)']
  rz = ckron(1+R, zt);
  W = solve( crossprod(rz)/nrow(R) );

  ##return sample moment conditions
  fmom <- function(vpar, means=TRUE){
    ##pricing kernel (vectorized)
    m1 = (zt %*% vpar[1:5])^2;#constant
    m1 = m1 - (zt %*% vpar[6:10])^2*rm1;
    if (pow>1) m1 = m1 + (zt %*% vpar[11:15])^2*rm1^2;
    if (pow>2) m1 = m1 - (zt %*% vpar[16:20])^2*rm1^3;
    ##pricing error (matrix)
    v = ckron((1+R)*c(m1) - 1, zt)
    if (means) v = colMeans(v);#(12)
    return(v)
  }

  ##for numeric derivatives
  library(numDeriv);
  ##evaluate parameter covariance matrix
  grd = jacobian(fmom, vp);#d
  dw = crossprod(grd, W);#d'W
  dwd = solve(dw %*% grd);#(d'W*d)^{-1}
  # dwd = solve(dw %*% grd, tol=1e-30);#(d'W*d)^{-1}
  ss = cov( fmom(vp, means=FALSE) );#S
  pcov = (dwd %*% (dw %*% tcrossprod(ss, dw)) %*% dwd) / nrow(zt);
  ##print estimates and standard errors
  temp = data.frame(par=vp, se=sqrt(diag(pcov)));
  temp$t_ratio = temp$par/temp$se;
  print(temp)
}

```

```

##average z
zbar = colMeans(zt);
print(zbar)
##plot time-varying parameters
par(mfcol=c(2,2));
pdx = seq.int(ncol(zt));
for (i in seq(0, pow)) {
  dt = (-1)^i * (zt %*% vp[pdx])^2;
  plot(dt, type="l", col="blue", xlab="", ylab=paste("d", i, "t", sep=""));
  dz = (-1)^i * crossprod(zbar, vp[pdx])^2;
  abline(h=dz, col=gray(0.5));
  ##delta method
  dzvar = 4*abs(dz)*crossprod(c(crossprod(zbar, pcov[pdx,pdx])), zbar);
  wald = dz^2 / dzvar;
  cat(sprintf("d(zbar)%it: %9.3f, s.e. = %9.3f, pval = %4.3f", i, dz, sqrt(dzvar),
  pchisq(wald, 0.05, lower.tail=FALSE)), "\n");
  pdx = pdx + ncol(zt);
}

##decompose hansen-jagannathan distance
pt = rz %*% crossprod(W, fmom(vp));
n = length(pt);#undo df correction
cat(sprintf("mean(pt) = %7.4f, sd(pt) = %7.4f, dist = %7.4f", mean(pt),
sqrt(var(pt)*(n-1)/n), sqrt(gmm$objective)), "\n");
}
##---main
#data();
#tab3a(1L);
#tab3b(1L);
#tab3a(2L);
tab3b(2L);
#tab3a(3L);
#tab3b(3L);

```

R-code for Table 4

```
##clear
rm(list=ls());
graphics.off();
options(warn=1, htmlhelp=TRUE);

##from MASS (but stripped arg checking)
ginv = function (X, tol = sqrt(.Machine$double.eps)) {
  sv = svd(X);
  pos = (sv$d > max(tol*sv$d[1],0));
  if (all(pos)) sv$v %*% (1/sv$d * t(sv$u))
  else if (!any(pos)) array(0, dim(X)[2:1])
  else sv$v[,pos,drop=FALSE] %*% ((1/sv$d[pos]) * t(sv$u[,pos,drop=FALSE]))
}

##merge and write data file (monthly fractions)
data = function(fnam="dat.csv") {

  ##17 industry portfolio returns (monthly %)
  p17 = read.table("17_Industry_Portfolios1.txt", header=TRUE);
  p17[, -1] = p17[, -1]/100;

  ##3 month treasury bill (annualized %)
  tb3 = read.table("TB31.txt", header=TRUE);
  tb3[, -1] = tb3[, -1]/12;
  ##fama-french factors (monthly %)
  ff = read.table("F-F_Research_Data_Factors1.txt", header=TRUE);
  ff[, -1] = ff[, -1]/100;

  ##Human capital
  hc = read.table("HC1.txt", header=TRUE);
  rl = hc$Rl # net return

  one <- rep(1,390);
  ##dividend yield (monthly fractions)
  dy <- read.table("dividend yield1.txt", header=TRUE);
  ##combine all regressors into one data frame
  dat <- data.frame(one, ff, rmrf = ff$rmrf, rm = ff$rmrf + ff$RF, divyld = dy$Y,
  yldspr = tb3[,2] - ff$RF, tb = ff$RF, rl);

  ##write to file
  write.table(p17, "p17.csv", sep=",", row.names=FALSE);
  write.table(dat, "zt.csv", sep=",", row.names=FALSE);
}

##column-wise kronecker product
ckron = function(A, B) {
  stopifnot(nrow(A)==nrow(B))
```

```

ca = ncol(A);
cb = ncol(B);
AB = matrix(NA, nrow(A), ca*cb);
jdx = seq.int(cb);
for (j in seq.int(ca)) {
  AB[,jdx] = A[,j]*B;
  jdx = jdx + cb;
}
return(AB)
}

gmm4 = function(vp, R, rm1, r11, zt, pow=3L, method=c("optim", "nlminb"),
control=list()) {
  method = match.arg(method);
  ##optimal weighting matrix using E[(R*zt)(R*zt)']
  W <- solve( crossprod( ckron(1+R, zt) )/nrow(R) );
  ##check only once
  stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1), nrow(R)==length(r11));
  ##gmm objective function to minimize
  objf <- function(vpar){
    ##pricing kernel (vectorized)
    m1 = (zt %*% vpar[1:5])^2;#constant
    m1 = m1 - (zt %*% vpar[6:10])^2*rm1;
    m1 = m1 - (zt %*% vpar[11:15])^2*r11;
    if (pow>1) {
      m1 = m1 + (zt %*% vpar[16:20])^2*rm1^2;
      m1 = m1 + (zt %*% vpar[21:25])^2*r11^2;
    }
    if (pow>2) {
      m1 = m1 - (zt %*% vpar[26:30])^2*rm1^3;
      m1 = m1 - (zt %*% vpar[31:35])^2*r11^3;
    }
    ##pricing error (matrix)
    v = ckron((1+R)*c(m1) - 1, zt);
    g = colMeans(v);#(12)
    return( crossprod(g, crossprod(W, g)) )
  }

  if (method=="optim") optim(vp, objf, control=control) else nlminb(vp, objf,
control=control)
}

##optimization with human capital
tab4a = function(pow=3L) {
  ##read data
  y = read.csv("p17.csv"); #R{t+1}
  y$Date = NULL;
  z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
  R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}

```

```

zt = as.matrix(z0[-nrow(z0),]); #z{t}
rm = read.csv("zt.csv")["rm"];
rm1 = as.matrix(rm[-1,]) #R{m,t+1}
rl = read.csv("zt.csv")[,c("rl")];
rl1 = as.matrix(rl[-1]) #R{1,t+1}
##fit gmm with hansen-jaganathan fixed weights
vp = rep(0.1, ncol(z0)*(1+2*pow));#starting values
##polytope
gmm = gmm4(vp, R, rm1, rl1, zt, pow, method="optim",
control=list(maxit=90000));
cat(sprintf("fmin = %13.9f, info=%i, iter=%i", gmm$value, gmm$convergence,
gmm$counts[1]), "\n");
##quasi-newton
gmm = gmm4(gmm$par, R, rm1, rl1, zt, pow, method="nlminb",
control=list(trace=0, eval.max=90000, iter.max=90000));
cat(sprintf("fmin = %13.9f, info=%i, mesg=%s", gmm$objective, gmm$convergence,
gmm$message), "\n");
save(gmm, file=paste("tab4", pow, ".Rdata", sep=""));
print(gmm$par);
}

##test parameters at means
tab4b = function(pow=3L) {
##read data
y = read.csv("p17.csv"); #R{t+1}
y$Date = NULL;
z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
zt = as.matrix(z0[-nrow(z0),]); #z{t}
rm = read.csv("zt.csv")["rm"];
rm1 = as.matrix(rm[-1,]) #R{m,t+1}
rl = read.csv("zt.csv")[,c("rl")];
rl1 = as.matrix(rl[-1]) #R{1,t+1}
##check only once
stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1), nrow(R)==length(rl1));

##read estimated parameter values
load(paste("tab4", pow, ".Rdata", sep=""));#gmm
vp = gmm$par;
##optimal weighting matrix using E[(R*zt)(R*zt)']
rz = ckron(1+R, zt);
W = solve( crossprod(rz)/nrow(R) );

##return sample moment conditions
fmom <- function(vpar, means=TRUE){
##pricing kernel (vectorized)
m1 = (zt %*% vpar[1:5])^2;#constant
m1 = m1 - (zt %*% vpar[6:10])^2*rm1;
m1 = m1 - (zt %*% vpar[11:15])^2*rl1;
if (pow>1) {

```

```

    m1 = m1 + (zt %*% vpar[16:20])^2*rm1^2;
    m1 = m1 + (zt %*% vpar[21:25])^2*rl1^2;
  }
  if (pow>2) {
    m1 = m1 - (zt %*% vpar[26:30])^2*rm1^3;
    m1 = m1 - (zt %*% vpar[31:35])^2*rl1^3;
  }
  ##pricing error (matrix)
  v = ckron((1+R)*c(m1) - 1, zt);
  if (means) v = colMeans(v);#(12)
  return(v)
}

##for numeric derivatives
library(numDeriv);
##evaluate parameter covariance matrix
grd = jacobian(fmom, vp);#d
dw = crossprod(grd, W);#d'W
#dwd = solve(dw %*% grd, tol=.Machine$double.eps);#(d'W*d)^{-1}
dwd = solve(dw %*% grd, tol=1e-30);#(d'W*d)^{-1}
#dwd = try(solve(dw %*% grd), silent=FALSE);#(d'W*d)^{-1}
if (inherits(dwd, "try-error")) dwd = ginv(dw %*% grd)
ss = cov( fmom(vp, means=FALSE) );#S
pcov = (dwd %*% (dw %*% tcrossprod(ss, dw)) %*% dwd) / nrow(zt);
##print estimates and standard errors
temp = data.frame(par=vp, se=sqrt(diag(pcov)));
temp$st_ratio = temp$par/temp$se;
print(temp)

##average z
zbar = colMeans(zt);
print(zbar)
tvpar = function(dz, pcov, dnam) {#delta method
  dzvar = 4 * abs(dz) * crossprod(c(crossprod(zbar, pcov)), zbar);
  wald = dz^2 / dzvar;
cat(sprintf("d(zbar)%s: %8.3f, s.e. = %9.3f, pval = %4.3f", dnam, dz, sqrt(dzvar),
pchisq(wald, 0.05,lower.tail=FALSE)), "\n");
}
##plot time-varying parameters
par(mfcol=c(2,2));
pdx = seq.int(ncol(zt));
##constant
dt = (zt %*% vp[pdx])^2;
plot(dt, type="l", col="blue", xlab="", ylab="d0t");
dz = crossprod(zbar, vp[pdx])^2;
abline(h=dz, col=gray(0.5));
tvpar(dz, pcov[pdx,pdx], "_0t");
for (i in seq.int(pow)) {
  ##r{market}
  pdx = pdx + ncol(zt);

```



```

dt = (-1)^i * (zt %*% vp[pdx])^2;
plot(dt, type="l", col="blue", xlab="", ylab=paste("d", i, "t", sep=""));
dz = (-1)^i * crossprod(zbar, vp[pdx])^2;
abline(h=dz, col="blue", lty="dashed");
tvar(dz, pcov[pdx,pdx], paste("_", i, "m", sep=""));
##r{labor}
pdx = pdx + ncol(zt);
dt = (-1)^i * (zt %*% vp[pdx])^2;
lines(dt, col="red");
dz = (-1)^i * crossprod(zbar, vp[pdx])^2;
abline(h=dz, col="red", lty="dashed");
tvar(dz, pcov[pdx,pdx], paste("_", i, "l", sep=""));
}

##decompose hansen-jagannathan distance
pt = rz %*% crossprod(W, fmom(vp));
n = length(pt);#undo df correction
cat(sprintf("mean(pt) = %7.4f, sd(pt) = %7.4f, dist = %7.4f", mean(pt),
sqrt(var(pt)*(n-1)/n), sqrt(gmm$objective)), "\n");
}

##pricing kernel surface
fig1 = function() {
library(lattice);
##read data
y = read.csv("p17.csv"); #R{t+1}
y$Date = NULL;
z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
##evaluate at means of z0
zbar = colMeans(z0);
R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
zt = as.matrix(z0[-nrow(z0),]); #z{t}
rm = read.csv("zt.csv")["rm"];
rm1 = as.matrix(rm[-1,]) #R{m,t+1}
rl = read.csv("zt.csv")[,c("rl")];
rl1 = as.matrix(rl[-1]) #R{1,t+1}

stopifnot(length(zbar)==5);

##evaluate pricing kernel
sdf <- function(vpar, rm1, rl1, pow=3L){
m1 = crossprod(zbar, vpar[1:5])^2;#constant
m1 = m1 - crossprod(zbar, vpar[6:10])^2*rm1;
m1 = m1 - crossprod(zbar, vpar[11:15])^2*rl1;
if (pow>1) {
m1 = m1 + (zbar %*% vpar[16:20])^2*rm1^2;
m1 = m1 + (zbar %*% vpar[21:25])^2*rl1^2;
}
if (pow>2) {
m1 = m1 - (zbar %*% vpar[26:30])^2*rm1^3;
}
}

```

```

    m1 = m1 - (zbar %*% vpar[31:35])^2*r11^3;
  }
  return(m1)
}

x = seq(min(rm1), max(rm1), len=20);#rm
y = seq(min(r11), max(r11), len=20);#rg
g = expand.grid(x = x, y = y);
for (pow in seq.int(3)) {
  load(paste("tab4", pow, ".Rdata", sep=""));#gmm
  g[[paste("pow", pow, sep="")] = sdf(gmm$par, g$x, g$y, pow)
}
#print(wireframe(pow1+pow2+pow3 ~ x*y, g, outer=TRUE, distance=0,
col=gray(0.7), screen=list(z=20, x=-50), colorkey=FALSE, drape=TRUE,
default.scales=list(distance=c(1,1,1), arrows=FALSE),
lattice.options=list(as.table=FALSE), xlab="market", ylab="labor", zlab="kernel"))
print(wireframe(pow3 ~ x*y, g, outer=TRUE, distance=0, col=gray(0.7),
screen=list(z=20, x=-50), colorkey=FALSE, drape=TRUE,
default.scales=list(distance=c(1,1,1), arrows=FALSE),
lattice.options=list(as.table=FALSE), xlab="market", ylab="labor", zlab="m"))
}

#data()
#tab4a(1L);
#tab4b(1L);
#tab4a(2L);
#tab4b(2L);
#tab4a(3L);
#tab4b(3L);
fig1()

```

R-code for Table 5

```
##clear
rm(list=ls());
graphics.off();
options(warn=1, htmlhelp=TRUE);

##from MASS (but stripped arg checking)
ginv = function (X, tol = sqrt(.Machine$double.eps)) {
  sv = svd(X);
  pos = (sv$d > max(tol*sv$d[1],0));
  if (all(pos)) sv$v %*% (1/sv$d * t(sv$u))
  else if (!any(pos)) array(0, dim(X)[2:1])
  else sv$v[,pos,drop=FALSE] %*% ((1/sv$d[pos]) * t(sv$u[,pos,drop=FALSE]))
}

##merge and write data file (monthly fractions)
data = function(fnam="dat.csv") {

  ##17 industry portfolio returns (monthly %)
  p17 = read.table("17_Industry_Portfolios1.txt", header=TRUE);
  p17[,-1] = p17[,-1]/100;

  ##3 month treasury bill (annualized %)
  tb3 = read.table("TB31.txt", header=TRUE);
  tb3[,-1] = tb3[,-1]/12;
  ##fama-french factors (monthly %)
  ff = read.table("F-F_Research_Data_Factors1.txt", header=TRUE);
  ff[,-1] = ff[,-1]/100;

  ##Human capital
  hc = read.table("HC1.txt", header=TRUE);
  rl = hc$Rl      # net return

  ## human capital house price index
  HP = read.table("house_price_index1.txt", header = TRUE);
  hi = HP$real_return;

  one <- rep(1,390);
  ##dividend yield (monthly fractions)
  dy <- read.table("dividend_yield1.txt", header=TRUE);
  ##combine all regressors into one data frame
  dat <- data.frame(one, ff, rmrf = ff$rmrf, rm = ff$rmrf + ff$RF, divyld = dy$Y,
yldspr = tb3[,2] - ff$RF, tb = ff$RF, rl, hi);

  ##write to file
  write.table(p17, "p17.csv", sep=",", row.names=FALSE);
  write.table(dat, "zt.csv", sep=",", row.names=FALSE);
}
```

```

##column-wise kronecker product
ckron = function(A, B) {
  stopifnot(nrow(A)==nrow(B))
  ca = ncol(A);
  cb = ncol(B);
  AB = matrix(NA, nrow(A), ca*cb);
  jdx = seq.int(cb);
  for (j in seq.int(ca)) {
    AB[,jdx] = A[,j]*B;
    jdx = jdx + cb;
  }
  return(AB)
}

gmm4 = function(vp, R, rm1, rh, zt, pow=3L, method=c("optim", "nlminb"),
control=list()) {
  method = match.arg(method);
  ##optimal weighting matrix using E[(R*zt)(R*zt)']
  W <- solve( crossprod( ckron(1+R, zt) )/nrow(R) );
  ##check only once
  stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1), nrow(R)==length(rh));
  ##gmm objective function to minimize
  objf <- function(vpar){
    ##pricing kernel (vectorized)
    m1 = (zt %*% vpar[1:5])^2;#constant
    m1 = m1 - (zt %*% vpar[6:10])^2*rm1;
    m1 = m1 - (zt %*% vpar[11:15])^2*rh;

    if (pow>1) {
      m1 = m1 + (zt %*% vpar[16:20])^2*rm1^2;
      m1 = m1 + (zt %*% vpar[21:25])^2*rh^2
    }
    if (pow>2) {
      m1 = m1 - (zt %*% vpar[26:30])^2*rm1^3;
      m1 = m1 - (zt %*% vpar[31:35])^2*rh^3;
    }
    ##pricing error (matrix)
    v = ckron((1+R)*c(m1) - 1, zt);
    g = colMeans(v);#(12)
    return( crossprod(g, crossprod(W, g)) )
  }

  if (method=="optim") optim(vp, objf, control=control) else nlminb(vp, objf,
control=control)
}

##optimization with human capital
tab4a = function(pow=3L) {

```

```

##read data
y = read.csv("p17.csv"); #R{t+1}
y$Date = NULL;
z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
zt = as.matrix(z0[-nrow(z0),]); #z{t}
rm = read.csv("zt.csv")["rm"];
rm1 = as.matrix(rm[-1,]) #R{m,t+1}
#rl = read.csv("zt.csv")[,c("rl")];
#rl1 = as.matrix(rl[-1]) #R{1,t+1}
hi = read.csv("zt.csv")[,c("hi")];
rh = as.matrix(hi[-1]) #R{h,t+1}
##fit gmm with hansen-jaganathan fixed weights
vp = rep(0.1, ncol(z0)*(1+2*pow));#starting values
##polytope
gmm = gmm4(vp, R, rm1,rh, zt, pow, method="optim", control=list(maxit=90000));
cat(sprintf("fmin = %13.9f, info=%i, iter=%i", gmm$value, gmm$convergence,
gmm$counts[1]), "\n");
##quasi-newton
gmm = gmm4(gmm$par, R, rm1, rh, zt, pow, method="nlminb",
control=list(trace=0, eval.max=90000, iter.max=90000));
cat(sprintf("fmin = %13.9f, info=%i, mesg=%s", gmm$objective, gmm$convergence,
gmm$message), "\n");
save(gmm, file=paste("tab4HPI", pow, ".Rdata", sep=""));
print(gmm$par);
}

```

```

##test parameters at means
tab4b = function(pow=3L) {
##read data
y = read.csv("p17.csv"); #R{t+1}
y$Date = NULL;
z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
zt = as.matrix(z0[-nrow(z0),]); #z{t}
rm = read.csv("zt.csv")["rm"];
rm1 = as.matrix(rm[-1,]) #R{m,t+1}
#rl = read.csv("zt.csv")[,c("rl")];
#rl1 = as.matrix(rl[-1]) #R{1,t+1}
hi = read.csv("zt.csv")[,c("hi")];
rh = as.matrix(hi[-1]) #R{h,t+1}
##check only once
stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1), nrow(R)==length(rh));

##read estimated parameter values
load(paste("tab4HPI", pow, ".Rdata", sep=""));#gmm
vp = gmm$par;
##optimal weighting matrix using  $E[(R*zt)(R*zt)']$ 
rz = kron(1+R, zt);
W = solve( crossprod(rz)/nrow(R) );

```

```

##return sample moment conditions
fmom <- function(vpar, means=TRUE){
  ##pricing kernel (vectorized)
  m1 = (zt %*% vpar[1:5])^2;#constant
  m1 = m1 - (zt %*% vpar[6:10])^2*rm1;
  m1 = m1 - (zt %*% vpar[11:15])^2*rh;

  if (pow>1) {
    m1 = m1 + (zt %*% vpar[16:20])^2*rm1^2;
    m1 = m1 + (zt %*% vpar[21:25])^2*rh^2
  }
  if (pow>2) {
    m1 = m1 - (zt %*% vpar[26:30])^2*rm1^3;
    m1 = m1 - (zt %*% vpar[31:35])^2*rh^3;
  }
  ##pricing error (matrix)
  v = ckron((1+R)*c(m1) - 1, zt);
  if (means) v = colMeans(v);#(12)
  return(v)
}
##for numeric derivatives
library(numDeriv);
##evaluate parameter covariance matrix
grd = jacobian(fmom, vp);#d
dw = crossprod(grd, W);#d'W
# dwd = solve(dw %*% grd, tol=Machine$double.eps);#(d'W*d)^{-1}
#dwd = try(solve(dw %*% grd), silent=FALSE);#(d'W*d)^{-1}
dwd = solve(dw %*% grd, tol=1e-30);#(d'W*d)^{-1}
if (inherits(dwd, "try-error")) dwd = ginv(dw %*% grd)
ss = cov( fmom(vp, means=FALSE) );#S
pcov = (dwd %*% (dw %*% tcrossprod(ss, dw)) %*% dwd) / nrow(zt);
##print estimates and standard errors
temp = data.frame(par=vp, se=sqrt(diag(pcov)));
temp$t_ratio = temp$par/temp$se;
print(temp)

##average z
zbar = colMeans(zt);
print(zbar)
tvpar = function(dz, pcov, dnam) {#delta method
  dzvar = 4 * abs(dz) * crossprod(c(crossprod(zbar, pcov)), zbar);
  wald = dz^2 / dzvar;
cat(sprintf("d(zbar)%s: %8.3f, s.e. = %9.3f, pval = %4.3f", dnam, dz, sqrt(dzvar),
pchisq(wald, 0.05, lower.tail=FALSE)), "\n");
}
##plot time-varying parameters
par(mfcol=c(2,2));
pdx = seq.int(ncol(zt));
##constant

```

```

dt = (zt %*% vp[pdx])^2;
plot(dt, type="l", col="blue", xlab="", ylab="d0t");
dz = crossprod(zbar, vp[pdx])^2;
abline(h=dz, col=gray(0.5));
tvar(dz, pcov[pdx,pdx], "_0t");
for (i in seq.int(pow)) {
  ##r{market}
  pdx = pdx + ncol(zt);
  dt = (-1)^i * (zt %*% vp[pdx])^2;
  plot(dt, type="l", col="blue", xlab="", ylab=paste("d", i, "t", sep=""));
  dz = (-1)^i * crossprod(zbar, vp[pdx])^2;
  abline(h=dz, col="blue", lty="dashed");
  tvar(dz, pcov[pdx,pdx], paste("_", i, "m", sep=""));

  ##r{house}
  pdx = pdx + ncol(zt);
  dt = (-1)^i * (zt %*% vp[pdx])^2;
  lines(dt, col="green");
  dz = (-1)^i * crossprod(zbar, vp[pdx])^2;
  abline(h=dz, col="red", lty="dashed");
  tvar(dz, pcov[pdx,pdx], paste("_", i, "h", sep=""));
}

##decompose hansen-jagannathan distance
pt = rz %*% crossprod(W, fmom(vp));
n = length(pt);#undo df correction
cat(sprintf("mean(pt) = %7.4f, sd(pt) = %7.4f, dist = %7.4f", mean(pt),
sqrt(var(pt)*(n-1)/n), sqrt(gmm$objective)), "\n");
}

##pricing kernel surface
fig1 = function() {
  library(lattice);
  ##read data
  y = read.csv("p17.csv"); #R{t+1}
  y$Date = NULL;
  z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
  ##evaluate at means of z0
  zbar = colMeans(z0);
  R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
  zt = as.matrix(z0[-nrow(z0),]); #z{t}
  rm = read.csv("zt.csv")["rm"];
  rm1 = as.matrix(rm[-1,]) #R{m,t+1}
  hi = read.csv("zt.csv")[,c("hi")];
  rh = as.matrix(hi[-1]) #R{h,t+1}

  stopifnot(length(zbar)==5);

  ##evaluate pricing kernel
  sdf <- function(vpar, rm1, rh, pow=3L){

```

```

m1 = crossprod(zbar, vpar[1:5])^2;#constant
m1 = m1 - crossprod(zbar, vpar[6:10])^2*rm1;
m1 = m1 - crossprod(zbar, vpar[11:15])^2*rh;
if (pow>1) {
  m1 = m1 + (zbar %*% vpar[16:20])^2*rm1^2;
  m1 = m1 + (zbar %*% vpar[21:25])^2*rh^2;
}
if (pow>2) {
  m1 = m1 - (zbar %*% vpar[26:30])^2*rm1^3;
  m1 = m1 - (zbar %*% vpar[31:35])^2*rh^3;
}
return(m1)
}

x = seq(min(rm1), max(rm1), len=20);#rm
y = seq(min(rh), max(rh), len=20);#rg
g = expand.grid(x = x, y = y);
for (pow in seq.int(3)) {
  load(paste("tab4HPI", pow, ".Rdata", sep=""));#gmm
  g[[paste("pow", pow, sep="")] = sdf(gmm$par, g$x, g$y, pow)
}
#print(wireframe(pow1+pow2+pow3 ~ x*y, g, outer=TRUE, distance=0,
col=gray(0.7), screen=list(z=20, x=-50), colorkey=FALSE, drape=TRUE,
default.scales=list(distance=c(1,1,1), arrows=FALSE),
lattice.options=list(as.table=FALSE), xlab="market", ylab="labor", zlab="kernel"))
print(wireframe(pow3 ~ x*y, g, outer=TRUE, distance=0, col=gray(0.7),
screen=list(z=20, x=-50), colorkey=FALSE, drape=TRUE,
default.scales=list(distance=c(1,1,1), arrows=FALSE),
lattice.options=list(as.table=FALSE), xlab="market", ylab="house", zlab="m"))
}

#data();
#tab4a(1L);
#tab4b(1L);
#tab4a(2L);
#tab4b(2L);
#tab4a(3L);
#tab4b(3L);
fig1()

```


R-code for Table 6

```
##clear
rm(list=ls());
graphics.off();
options(warn=1, htmlhelp=TRUE);

##merge and write data file (monthly fractions)
data = function(fnam="dat.csv") {

  ##17 industry portfolio returns (monthly %)
  p17 = read.table("17_Industry_Portfolios1.txt", header=TRUE);
  p17[, -1] = p17[, -1]/100;

  ##3 month treasury bill (annualized %)
  tb3 = read.table("TB31.txt", header=TRUE);
  tb3[, -1] = tb3[, -1]/12;
  ##fama-french factors (monthly %)
  ff = read.table("F-F_Research_Data_Factors1.txt", header=TRUE);
  ff[, -1] = ff[, -1]/100;

  ##Human capital
  hc = read.table("HC1.txt", header=TRUE);
  rl = hc$Rl    # net return

  ## human capital house price index

  HP = read.table("house_price_index1.txt", header = TRUE);
  hi = HP$real_return;

  one <- rep(1,558);
  ##dividend yield (monthly fractions)
  dy <- read.table("dividend yield1.txt", header=TRUE);
  ##combine all regressors into one data frame
  dat <- data.frame(one, ff, rmrf = ff$rmrf, rm = ff$rmrf + ff$RF, divyld = dy$Y,
  yldspr = tb3[,2] - ff$RF, tb = ff$RF, rl, hi);

  ##write to file
  write.table(p17, "p17.csv", sep=",", row.names=FALSE);
  write.table(dat, "zt.csv", sep=",", row.names=FALSE);
}

##column-wise kronecker product
ckron = function(A, B) {
  stopifnot(nrow(A)==nrow(B))
  ca = ncol(A);
  cb = ncol(B);
  AB = matrix(NA, nrow(A), ca*cb);
  jdx = seq.int(cb);
```

```

for (j in seq.int(ca)) {
  AB[,jdx] = A[,j]*B;
  jdx = jdx + cb;
}
return(AB)
}

gmm4 = function(vp, R, rm1, r11, rh, zt, pow=3L, method=c("optim", "nlminb"),
control=list()) {
  method = match.arg(method);
  ##optimal weighting matrix using E[(R*zt)(R*zt)']
  W <- solve( crossprod( ckron(1+R, zt) )/nrow(R) );
  ##check only once
  stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1), nrow(R)==length(r11),
nrow(R)==length(rh));
  ##gmm objective function to minimize
  objf <- function(vpar){
    ##pricing kernel (vectorized)
    m1 = (zt %*% vpar[1:5])^2;#constant
    m1 = m1 - (zt %*% vpar[6:10])^2*rm1;
    m1 = m1 - (zt %*% vpar[11:15])^2*r11;
    m1 = m1 - (zt %*% vpar[16:20])^2*rh;
    if (pow>1) {
      m1 = m1 + (zt %*% vpar[21:25])^2*rm1^2;
      m1 = m1 + (zt %*% vpar[26:30])^2*r11^2;
      m1 = m1 + (zt %*% vpar[31:35])^2*rh^2
    }
    if (pow>2) {
      m1 = m1 - (zt %*% vpar[36:40])^2*rm1^3;
      m1 = m1 - (zt %*% vpar[41:45])^2*r11^3;
      m1 = m1 - (zt %*% vpar[46:50])^2*rh^3;
    }
    ##pricing error (matrix)
    v = ckron((1+R)*c(m1) - 1, zt);
    g = colMeans(v);#(12)
    return( crossprod(g, crossprod(W, g)) )
  }

  if (method=="optim") optim(vp, objf, control=control) else nlminb(vp, objf,
control=control)
}

##optimization with human capital
tab4a = function(pow=3L) {
  ##read data
  y = read.csv("p17.csv"); #R{t+1}
  y$Date = NULL;
  z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
  R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}

```

```

zt = as.matrix(z0[-nrow(z0),]); #z{t}
rm = read.csv("zt.csv")["rm"];
rm1 = as.matrix(rm[-1,]) #R{m,t+1}
rl = read.csv("zt.csv")[,c("rl")];
rl1 = as.matrix(rl[-1]) #R{1,t+1}
hi = read.csv("zt.csv")[,c("hi")];
rh = as.matrix(hi[-1]) #R{h,t+1}
##fit gmm with hansen-jaganathan fixed weights
vp = rep(0.1, ncol(z0)*(1+3*pow));#starting values
##polytope
gmm = gmm4(vp, R, rm1, rl1, rh, zt, pow, method="optim",
control=list(maxit=90000));
cat(sprintf("fmin = %13.9f, info=%i, iter=%i", gmm$value, gmm$convergence,
gmm$count[1]), "\n");
##quasi-newton
gmm = gmm4(gmm$par, R, rm1, rl1, rh, zt, pow, method="nlminb",
control=list(trace=0, eval.max=90000, iter.max=90000));
cat(sprintf("fmin = %13.9f, info=%i, mesg=%s", gmm$objective, gmm$convergence,
gmm$message), "\n");
save(gmm, file=paste("tab4", pow, ".Rdata", sep=""));
print(gmm$par);
}

```

```

##test parameters at means
tab4b = function(pow=3L) {
##read data
y = read.csv("p17.csv"); #R{t+1}
y$Date = NULL;
z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
zt = as.matrix(z0[-nrow(z0),]); #z{t}
rm = read.csv("zt.csv")["rm"];
rm1 = as.matrix(rm[-1,]) #R{m,t+1}
rl = read.csv("zt.csv")[,c("rl")];
rl1 = as.matrix(rl[-1]) #R{1,t+1}
hi = read.csv("zt.csv")[,c("hi")];
rh = as.matrix(hi[-1]) #R{h,t+1}
##check only once
stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1), nrow(R)==length(rl1),
nrow(R)==length(rh));

```

```

##read estimated parameter values
load(paste("tab4", pow, ".Rdata", sep=""));#gmm
vp = gmm$par;
##optimal weighting matrix using  $E[(R*zt)(R*zt)']$ 
rz = ckron(1+R, zt);
W = solve( crossprod(rz)/nrow(R) );

```

```

##return sample moment conditions
fmom <- function(vpar, means=TRUE){

```

```

##pricing kernel (vectorized)
m1 = (zt %*% vpar[1:5])^2;#constant
m1 = m1 - (zt %*% vpar[6:10])^2*rm1;
m1 = m1 - (zt %*% vpar[11:15])^2*r11;
m1 = m1 - (zt %*% vpar[16:20])^2*rh;
if (pow>1) {
  m1 = m1 + (zt %*% vpar[21:25])^2*rm1^2;
  m1 = m1 + (zt %*% vpar[26:30])^2*r11^2;
  m1 = m1 + (zt %*% vpar[31:35])^2*rh^2
}
if (pow>2) {
  m1 = m1 - (zt %*% vpar[36:40])^2*rm1^3;
  m1 = m1 - (zt %*% vpar[41:45])^2*r11^3;
  m1 = m1 - (zt %*% vpar[46:50])^2*rh^3;
}
}
##pricing error (matrix)
v = ckron((1+R)*c(m1) - 1, zt);
if (means) v = colMeans(v);#(12)
return(v)
}
##for numeric derivatives
library(numDeriv);
library(MASS);
##evaluate parameter covariance matrix
grd = jacobian(fmom, vp);#d
dw = crossprod(grd, W);#d'W
# dwd = solve(dw %*% grd, tol=.Machine$double.eps);#(d'W*d)^{-1}
dwd = try(solve(dw %*% grd), silent=FALSE);#(d'W*d)^{-1}
if (inherits(dwd, "try-error")) dwd = ginv(dw %*% grd)
ss = cov( fmom(vp, means=FALSE) );#S
pcov = (dwd %*% (dw %*% tcrossprod(ss, dw)) %*% dwd) / nrow(zt);
##print estimates and standard errors
temp = data.frame(par=vp, se=sqrt(diag(pcov)));
temp$t_ratio = temp$par/temp$se;
print(temp)

##average z
zbar = colMeans(zt);
print(zbar)
tvp = function(dz, pcov, dnam) {#delta method
  dzvar = 4 * abs(dz) * crossprod(c(crossprod(zbar, pcov)), zbar);
  wald = dz^2 / dzvar;
cat(sprintf("d(zbar)%s: %8.3f, s.e. = %9.3f, pval = %4.3f", dnam, dz, sqrt(dzvar),
pchisq(wald, 0.05, lower.tail=FALSE)), "\n");
}
##plot time-varying parameters
par(mfcol=c(2,2));
pdx = seq.int(ncol(zt));
##constant
dt = (zt %*% vp[pdx])^2;

```

```

plot(dt, type="l", col="blue", xlab="", ylab="d0t");
dz = crossprod(zbar, vp[pdx])^2;
abline(h=dz, col=gray(0.5));
tvpar(dz, pcov[pdx,pdx], "_0t");
for (i in seq.int(pow)) {
  ##r{market}
  pdx = pdx + ncol(zt);
  dt = (-1)^i * (zt %*% vp[pdx])^2;
  plot(dt, type="l", col="blue", xlab="", ylab=paste("d", i, "t", sep=""));
  dz = (-1)^i * crossprod(zbar, vp[pdx])^2;
  abline(h=dz, col="blue", lty="dashed");
  tvpar(dz, pcov[pdx,pdx], paste("_", i, "m", sep=""));
  ##r{labor}
  pdx = pdx + ncol(zt);
  dt = (-1)^i * (zt %*% vp[pdx])^2;
  lines(dt, col="red");
  dz = (-1)^i * crossprod(zbar, vp[pdx])^2;
  abline(h=dz, col="red", lty="dashed");
  tvpar(dz, pcov[pdx,pdx], paste("_", i, "l", sep=""));
  ##r{house}
  pdx = pdx + ncol(zt);
  dt = (-1)^i * (zt %*% vp[pdx])^2;
  lines(dt, col="green");
  dz = (-1)^i * crossprod(zbar, vp[pdx])^2;
  abline(h=dz, col="green", lty="dashed");
  tvpar(dz, pcov[pdx,pdx], paste("_", i, "h", sep=""));
}

##decompose hansen-jagannathan distance
pt = rz %*% crossprod(W, fmom(vp));
n = length(pt);#undo df correction
cat(sprintf("mean(pt) = %7.4f, sd(pt) = %7.4f, dist = %7.4f", mean(pt),
sqrt(var(pt)*(n-1)/n), sqrt(gmm$objective)), "\n");
}

##pricing kernel surface
fig1 = function() {
  library(lattice);
  ##read data
  y = read.csv("p17.csv"); #R{t+1}
  y$Date = NULL;
  z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
  ##evaluate at means of z0
  zbar = colMeans(z0);
  R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
  zt = as.matrix(z0[-nrow(z0),]); #z{t}
  rm = read.csv("zt.csv")["rm"];
  rm1 = as.matrix(rm[-1,]) #R{m,t+1}
  rl = read.csv("zt.csv")[,c("rl")];
  rl1 = as.matrix(rl[-1]) #R{1,t+1}
}

```

```

stopifnot(length(zbar)==5);

##evaluate pricing kernel
sdf <- function(vpar, rm1, rl1, pow=3L){
  m1 = crossprod(zbar, vpar[1:5])^2;#constant
  m1 = m1 - crossprod(zbar, vpar[6:10])^2*rm1;
  m1 = m1 - crossprod(zbar, vpar[11:15])^2*rl1;
  if (pow>1) {
    m1 = m1 + (zbar %*% vpar[16:20])^2*rm1^2;
    m1 = m1 + (zbar %*% vpar[21:25])^2*rl1^2;
  }
  if (pow>2) {
    m1 = m1 - (zbar %*% vpar[26:30])^2*rm1^3;
    m1 = m1 - (zbar %*% vpar[31:35])^2*rl1^3;
  }
  return(m1)
}

x = seq(min(rm1), max(rm1), len=20);#rm
y = seq(min(rl1), max(rl1), len=20);#rg
g = expand.grid(x = x, y = y);
for (pow in seq.int(3)) {
  load(paste("tab4", pow, ".Rdata", sep=""));#gmm
  g[[paste("pow", pow, sep="")]] = sdf(gmm$par, g$x, g$y, pow)
}
#print(wireframe(pow1+pow2+pow3 ~ x*y, g, outer=TRUE, distance=0,
col=gray(0.7), screen=list(z=20, x=-50), colorkey=FALSE, drape=TRUE,
default.scales=list(distance=c(1,1,1), arrows=FALSE),
lattice.options=list(as.table=FALSE), xlab="market", ylab="labor", zlab="kernel"))
print(wireframe(pow2 ~ x*y, g, outer=TRUE, distance=0, col=gray(0.7),
screen=list(z=20, x=-50), colorkey=FALSE, drape=TRUE,
default.scales=list(distance=c(1,1,1), arrows=FALSE),
lattice.options=list(as.table=FALSE), xlab="market", ylab="labor", zlab="kernel"))
}
#data();
#tab4a(1L);
#tab4b(1L);
#tab4a(2L);
#tab4b(2L);
#tab4a(3L);
#tab4b(3L)

```

R-code for Table 7

```
##clear
rm(list=ls());
graphics.off();
options(warn=1, htmlhelp=TRUE);

##merge and write data file (monthly fractions)
data = function(fnam="dat3.csv") {

  ##17 industry portfolio returns (monthly %)
  p17 = read.table("17_Industry_Portfolios1.txt", header=TRUE);
  p17[, -1] = p17[, -1]/100;

  ##3 month treasury bill (annualized %)
  tb3 = read.table("TB31.txt", header=TRUE);
  tb3[, -1] = tb3[, -1]/12;
  ##fama-french factors (monthly %)
  ff = read.table("F-F_Research_Data_Factors1.txt", header=TRUE);
  ff[, -1] = ff[, -1]/100;

  ##fama-french momentum factor (monthly %)
  Mom = read.table("F-F_Momentum_Factor1.txt", header=TRUE);
  Mom[, -1] = Mom[, -1]/100;

  ##Human capital
  hc = read.table("HC1.txt", header=TRUE);
  rl = hc$Rl      # net return

  one <- rep(1,390);
  ##dividend yield (monthly fractions)
  dy <- read.table("dividend yield1.txt", header=TRUE);
  ##combine all regressors into one data frame
  dat <- data.frame(one, ff, Mom, rmrf = ff$rmrf, rm = ff$rmrf + ff$RF, divyld =
  dy$Y, yldspr = tb3[,2] - ff$RF, tb = ff$RF, rl);

  ##write to file
  write.table(p17, "p17.csv", sep=",", row.names=FALSE);
  write.table(dat, "zt.csv", sep=",", row.names=FALSE);
}

##column-wise kronecker product
ckron = function(A, B) {
  stopifnot(nrow(A)==nrow(B))
  ca = ncol(A);
  cb = ncol(B);
  AB = matrix(NA, nrow(A), ca*cb);
  jdx = seq.int(cb);
  for (j in seq.int(ca)) {
    AB[,jdx] = A[,j]*B;
```

```

    jdx = jdx + cb;
  }
  return(AB)
}

#-----Fama-French Factors Only-----
-----
##with fama-french factors
gmm5 = function(vp, R, rm1, r11, ff1, zt,pow=3L, method=c("optim", "nlminb"),
control=list()) {
  method = match.arg(method);
  ##optimal weighting matrix using E[(R*zt)(R*zt)']
  W <- solve( crossprod( kron(1+R, zt) )/nrow(R) );
  ##check only once
  stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1), nrow(R)==length(r11),
nrow(R)==nrow(ff1));
  ##gmm objective function to minimize
  objf <- function(vpar){
    nz = ncol(zt);
    ##pricing kernel (vectorized)
    idx = seq.int(nz);
    m1 = (zt %*% vpar[idx])^2;#constant
    if (pow==0) {
      for (j in seq.int(ncol(ff1))) {
        idx = idx + nz;
        m1 = m1 + (zt %*% vpar[idx])*ff1[,j];
      }
    } else {
      ##drop rmrf
      idx = idx + nz;
      m1 = m1 + (zt %*% vpar[idx])*ff1[,2];
      idx = idx + nz;
      m1 = m1 + (zt %*% vpar[idx])*ff1[,3];

      ##linear
      idx = idx + nz;
      m1 = m1 - (zt %*% vpar[idx])^2*rm1;
      idx = idx + nz;
      m1 = m1 - (zt %*% vpar[idx])^2*r11;
      if (pow>1) {
        idx = idx + nz;
        m1 = m1 + (zt %*% vpar[idx])^2*rm1^2;
        idx = idx + nz;
        m1 = m1 + (zt %*% vpar[idx])^2*r11^2;
      }
      if (pow>2) {
        idx = idx + nz;
        m1 = m1 - (zt %*% vpar[idx])^2*rm1^3;
        idx = idx + nz;
        m1 = m1 - (zt %*% vpar[idx])^2*r11^3;
      }
    }
  }
}

```



```

    }
  }
  ##pricing error (matrix)
  v = ckron((1+R)*c(m1) - 1, zt);
  g = colMeans(v);#(12)
  return( crossprod(g, crossprod(W, g)) )
}

if (method=="optim") optim(vp, objf, control=control) else nlminb(vp, objf,
control=control)
}

##optimization with fama-french factors
tab5a = function(pow=3L) {
  ##read data
  y = read.csv("p17.csv"); #R{t+1}
  y$Date = NULL;
  z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
  R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
  zt = as.matrix(z0[-nrow(z0),]); #z{t}
  rm = read.csv("zt.csv")["rm"];
  rm1 = as.matrix(rm[-1,]) #R{m,t+1}
  rl = read.csv("zt.csv")[,c("rl")];
  rl1 = as.matrix(rl[-1]) #R{1,t+1}
  ff = read.csv("zt.csv")[,c("rmrf", "SMB", "HML")];
  ff1 = as.matrix(ff[-1,]);
  ##fit gmm with hansen-jaganathan fixed weights
  vp = rep(0.1, if (pow) ncol(zt)*(4+2*pow) else 5*ncol(zt));#starting values
  ##polytope
  gmm = gmm5(vp, R, rm1, rl1, ff1, zt, pow, method="optim",
control=list(maxit=90000));
  cat(sprintf("fmin = %13.9f, info=%i, iter=%i", gmm$value, gmm$convergence,
gmm$counts[1]), "\n");
  ##quasi-newton
  gmm = gmm5(gmm$par, R, rm1, rl1, ff1, zt, pow, method="nlminb",
control=list(trace=0, eval.max=90000, iter.max=90000));
  cat(sprintf("fmin = %13.9f, info=%i, iter=%i, mesg=%s", gmm$objective,
gmm$convergence, gmm$iterations, gmm$message), "\n");
  save(gmm, file=paste("tab5", pow, ".Rdata", sep=""));
  print(gmm$par);
}

##test parameters at means
tab5b = function(pow=3L) {
  ##read data
  y = read.csv("p17.csv"); #R{t+1}
  y$Date = NULL;
  z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
  R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
  zt = as.matrix(z0[-nrow(z0),]); #z{t}

```

```

rm = read.csv("zt.csv")["rm"];
rm1 = as.matrix(rm[-1,]) #R{m,t+1}
r1 = read.csv("zt.csv")[,c("r1")];
r11 = as.matrix(r1[-1]) #R{1,t+1}
ff = read.csv("zt.csv")[,c("rmrf", "SMB", "HML")];
ff1 = as.matrix(ff[-1,]);
nz = ncol(zt);
##check only once
stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1), nrow(R)==length(r11),
nrow(R)==nrow(ff1));

##read estimated parameter values
load(paste("tab5", pow, ".Rdata", sep=""));#gmm
vp = gmm$par;
##optimal weighting matrix using E[(R*zt)(R*zt)']
rz = ckron(1+R, zt);
W = solve( crossprod(rz)/nrow(y) );

##return sample moment conditions
fmom <- function(vpar, means=TRUE){
nz = ncol(zt);
##pricing kernel (vectorized)
idx = seq.int(nz);
m1 = (zt %*% vpar[idx])^2;#constant
if (pow==0) {
for (j in seq.int(ncol(ff1))) {
idx = idx + nz;
m1 = m1 + (zt %*% vpar[idx])*ff1[,j];
}
} else {
##drop rmrf
idx = idx + nz;
m1 = m1 + (zt %*% vpar[idx])*ff1[,2];
idx = idx + nz;
m1 = m1 + (zt %*% vpar[idx])*ff1[,3];

##linear
idx = idx + nz;
m1 = m1 - (zt %*% vpar[idx])^2*rm1;
idx = idx + nz;
m1 = m1 - (zt %*% vpar[idx])^2*r11;
if (pow>1) {
idx = idx + nz;
m1 = m1 + (zt %*% vpar[idx])^2*rm1^2;
idx = idx + nz;
m1 = m1 + (zt %*% vpar[idx])^2*r11^2;
}
if (pow>2) {
idx = idx + nz;
m1 = m1 - (zt %*% vpar[idx])^2*rm1^3;
}
}
}

```

```

    idx = idx + nz;
    m1 = m1 - (zt %*% vpar[idx])^2*r11^3;
  }
}
###pricing error (matrix)
v = ckron((1+R)*c(m1) - 1, zt);
if (means) v = colMeans(v);#(12)
return(v)
}

###for numeric derivatives
library(numDeriv);
library(MASS);
###evaluate parameter covariance matrix
grd = jacobian(fmom, vp);#d
dw = crossprod(grd, W);#d'W
# dwd = solve(dw %*% grd, tol=.Machine$double.eps);#(d'W*d)^{-1}
dwd = try(solve(dw %*% grd), silent=FALSE);#(d'W*d)^{-1}
if (inherits(dwd, "try-error")) dwd = ginv(dw %*% grd)
ss = cov( fmom(vp, means=FALSE) );#S
pcov = (dwd %*% (dw %*% tcrossprod(ss, dw)) %*% dwd) / nrow(zt);
###print estimates and standard errors
temp = data.frame(par=vp, se=sqrt(diag(pcov)));
temp$t_ratio = temp$par/temp$se;
print(temp)

###average z
zbar = colMeans(zt);
print(zbar)
tvpar = function(dz, pcov, dnam, quad=FALSE) {#delta method
  dzvar = crossprod(c(crossprod(zbar, pcov)), zbar);
  if (quad) dzvar = 4 * abs(dz) * dzvar
  wald = dz^2 / dzvar;
cat(sprintf("d(zbar)%s: %8.3f, s.e. = %9.3f, pval = %4.3f", dnam, dz, sqrt(dzvar),
pchisq(wald, 0.05, lower.tail=FALSE)), "\n");
}

###plot time-varying parameters
par(mfcol=c(3,2), mar=c(2, 2, 1, 1), mgp=c(1, 0.2, 0), tcl=-0.2);
###constant
pdx = seq.int(nz);
dt = (zt %*% vp[pdx])^2;
plot(dt, type="l", col="blue", xlab="", ylab="d0t");
dz = crossprod(zbar, vp[pdx])^2;
abline(h=dz, col=gray(0.5));
tvpar(dz, pcov[pdx,pdx], "_0t", quad=TRUE);
if (pow==0) {
  dnam = c("_rm", "smb", "hml");
  for (j in seq_along(dnam)) {
    pdx = pdx + nz;

```

```

dt = zt %*% vp[pdx];
plot(dt, type="l", col="blue", xlab="", ylab=paste("d", dnam[j], sep=""));
dz = crossprod(zbar, vp[pdx]);
abline(h=dz, col=gray(0.5));
tvar(dz, pcov[pdx,pdx], dnam[j]);
}
} else {
dnam = c("smb", "hml");
for (j in seq_along(dnam)) {
pdx = pdx + nz;
dt = zt %*% vp[pdx];
plot(dt, type="l", col="blue", xlab="", ylab=paste("d", dnam[j], sep=""));
dz = crossprod(zbar, vp[pdx]);
abline(h=dz, col=gray(0.5));
tvar(dz, pcov[pdx,pdx], dnam[j]);
}
for (i in seq.int(pow)) {
###{market}
pdx = pdx + nz;
dt = (-1)^i * (zt %*% vp[pdx])^2;
plot(dt, type="l", col="blue", xlab="", ylab=paste("d", i, "t", sep=""));
dz = (-1)^i * crossprod(zbar, vp[pdx])^2;
abline(h=dz, col="blue", lty="dashed");
tvar(dz, pcov[pdx,pdx], paste("_", i, "m", sep=""), quad=TRUE);
###{labor}
pdx = pdx + nz;
dt = (-1)^i * (zt %*% vp[pdx])^2;
lines(dt, col="red");
dz = (-1)^i * crossprod(zbar, vp[pdx])^2;
abline(h=dz, col="red", lty="dashed");
tvar(dz, pcov[pdx,pdx], paste("_", i, "l", sep=""), quad=TRUE);
}
}
##decompose hansen-jagannathan distance
pt = rz %*% crossprod(W, fmom(vp));
n = length(pt);#undo df correction
cat(sprintf("mean(pt) = %7.4f, sd(pt) = %7.4f, dist = %7.4f", mean(pt),
sqrt(var(pt)*(n-1)/n), sqrt(gmm$objective)), "\n");
}

###hansen-jagannathan bounds
fig3 = function() {
##read data
y = read.csv("p17.csv"); #R{t+1}
y$Date = NULL;
z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
zbar = colMeans(z0);
R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
zt = as.matrix(z0[-nrow(z0),]); #z{t}
rm = read.csv("zt.csv")["rm"];

```

```

rm1 = as.matrix(rm[-1,]) #R{m,t+1}
r1 = read.csv("zt.csv")[,c("r1")];
r11 = as.matrix(r1[-1]) #R{1,t+1}
ff = read.csv("zt.csv")[,c("rmrf", "SMB", "HML", "Mom")];
ff1 = as.matrix(ff[-1,]);
##evaluate at means

##bounds
invc = solve( var(R) );
rbar = 1+colMeans(R);#gross
mbar = seq(from=0.97, to=1.24, length.out=500);#E[m]
ones = rep(1, length(rbar));#price vector
hjbd = function(x, p, w, xbar) {
  err = p - x*xbar;
  sqrt(crossprod(err, w %*% err)[1,1])
}
msdv = sapply(mbar, hjbd, p=ones, w=invc, xbar=rbar, USE.NAMES=FALSE);

##evaluate pricing kernel
sdf <- function(vpar, rm1, r11=NULL, pow=3L){
  stopifnot(pow==1 || pow==2 || pow==3);
  nz = length(zbar);
  idx = seq.int(nz);
  m1 = crossprod(zbar, vpar[idx])^2;#constant
  idx = idx + nz;
  d1m = -crossprod(zbar, vpar[idx])^2

  m1 = m1 - d1m*rm1;
  if (!is.null(r11)) {
    idx = idx + nz;
    m1 = m1 - crossprod(zbar, vpar[idx])^2*r11;
  }
  if (pow>1) {
    idx = idx + nz;
    m1 = m1 + (zbar %*% vpar[idx])^2*rm1^2;
    if (!is.null(r11)) {
      idx = idx + nz;
      m1 = m1 + (zbar %*% vpar[idx])^2*r11^2;
    }
  }
  if (pow>2) {
    idx = idx + nz;
    m1 = m1 - (zbar %*% vpar[idx])^2*rm1^3;
    if (!is.null(r11)) {
      idx = idx + nz;
      m1 = m1 - (zbar %*% vpar[idx])^2*r11^3;
    }
  }
  return(m1)
}

```

```

}
##fama-french pricing kernel
sdf_ff <- function(vpar, ff1) {
  nz = length(zbar);
  idx = seq.int(nz);
  m1 = crossprod(zbar, vpar[idx])^2;#constant
  for (j in seq.int(ncol(ff1))) {
    idx = idx + nz;
    m1 = m1 + crossprod(zbar, vpar[idx])*ff1[,j];
  }
  return(m1)
}
##add (mean, sd) pair to plot
padd = function(fnam, pnam, pch, ..., r11=NULL, pow=3L, isff1=FALSE) {
  load(fnam);#gmm
  m1 = if (isff1) sdf_ff(gmm$par, ff1) else sdf(gmm$par, rm1, r11, pow)
  mbar = mean(m1);
  msdv = sd(m1);
print(c(mbar, msdv))
  points(mbar, msdv, pch=pch, ...);
  text(mbar, msdv, pnam, pos=4, offset=0.2, xpd=TRUE, ...);
}

plot(mbar, msdv, type="l", ylim=c(0, 1.5), col=gray(0.5), xlab="mean",
ylab="std.dev.");
##exclude human capital
padd("tab31.Rdata", "pow=1", pch=21, col="blue", cex=0.8, pow=1L);
padd("tab32.Rdata", "pow=2", pch=21, col="green", cex=0.8, pow=2L);
padd("tab33.Rdata", "pow=3", pch=21, col="red", cex=0.8, pow=3L);
padd("tab50.Rdata", "fama-french", pch=21, col="brown", cex=0.8, isff1=TRUE);
##include human capital
padd("tab41.Rdata", "hc=1", pch=22, col="blue", cex=0.8, rg=rg, pow=1L);
padd("tab42.Rdata", "hc=2", pch=22, col="green", cex=0.8, rg=rg, pow=2L);
padd("tab43.Rdata", "hc=3", pch=22, col="red", cex=0.8, rg=rg, pow=3L);

}

#data();
#tab5a(0L);
tab5b(3L);

#fig3()
R-code for Table 8

##clear
rm(list=ls());
graphics.off();
options(warn=1, htmlhelp=TRUE);

##merge and write data file (monthly fractions)

```

```

data = function(fnam="dat.csv") {

  ##17 industry portfolio returns (monthly %)
  p17 = read.table("17_Industry_Portfolios1.txt", header=TRUE);
  p17[,-1] = p17[,-1]/100;

  ##3 month treasury bill (annualized %)
  tb3 = read.table("TB31.txt", header=TRUE);
  tb3[,-1] = tb3[,-1]/12;
  ##fama-french factors (monthly %)
  ff = read.table("F-F_Research_Data_Factors1.txt", header=TRUE);
  ff[,-1] = ff[,-1]/100;

  ##fama-french momentum factor (monthly %)
  Mom = read.table("F-F_Momentum_Factor1.txt", header=TRUE);
  Mom[,-1] = Mom[,-1]/100;

  ##Human capital
  hc = read.table("HC1.txt", header=TRUE);
  rl = hc$R1    # net return

  ## human capital house price index

  HP = read.table("house_price_index1.txt", header = TRUE);
  hi = HP$real_return;

  one <- rep(1,390);
  ##dividend yield (monthly fractions)
  dy <- read.table("dividend_yield1.txt", header=TRUE);
  ##combine all regressors into one data frame
  dat <- data.frame(one, ff,Mom, rmrf = ff$rmrf, rm = ff$rmrf + ff$RF, divyld =
  dy$Y, yldspr = tb3[,2] - ff$RF, tb = ff$RF, rl, hi);

  ##write to file
  write.table(p17, "p17.csv", sep=",", row.names=FALSE);
  write.table(dat, "zt.csv", sep=",", row.names=FALSE);
}

##column-wise kronecker product
ckron = function(A, B) {
  stopifnot(nrow(A)==nrow(B))
  ca = ncol(A);
  cb = ncol(B);
  AB = matrix(NA, nrow(A), ca*cb);
  jdx = seq.int(cb);
  for (j in seq.int(ca)) {
    AB[,jdx] = A[,j]*B;
    jdx = jdx + cb;
  }
  return(AB)
}

```

```

}

#-----Fama-French Factors Only-----
-----
##with fama-french factors
gmm5 = function(vp, R, rm1, r11, ff1, zt,pow=3L, method=c("optim", "nlminb"),
control=list()) {
  method = match.arg(method);
  ##optimal weighting matrix using E[(R*zt)(R*zt)']
  W <- solve( crossprod( kron(1+R, zt )/nrow(R) );
  ##check only once
  stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1), nrow(R)==length(r11),
nrow(R)==nrow(ff1));
  ##gmm objective function to minimize
  objf <- function(vpar){
    nz = ncol(zt);
    ##pricing kernel (vectorized)
    idx = seq.int(nz);
    m1 = (zt %*% vpar[idx])^2;#constant
    if (pow==0) {
      for (j in seq.int(ncol(ff1))) {
        idx = idx + nz;
        m1 = m1 + (zt %*% vpar[idx])*ff1[,j];
      }
    } else {
      ##drop rmrf
      idx = idx + nz;
      m1 = m1 + (zt %*% vpar[idx])*ff1[,2];
      idx = idx + nz;
      m1 = m1 + (zt %*% vpar[idx])*ff1[,3];
      idx = idx + nz;
      m1 = m1 + (zt %*% vpar[idx])*ff1[,4];
      ##linear
      idx = idx + nz;
      m1 = m1 - (zt %*% vpar[idx])^2*rm1;
      idx = idx + nz;
      m1 = m1 - (zt %*% vpar[idx])^2*r11;

      if (pow>1) {
        idx = idx + nz;
        m1 = m1 + (zt %*% vpar[idx])^2*rm1^2;
        idx = idx + nz;
        m1 = m1 + (zt %*% vpar[idx])^2*r11^2;
      }
    }
    if (pow>2) {
      idx = idx + nz;
      m1 = m1 - (zt %*% vpar[idx])^2*rm1^3;
      idx = idx + nz;
    }
  }
}

```



```

    m1 = m1 - (zt %*% vpar[idx])^2*r11^3;

  }
}
###pricing error (matrix)
v = ckron((1+R)*c(m1) - 1, zt);
g = colMeans(v);#(12)
return( crossprod(g, crossprod(W, g)) )
}

if (method=="optim") optim(vp, objf, control=control) else nlminb(vp, objf,
control=control)
}

##optimization with fama-french factors
tab5a = function(pow=3L) {
  ##read data
  y = read.csv("p17.csv"); #R{t+1}
  y$Date = NULL;
  z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
  R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
  zt = as.matrix(z0[-nrow(z0),]); #z{t}
  rm = read.csv("zt.csv")["rm"];
  rm1 = as.matrix(rm[-1,]) #R{m,t+1}
  rl = read.csv("zt.csv")[,c("rl")];
  rl1 = as.matrix(rl[-1]) #R{1,t+1}
  ff = read.csv("zt.csv")[,c("rmrf", "SMB", "HML", "Mom")];
  ff1 = as.matrix(ff[-1,]);

  ##fit gmm with hansen-jaganathan fixed weights
  vp = rep(0.1, if (pow) ncol(zt)*(4+2*pow) else 5*ncol(zt));#starting values
  ##polytope
  gmm = gmm5(vp, R, rm1, rl1, ff1, zt, pow, method="optim",
control=list(maxit=90000));
cat(sprintf("fmin = %13.9f, info=%i, iter=%i", gmm$value, gmm$convergence,
gmm$counts[1]), "\n");
  ##quasi-newton
  gmm = gmm5(gmm$par, R, rm1, rl1, ff1, zt, pow, method="nlminb",
control=list(trace=0, eval.max=90000, iter.max=90000));
cat(sprintf("fmin = %13.9f, info=%i, iter=%i, mesg=%s", gmm$objective,
gmm$convergence, gmm$iterations, gmm$message), "\n");
  save(gmm, file=paste("tab5", pow, ".Rdata", sep=""));
print(gmm$par);
}

##test parameters at means
tab5b = function(pow=3L) {
  ##read data
  y = read.csv("p17.csv"); #R{t+1}

```

```

y$Date = NULL;
z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
zt = as.matrix(z0[-nrow(z0),]); #z{t}
rm = read.csv("zt.csv")["rm"];
rm1 = as.matrix(rm[-1,]) #R{m,t+1}
rl = read.csv("zt.csv")[,c("rl")];
rl1 = as.matrix(rl[-1]) #R{1,t+1}
ff = read.csv("zt.csv")[,c("rmrf", "SMB", "HML", "Mom")];
ff1 = as.matrix(ff[-1,]);
nz = ncol(zt);
##check only once
stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1), nrow(R)==length(rl1),
nrow(R)==nrow(ff1));

##read estimated parameter values
load(paste("tab5", pow, ".Rdata", sep=""));#gmm
vp = gmm$par;
##optimal weigting matrix using E[(R*zt)(R*zt)']
rz = kron(1+R, zt);
W = solve( crossprod(rz)/nrow(y) );

##return sample moment conditions
fmom <- function(vpar, means=TRUE){
nz = ncol(zt);
##pricing kernel (vectorized)
idx = seq.int(nz);
m1 = (zt %*% vpar[idx])^2;#constant
if (pow==0) {
for (j in seq.int(ncol(ff1))) {
idx = idx + nz;
m1 = m1 + (zt %*% vpar[idx])*ff1[,j];
}
} else {
##drop rmrf
idx = idx + nz;
m1 = m1 + (zt %*% vpar[idx])*ff1[,2];
idx = idx + nz;
m1 = m1 + (zt %*% vpar[idx])*ff1[,3];
idx = idx + nz;
m1 = m1 + (zt %*% vpar[idx])*ff1[,4];
##linear
idx = idx + nz;
m1 = m1 - (zt %*% vpar[idx])^2*rm1;
idx = idx + nz;
m1 = m1 - (zt %*% vpar[idx])^2*rl1;

if (pow>1) {
idx = idx + nz;
m1 = m1 + (zt %*% vpar[idx])^2*rm1^2;
}
}
}

```

```

    idx = idx + nz;
    m1 = m1 + (zt %*% vpar[idx])^2*r1^2;

  }
  if (pow>2) {
    idx = idx + nz;
    m1 = m1 - (zt %*% vpar[idx])^2*rm1^3;
    idx = idx + nz;
    m1 = m1 - (zt %*% vpar[idx])^2*r1^3;

  }
}
##pricing error (matrix)
v = ckron((1+R)*c(m1) - 1, zt);
if (means) v = colMeans(v);#(12)
return(v)
}

##for numeric derivatives
library(numDeriv);
library(MASS);
##evaluate parameter covariance matrix
grd = jacobian(fmom, vp);#d
dw = crossprod(grd, W);#d'W
#dwd = solve(dw %*% grd, tol=.Machine$double.eps);#(d'W*d)^{-1}
dwd = try(solve(dw %*% grd), silent=FALSE);#(d'W*d)^{-1}
if (inherits(dwd, "try-error")) dwd = ginv(dw %*% grd)
ss = cov( fmom(vp, means=FALSE) );#S
pcov = (dwd %*% (dw %*% tcrossprod(ss, dw)) %*% dwd) / nrow(zt);
##print estimates and standard errors
temp = data.frame(par=vp, se=sqrt(diag(pcov)));
temp$t_ratio = temp$par/temp$se;
print(temp)

##average z
zbar = colMeans(zt);
print(zbar)
tvpar = function(dz, pcov, dnam, quad=FALSE) {#delta method
  dzvar = crossprod(c(crossprod(zbar, pcov)), zbar);
  if (quad) dzvar = 4 * abs(dz) * dzvar
  wald = dz^2 / dzvar;
cat(sprintf("d(zbar)%s: %8.3f, s.e. = %9.3f, pval = %4.3f", dnam, dz, sqrt(dzvar),
pchisq(wald, 0.05, lower.tail=FALSE)), "\n");
}

##plot time-varying parameters
par(mfcol=c(3,2), mar=c(2, 2, 1, 1), mgp=c(1, 0.2, 0), tcl=-0.2);
##constant
pdx = seq.int(nz);
dt = (zt %*% vp[pdx])^2;

```

```

plot(dt, type="l", col="blue", xlab="", ylab="d0t");
dz = crossprod(zbar, vp[pdx])^2;
abline(h=dz, col=gray(0.5));
tvpar(dz, pcov[pdx,pdx], "_0t", quad=TRUE);
if (pow==0) {
  dnam = c("_rm", "smb", "hml", "Mom");
  for (j in seq_along(dnam)) {
    pdx = pdx + nz;
    dt = zt %*% vp[pdx];
    plot(dt, type="l", col="blue", xlab="", ylab=paste("d", dnam[j], sep=""));
    dz = crossprod(zbar, vp[pdx]);
    abline(h=dz, col=gray(0.5));
    tvpar(dz, pcov[pdx,pdx], dnam[j]);
  }
} else {
  dnam = c("smb", "hml", "Mom");
  for (j in seq_along(dnam)) {
    pdx = pdx + nz;
    dt = zt %*% vp[pdx];
    plot(dt, type="l", col="blue", xlab="", ylab=paste("d", dnam[j], sep=""));
    dz = crossprod(zbar, vp[pdx]);
    abline(h=dz, col=gray(0.5));
    tvpar(dz, pcov[pdx,pdx], dnam[j]);
  }
}
for (i in seq.int(pow)) {
  ##r{market}
  pdx = pdx + nz;
  dt = (-1)^i * (zt %*% vp[pdx])^2;
  plot(dt, type="l", col="blue", xlab="", ylab=paste("d", i, "t", sep=""));
  dz = (-1)^i * crossprod(zbar, vp[pdx])^2;
  abline(h=dz, col="blue", lty="dashed");
  tvpar(dz, pcov[pdx,pdx], paste("_", i, "m", sep=""), quad=TRUE);
  ##r{labor}
  pdx = pdx + nz;
  dt = (-1)^i * (zt %*% vp[pdx])^2;
  lines(dt, col="red");
  dz = (-1)^i * crossprod(zbar, vp[pdx])^2;
  abline(h=dz, col="red", lty="dashed");
  tvpar(dz, pcov[pdx,pdx], paste("_", i, "l", sep=""), quad=TRUE);
}
}
##decompose hansen-jagannathan distance
pt = rz %*% crossprod(W, fmom(vp));
n = length(pt);#undo df correction
cat(sprintf("mean(pt) = %7.4f, sd(pt) = %7.4f, dist = %7.4f", mean(pt),
sqrt(var(pt)*(n-1)/n), sqrt(gmm$objective)), "\n");
}

```

```
#data();  
#tab5a(0L);  
#tab5b(0L);  
#tab5a(2L);  
#tab5b(2L);  
#tab5a(3L);  
tab5b(3L);
```

R-code for Table 9

```
##clear
rm(list=ls());
graphics.off();
options(warn=1, htmlhelp=TRUE);

##merge and write data file (monthly fractions)
data = function(fnam="dat.csv") {

  ##17 industry portfolio returns (monthly %)
  p17 = read.table("17_Industry_Portfolios1.txt", header=TRUE);
  p17[,-1] = p17[,-1]/100;

  ##3 month treasury bill (annualized %)
  tb3 = read.table("TB31.txt", header=TRUE);
  tb3[,-1] = tb3[,-1]/12;
  ##fama-french factors (monthly %)
  ff = read.table("F-F_Research_Data_Factors1.txt", header=TRUE);
  ff[,-1] = ff[,-1]/100;

  ##fama-french momentum factor (monthly %)
  Mom = read.table("F-F_Momentum_Factor1.txt", header=TRUE);
  Mom[,-1] = Mom[,-1]/100;

  ##Human capital
  hc = read.table("HC1.txt", header=TRUE);
  rl = hc$R1      # net return

  ## human capital house price index

  HP = read.table("house_price_index1.txt", header = TRUE);
  hi = HP$real_return;

  one <- rep(1,390);
  ##dividend yield (monthly fractions)
  dy <- read.table("dividend_yield1.txt", header=TRUE);
  ##combine all regressors into one data frame
  dat <- data.frame(one, ff,Mom, rmrf = ff$rmrf, rm = ff$rmrf + ff$RF, divyld =
dy$Y, yldspr = tb3[,2] - ff$RF, tb = ff$RF, rl, hi);

  ##write to file
  write.table(p17, "p17.csv", sep="," , row.names=FALSE);
  write.table(dat, "zt.csv", sep="," , row.names=FALSE);
}

##column-wise kronecker product
ckron = function(A, B) {
  stopifnot(nrow(A)==nrow(B))
  ca = ncol(A);
```

```

cb = ncol(B);
AB = matrix(NA, nrow(A), ca*cb);
jdx = seq.int(cb);
for (j in seq.int(ca)) {
  AB[,jdx] = A[,j]*B;
  jdx = jdx + cb;
}
return(AB)
}

#-----Fama-French Factors Only-----
-----
##with fama-french factors
gmm5 = function(vp, R, rm1, rh, ff1, zt,pow=3L, method=c("optim", "nlminb"),
control=list()) {
  method = match.arg(method);
  ##optimal weighting matrix using E[(R*zt)(R*zt)']
  W <- solve( crossprod( ckron(1+R, zt) )/nrow(R) );
  ##check only once
  stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1), nrow(R)==nrow(ff1),
nrow(R)==nrow(rh));
  ##gmm objective function to minimize
  objf <- function(vpar){
    nz = ncol(zt);
    ##pricing kernel (vectorized)
    idx = seq.int(nz);
    m1 = (zt %*% vpar[idx])^2;#constant
    if (pow==0) {
      for (j in seq.int(ncol(ff1))) {
        idx = idx + nz;
        m1 = m1 + (zt %*% vpar[idx])*ff1[,j];
      }
    } else {
      ##drop rmrf
      idx = idx + nz;
      m1 = m1 + (zt %*% vpar[idx])*ff1[,2];
      idx = idx + nz;
      m1 = m1 + (zt %*% vpar[idx])*ff1[,3];
      idx = idx + nz;
      m1 = m1 + (zt %*% vpar[idx])*ff1[,4];
      ##linear
      idx = idx + nz;
      m1 = m1 - (zt %*% vpar[idx])^2*rm1;
      idx = idx + nz;
      m1 = m1 - (zt %*% vpar[idx])^2*rh;
      if (pow>1) {
        idx = idx + nz;
        m1 = m1 + (zt %*% vpar[idx])^2*rm1^2;
        idx = idx + nz;
        m1 = m1 + (zt %*% vpar[idx])^2*rh^2
      }
    }
  }
}

```

```

    }
    if (pow>2) {
      idx = idx + nz;
      m1 = m1 - (zt %*% vpar[idx])^2*rm1^3;
      idx = idx + nz;
      m1 = m1 - (zt %*% vpar[idx])^2*rh^3;
    }
  }
  ##pricing error (matrix)
  v = ckron((1+R)*c(m1) - 1, zt);
  g = colMeans(v);#(12)
  return( crossprod(g, crossprod(W, g)) )
}

if (method=="optim") optim(vp, objf, control=control) else nlminb(vp, objf,
control=control)
}

##optimization with fama-french factors
tab5a = function(pow=3L) {
  ##read data
  y = read.csv("p17.csv"); #R{t+1}
  y$Date = NULL;
  z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
  R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
  zt = as.matrix(z0[-nrow(z0),]); #z{t}
  rm = read.csv("zt.csv")["rm"];
  rm1 = as.matrix(rm[-1,]) #R{m,t+1}
  ff = read.csv("zt.csv")[,c("rmrf", "SMB", "HML", "Mom")];
  ff1 = as.matrix(ff[-1,]);
  hi = read.csv("zt.csv")[,c("hi")];
  rh = as.matrix(hi[-1]) #R{h,t+1}
  ##fit gmm with hansen-jaganathan fixed weights
  vp = rep(0.1, if (pow) ncol(zt)*(4+2*pow) else 5*ncol(zt));#starting values
  ##polytope
  gmm = gmm5(vp, R, rm1, rh, ff1, zt, pow, method="optim",
control=list(maxit=90000));
  cat(sprintf("fmin = %13.9f, info=%i, iter=%i", gmm$value, gmm$convergence,
gmm$counts[1]), "\n");
  ##quasi-newton
  gmm = gmm5(gmm$par, R, rm1, rh, ff1, zt, pow, method="nlminb",
control=list(trace=0, eval.max=90000, iter.max=90000));
  cat(sprintf("fmin = %13.9f, info=%i, iter=%i, mesg=%s", gmm$objective,
gmm$convergence, gmm$iterations, gmm$message), "\n");
  save(gmm, file=paste("tab5", pow, ".Rdata", sep=""));
  print(gmm$par);
}

##test parameters at means
tab5b = function(pow=3L) {

```



```

##read data
y = read.csv("p17.csv"); #R{t+1}
y$Date = NULL;
z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
zt = as.matrix(z0[-nrow(z0),]); #z{t}
rm = read.csv("zt.csv")["rm"];
rm1 = as.matrix(rm[-1,]) #R{m,t+1}
ff = read.csv("zt.csv")[,c("rmrf", "SMB", "HML", "Mom")];
ff1 = as.matrix(ff[-1,]);
hi = read.csv("zt.csv")[,c("hi")];
rh = as.matrix(hi[-1]) #R{h,t+1}
nz = ncol(zt);
##check only once
stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1), nrow(R)==nrow(ff1),
nrow(R)==nrow(rh));

##read estimated parameter values
load(paste("tab5", pow, ".Rdata", sep=""));#gmm
vp = gmm$par;
##optimal weighting matrix using E[(R*zt)(R*zt)']
rz = ckron(1+R, zt);
W = solve( crossprod(rz)/nrow(y) );

##return sample moment conditions
fmom <- function(vpar, means=TRUE){
nz = ncol(zt);
##pricing kernel (vectorized)
idx = seq.int(nz);
m1 = (zt %*% vpar[idx])^2;#constant
if (pow==0) {
for (j in seq.int(ncol(ff1))) {
idx = idx + nz;
m1 = m1 + (zt %*% vpar[idx])*ff1[,j];
}
} else {
##drop rmrf
idx = idx + nz;
m1 = m1 + (zt %*% vpar[idx])*ff1[,2];
idx = idx + nz;
m1 = m1 + (zt %*% vpar[idx])*ff1[,3];
idx = idx + nz;
m1 = m1 + (zt %*% vpar[idx])*ff1[,4];
##linear
idx = idx + nz;
m1 = m1 - (zt %*% vpar[idx])^2*rm1;
idx = idx + nz;
m1 = m1 - (zt %*% vpar[idx])^2*rh;
if (pow>1) {
idx = idx + nz;
}
}
}

```

```

    m1 = m1 + (zt %*% vpar[idx])^2*rm1^2;
    idx = idx + nz;
    m1 = m1 + (zt %*% vpar[idx])^2*rh^2
  }
  if (pow>2) {
    idx = idx + nz;
    m1 = m1 - (zt %*% vpar[idx])^2*rm1^3;
    idx = idx + nz;
    m1 = m1 - (zt %*% vpar[idx])^2*rh^3;
  }
}
##pricing error (matrix)
v = ckron((1+R)*c(m1) - 1, zt);
if (means) v = colMeans(v);#(12)
return(v)
}

##for numeric derivatives
library(numDeriv);
library(MASS);
##evaluate parameter covariance matrix
grd = jacobian(fmom, vp);#d
dw = crossprod(grd, W);#d'W
# dwd = solve(dw %*% grd, tol=Machine$double.eps);#(d'W*d)^{-1}
dwd = try(solve(dw %*% grd), silent=FALSE);#(d'W*d)^{-1}
if (inherits(dwd, "try-error")) dwd = ginv(dw %*% grd)
ss = cov( fmom(vp, means=FALSE) );#S
pcov = (dwd %*% (dw %*% tcrossprod(ss, dw)) %*% dwd) / nrow(zt);
##print estimates and standard errors
temp = data.frame(par=vp, se=sqrt(diag(pcov)));
temp$t_ratio = temp$par/temp$se;
print(temp)

##average z
zbar = colMeans(zt);
print(zbar)
tvar = function(dz, pcov, dnam, quad=FALSE) {#delta method
  dzvar = crossprod(c(crossprod(zbar, pcov)), zbar);
  if (quad) dzvar = 4 * abs(dz) * dzvar
  wald = dz^2 / dzvar;
cat(sprintf("d(zbar)%s: %8.3f, s.e. = %9.3f, pval = %4.3f", dnam, dz, sqrt(dzvar),
pchisq(wald, 0.05, lower.tail=FALSE)), "\n");
}

##plot time-varying parameters
par(mfcol=c(3,2), mar=c(2, 2, 1, 1), mgp=c(1, 0.2, 0), tcl=-0.2);
##constant
pdx = seq.int(nz);
dt = (zt %*% vp[pdx])^2;
plot(dt, type="l", col="blue", xlab="", ylab="d0t");

```

```

dz = crossprod(zbar, vp[pdx])^2;
abline(h=dz, col=gray(0.5));
tvpar(dz, pcov[pdx,pdx], "_0t", quad=TRUE);
if (pow==0) {
  dnam = c("_rm", "smb", "hml", "Mom");
  for (j in seq_along(dnam)) {
    pdx = pdx + nz;
    dt = zt %*% vp[pdx];
    plot(dt, type="l", col="blue", xlab="", ylab=paste("d", dnam[j], sep=""));
    dz = crossprod(zbar, vp[pdx]);
    abline(h=dz, col=gray(0.5));
    tvpar(dz, pcov[pdx,pdx], dnam[j]);
  }
} else {
  dnam = c("smb", "hml", "Mom");
  for (j in seq_along(dnam)) {
    pdx = pdx + nz;
    dt = zt %*% vp[pdx];
    plot(dt, type="l", col="blue", xlab="", ylab=paste("d", dnam[j], sep=""));
    dz = crossprod(zbar, vp[pdx]);
    abline(h=dz, col=gray(0.5));
    tvpar(dz, pcov[pdx,pdx], dnam[j]);
  }
}
for (i in seq.int(pow)) {
  ##r{ market }
  pdx = pdx + nz;
  dt = (-1)^i * (zt %*% vp[pdx])^2;
  plot(dt, type="l", col="blue", xlab="", ylab=paste("d", i, "t", sep=""));
  dz = (-1)^i * crossprod(zbar, vp[pdx])^2;
  abline(h=dz, col="blue", lty="dashed");
  tvpar(dz, pcov[pdx,pdx], paste("_", i, "m", sep=""), quad=TRUE);
  ##r{ house price }
  pdx = pdx + nz;
  dt = (-1)^i * (zt %*% vp[pdx])^2;
  lines(dt, col="red");
  dz = (-1)^i * crossprod(zbar, vp[pdx])^2;
  abline(h=dz, col="green", lty="dashed");
  tvpar(dz, pcov[pdx,pdx], paste("_", i, "h", sep=""), quad=TRUE);
}
}
##decompose hansen-jagannathan distance
pt = rz %*% crossprod(W, fmom(vp));
n = length(pt);#undo df correction
cat(sprintf("mean(pt) = %7.4f, sd(pt) = %7.4f, dist = %7.4f", mean(pt),
sqrt(var(pt)*(n-1)/n), sqrt(gmm$objective)), "\n");
}

#data();
#tab5a(0L);

```

#tab5b(0L);
#tab5a(2L);
#tab5b(2L);
#tab5a(3L);
tab5b(3L);

R-code for Table 10

```
##clear
rm(list=ls());
graphics.off();
options(warn=1, htmlhelp=TRUE);

##merge and write data file (monthly fractions)
data = function(fnam="dat.csv") {

  ##17 industry portfolio returns (monthly %)
  p17 = read.table("17_Industry_Portfolios1.txt", header=TRUE);
  p17[,-1] = p17[,-1]/100;

  ##3 month treasury bill (annualized %)
  tb3 = read.table("TB31.txt", header=TRUE);
  tb3[,-1] = tb3[,-1]/12;
  ##fama-french factors (monthly %)
  ff = read.table("F-F_Research_Data_Factors1.txt", header=TRUE);
  ff[,-1] = ff[,-1]/100;

  ##fama-french momentum factor (monthly %)
  Mom = read.table("F-F_Momentum_Factor1.txt", header=TRUE);
  Mom[,-1] = Mom[,-1]/100;

  ##Human capital
  hc = read.table("HC1.txt", header=TRUE);
  rl = hc$R1    # net return

  ## human capital house price index

  HP = read.table("house_price_index1.txt", header = TRUE);
  hi = HP$real_return;

  one <- rep(1,390);
  ##dividend yield (monthly fractions)
  dy <- read.table("dividend_yield1.txt", header=TRUE);
  ##combine all regressors into one data frame
  dat <- data.frame(one, ff,Mom, rmrf = ff$rmrf, rm = ff$rmrf + ff$RF, divyld =
dy$Y, yldspr = tb3[,2] - ff$RF, tb = ff$RF, rl, hi);

  ##write to file
  write.table(p17, "p17.csv", sep="," , row.names=FALSE);
  write.table(dat, "zt.csv", sep="," , row.names=FALSE);
}

##column-wise kronecker product
ckron = function(A, B) {
  stopifnot(nrow(A)==nrow(B))
```

```

ca = ncol(A);
cb = ncol(B);
AB = matrix(NA, nrow(A), ca*cb);
jdx = seq.int(cb);
for (j in seq.int(ca)) {
  AB[,jdx] = A[,j]*B;
  jdx = jdx + cb;
}
return(AB)
}

#-----Fama-French Factors Only-----
-----
##with fama-french factors
gmm5 = function(vp, R, rm1, r11, rh, ff1, zt,pow=3L, method=c("optim", "nlminb"),
control=list()) {
  method = match.arg(method);
  ##optimal weighting matrix using E[(R*zt)(R*zt)']
  W <- solve( crossprod( ckron(1+R, zt) )/nrow(R) );
  ##check only once
  stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1), nrow(R)==length(r11),
nrow(R)==nrow(ff1), nrow(R)==nrow(rh));
  ##gmm objective function to minimize
  objf <- function(vpar){
    nz = ncol(zt);
    ##pricing kernel (vectorized)
    idx = seq.int(nz);
    m1 = (zt %*% vpar[idx])^2;#constant
    if (pow==0) {
      for (j in seq.int(ncol(ff1))) {
        idx = idx + nz;
        m1 = m1 + (zt %*% vpar[idx])*ff1[,j];
      }
    } else {
      ##drop rmrf
      idx = idx + nz;
      m1 = m1 + (zt %*% vpar[idx])*ff1[,2];
      idx = idx + nz;
      m1 = m1 + (zt %*% vpar[idx])*ff1[,3];
      idx = idx + nz;
      m1 = m1 + (zt %*% vpar[idx])*ff1[,4];
      ##linear
      idx = idx + nz;
      m1 = m1 - (zt %*% vpar[idx])^2*rm1;
      idx = idx + nz;
      m1 = m1 - (zt %*% vpar[idx])^2*r11;
      idx = idx + nz;
      m1 = m1 - (zt %*% vpar[idx])^2*rh;
    }
    if (pow>1) {
      idx = idx + nz;

```

```

    m1 = m1 + (zt %*% vpar[idx])^2*rm1^2;
    idx = idx + nz;
    m1 = m1 + (zt %*% vpar[idx])^2*r11^2;
    idx = idx + nz;
    m1 = m1 + (zt %*% vpar[idx])^2*rh^2
  }
  if (pow>2) {
    idx = idx + nz;
    m1 = m1 - (zt %*% vpar[idx])^2*rm1^3;
    idx = idx + nz;
    m1 = m1 - (zt %*% vpar[idx])^2*r11^3;
    idx = idx + nz;
    m1 = m1 - (zt %*% vpar[idx])^2*rh^3;
  }
}
##pricing error (matrix)
v = ckron((1+R)*c(m1) - 1, zt);
g = colMeans(v);#(12)
return( crossprod(g, crossprod(W, g)) )
}

if (method=="optim") optim(vp, objf, control=control) else nlminb(vp, objf,
control=control)
}

##optimization with fama-french factors
tab5a = function(pow=3L) {
  ##read data
  y = read.csv("p17.csv"); #R{t+1}
  y$Date = NULL;
  z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
  R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
  zt = as.matrix(z0[-nrow(z0),]); #z{t}
  rm = read.csv("zt.csv")["rm"];
  rm1 = as.matrix(rm[-1,]) #R{m,t+1}
  rl = read.csv("zt.csv")[,c("rl")];
  rl1 = as.matrix(rl[-1]) #R{1,t+1}
  ff = read.csv("zt.csv")[,c("rmrf", "SMB", "HML", "Mom")];
  ff1 = as.matrix(ff[-1,]);
  hi = read.csv("zt.csv")[,c("hi")];
  rh = as.matrix(hi[-1]) #R{h,t+1}
  ##fit gmm with hansen-jaganathan fixed weights
  vp = rep(0.1, if (pow) ncol(zt)*(4+3*pow) else 5*ncol(zt));#starting values
  ##polytope
  gmm = gmm5(vp, R, rm1, rl1, rh, ff1, zt, pow, method="optim",
control=list(maxit=90000));
cat(sprintf("fmin = %13.9f, info=%i, iter=%i", gmm$value, gmm$convergence,
gmm$counts[1]), "\n");
  ##quasi-newton

```

```

gmm = gmm5(gmm$par, R, rm1, r11, rh, ff1, zt, pow, method="nlminb",
control=list(trace=0, eval.max=90000, iter.max=90000));
cat(sprintf("fmin = %13.9f, info=%i, iter=%i, mesg=%s", gmm$objective,
gmm$convergence, gmm$iterations, gmm$message), "\n");
save(gmm, file=paste("tab5", pow, ".Rdata", sep=""));
print(gmm$par);
}

##test parameters at means
tab5b = function(pow=3L) {
##read data
y = read.csv("p17.csv"); #R{t+1}
y$Date = NULL;
z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
zt = as.matrix(z0[-nrow(z0),]); #z{t}
rm = read.csv("zt.csv")["rm"];
rm1 = as.matrix(rm[-1,]) #R{m,t+1}
r1 = read.csv("zt.csv")[,c("r1")];
r11 = as.matrix(r1[-1]) #R{1,t+1}
ff = read.csv("zt.csv")[,c("rmrf", "SMB", "HML", "Mom")];
ff1 = as.matrix(ff[-1,]);
hi = read.csv("zt.csv")[,c("hi")];
rh = as.matrix(hi[-1]) #R{h,t+1}
nz = ncol(zt);
##check only once
stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1), nrow(R)==length(r11),
nrow(R)==nrow(ff1), nrow(R)==nrow(rh));

##read estimated parameter values
load(paste("tab5", pow, ".Rdata", sep=""));#gmm
vp = gmm$par;
##optimal weighting matrix using E[(R*zt)(R*zt)']
rz = ckron(1+R, zt);
W = solve( crossprod(rz)/nrow(y) );

##return sample moment conditions
fmom <- function(vpar, means=TRUE){
nz = ncol(zt);
##pricing kernel (vectorized)
idx = seq.int(nz);
m1 = (zt %*% vpar[idx])^2;#constant
if (pow==0) {
for (j in seq.int(ncol(ff1))) {
idx = idx + nz;
m1 = m1 + (zt %*% vpar[idx])*ff1[,j];
}
} else {
##drop rmrf
idx = idx + nz;

```



```

m1 = m1 + (zt %*% vpar[idx])*ff1[,2];
idx = idx + nz;
m1 = m1 + (zt %*% vpar[idx])*ff1[,3];
idx = idx + nz;
m1 = m1 + (zt %*% vpar[idx])*ff1[,4];
###linear
idx = idx + nz;
m1 = m1 - (zt %*% vpar[idx])^2*rm1;
idx = idx + nz;
m1 = m1 - (zt %*% vpar[idx])^2*r11;
idx = idx + nz;
m1 = m1 - (zt %*% vpar[idx])^2*rh;
if (pow>1) {
  idx = idx + nz;
  m1 = m1 + (zt %*% vpar[idx])^2*rm1^2;
  idx = idx + nz;
  m1 = m1 + (zt %*% vpar[idx])^2*r11^2;
  idx = idx + nz;
  m1 = m1 + (zt %*% vpar[idx])^2*rh^2
}
if (pow>2) {
  idx = idx + nz;
  m1 = m1 - (zt %*% vpar[idx])^2*rm1^3;
  idx = idx + nz;
  m1 = m1 - (zt %*% vpar[idx])^2*r11^3;
  idx = idx + nz;
  m1 = m1 - (zt %*% vpar[idx])^2*rh^3;
}
}
}
###pricing error (matrix)
v = ckron((1+R)*c(m1) - 1, zt);
if (means) v = colMeans(v);#(12)
return(v)
}

###for numeric derivatives
library(numDeriv);
library(MASS);
###evaluate parameter covariance matrix
grd = jacobian(fmom, vp);#d
dw = crossprod(grd, W);#d'W
# dwd = solve(dw %*% grd, tol=.Machine$double.eps);#(d'W*d)^{-1}
dwd = try(solve(dw %*% grd), silent=FALSE);#(d'W*d)^{-1}
if (inherits(dwd, "try-error")) dwd = ginv(dw %*% grd)
ss = cov( fmom(vp, means=FALSE) );#S
pcov = (dwd %*% (dw %*% tcrossprod(ss, dw)) %*% dwd) / nrow(zt);
###print estimates and standard errors
temp = data.frame(par=vp, se=sqrt(diag(pcov)));
temp$t_ratio = temp$par/temp$se;
print(temp)

```

```

##average z
zbar = colMeans(zt);
print(zbar)
tvpar = function(dz, pcov, dnam, quad=FALSE) {#delta method
  dzvar = crossprod(c(crossprod(zbar, pcov)), zbar);
  if (quad) dzvar = 4 * abs(dz) * dzvar
  wald = dz^2 / dzvar;
cat(sprintf("d(zbar)%s: %8.3f, s.e. = %9.3f, pval = %4.3f", dnam, dz, sqrt(dzvar),
pchisq(wald, 0.05, lower.tail=FALSE)), "\n");
}

##plot time-varying parameters
par(mfcol=c(3,2), mar=c(2, 2, 1, 1), mgp=c(1, 0.2, 0), tcl=-0.2);
##constant
pdx = seq.int(nz);
dt = (zt %*% vp[pdx])^2;
plot(dt, type="l", col="blue", xlab="", ylab="d0t");
dz = crossprod(zbar, vp[pdx])^2;
abline(h=dz, col=gray(0.5));
tvpar(dz, pcov[pdx,pdx], "_0t", quad=TRUE);
if (pow==0) {
  dnam = c("_rm", "smb", "hml", "Mom");
  for (j in seq_along(dnam)) {
    pdx = pdx + nz;
    dt = zt %*% vp[pdx];
    plot(dt, type="l", col="blue", xlab="", ylab=paste("d", dnam[j], sep=""));
    dz = crossprod(zbar, vp[pdx]);
    abline(h=dz, col=gray(0.5));
    tvpar(dz, pcov[pdx,pdx], dnam[j]);
  }
} else {
  dnam = c("smb", "hml", "Mom");
  for (j in seq_along(dnam)) {
    pdx = pdx + nz;
    dt = zt %*% vp[pdx];
    plot(dt, type="l", col="blue", xlab="", ylab=paste("d", dnam[j], sep=""));
    dz = crossprod(zbar, vp[pdx]);
    abline(h=dz, col=gray(0.5));
    tvpar(dz, pcov[pdx,pdx], dnam[j]);
  }
}
for (i in seq.int(pow)) {
  ##r{ market}
  pdx = pdx + nz;
  dt = (-1)^i * (zt %*% vp[pdx])^2;
  plot(dt, type="l", col="blue", xlab="", ylab=paste("d", i, "t", sep=""));
  dz = (-1)^i * crossprod(zbar, vp[pdx])^2;
  abline(h=dz, col="blue", lty="dashed");
  tvpar(dz, pcov[pdx,pdx], paste("_", i, "m", sep=""), quad=TRUE);
  ##r{labor}
}

```

```

pdx = pdx + nz;
dt = (-1)^i * (zt %*% vp[pdx])^2;
lines(dt, col="red");
dz = (-1)^i * crossprod(zbar, vp[pdx])^2;
abline(h=dz, col="red", lty="dashed");
tvpvar(dz, pcov[pdx,pdx], paste("_", i, "I", sep=""), quad=TRUE);
##r{house price}
pdx = pdx + nz;
dt = (-1)^i * (zt %*% vp[pdx])^2;
lines(dt, col="red");
dz = (-1)^i * crossprod(zbar, vp[pdx])^2;
abline(h=dz, col="green", lty="dashed");
tvpvar(dz, pcov[pdx,pdx], paste("_", i, "h", sep=""), quad=TRUE);
}
}
##decompose hansen-jagannathan distance
pt = rz %*% crossprod(W, fmom(vp));
n = length(pt);#undo df correction
cat(sprintf("mean(pt) = %7.4f, sd(pt) = %7.4f, dist = %7.4f", mean(pt),
sqrt(var(pt)*(n-1)/n), sqrt(gmm$objective)), "\n");
}

#data();
#tab5a(0L);
#tab5b(0L);
#tab5a(2L);
#tab5b(2L);
#tab5a(3L);
tab5b(3L);

```

R-code for Table 12

```
##clear
rm(list=ls());
graphics.off();
options(warn=1, htmlhelp=TRUE);

##merge and write data file (monthly fractions)
data = function(fnam="dat3.csv") {

  ##17 industry portfolio returns (monthly %)
  p17 = read.table("17_Industry_Portfolios1.txt", header=TRUE);
  p17[,-1] = p17[,-1]/100;

  ##3 month treasury bill (annualized %)
  tb3 = read.table("TB31.txt", header=TRUE);
  tb3[,-1] = tb3[,-1]/12;
  ##fama-french factors (monthly %)
  ff = read.table("F-F_Research_Data_Factors1.txt", header=TRUE);
  ff[,-1] = ff[,-1]/100;

  ##Human capital
  hc = read.table("HC1.txt", header=TRUE);
  rl = hc$Rl    # net return

  one <- rep(1,390);
  ##dividend yield (monthly fractions)
  dy <- read.table("dividend yield1.txt", header=TRUE);
  ##combine all regressors into one data frame
  dat <- data.frame(one, ff, rrmrf = ff$rmrf, rm = ff$rmrf + ff$RF, divyld = dy$Y,
  yldspr = tb3[,2] - ff$RF, tb = ff$RF, rl);

  ##write to file
  write.table(p17, "p17.csv", sep=",", row.names=FALSE);
  write.table(dat, "zt.csv", sep=",", row.names=FALSE);
}

##column-wise kronecker product
ckron = function(A, B) {
  stopifnot(nrow(A)==nrow(B))
  ca = ncol(A);
  cb = ncol(B);
  AB = matrix(NA, nrow(A), ca*cb);
  jdx = seq.int(cb);
  for (j in seq.int(ca)) {
    AB[,jdx] = A[,j]*B;
    jdx = jdx + cb;
  }
  return(AB)
}
```

```

}

gmm6a = function(vp, R, rm1, zt, method=c("optim", "nlminb"), control=list()) {
  #stopifnot((length(vp)==2 && is.null(r1)) || (length(vp)==3 && !is.null(r1)));
  method = match.arg(method);
  ##optimal weigting matrix using E[(R*zt)(R*zt)']
  W <- solve( crossprod( ckron(1+R, zt) )/nrow(R) );
  ##check only once
  stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1));
  ##gmm objective function to minimize
  objf <- function(vpar){
    m1 = vpar[1]*(1+rm1)^vpar[2];
    ##pricing error
    v = ckron((1+R)*c(m1) - 1, zt);
    g = colMeans(v);#(12)
    return( crossprod(g, crossprod(W, g)) )
  }
  if (method=="optim") optim(vp, objf, control=control) else nlminb(vp, objf,
control=control)
}

```

```

##power utility kernel
tab6a = function(addhc=FALSE) {
  y = read.csv("p17.csv"); #R{t+1}
  y$Date = NULL;
  z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
  R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
  zt = as.matrix(z0[-nrow(z0),]); #z{t}
  rm = read.csv("zt.csv")["rm"];
  rm1 = as.matrix(rm[-1,]) #R{m,t+1}
  rl = read.csv("zt.csv")[,c("rl")];
  rl1 = as.matrix(rl[-1]) #R{1,t+1}
  ##estimate power utility parameters
  v0 = c(0.1, -0.1)
  ##quasi-newton
  gmm = gmm6a(v0, R, rm1, zt, method="nlminb");
  cat(sprintf("fmin = %13.9f, info=%i, iter=%i, mesg=%s", gmm$objective,
gmm$convergence, gmm$iterations, gmm$message), "\n");
  # save(gmm, file="tab60.Rdata");
  print(gmm$par);
  stopifnot(gmm$convergence==0);

```

```

##standard errors
##optimal weigting matrix using E[(R*zt)(R*zt)']
rz = ckron(1+R, zt);
W = solve( crossprod(rz)/nrow(R) );
##sample moment conditions
fmom <- function(vp, means=TRUE){
  m1 = vp[1]*(1+rm1)^vp[2];
  ##pricing error

```

```

v = ckron((1+R)*c(m1) - 1, zt);
if (means) v = colMeans(v);#(12)
return(v)
}

library(numDeriv);
##evaluate parameter covariance matrix
grd = jacobian(fmom, gmm$par);#d
dw = crossprod(grd, W);#d'W
dwd = solve(dw %*% grd);#(d'W*d)^{-1}
# dwd = solve(dw %*% grd, tol=1e-30);#(d'W*d)^{-1}
ss = cov( fmom(gmm$par, means=FALSE) );#S
pcov = (dwd %*% (dw %*% tcrossprod(ss, dw)) %*% dwd) / nrow(zt);
##print estimates and standard errors
temp = data.frame(par=gmm$par, se=sqrt(diag(pcov)));
temp$t_ratio = temp$par/temp$se;
temp$pvalue = 2*pnorm(abs(temp$t_ratio), lower.tail=FALSE);
print(temp)

##decompose hansen-jagannathan distance
pt = rz %*% crossprod(W, fmom(gmm$par));
n = length(pt);#undo df correction
cat(sprintf("mean(pt) = %7.4f, sd(pt) = %7.4f, dist = %7.4f", mean(pt),
sqrt(var(pt)*(n-1)/n), sqrt(gmm$objective)), "\n");
}

gmm6b = function(vp, R, rm1, r11, zt, method=c("optim", "nlminb"), control=list()) {
#stopifnot((length(vp)==2 && is.null(r11)) || (length(vp)==3 && !is.null(r11)));
method = match.arg(method);
##optimal weighting matrix using E[(R*zt)(R*zt)']
W <- solve( crossprod( ckron(1+R, zt) )/nrow(R) );
##check only once
stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1));
##gmm objective function to minimize
objf <- function(vpar){
m1 = vpar[1]*(1+vpar[3]*rm1+(1-vpar[3])*r11)^vpar[2];
##pricing error
v = ckron((1+R)*c(m1) - 1, zt);
g = colMeans(v);#(12)
return( crossprod(g, crossprod(W, g)) )
}
if (method=="optim") optim(vp, objf, control=control) else nlminb(vp, objf,
control=control)
}

##power utility kernel
tab6b = function() {
y = read.csv("p17.csv"); #R{t+1}
y$Date = NULL;

```

```

z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
zt = as.matrix(z0[-nrow(z0),]); #z{t}
rm = read.csv("zt.csv")["rm"];
rm1 = as.matrix(rm[-1,]) #R{m,t+1}
rl = read.csv("zt.csv")[,c("rl")];
rl1 = as.matrix(rl[-1]) #R{1,t+1}
##estimate power utility parameters
v0 = c(0.1, -0.1, 0.5)
##quasi-newton
gmm = gmm6b(v0, R, rm1, rl1, zt, method="nlminb");
cat(sprintf("fmin = %13.9f, info=%i, iter=%i, mesg=%s", gmm$objective,
gmm$convergence, gmm$iterations, gmm$message), "\n");
# save(gmm, file="tab60.Rdata");
print(gmm$par);
stopifnot(gmm$convergence==0);

##standard errors
##optimal weighting matrix using E[(R*zt)(R*zt)']
rz = kron(1+R, zt);
W = solve( crossprod(rz)/nrow(R) );
##sample moment conditions
fmom <- function(vp, means=TRUE){
  m1 = vp[1]*(1+vp[3]*rm1+(1-vp[3])*rl1)^vp[2];
  ##pricing error
  v = kron((1+R)*c(m1) - 1, zt);
  if (means) v = colMeans(v);#(12)
  return(v)
}

library(numDeriv);
##evaluate parameter covariance matrix
grd = jacobian(fmom, gmm$par);#d
dw = crossprod(grd, W);#d'W
dwd = solve(dw %*% grd);#(d'W*d)^{-1}
# dwd = solve(dw %*% grd, tol=1e-30);#(d'W*d)^{-1}
ss = cov( fmom(gmm$par, means=FALSE) );#S
pcov = (dwd %*% (dw %*% tcrossprod(ss, dw)) %*% dwd) / nrow(zt);
##print estimates and standard errors
temp = data.frame(par=gmm$par, se=sqrt(diag(pcov)));
temp$t_ratio = temp$par/temp$se;
temp$pvalue = 2*pnorm(abs(temp$t_ratio), lower.tail=FALSE);
print(temp)

##decompose hansen-jagannathan distance
pt = rz %*% crossprod(W, fmom(gmm$par));
n = length(pt);#undo df correction
cat(sprintf("mean(pt) = %7.4f, sd(pt) = %7.4f, dist = %7.4f", mean(pt),
sqrt(var(pt)*(n-1)/n), sqrt(gmm$objective)), "\n");
}

```

```
#tab6a());
tab6b());
```

R-code for Table 13

```
##clear
rm(list=ls());
graphics.off();
options(warn=1, htmlhelp=TRUE);

##merge and write data file (monthly fractions)
data = function(fnam="dat.csv") {

  ##17 industry portfolio returns (monthly %)
  p17 = read.table("17_Industry_Portfolios1.txt", header=TRUE);
  p17[,-1] = p17[,-1]/100;

  ##3 month treasury bill (annualized %)
  tb3 = read.table("TB31.txt", header=TRUE);
  tb3[,-1] = tb3[,-1]/12;
  ##fama-french factors (monthly %)
  ff = read.table("F-F_Research_Data_Factors1.txt", header=TRUE);
  ff[,-1] = ff[,-1]/100;

  ##Human capital
  hc = read.table("HC1.txt", header=TRUE);
  rl = hc$R1 # net return

  one <- rep(1,390);
  ##dividend yield (monthly fractions)
  dy <- read.table("dividend yield1.txt", header=TRUE);
  ##combine all regressors into one data frame
  dat <- data.frame(one, ff, rmrf = ff$rmrf, rm = ff$rmrf + ff$RF, divyld = dy$Y,
yldspr = tb3[,2] - ff$RF, tb = ff$RF, rl);

  ##write to file
  write.table(p17, "p17.csv", sep=",", row.names=FALSE);
  write.table(dat, "zt.csv", sep=",", row.names=FALSE);
}

##column-wise kronecker product
ckron = function(A, B) {
```



```

stopifnot(nrow(A)==nrow(B))
ca = ncol(A);
cb = ncol(B);
AB = matrix(NA, nrow(A), ca*cb);
jdx = seq.int(cb);
for (j in seq.int(ca)) {
  AB[,jdx] = A[,j]*B;
  jdx = jdx + cb;
}
return(AB)
}

```

```

##with human capital and shape restriction
gmm8 = function(vp, R, rm1, r11, zt, pow=3L, method=c("optim", "nlminb"),
control=list()) {
  stopifnot(pow==1 || pow==2 || pow==3);
  method = match.arg(method);
  ##optimal weigting matrix using E[(R*zt)(R*zt)']
  W <- solve( crossprod( ckron(1+R, zt) )/nrow(R) );
  ##check only once
  stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1), nrow(R)==length(r11));
  ##gmm objective function to minimize
  objf <- function(vpar){
    m1 = (zt %*% vpar[1:5])^2;#constant
    d1m = -(zt %*% vpar[6:10])^2;
    d1l = -(zt %*% vpar[11:15])^2;
    if (pow==1) {
      m1 = m1 + d1m*rm1 + d1l*r11;
    } else if (pow>1) {
      d2m = (zt %*% vpar[16:20])^2;
      d2l = (zt %*% vpar[21:25])^2;
      if (pow==2) {
        d1m = pmin(d1m, -2*d2m*rm1);
        d1l = pmin(d1l, -2*d2l*r11);
        m1 = m1 + d1m*rm1 + d1l*r11 + d2m*rm1^2 + d2l*r11^2;
      } else {#pow==3
        d3m = -(zt %*% vpar[26:30])^2;
        tmp = -(d1m + 2*d2m*rm1)/(3*rm1^2);
        idx = (d3m > tmp);
        d1m = ifelse(idx, pmin(d1m, -d2m*rm1), d1m);
        d3m = ifelse(idx, tmp, d3m);

        d3l = -(zt %*% vpar[31:35])^2;
        tmp = -(d1l + 2*d2l*r11)/(3*r11^2);
        idx = (d3l > tmp);
        d1l = ifelse(idx, pmin(d1l, -d2l*r11), d1l);
        d3l = ifelse(idx, tmp, d3l);
      }
    }
  }
}

```

```

    m1 = m1 + d1m*rm1 + d1l*r1l + d2m*rm1^2 + d2l*r1l^2 + d3m*rm1^3 +
d3l*r1l^3;
  }
}
m1 = pmax(m1, 0);#impose m1>=0
##pricing error (matrix)
v = ckron((1+R)*c(m1) - 1, zt);
g = colMeans(v);#(12)
return( crossprod(g, crossprod(W, g)) )
}

if (method=="optim") optim(vp, objf, control=control) else nlminb(vp, objf,
control=control)
}

##optimization with human capital and shape restriction
tab8a = function(pow=3L) {
  ##read data
  y = read.csv("p17.csv"); #R{t+1}
  y$Date = NULL;
  z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
  R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
  zt = as.matrix(z0[-nrow(z0),]); #z{t}
  rm = read.csv("zt.csv")["rm"];
  rm1 = as.matrix(rm[-1,]) #R{m,t+1}
  rl = read.csv("zt.csv")[,c("rl")];
  rl1 = as.matrix(rl[-1]) #R{1,t+1}
  ##fit gmm with hansen-jaganathan fixed weights
# vp = rep(0.1, ncol(z0)*(1+2*pow));#starting values
if (pow==1) {
  load(paste("tab4", pow, ".Rdata", sep=""));#gmm
} else {
  load(paste("tab8", pow-1, ".Rdata", sep=""));#gmm
  gmm$par = c(gmm$par, rep(0, 2*ncol(zt)));
}
##polytope
gmm = gmm8(gmm$par, R, rm1, rl1, zt, pow, method="optim",
control=list(maxit=90000));
cat(sprintf("fmin = %13.9f, info=%i, iter=%i", gmm$value, gmm$convergence,
gmm$counts[1]), "\n");
##quasi-newton
# gmm = gmm8(gmm$par, y, rm, rg, z0, pow, method="nlminb",
control=list(trace=0, eval.max=90000, iter.max=90000));
#cat(sprintf("fmin = %13.9f, info=%i, mesg=%s", gmm$objective,
gmm$convergence, gmm$message), "\n");
save(gmm, file=paste("tab8", pow, ".Rdata", sep=""));
print(gmm$par);
}

##test parameters at means

```

```

tab8b = function(pow=3L) {
  stopifnot(pow==1 || pow==2 || pow==3);
  ##read data
  y = read.csv("p17.csv"); #R{t+1}
  y$Date = NULL;
  z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
  R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
  zt = as.matrix(z0[-nrow(z0),]); #z{t}
  rm = read.csv("zt.csv")["rm"];
  rm1 = as.matrix(rm[-1,]) #R{m,t+1}
  rl = read.csv("zt.csv")[,c("r1")];
  rl1 = as.matrix(rl[-1]) #R{1,t+1}
  ##check only once
  stopifnot(nrow(R)==nrow(zt), nrow(R)==length(rm1), nrow(R)==length(rl1));

  ##read estimated parameter values
  load(paste("tab8", pow, ".Rdata", sep=""));#gmm
  vpar = gmm$par;

  ##optimal weighting matrix using E[(R*zt)(R*zt)']
  rz = kron(1+R, zt);
  W = solve( crossprod(rz)/nrow(R) );

  ##return sample moment conditions
  fmom <- function(vpar, means=TRUE){
    m1 = (zt %*% vpar[1:5])^2;#constant
    d1m = -(zt %*% vpar[6:10])^2;
    d1l = -(zt %*% vpar[11:15])^2;
    if (pow==1) {
      m1 = m1 + d1m*rm1 + d1l*rl1;
    } else if (pow>1) {
      d2m = (zt %*% vpar[16:20])^2;
      d2l = (zt %*% vpar[21:25])^2;
      if (pow==2) {
        d1m = pmin(d1m, -2*d2m*rm1);
        d1l = pmin(d1l, -2*d2l*rl1);
        m1 = m1 + d1m*rm1 + d1l*rl1 + d2m*rm1^2 + d2l*rl1^2;
      } else {#pow==3
        d3m = -(zt %*% vpar[26:30])^2;
        tmp = -(d1m + 2*d2m*rm1)/(3*rm1^2);
        idx = (d3m > tmp);
        d1m = ifelse(idx, pmin(d1m, -d2m*rm1), d1m);
        d3m = ifelse(idx, tmp, d3m);

        d3l = -(zt %*% vpar[31:35])^2;
        tmp = -(d1l + 2*d2l*rl1)/(3*rl1^2);
        idx = (d3l > tmp);
        d1l = ifelse(idx, pmin(d1l, -d2l*rl1), d1l);
        d3l = ifelse(idx, tmp, d3l);
      }
    }
  }
}

```

```

    m1 = m1 + d1m*rm1 + d1l*r1l + d2m*rm1^2 + d2l*r1l^2 + d3m*rm1^3 +
d3l*r1l^3;
  }
}

m1 = pmax(m1, 0);#impose m1>=0
##pricing error (matrix)
v = ckron((1+R)*c(m1) - 1, zt);
if (means) v = colMeans(v);#(12)
return(v)
}

##for numeric derivatives
library(numDeriv);
library(MASS);
print(vp)
##evaluate parameter covariance matrix
grd = jacobian(fmom, vp);#d
dw = crossprod(grd, W);#d'W

# dwd = solve(dw %*% grd, tol=Machine$double.eps);#(d'W*d)^{-1}
dwd = try(solve(dw %*% grd), silent=FALSE);#(d'W*d)^{-1}
if (inherits(dwd, "try-error")) dwd = ginv(dw %*% grd)
ss = cov( fmom(vp, means=FALSE) );#S
pcov = (dwd %*% (dw %*% tcrossprod(ss, dw)) %*% dwd) / nrow(zt);
##print estimates and standard errors
temp = data.frame(par=vp, se=sqrt(diag(pcov)));
temp$st_ratio = temp$par/temp$se;
print(temp)

##average z
zbar = colMeans(zt);
print(zbar)
tvar = function(dz, pcov, dnam) {#delta method
  dzvar = 4 * abs(dz) * crossprod(c(crossprod(zbar, pcov)), zbar);
  wald = dz^2 / dzvar;
cat(sprintf("d(zbar)%s: %8.3f, s.e. = %9.3f, pval = %4.3f", dnam, dz, sqrt(dzvar),
pchisq(wald, 0.05, lower.tail=FALSE)), "\n");
}
##plot time-varying parameters
par(mfcol=c(2,2));
pdx = seq.int(ncol(zt));
##constant
dt = (zt %*% vp[pdx])^2;
plot(dt, type="l", col="blue", xlab="", ylab="d0t");
dz = crossprod(zbar, vp[pdx])^2;
abline(h=dz, col=gray(0.5));
tvar(dz, pcov[pdx,pdx], "_0t");
for (i in seq.int(pow)) {
  ##r{market}

```

```

pdx = pdx + ncol(zt);
dt = (-1)^i * (zt %*% vp[pdx])^2;
plot(dt, type="l", col="blue", xlab="", ylab=paste("d", i, "t", sep=""));
dz = (-1)^i * crossprod(zbar, vp[pdx])^2;
abline(h=dz, col="blue", lty="dashed");
tvpair(dz, pcov[pdx,pdx], paste("_", i, "m", sep=""));
##r{labor}
pdx = pdx + ncol(zt);
dt = (-1)^i * (zt %*% vp[pdx])^2;
lines(dt, col="red");
dz = (-1)^i * crossprod(zbar, vp[pdx])^2;
abline(h=dz, col="red", lty="dashed");
tvpair(dz, pcov[pdx,pdx], paste("_", i, "l", sep=""));
}

```

```

##decompose hansen-jagannathan distance
pt = rz %*% crossprod(W, fmom(vp));
n = length(pt);#undo df correction
# cat(sprintf("mean(pt) = %7.4f, sd(pt) = %7.4f, dist = %7.4f", mean(pt),
sqrt(var(pt)*(n-1)/n), sqrt(gmm$objective)), "\n");
cat(sprintf("mean(pt) = %7.4f, sd(pt) = %7.4f, dist = %7.4f", mean(pt),
sqrt(var(pt)*(n-1)/n), sqrt(gmm$value)), "\n");
}

```

```

##restricted pricing kernel surface
fig2 = function() {
library(lattice);
##read data
y = read.csv("p17.csv");
y$Date = NULL;
z0 = read.csv("zt.csv")[,c("one", "rmrf", "divyld", "yldspr", "tb")];
##evaluate at means of z0
zbar = colMeans(z0);
R = as.matrix(cbind(y[-1,], z0$tb[-1]));#R{t+1}
zt = as.matrix(z0[-nrow(z0),]); #z{t}
rm = read.csv("zt.csv")["rm"];
rm1 = as.matrix(rm[-1,]) #R{m,t+1}
rl = read.csv("zt.csv")[,c("rl")];
rl1 = as.matrix(rl[-1]) #R{1,t+1}

```

```

stopifnot(length(zbar)==5);

```

```

##evaluate pricing kernel
sdf <- function(vpar, rm1, rl1, pow=3L){
stopifnot(pow==1 || pow==2 || pow==3);
m1 = crossprod(zbar, vpar[1:5])^2;#constant
d1m = -crossprod(zbar, vpar[6:10])^2;
d1l = -crossprod(zbar, vpar[11:15])^2;
if (pow==1) {

```

```

    m1 = m1 + d1m*rm1 + d1l*r1l;
  } else if (pow>1) {
    d2m = crossprod(zbar, vpar[16:20])^2;
    d2l = crossprod(zbar, vpar[21:25])^2;
    if (pow==2) {
      d1m = pmin(d1m, -2*d2m*rm1);
      d1l = pmin(d1l, -2*d2l*r1l);
      m1 = m1 + d1m*rm1 + d1l*r1l + d2m*rm1^2 + d2l*r1l^2;
    } else {#pow==3
      d3m = -crossprod(zbar, vpar[26:30])^2;
      tmp = -(d1m + 2*d2m*rm1)/(3*rm1^2);
      idx = (c(d3m) > tmp);
      d1m = ifelse(idx, pmin(d1m, -d2m*rm1), d1m);
      d3m = ifelse(idx, tmp, d3m);

      d3l = -crossprod(zbar, vpar[31:35])^2;
      tmp = -(d1l + 2*d2l*r1l)/(3*r1l^2);
      idx = (c(d3l) > tmp);
      d1l = ifelse(idx, pmin(d1l, -d2l*r1l), d1l);
      d3l = ifelse(idx, tmp, d3l);

      m1 = m1 + d1m*rm1 + d1l*r1l + d2m*rm1^2 + d2l*r1l^2 + d3m*rm1^3 +
d3l*r1l^3;
    }
  }
  return( pmax(m1, 0) );#impose m1>=0
}

x = seq(min(rm1), max(rm1), len=20);#rm1
y = seq(min(r1l), max(r1l), len=20);#r1l
g = expand.grid(x = x, y = y);
for (pow in seq.int(3)) {
  load(paste("tab8", pow, ".Rdata", sep=""));#gmm
  g[[paste("pow", pow, sep="")] ] = sdf(gmm$par, g$x, g$y, pow)
}
print(wireframe(pow3 ~ x*y, g, outer=TRUE, distance=0, col=gray(0.7),
screen=list(z=20, x=-40), colorkey=FALSE, drape=TRUE,
default.scales=list(distance=c(1,1,1), arrows=FALSE),
lattice.options=list(as.table=FALSE), xlab="market", ylab="labor", zlab="m"))

```