# Equivalent Random Analysis of a Buffered Optical Switch with General Interarrival Times

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***Abstract*—We propose an approximate analytic model of an optical switch with fibre delay lines and wavelength converters by employing Equivalent Random Theory. General arrival traffic is modelled by means of Gamma-distributed interarrival times. The analysis is formulated in terms of virtual traffic flows within the optical switch from which we derive expressions for burst blocking probability, fibre delay line occupancy and mean delay. Emphasis is on approximations that give good numerical efficiency so that the method can be useful for formulating dimensioning problems for large-scale networks. Numerical solution values from the proposed analysis method compare well with results from a discrete-event simulation of an optical burst switch.**

## I. Introduction

In recent years, considerable research effort has focused on developing efficient Optical Burst Switching (OBS) and Optical Packet Switching (OPS) architectures and on performance improvements by way of contention-resolution schemes [1], [2] and optimised burst aggregation algorithms [3], [4]. Although the technologies are maturing to the extent that test-beds have been built [5], [6] and it seems likely that OBS and OPS may be deployed in the medium-term, there remains a need to resolve pertinent network design, dimensioning and cost-optimisation challenges to enable network deployment. To this end, efficient analysis methods for OBS/OPS node and network performance evaluation are desirable and considerable attention is now focused there [7]. In particular, the analysis of wavelength conversion schemes and fibre delay lines (FDLs), as two of the main contention-resolution components of the switch, is receiving attention.

The addition of wavelength converters to the switch reduces contention at output ports by enabling a packet arriving on one wavelength channel to be directed to an alternative free wavelength channel at the output. In performance evaluation studies, there may be assumed restrictions on the number of available wavelength converters and on the sharing strategy. Additionally, there may be restrictions on the range of conversion between one wavelength and another, due to limiting physical properties of the conversion devices [8], [9].

The addition of FDLs to the switch has also been shown to achieve a substantial reduction in packet loss (by orders of magnitude in some cases [10]) by selectively delaying packets in order to reduce contention for outgoing channels.

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Our focus in this paper is on the analysis of burst/packet loss and delay in OBS/OPS nodes with FDLs and unrestricted wavelength conversion. We develop an approximate model of switch performance, for general offered traffic, by applying circuit-switching analysis methods to model the switch output port and associated FDLs. Our goal is an efficient model that can accurately account for likely traffic characteristics within an OBS/OPS network so that the node model may be applied to modelling/dimensioning large networks of optical switches.

There are several existing approaches to performance evaluation of optical nodes with buffering functionality implemented with FDLs. In [11], Callegati presents a framework for evaluating the blocking probability for asynchronous variable length bursts and models a single FDL as a queue with balking. A similar approach has been adopted by Lu & Mark in [10], where the overall system behaviour is characterised as a multi-dimensional continuous-time Markov chain. They develop an asymptotic approximation based on the $M/M/k$ queue with balking, when arriving bursts are short, and an $M/M/k/m$ queue for long bursts. An exact Markov chain analysis is also provided by Rogiest et al. [12] and an analysis for correlated arrivals is considered in [13]. In [14], Fan et al. model buffers as $M/M/k/m$ queues and provide bounds on the loss probabilities for classless and prioritised bursts. Gauger [15] investigates the influence of the combination of wavelength converters and FDL buffers in OBS, through simulation. The performance of several scenarios of feed-back and feed-forward FDL schemes are evaluated.

Previous work on performance evaluation of FDLs has largely assumed that burst interarrival times are exponentially distributed. Recently, Mountrouidou and Perros have studied burst aggregation algorithms at ingress nodes and propose that this assumption is not accurate [16]. Burst interarrival times are shown to be Gaussian or Erlang distributed, depending on the burst aggregation method and the packet arrival process at the aggregator. As for burst length distribution, Gauger [17] has found from simulation that performance is relatively insensitive to burst length distribution. Rostami and Wolisz [18], through analysis, also show that burst length distribution has little impact on performance, concluding that assuming exponentially-distributed burst lengths is appropriate.

This previous work leads us to consider a modelling framework for generally-distributed arrivals and exponentially-distributed burst lengths and we base our analysis on the

$GI/M/N/N$ loss system. We have chosen gamma-distributed interarrivals as a concrete case of a general independent ($GI$) traffic arrival process, although our analysis method is applicable for any renewal-type traffic with known interarrival distribution. Gamma-distributed interarrivals allow a full range of interarrival-time variance to be modelled (both 'smooth' and 'peaked' traffic). It is known that variance of ingress traffic can range widely, depending on the burst aggregation method employed [16] and the traffic offered to the aggregator. We note that some popular traffic models, such as the Interrupted Poisson Process [21], do not allow representation of low variances (smooth traffic). Additionally, the gamma distribution can be parameterised to correspond exactly to the exponential distribution, allowing comparison of results to Poisson traffic, the most commonly assumed traffic type.

The modelling approach in the current paper identifies virtual traffic flows, between the output channels and FDLs, modelling the node as a network of relatively simple queuing systems. This differs from previous work, as outlined above, which has focused mainly on direct evaluation of more complex single-queue systems. We make use of existing results for calculating overflow and carried traffic characteristics in loss systems, by way of Equivalent Random Theory (ERT) [19] and Brandt and Brandt's work on the $GI/M/N/N$ system [20]. Our approach most closely relates to Reviriego et al. [9], where overflow analysis is applied to evaluate blocking, for Poisson arrivals, for a limited number of shared wavelength converters in an OBS node without FDLs. We do not consider the added complexity of converter sharing in the present work.

## II. Switch Architecture

The system under study (Fig. 1) is an optical burst switching node with wavelength conversion and feed-forward FDLs at the output ports. It is assumed that the range of conversion from one wavelength to another is unrestricted and that there are as many converters at an output port as there are wavelength channels, that is, 'full' wavelength conversion is available. We note that, although we deal with the case of burst switching, the model we develop may also be applied to an optical packet switch with feed-forward buffers.

Our model focuses on the analysis of the blocking probability and mean delay at an output port with $N$ channels and a bank of FDLs containing $K$ FDL units. Each FDL unit is a single fibre offering a constant delay time of $D_k$ seconds, $k \in \{1, 2, \ldots, K\}$. Delay times of the units are each a multiple of a base delay time $C$ so that $D_k = kC$. There is also a direct channel from the switch to the output unit, with delay $D_0 \approx 0$. Additionally, each fibre may be wavelength division multiplexed carrying multiple wavelengths simultaneously with FDL unit $k$ supporting $L_k$ wavelength channels. The total number of wavelength channels provided by the bank of FDLs is $L = \sum_k L_k$.

A controller in the switch coordinates scheduling of the channels and FDLs. If none of the $N$ output channels is available for the duration of a burst arriving at a time $t$, an attempt is made to simultaneously schedule a free FDL (of

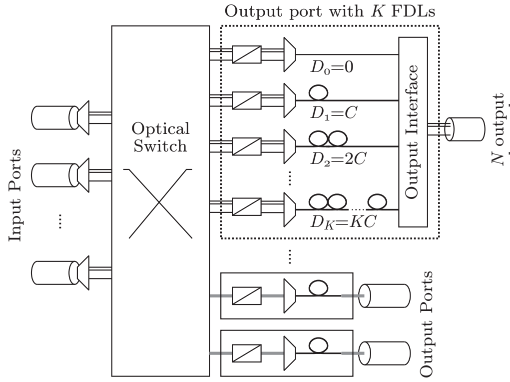


Fig. 1. Optical Switch Under Study

delay length $D_k$) and any output channel that will become free at time $t + D_k$. The scheduler first attempts the procedure using FDL unit 1, offering delay $D_1$, and iterates in sequence through all $K$ FDLs until a feasible schedule is found. If none of the available FDL delay times can resolve the schedule, then the burst is blocked (lost). We next develop a traffic model which represents an approximate analogue of the switch resource scheduling behaviour just described.

## III. Output Port Traffic Model

We assume that the aggregate traffic arriving to the output port is of general renewal type ($GI$ traffic) and burst lengths are taken to be exponentially distributed. Thus, the probability of blocking at the output port could be estimated, in the first instance, by analysing blocking in an $GI/M/N/N$ system, where $N$ is the number of output channels. A single $GI/M/N/N$ model would, of course, not take into account the coordinated scheduling of output channels and FDLs in the actual system, which tends to correlate burst arrivals at the output channels in a manner that gives a reduction in blocking compared to that of a $GI/M/N/N$ system. Our modelling aim is to approximate the improvement given by the FDLs without resorting to a detailed analysis of the traffic correlations involved. We model FDL behaviour as an additional $GI/M/L/L$ blocking system and develop a model of virtual flows (Fig. 2) that approximates the overall output port scheduling behaviour.

We make the observation that traffic which is potentially blocked by the output channels, before the scheduler attempts to resolve conflicts by delaying bursts in the FDLs, may be approximated as a (virtual) overflow traffic from an $GI/M/N/N$ system representing the group of output channels. This overflow is indicated in Fig. 2 as flow $\hat{\mathcal{F}}$. We then consider this overflow traffic as forming offered traffic to an independent $GI/M/L/L$ system representing the bank of FDL.

We justify this lumped model of the FDL bank by observing that each FDL $k$, consisting of a group of $L_k$ channels, may be approximately modelled as an $GI/M/L_k/L_k$ system. As traffic offered to the output channels is assumed renewal, then so is the overflow $\hat{\mathcal{F}}$ [19] and as the scheduler first attempts to

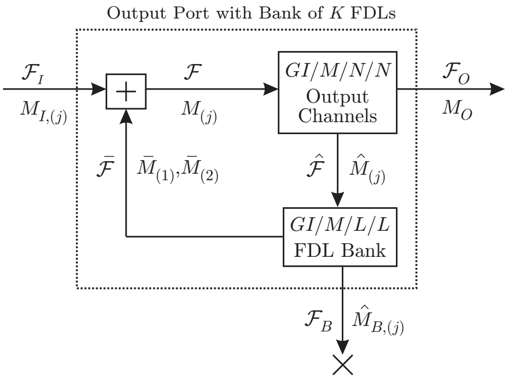


Fig. 2. Virtual Flow Model of Output Port with FDLs

resolve a conflict with FDL 1, we may consider FDL 1 as an independent loss system offered all overflow traffic from the group of $N$ channels. FDL 1 is itself a group of $L_1$ channels and, when all $L_1$ channels are occupied, the scheduler cannot resolve a conflict using FDL 1 and instead attempts to resolve it with the delay offered by FDL 2. Thus we can view FDL 1 as generating its own renewal overflow traffic $\hat{\mathcal{F}}_1$ which in turn is offered to FDL 2, and so on down the chain of $K$ FDLs, with each FDL $k$ producing overflow which is offered to FDL $k+1$. These virtual traffic flows within the FDL bank are depicted in Fig. 3. Overflow $\hat{\mathcal{F}}_K = \mathcal{F}_B$ from the final FDL represents the actual overflow from the output port. This traffic flow, $\mathcal{F}_B$, is lost from the system (blocked). For the purposes of calculating overflow (blocking) from the FDL bank, we may combine this cascade of overflowing loss systems as a single $GI/M/L/L$ system, where $L$ is the aggregate number of channels in the bank. To calculate mean delay, we resolve the occupancy in each of the $K$ FDLs.

To complete the flow model, we consider the combined traffic carried by all FDLs in the bank as a traffic flow that is offered again (notionally) to the output channels, at some time in the future. This total carried traffic flow from the FDLs, $\bar{\mathcal{F}}$, competes with the input traffic flow ($\mathcal{F}_I$) for the output channels at that future time. We neglect time correlations between these flows and identify an effective (virtual) flow $\mathcal{F}$ that is the aggregation of the input flow ($\mathcal{F}_I$) and the FDL carried traffic ($\bar{\mathcal{F}}$) that is fed back to the input of the channel model. We emphasise that there is no such feedback path in the actual system. We have adopted it solely to capture the balance of flows in our modelling analogue. As the feedback traffic is not renewal, neither is the aggregated input traffic ($\mathcal{F}$). For the purpose of formulating an approximate model, we assume that the feedback flow ($\bar{\mathcal{F}}$) is small in comparison with ($\mathcal{F}_I$) and so the renewal nature of $\mathcal{F}$ is assumed to be undisturbed.

We characterise the various traffic flows in the model using the notion of an infinite server (or 'infinite trunk group') [21], whereby a traffic flow is described in terms of the moments of the channel occupancy distribution in a $GI/M/\infty$ system when offered an identical traffic flow. The channel occupancy distribution may be classes as being 'peaked', when the variance $V$ is greater than the mean $M$, or 'smooth' when the variance is less than the mean. The 'peakedness' of the traffic is denoted as $Z = V/M$. The mean of the occupancy distribution is termed 'traffic intensity'. We summarise the main flows in the model and identify the traffic moments of interest, below. We identify either the central or factorial moments of the flows depending on which representation is the most convenient in the analysis that follows (Section IV).

- $\mathcal{F}_I$ is the actual traffic flow offered to the output port. It is assumed to be renewal, that is, burst interarrival times are independent and identically distributed. The factorial moments of this traffic flow are denoted $M_{I,(j)}, j \in \mathbb{N}$.
- $\mathcal{F}_O$ is the actual carried traffic from the node, with traffic intensity $M_O$.
- $\mathcal{F}_B$ is the total actual blocked traffic from the node, with factorial moments denoted $\hat{M}_{B,(j)}, j \in \mathbb{N}$.
- $\hat{\mathcal{F}}$ is the virtual overflow traffic from the $GI/M/N/N$ system. This flow constitutes the traffic that must either be delayed and scheduled on output channels for transmission at a later time, or else blocked if there is no feasible schedule. The factorial moments of the flow are denoted $\hat{M}_{(j)}, j \in \mathbb{N}$.
- $\bar{\mathcal{F}}$ is the carried traffic from the $GI/M/L/L$ system. This flow represents the traffic that is successfully scheduled to be delayed in the FDL bank and subsequently carried by the output channels. The first and second factorial moments of the flow are denoted $\bar{M}_{(1)}$ and $\bar{M}_{(2)}$ respectively.
- $\mathcal{F}$ is the effective total offered traffic at the output channels. This consists of the actual offered traffic to the node plus the traffic flow generated by previously delayed traffic from the FDL bank. It is assumed to be renewal with factorial moments $M_{(j)}, j \in \mathbb{N}$.
- With respect to flows within the FDL bank (Fig. 3), $\hat{\mathcal{F}}_k$ is overflow traffic from FDL $k$, with mean and variance $\hat{M}_k$ and $\hat{V}_k$. The mean of the channel occupancy in FDL $k$ is denoted $\bar{M}_k$.

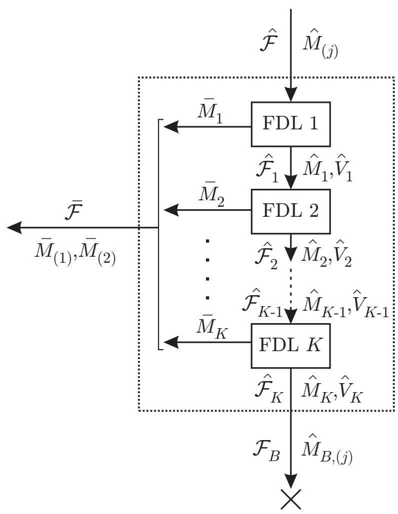


Fig. 3. Virtual Cascading Overflows Within FDL Bank

We next analyse the model of Fig. 2 to resolve the moments of the flows identified above. Having done so, we may estimate the burst blocking probability at the output port and then, by resolving the flows of Fig. 3, we may estimate the mean delay experienced by a packet transiting through the port.

## IV. Model Analysis

To resolve blocking probability we require the mean of flow $\mathcal{F}_B$. We first resolve the effective input traffic flow, $\mathcal{F}$, from which calculation of the other flows follow. Although only the mean of flow $\mathcal{F}_B$ is required, we include higher moments of the flows in calculations in order to achieve an accurate estimate.

### A. *Offered Traffic*

We model the offered traffic flow $\mathcal{F}_I$ as having interarrival times distributed according to a gamma distribution. This characterisation enables performance for a full range of offered-traffic peakedness to be examined. We note, however, that the methods that follow allow any independent interarrival time distribution to be represented.

In order to apply the gamma distribution in our analysis, we need to first derive the relationship between the parameters of the distribution and the moments of the traffic, that is, the moments of the occupancy distribution in an infinite trunk group with exponential holding times, when offered traffic with gamma-distributed interarrivals.

It is known [20] that the factorial moments of the traffic, denoted $M_{(j)}$ here, may be expressed in terms of the interarrival time distribution for a renewal arrival process as

$$M_{(j)} = \frac{1}{\mu E[\tau]} \cdot \prod_{i=1}^{j-1} \frac{i\, F^*(i\mu)}{1 - F^*(i\mu)}, \quad j \in \mathbb{N}, \tag{1}$$

where $F^*(\cdot)$ denotes the Laplace-Stieltjes transform (LST) of the interarrival cdf, $\mu$ is the parameter of the exponentially distributed holding times in the infinite trunk group and $E[\tau]$ is the mean interarrival time. In our analysis, we will also require expressions for the first two moments of the traffic in terms of the interarrival time distribution, and we derive these as follows. Let $\tau$ be the random variable denoting the interarrival time where $\tau$ has a gamma distribution, that is, its probability density function $f_\tau(t)$ is given by

$$f_\tau(t) = \frac{\theta^{-k}\, t^{k-1}\, e^{-t/\theta}}{\Gamma(k)} \quad t \geq 0 \tag{2}$$

where $k > 0$ is the shape parameter, $\theta > 0$ is the scale parameter and $\Gamma(k)$ is the gamma function. The LST $F^*(s)$ of the corresponding cumulative distribution function $F_\tau(t)$ is given as

$$F^*(s) = \int_0^\infty e^{-st} f_\tau(t)\, dt = (1 + \theta s)^{-k} \tag{3}$$

from which the first moment of the interarrival time $\tau$ is

$$E[\tau] = -\Big[\frac{dF^*(s)}{ds}\Big]_{s=0} = \theta k. \tag{4}$$

We now wish to find values of the parameters $\theta$ and $k$ such that traffic with interarrival time $\tau$ arriving to an infinite trunk group has a given mean intensity $M$ and peakedness $Z$. From (1) and (3) we may calculate the first two factorial moments of the traffic as

$$M_{(1)} = \frac{1}{\mu E[\tau]} = M \tag{5}$$

$$M_{(2)} = \frac{1}{\mu E[\tau]} \cdot \frac{(1+\theta\mu)^{-k}}{1 - (1+\theta\mu)^{-k}} = \frac{M}{(1 + \frac{1}{Mk})^k - 1}. \tag{6}$$

The mean and peakedness expressed in terms of the factorial moments of the offered traffic are

$$M = M_{(1)} \quad \text{and} \quad Z = 1 - M_{(1)} + M_{(2)}/M_{(1)}, \tag{7}$$

and so we may relate the mean and peakedness of the traffic to the gamma distribution parameters by the equations:

$$\theta = \frac{1}{M\mu k} \tag{8}$$

$$Z = 1 - M + \frac{1}{(1 + \frac{1}{Mk})^k - 1} \tag{9}$$

Given desired values of mean $M$ and peakedness $Z$ of the offered traffic, we may solve (9) numerically to yield corresponding values of $k$ and $\theta$.

It is also useful to derive the bounds on traffic peakedness $Z$ for gamma interarrivals. From (9) we see that, as $k \to 0$, $Z \to \infty$, so there is no upper bound. To find the lower bound on $Z$, we compute the limit of $(1 + \frac{1}{Mk})^k$ as $k \to \infty$. This limit has the indeterminate from $1^\infty$ but we may transform to the form $0^0$ and apply l'Hôpital's rule to find

$$\lim_{k\to\infty} \Big(1 + \frac{1}{Mk}\Big)^k = e^{1/M} \tag{10}$$

and so the lower bound on $Z$ is given as

$$Z_{min} = 1 - M + (e^{1/M} - 1)^{-1} \tag{11}$$

This limit is identical to the general result [20], so we may conclude that there is no restriction on the range of peakedness we may examine using gamma-distributed interarrivals.

### B. *Overflow and Carried Traffics*

We wish to characterise the overflow traffic from the $GI/M/N/N$ system, representing the output port channels (Fig. 2). Let us assume initially that there is no feedback flow $\bar{\mathcal{F}}$ and so the effective offered flow $\mathcal{F}$ is equal to the actual gamma-distributed offered flow $\mathcal{F}_I$. We may calculate the factorial moments of the overflow $\hat{\mathcal{F}}$, from Potter's formula [20], as

$$\frac{1}{\hat{M}_{(k)}} = \sum_{l=0}^{N} \binom{N}{l} \frac{(k+l-1)!}{(k-1)!\, M_{(l+k)}}, \quad k \in \mathbb{N}, \tag{12}$$

where $M_{(j)}, j \in \mathbb{N}$ are the factorial moments of the offered traffic $\mathcal{F}$, which may be computed from (1) given the LST of the gamma distribution from (3).

In a similar manner, we may compute the factorial moments of the overflow $\mathcal{F}_B$ from the FDL bank, given the factorial moments of the offered traffic, which in this case is the flow $\hat{\mathcal{F}}$ with factorial moments $\hat{M}_{(j)}, j \in \mathbb{N}$ computed by (12):

$$\frac{1}{\hat{M}_{B,(k)}} = \sum_{l=0}^{L} \binom{L}{l} \frac{(k+l-1)!}{(k-1)!\,\hat{M}_{(l+k)}}, \quad k \in \mathbb{N}. \tag{13}$$

We have calculated the overflow moments when the feedback traffic $\bar{\mathcal{F}}$ is neglected. To accurately estimate $\mathcal{F}$ (and subsequently all other flows in the model) we account for the additional feedback traffic as follows. Given an estimate of the moments of $\mathcal{F}$, we may calculate the first two moments of the carried traffic $\bar{\mathcal{F}}$ using Brandt's calculation [20], where the offered traffic in this case is again the overflow $\hat{\mathcal{F}}$ with factorial moments $\hat{M}_{(j)}$ given by (12), that is,

$$\begin{aligned}\bar{M}_{(2)} &= \bar{M}_{(1)} \frac{\hat{M}_{(2)}}{\hat{M}_{(1)}} \\ &- \hat{M}_{B,(1)}\hat{M}_{B,(2)} \sum_{l=1}^{L} \binom{L}{l} \frac{l!}{\hat{M}_{(l+1)}} \sum_{m=1}^{l} \left( \frac{m\hat{M}_{(m)}}{\hat{M}_{(m+1)}} + 1 \right),\end{aligned} \tag{14}$$

where $\bar{M}_{(1)} = \hat{M}_{(1)} - \hat{M}_{B,(1)}$, by the conservation principle, and $\hat{M}_{B,(1)}$ and $\hat{M}_{B,(2)}$ are given by (13). We note that (14) gives the required moments of the "freed" carried traffic as distinct from the moments of channel occupancy, provided by the usual equivalent random methods [19].

Having calculated the moments of the feedback traffic, we now make the assumption that $\mathcal{F}$ may be estimated as begin gamma-distributed traffic with moments determined as follows. The mean of $\mathcal{F}$ may be calculated simply as the sum of the means of $\bar{\mathcal{F}}$ and the actual offered traffic $\mathcal{F}_I$, that is,

$$M_{(1)} = \bar{M}_{(1)} + M_{I,(1)}. \tag{15}$$

We make the assumption that $\mathcal{F}$ and $\bar{\mathcal{F}}$ are independent traffic streams and so the variance of $\bar{\mathcal{F}}$ may similarly be estimated as the sum of the variances of $\bar{\mathcal{F}}$ and $\mathcal{F}_I$ or, in terms of the factorial moments, we may derive

$$M_{(2)} = 2M_{I,(1)}\bar{M}_{(1)} + M_{I,(2)} + \bar{M}_{(2)}. \tag{16}$$

Given the first two moments of $\mathcal{F}$, which we have assumed remains gamma-distributed, we may calculate further moments by calculating the distribution parameters $k, \theta$ from (8) and (9), calculating the distribution's LST from (3) and then calculating higher factorial moments from (1).

We now have a set of open-form equations relating the factorial moments of all flows from which we may form an iterative algorithm to resolve the blocking probability.

### C. Resolving Blocking Probability

To resolve the factorial moments of the effective offered traffic $\mathcal{F}$ we first calculate overflow and carried traffic moments assuming no feedback flow $\bar{\mathcal{F}}$. This yields an approximation for the moments of $\bar{\mathcal{F}}$ from which a new estimate for $\mathcal{F}$ may be calculated. We then iterate this calculation until the first two moments of $\mathcal{F}$ are within a desired $\epsilon$ over two successive iterations. We note from [20] that the complexity of calculation of overflow and carried traffic moments in (12), (13) and (14) is $\mathcal{O}(C)$, where $C$ is the number of channels, and thus our simple iterative method has good efficiency. (We have found the algorithm to converge rapidly for a range of test cases, although we do not have a convergence proof.) Given the solution values for the moments of $\mathcal{F}$, we have a solution value for the first moment of the node overflow traffic $\hat{M}_{B,(1)}$ from (13) and so the burst blocking probability at the node may be calculated as

$$\boxed{B = \hat{M}_{B,(1)}/M_{I,(1)}} \tag{17}$$

### D. Resolving Mean Delay

Delay in the system occurs when FDLs are employed by the scheduler to resolve contention at the output channels. To estimate the mean delay we first resolve the mean and variance of the offered traffic to each of the $K$ FDL units (Fig. 3). Having done so, we may then resolve the mean occupancy of each FDL, $\bar{M}_k$, from which, given a set of FDL delay times $\{D_k\}$, we may approximate the mean delay in the system.

We denote the mean and variance of the overflow from FDL $k$ as $\hat{M}_k$ and $\hat{V}_k$ respectively, as per Fig. 3. As the overflow from an FDL is the offered traffic to the next FDL in the chain, the offered traffic to FDL $k$ has mean and variance $\hat{M}_{k-1}$ and $\hat{V}_{k-1}$.

Having solved for the factorial moments $\hat{M}_{(k)}$ of the overflow from the group of $N$ channels in the previous subsection, the mean and variance of the traffic offered to the first FDL in the FDL bank are given as

$$\hat{M} = \hat{M}_{(1)} \tag{18}$$

$$\hat{V} = \hat{M}_{(1)} - \hat{M}_{(1)}^2 + \hat{M}_{(2)}. \tag{19}$$

We now wish to resolve the mean and variance of the overflow from FDL 1, $\hat{M}_1$ and $\hat{V}_1$ respectively, when it is offered traffic $\hat{M}, \hat{V}$. We employ Equivalent Random Theory (ERT) to resolve $\hat{M}_1, \hat{V}_1$ [19]. Having done so, $\hat{M}_1, \hat{V}_1$ becomes offered traffic to FDL 2 and, assuming independence between flows, reapplying ERT resolves $\hat{M}_2, \hat{V}_2$ and so on down the chain of $K$ FDLs.

We show the solution for an arbitrary FDL $k$ receiving traffic $\hat{M}_{k-1}, \hat{V}_{k-1}$ and producing overflow $\hat{M}_k, \hat{V}_k$. With this solution and $\hat{M}_0, \hat{V}_0$ given by $\hat{M}, \hat{V}$ respectively, we may iterate for all $K$ FDLs in the bank. The details of the method follow.

In Equivalent Random Theory, a virtual group of size $N^*$ is offered virtual Poisson traffic of intensity $A^*$ which produces an overflow mean and variance which may be matched, given appropriate values of $N^*$ and $A^*$, to the given (actual) mean and variance. This overflow traffic is the offered traffic to the actual group. The problem reduces to finding the $A^*$ and $N^*$ group whose overflow matches the required actual (peaked) offered traffic. Having resolved $A^*$ and $N^*$, the mean and variance of the overflow (and carried traffic) from the actual group may be resolved using the equivalent overflow model of Fig. 4.

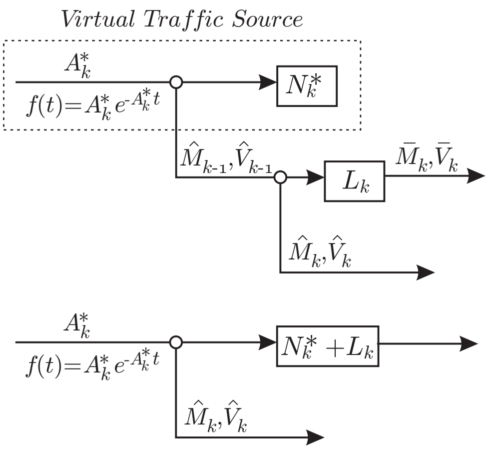


Fig. 4. Moment-Matching Equivalent Overflow System For Analysis of Individual FDL Units

From Fig. 4, for FDL $k$, expressions for the mean and variance of the actual overflow, in terms of the virtual group size $N_k^*$, the virtual offered intensity $A_k^*$ and the actual group size $L_k$ are given by the equivalent system as

$$\hat{M}_k = A_k^* \cdot E(A_k^*, L_k + N_k^*) \tag{20}$$

$$\hat{V}_k = \hat{M}_k \left( 1 - \hat{M}_k + \frac{A_k^*}{L_k + N_k^* + 1 - A_k^* + \hat{M}_k} \right). \tag{21}$$

$A_k^*$ and $N_k^*$ are given implicitly in terms of $\hat{M}_{k-1}$ and $\hat{V}_{k-1}$, the previously calculated mean and variance of the overflow from the virtual source, as

$$\hat{M}_{k-1} = A_k^* \cdot E(A_k^*, N_k^*) \tag{22}$$

$$\hat{V}_{k-1} = \hat{M}_{k-1} \left( 1 - \hat{M}_{k-1} + \frac{A_k^*}{N_k^* + 1 + \hat{M}_{k-1} - A_k^*} \right). \tag{23}$$

From (22) and (23), $N_k^*$ may be written in terms of $A_k^*$ and known constants $\hat{M}_{k-1}$ and $\hat{V}_{k-1}$ as

$$N_k^* = A_k^* \left( \frac{\hat{M}_{k-1} + \hat{V}_{k-1}/\hat{M}_{k-1}}{\hat{M}_{k-1} + \hat{V}_{k-1}/\hat{M}_{k-1} - 1} \right) - \hat{M}_{k-1} - 1 \tag{24}$$

and so, from (22), we have a function of a single variable $A_k^*$,

$$f(A_k^*) = \hat{M}_{k-1} - A_k^* \cdot E(A_k^*, N_k^*) = 0, \tag{25}$$

which may be solved for $A_k^*$ as a numerical root finding problem. We may choose an initial solution for the numerical solution from Rapp's approximation [19] for an overflow system:

$$A^* \approx V + 3Z(Z-1)$$
$$N^* \approx \frac{A^*(M+Z)}{M+Z-1} - M - 1$$

where $Z$ is the peakedness.

We note that, in the numerical method, the values of $N^*$ must be allowed to take non-integer values for a solution to be found. The usual recurrent evaluation method for the Erlang B formula

$$E(A, k+1) = \frac{A \cdot E(A,k)}{k + 1 + A \cdot E(A,k)}, \quad E(A,0) = 1 \tag{26}$$

is extended using Szybicky's approximation [22] which gives the blocking probability for real-valued $0 \le N \le 2$ as

$$E_s(A, n) \approx \frac{(2-n)A + A^2}{n + 2A + A^2} \quad n \in \text{real interval } [0, 2].$$

For a given positive real-valued $N = \lfloor N \rfloor + (N - \lfloor N \rfloor)$, where $N$ may be $\ge 2$, we first evaluate

$$E(A, N - \lfloor N \rfloor) = E_s(A, N - \lfloor N \rfloor).$$

and then (from (26)) form the recursion

$$E(A, k + 1 + (N - \lfloor N \rfloor)) = \frac{A \cdot E(A, k + (N - \lfloor N \rfloor))}{k + 1 + (N - \lfloor N \rfloor) + A \cdot E(A, k + (N - \lfloor N \rfloor))}$$

where, for $k = 0$

$$E(A, 0 + N - \lfloor N \rfloor) = E_s(A, N - \lfloor N \rfloor).$$

Iterating for $k = 0, 1, \ldots, \lfloor N \rfloor - 1$ gives the final value of $E(A, N)$, for positive real-valued $N$.

We have solved (25) for $A_k^*$, and thus $N_k^*$ is given by equation (24). The mean and variance of the overflow traffic from FDL $k$ are then given by equations (20) and (21) respectively. We now have the mean of the carried traffic from FDL $k$ as

$$\bar{M}_k = \hat{M}_{k-1} - \hat{M}_k.$$

With this solution for FDL $k$, and $\hat{M}_0, \hat{V}_0$ given by $\hat{M}, \hat{V}$, we may solve for all $k \in \{1, 2, \ldots, K\}$ iteratively. The average burst delay $D$ at the output port is then given as

$$\boxed{D = \sum_{k \in \{1,\ldots,K\}} \frac{\bar{M}_k}{M_O} D_k} \tag{27}$$

where $M_O$ is the mean of the carried traffic from the port, which may be calculated, by the conservation principle, as:

$$M_O = M_{(1)} - \hat{M}_{(1)}.$$

## V. Results and Analysis

We compare analytic results for blocking $B$ and mean delay $D$ with results from a discrete-event simulation of an OBS node implemented in Opnet Modeler™ [23]. Two different node configurations are considered, Scenario I: a node with 10 output channels and 2 FDLs and Scenario II: a node with 40 output channels and 5 FDLs. In both cases, each FDL carries a single wavelength.

Our discrete-event simulator models the full details of the output channel and FDL scheduling. The channel scheduler implements Latest Available Unscheduled Channel (LAUC) on both the output channels and the FDLs. When there is no output channel available for an arriving burst, coordination of output channel and FDL scheduling is of the "PreRes" type

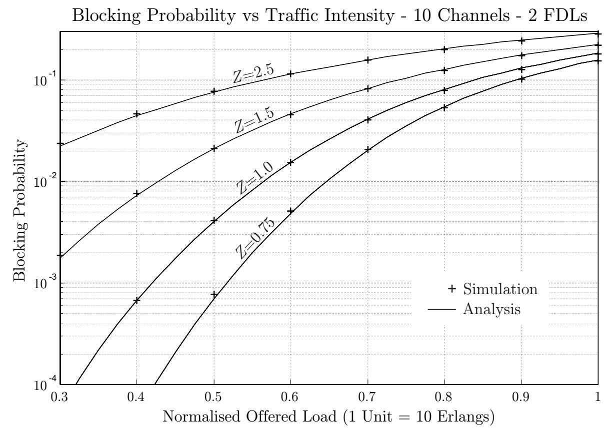


Fig. 5. Blocking Probability - Simulation vs Analysis - Scenario I

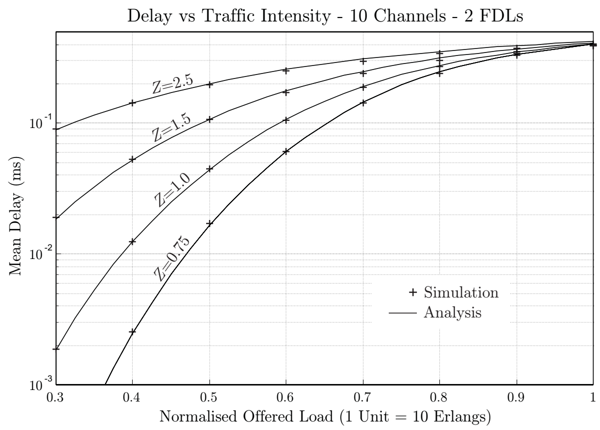


Fig. 7. Mean Delay - Simulation vs Analysis - Scenario I

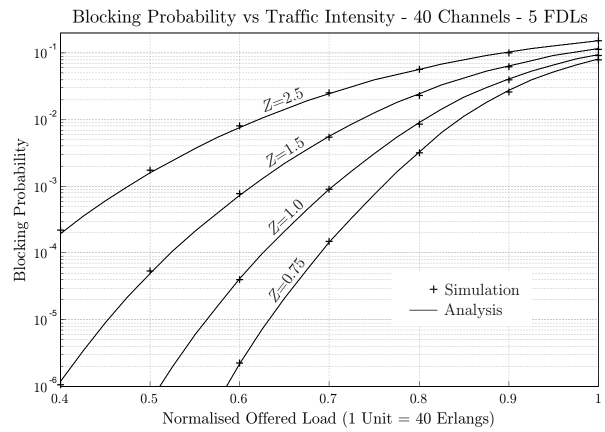


Fig. 6. Blocking Probability - Simulation vs Analysis - Scenario II

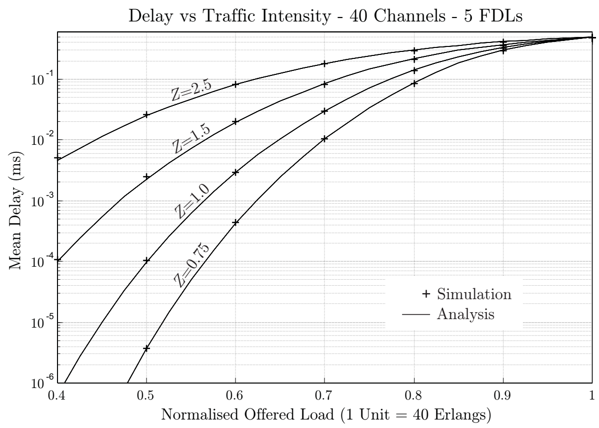


Fig. 8. Mean Delay - Simulation vs Analysis - Scenario II

[15]. In this scheme a schedule is sought simultaneously for future availability of an output channel and FDL. Also, the simulator implements full wavelength conversion at the output port.

Burst interarrival times are gamma-distributed and burst lengths are exponentially distributed of mean length 1ms. We note that our simulator packet generator is parameterised by the gamma-distribution parameters ($k$, $\theta$) while our analytic model is parameterised by the factorial moments of gamma-distributed traffic offered to a virtual $GI/M/\infty$ group, however, we may match simulation setup with analytic model input values by evaluating our previously derived relations (8), (9) to give the appropriate ($k$, $\theta$) to generate a given mean and peakedness of offered traffic.

The FDL base delay time is chosen as $C = 2$ms. It has been shown in [15] that, when $C$ is shorter than the average burst length, the FDLs are less effective and blocking increases due to increased overlap between bursts at the output channels. When $C$ is increased beyond the average burst length, burst blocking settles to a near constant value for a given load. $C$ should not be too large, as fibre lengths in FDLs become unfeasibly long and delay increases. We set $C$ to be twice the average burst length as a trade-off.

The simulations were executed such that the confidence interval for all points is better than $\pm$ 1% at a confidence level of 99%. These intervals are small and omitted from result plots for clarity.

We compare the blocking probability $B$ for Scenarios I and II, over a range of offered load intensities ($M_{I,(1)}$), in Fig. 5 and Fig. 6 respectively. Load values are shown normalised with respect to the number of output channels $N$. Results from our analytic model in both scenarios compare favourably with simulation over the range of offered load examined. The error in the analytic results, when compared to simulation, is tabulated in Table 1. We compare results for mean delay $D$ for Scenarios I and II in Fig. 7 and Fig. 8 respectively.

Fig. 9 illustrates the effect of increasing peakedness on blocking probability, for a fixed mean traffic intensity. For higher mean loads, it can be seen that blocking probability increases quite strongly with peakedness and thus the peakedness of the offered traffic is an important factor to consider in determining system performance.

## VI. Conclusion

We have developed a relatively simple approximate model for the analysis of an OBS node with FDLs by applying circuit

TABLE I
ESTIMATED AVERAGE ERRORS IN ANALYTIC RESULTS

| | **Offered Peakedness** | **Blocking % Error** | | **Delay % Error** | |
|---|---|---|---|---|---|
| | $(Z)$ | % | $\sigma$ | % | $\sigma$ |
| **Scenario I** | 0.75 | -2.7 | 5.3 | 0.2 | 2.2 |
| 10 Channels | 1.00 | 0.6 | 1.9 | 1.5 | 1.4 |
| 2 FDLs | 1.50 | 0.7 | 2.4 | 2.5 | 1.6 |
| | 2.50 | -0.1 | 2.2 | 3.1 | 1.8 |
| **Scenario II** | 0.75 | 1.0 | 4.2 | 1.3 | 2.0 |
| 40 Channels | 1.00 | 3.2 | 1.6 | 2.8 | 1.1 |
| 5 FDLs | 1.50 | 0.9 | 4.5 | 0.9 | 4.6 |
| | 2.50 | -4.1 | 3.9 | -2.0 | 3.4 |

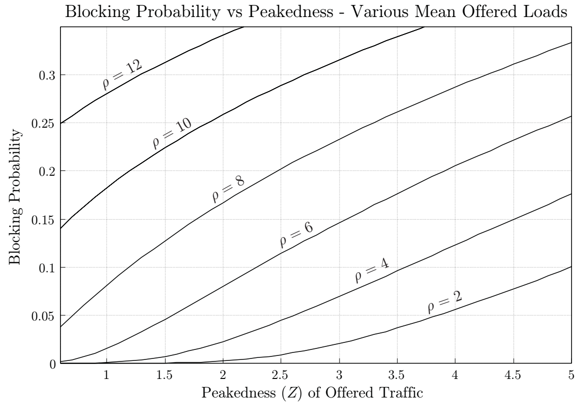


Fig. 9. Variation in Blocking with Peakedness (Scenario I)

switching analysis methods in a novel way, by allowing a feedback path between groups of channels. Our overall aim is to produce a relatively simple model, of good accuracy and good numerical efficiency, that may easily be extended to modelling and dimensioning of large networks of optical switches. We note the potential usefulness of modelling smooth, as well as peaked, offered traffic. As carried traffic from a group of channels is generally smoother than offered traffic, in network models smooth offered traffic may be encountered at some point on a transmission path, even if traffic is peaked at the ingress point. The traffic peakedness will also vary with the burst aggregation mechanism in use. We also note that the model may be used to evaluate performance under any $GI$ traffic stream that may be expressed in terms of an interarrival distribution (e.g. Gaussian interarrivals). As future work, an extension to the model will be to include limited numbers of shared wavelength converters and to investigate dimensioning optimisation based on this extended model. We also plan to explore the accuracy of the method for modelling a network of switches where ingress traffic is generated by various burst aggregation schemes.

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