An Efficient Nonlinear Circuit Simulation Technique

Emira Dautbegović, *Student Member, IEEE*, Marissa Condon, *Member, IEEE*, and Conor Brennan, *Member, IEEE*

***Abstract*—This paper proposes a novel method for the analysis and simulation of integrated circuits (ICs) with the potential to greatly shorten the IC design cycle. The circuits are assumed to be subjected to input signals that have widely separated rates of variation, e.g., in communication systems, an RF carrier modulated by a low-frequency information signal. The proposed technique involves two stages. Initially, a particular order result for the circuit response is obtained using a multiresolution collocation scheme involving cubic spline wavelet decomposition. A more accurate solution is then obtained by adding another layer to the wavelet series approximation. However, the novel technique presented here enables the reuse of results acquired in the first stage to obtain the second-stage result. Therefore, vast gains in efficiency are obtained. Furthermore, a nonlinear model-order reduction technique can readily be used in both stages making the calculations even more efficient. Results will highlight the efficacy of the proposed approach.**

***Index Terms*—Integrated-circuit (IC) design, model-order reduction, nonlinear circuit simulation, wavelet collocation scheme.**

I. Introduction

IN THE initial stage of a design cycle, the circuit designer is interested in the overall functional behavior of the designed circuit, i.e., will the integrity of desired logical states be preserved at the output? In order to ascertain this, the designer needs to perform numerous simulations before settling on a final design. Any change in the requirements for the circuit design will necessitate the simulation process to restart from the beginning. Furthermore, the complexity of today's integrated circuits (ICs) is such that these simulations are computationally expensive both in terms of time and computer resources. The overall result is a prolonged design cycle that is economically unacceptable. Hence, there is a need for a simulation technique that enables the designer to obtain the circuit response with the desired accuracy and within a reasonable time frame. Ideally, the first phase of the design process should involve obtaining a rough initial result for the circuit response to verify the functionality of the design. In the second phase, when a higher degree of accuracy for fine tuning the designed IC is sought, the possibility of reusing results from the first phase would yield huge gains in the efficiency of a simulation, thereby leading to major savings in design time and ultimately reducing the cost of the designed IC.

Harmonic balance [1]–[3] and time-domain integration [4] are the two most widely employed circuit simulation techniques in circuit simulators for the analysis of high-frequency nonlinear circuits. Harmonic balance is most effective for periodic or quasi-periodic steady-state analysis of mildly nonlinear circuits and, thus, is of limited use for the complex modulation formats encountered in today's high-speed systems or for systems involving strong nonlinearities. Time-domain integration, on the other hand, is only practical for baseband systems. For the simulation of circuits with digitally modulated high-frequency carriers with long bit sequences, time-domain integration is excessively slow. As a result, there is a need for some form of a general-purpose technique that can simulate state-of-the-art systems that are subject to transient high-frequency signals or complex modulated RF carriers.

Several envelope transient analysis approaches have recently been proposed, whereby a mixed-mode technique is implemented [5], [6]. The essence of these approaches is that the slowly varying envelope of a signal is treated by time-domain integration and that harmonic balance treats the high-frequency carrier. However, existing techniques have limitations, e.g., restrictions in the bandwidth of the excitation signal [5] and the limitations of harmonic balance with respect to strong nonlinearities. In [7], Roychowdhury proposes converting the differential-algebraic equations that describe the circuit to multitime partial differential equations and applying time-domain methods directly to solve the resultant systems. Pedro and Carvalho [8] also employ a multitime partial differential-equation approach, but uses a combination of harmonic balance and time-domain integration to solve the resultant system.

The basic technique presented in this paper is a variation and improvement of the multitime partial differential-equation approach presented by Condon and Dautbegović in [9]. A modification of the wavelet-based collocation approach proposed by Cai and Wang in [10] forms the core of the technique and, unlike Christoffersen and Steer [11], the cubic spline wavelet basis is employed to solve the multitime partial differential-equation representation of the system rather than the original ordinary differential-equation representation. However, the technique presented in [9] is greatly enhanced in this paper, yielding considerable gains in efficiency in two respects.

Firstly, a nonlinear model reduction process similar to that in [12] is employed within the proposed envelope simulation technique to obtain very high standalone simulation efficiencies, as shown in [13]. This paper compares, for test systems, the result that is obtained with a full wavelet system, as is employed in [9], to the result that is obtained when the model reduction strategy is utilized. The accuracy will be seen to be excellent while significant gains in computational speed are achieved. A result is also given when a lower order wavelet scheme is employed. The results will confirm that, for comparable computation times, significant gains in accuracy may be achieved by employing the

Manuscript received April 18, 2004; revised August 16, 2004. This work was supported by IBM under the Ph.D. Fellowship Program.

The authors are with the School of Electronic Engineering, Dublin City University, Dublin 9, Ireland (e-mail: dautbego@eeng.dcu.ie; marissa.condon@dcu.ie; brennanc@eeng.dcu.ie).

Digital Object Identifier 10.1109/TMTT.2004.840627

approach proposed in this paper as opposed to simply using a lower order full wavelet scheme.

Secondly, in this paper, a further step toward a more accurate simulation technique with even greater efficiency within the overall design cycle has been proposed. The crucial step introduced in this paper involves utilizing the multiresolution nature of wavelet systems [14]. Rather than recalculating a complete set of new coefficients for a higher degree accuracy approximation, it utilizes the coefficients calculated from a previous simulation that involved a lower order approximation. Each time a new layer is added in the wavelet series approximation, only the coefficients for that layer need to be calculated. The process may be repeated by adding more layers until the required degree of accuracy is achieved. The excellent simulation results obtained for test circuits show the potential of the proposed technique as a modern design simulation tool.

The remainder of this paper is organized as follows. Section II gives a short presentation of the multitime partial differential-equation approach. Sections III and IV describe the technique for obtaining the rough initial solution of the first stage. Section V further extends this contribution by presenting the mathematical basis for an approximation with a higher degree of accuracy incorporating results from an approximation with a lower degree of accuracy. Finally, the simulation results obtained for sample circuits using the proposed technique are presented in Section VI.

II. Multitime Partial Differential-Equation Approach

Consider a signal $x(t)$ that is composed of a carrier modulated by an envelope where the envelope signal is assumed to be uncorrelated with the carrier. The signal may be represented in two independent time variables as

$$x(t) = \hat{x}(t_1, t_2). \tag{1}$$

t_1 relates to the low-frequency envelope and t_2 relates to the high-frequency carrier.

Now consider a general nonlinear circuit described by

$$\dot{\boldsymbol{x}}(t) = f\big(x(t)\big) + \boldsymbol{b}(t) \tag{2}$$

where $\boldsymbol{b}(t)$ is the excitation vector, $\boldsymbol{x}(t)$ are the state variables, and f is a nonlinear function. The corresponding multitime partial differential-equation system can be written as shown in [7] as

$$\frac{\partial \hat{\boldsymbol{x}}}{\partial t_1} + \frac{\partial \hat{\boldsymbol{x}}}{\partial t_2} = \boldsymbol{f}\big(\hat{\boldsymbol{x}}(t_1, t_2)\big) + \hat{\boldsymbol{b}}(t_1, t_2). \tag{3}$$

This multitime partial differential equation can be solved using exclusively time-domain approaches as employed by Roychowdhury [7] or using a combination of time-domain integration for the envelope and harmonic balance for the carrier, as in [8]. However, for strongly nonlinear circuits, the use of harmonic balance for the inner loop can prove limited. Another approach for solving (3) using a pseudowavelet collocation method derived from that proposed by Cai and Wang [10] has been suggested by the authors [9]. In Section III, a brief overview of the wavelet collocation scheme is given.

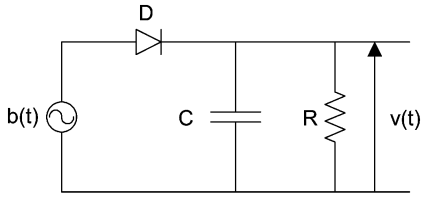


Fig. 1. Diode rectifier circuit.

III. Wavelet Collocation Scheme

The technique involves approximating the unknown function $\hat{x}(t_1, t_2)$ with a wavelet series in the t_2 dimension where t_2 is scaled such that $t_2 \in [0, L], L > 4$, i.e.,

$$\begin{aligned}
&\hat{x}_J(t_1, t_2) \\
&\quad = \bar{x}_{-1,-3}(t_1)\eta_1(t_2) + \bar{x}_{-1,-2}(t_1)\eta_2(t_2) \\
&\qquad + \bar{x}_{-1,-1}(t_1)\varphi_b(t_2) \\
&\qquad + \sum_{k=0}^{L-4} \bar{x}_{-1,k}(t_1)\varphi_k(t_2) + \bar{x}_{-1,L-3}(t_1)\varphi_b(L - t_2) \\
&\qquad + \sum_{j=0}^{J-1} \sum_{k=-1}^{n_j-2} \bar{x}_{j,k}(t_1)\psi_{j,k}(t_2) \\
&\qquad + \bar{x}_{-1,L-2}(t_1)\eta_2(L - t_2) + \bar{x}_{-1,L-1}(t_1)\eta_1(L - t_2) \\
&\quad = \sum_{k=1}^{N} \bar{x}_k(t_1)\Psi_k(t_2)
\end{aligned} \tag{4}$$

where $\phi(t)$ and $\psi(t)$ are scaling and wavelet functions, respectively, and $\eta(t)$ are spline functions introduced to approximate boundary nonhomogeneities, as described in [10]. $\bar{x}(t_1)$ are the unknown coefficients, which are a function of t_1 only. The total number of unknown coefficients is $N = 2^J L + 3$, where J determines the level of wavelet coefficients to be taken into account when approximating $\hat{x}(t_1, t_2)$. From this point forward, $\Psi_k(t)$ shall be referred to as wavelets where it is understood that these comprise the scaling functions $\phi(t)$, the wavelet functions $\psi(t)$, and the nonhomogeneity functions $\eta(t)$.

As proposed by Condon and Dautbegović in [9], (3) is then collocated on collocation points in t_2 to result in a semidiscretized equation system. The interpolation points are those as chosen in [10]. At this juncture, a nonlinear model reduction strategy is employed. The particular model reduction strategy chosen is based on that presented by Gunupudi and Nakhla in [12]. Thus, instead of solving an Nth-order system at each time step to obtain the unknown state variables and output quantity $y(t)$, a reduced-order system of transformed coefficients is solved. Once the transformed coefficients are determined for the entire time range of interest, the original N coefficients, $\bar{x}(t_1)$, and, consequently, the value of the state variables and output quantity $y(t)$ may be obtained in one single post-processing step. The solution process with the applied reduction scheme is described in detail in the Section IV.

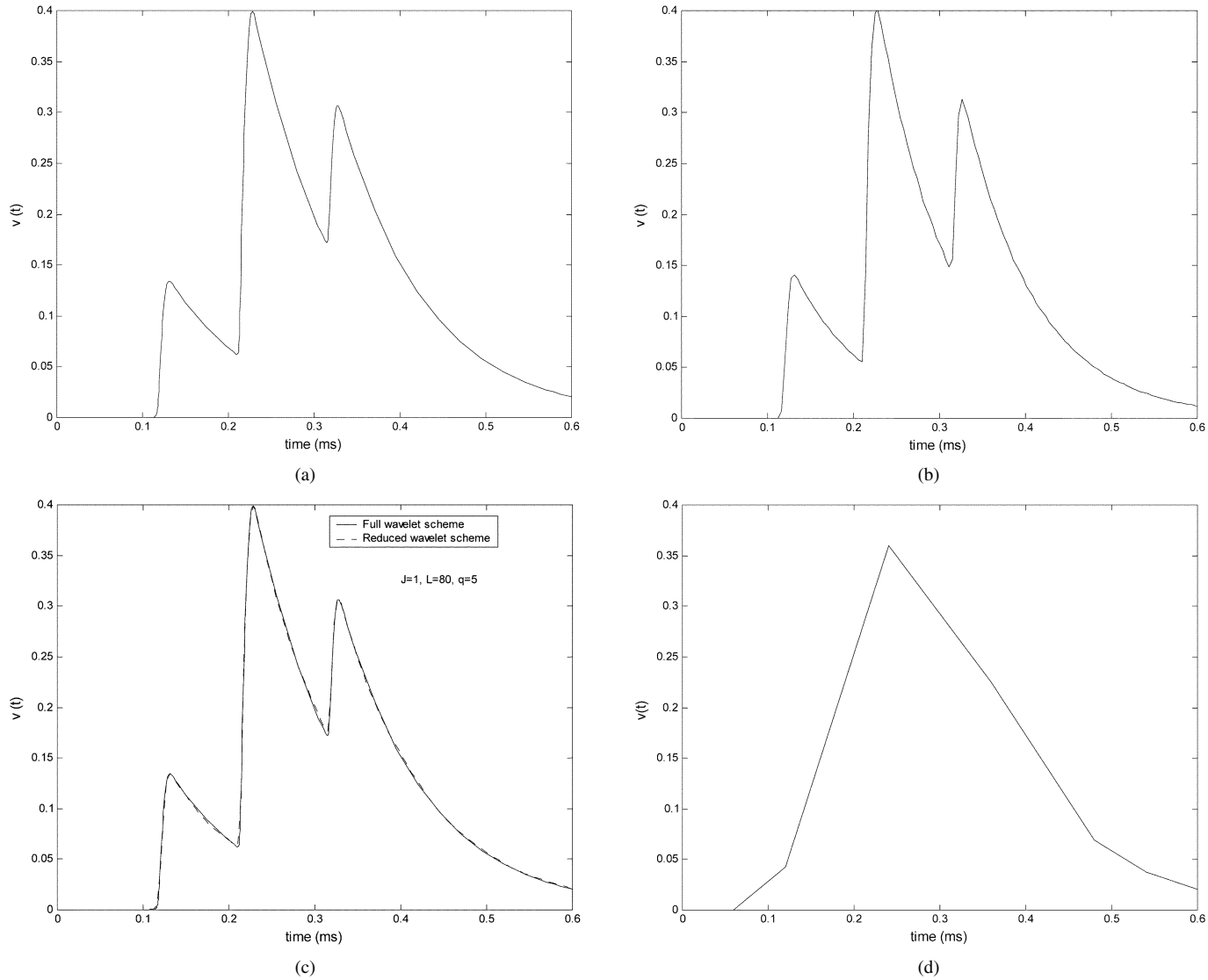


Fig. 2. (a) Result from ODE solver with a very short time step. (b) Result from a full wavelet scheme $(J = 1, L = 80)$. (c) Result from a full wavelet scheme $(J = 1, L = 80)$ with model-order reduction applied $(q = 5)$. (d) Result with lower order full wavelet scheme $(J = 0, L = 5)$.

IV. Model Reduction Technique

The expression in (4), if written for all collocation points in t_2, may be expressed as follows at a specific point in time t_1:

$$\hat{\boldsymbol{x}}_{JN}(t_1) = \boldsymbol{E}\bar{\boldsymbol{x}}(t_1) \tag{5}$$

where $\boldsymbol{E}$ is a constant N-dimensional square matrix whose columns comprise the values of the N wavelet functions $\Psi_k(t_2)$ at the N collocation points. The matrix is evaluated once at the outset of the algorithm. $\hat{\boldsymbol{x}}_{JN}(t_1)$ is an N-dimensional column vector of the unknown state variables and $\bar{\boldsymbol{x}}(t_1)$ is an N-dimensional column vector of the unknown wavelet coefficients at the collocation points in t_2 at a specific instant in t_1.

Substitution of (4) and (5) in (3) yields

$$\boldsymbol{E}\frac{d\bar{\boldsymbol{x}}}{dt_1} = -\boldsymbol{D}\bar{\boldsymbol{x}} + \boldsymbol{f}_N(\bar{\boldsymbol{x}}) + \boldsymbol{b}_N \tag{6}$$

where $\boldsymbol{D}$ is an N-dimensional matrix whose columns are formed from the derivatives of the wavelet functions in (4) evaluated at each of the N collocation points in $\boldsymbol{t}_2$. Again, $\boldsymbol{D}$ is evaluated only once at the outset of the algorithm. $\boldsymbol{f}_N$ and $\boldsymbol{b}_N$ are column vectors comprising the values of f and b at the collocation points.

At this point, the vector of coefficients $\bar{\boldsymbol{x}}(t_1)$ is expanded in a Taylor series as follows:

$$\bar{\boldsymbol{x}}(t_1) = \sum_{i=0}^{\infty} \boldsymbol{a_i}\left(t_1 - t_1^0\right)^i \tag{7}$$

where t_1^0 is the initial time and where the coefficients $\boldsymbol{a_i}$ may be computed recursively as in [12].

A Krylov space is formed for $\boldsymbol{a_i}$ as follows:

$$\boldsymbol{K} = [\boldsymbol{a}_0 \quad \boldsymbol{a}_1 \quad \cdots \quad \boldsymbol{a_q}] \tag{8}$$

where q is the order of the reduced system and is significantly less than N.

An orthogonal decomposition of $\boldsymbol{K}$ results in

$$\boldsymbol{K} = \boldsymbol{Q}R \tag{9}$$

where $\boldsymbol{Q^T Q = I_q}$. $\boldsymbol{I_q}$ is the q-dimensional identity matrix. $\boldsymbol{Q}$ is then employed to perform a congruent transformation of (5) as follows:

$$\bar{\boldsymbol{x}} = \boldsymbol{Q}\hat{\bar{\boldsymbol{x}}}. \tag{10}$$

Thus, a new reduced equation system is formed as

$$\boldsymbol{Q^T E Q}\frac{d\hat{\bar{\boldsymbol{x}}}}{dt_1} = -\boldsymbol{Q^T D Q}\hat{\bar{\boldsymbol{x}}} + \boldsymbol{Q^T f_N}(\boldsymbol{Q}\hat{\bar{\boldsymbol{x}}}) + \boldsymbol{Q^T b_N}$$

or

$$\hat{\boldsymbol{E}}\frac{d\hat{\bar{\boldsymbol{x}}}}{dt_1} = -\hat{\boldsymbol{D}}\hat{\bar{\boldsymbol{x}}} + \boldsymbol{Q^T f_N}(\boldsymbol{Q}\hat{\bar{\boldsymbol{x}}}) + \hat{\boldsymbol{b}}_{\boldsymbol{N}} \tag{11}$$

where $\hat{\boldsymbol{E}} = \boldsymbol{Q^T E Q}, \hat{\boldsymbol{D}} = \boldsymbol{Q^T D Q}$ and $\hat{\boldsymbol{b}}_{\boldsymbol{N}} = \boldsymbol{Q^T b_N}$.

This new system, i.e., (11), of dimension q may then be solved to determine $\hat{\bar{\boldsymbol{x}}}$ over the entire time domain of interest. A trapezoidal-rule integration scheme is employed because of its superior stability qualities. Once the q coefficients $\hat{\bar{\boldsymbol{x}}}$ have been determined, $\bar{\boldsymbol{x}}(t_1)$ and, consequently, $\hat{\boldsymbol{x}}_{JN}(t_1) = \boldsymbol{E}\bar{\boldsymbol{x}}(t_1)$, may be obtained in one single post-processing step. Thus, the above solution process is significantly more efficient than solving directly for $\hat{\boldsymbol{x}}_{JN}(t_1)$ at each time step, as was done in [9].

V. Formation of an Approximation of a Higher Degree of Accuracy

Assume that a preliminary circuit response is obtained by applying the technique presented in Section IV. If a response with a higher degree of accuracy is now required, the wavelet series approximating the unknown function $\hat{x}(t_1, t_2)$ can be expanded for another layer, i.e.,

$$\hat{x}_{J_1}(t_1, t_2) = \sum_{k=1}^{N_1} \bar{x}_k(t_1)\Psi_k(t_2) \tag{12}$$

where $J_1 = J+1$ and the total number of unknown coefficients is now $N_1 = 2^{J_1}L + 3$. At this point, two options are available.

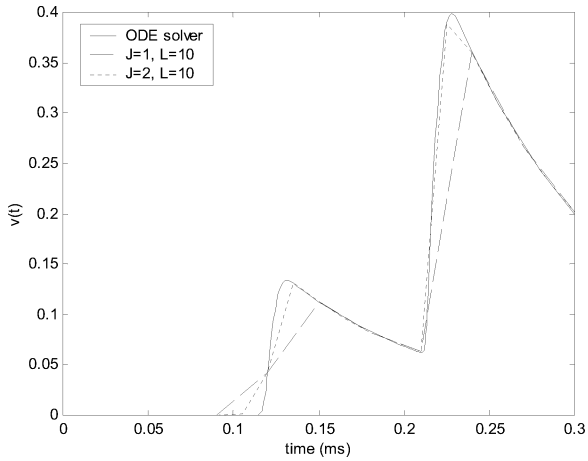
Firstly, the method proposed in Section IV can be implemented from scratch to obtain the circuit response. The size of the ordinary differential equations (ODEs) system to be solved is increased from $N = 2^J L + 3$ to $N_1 = 2^{J_1}L + 3 = 2^{J+1}L + 3$ and, consequently, the computational requirements for obtaining the required solution are also increased.

Alternatively, the following approach may be applied to obtain the circuit response with increased accuracy. First, write (12) as

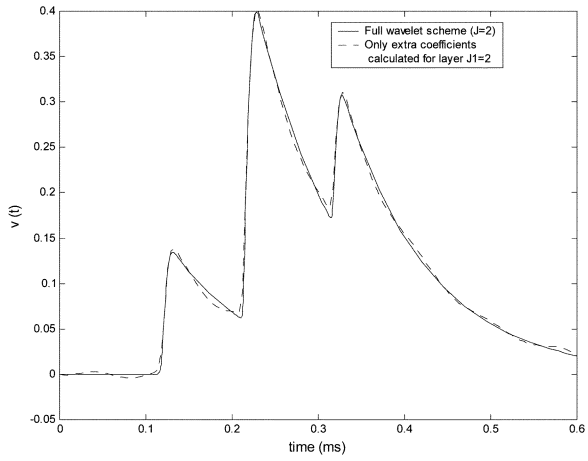
$$\begin{aligned}\hat{x}_{J_1}(t_1, t_2) &= \sum_{k=1}^{N_1} \bar{x}_k(t_1)\Psi_k(t_2) \\ &= \sum_{k=1}^{N} \bar{x}_k(t_1)\Psi_k(t_2) + \sum_{k=N+1}^{N_1} \bar{x}_k(t_1)\Psi_k(t_2)\end{aligned} \tag{13}$$

or, after setting $M = N_1 - N = 2^J L$, the wavelet series approximating the unknown function $\hat{x}(t_1, t_2)$ can be written as

$$\hat{x}_{J_1}(t_1, t_2) = \sum_{k=1}^{N} \bar{x}_k(t_1)\Psi_k(t_2) + \sum_{m=1}^{M} \bar{x}_{N+m}(t_1)\Psi_{N+m}(t_2).$$



(a)



(b)

Fig. 3. (a) Accuracy improved by adding an extra layer ($J = 2$) in wavelet series approximation. (b) Result from the proposed new higher order technique after adding an extra layer ($J_1 = 2$) in wavelet series approximation.

(14)

The first term in (14) depends solely on coefficients from previous layers. The values for these coefficients at the collocation points up to the layer J are already known from previous calculations and any additional required values can be obtained using an interpolation technique [15]. The second term in (14) consists solely of coefficients from the added layer and, thus, they need to be calculated.

Now, for presentation purposes, consider the following notation:

$$\bar{x}_k(t_1) = \bar{c}_k(t_1), \qquad k = 1, \ldots, N \tag{15}$$

and

$$\bar{x}_k(t_1) = \bar{g}_m(t_1) \quad k = N+1, \ldots, N+M, \qquad m = 1, \ldots, M. \tag{16}$$

Thus, the wavelet series approximating the unknown function $\hat{x}(t_1, t_2)$ can be written as

$$\hat{x}_{J_1}(t_1, t_2) = \sum_{k=1}^{N} \bar{c}_k(t_1)\Psi_k(t_2) + \sum_{m=1}^{M} \bar{g}_m(t_1)\Psi_{N+m}(t_2). \tag{17}$$

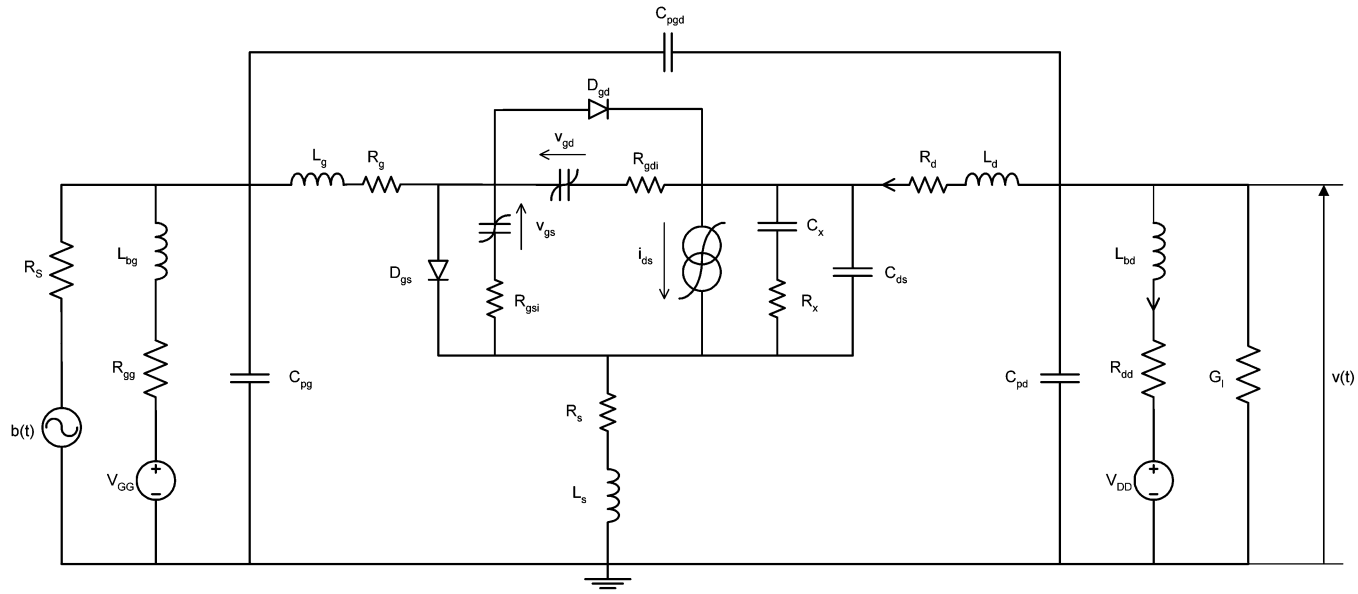


Fig. 4. MESFET amplifier.

The expression in (17), if written for the M collocation points of the added layer in t_2, may be expressed as follows at a specific point in time t_1:

$$\hat{\boldsymbol{x}}_{J_1M}(t_1) = \boldsymbol{E_0}\bar{\boldsymbol{c}}(t_1) + \boldsymbol{E_1}\bar{\boldsymbol{g}}(t_1) \tag{18}$$

where $\bar{\boldsymbol{g}}(t_1)$ is an M-dimensional column vector of the *unknown* wavelet coefficients of layer J_1. $\bar{\boldsymbol{c}}(t_1)$ is an N-dimensional column vector of the *known* wavelet coefficients at the collocation points in t_2 at a specific instant in t_1, and its entries are either already known directly or may be obtained as interpolated values for any time t_1. $\boldsymbol{E}_0$ is a constant $M \times N$-dimensional matrix whose columns comprise the values of the N wavelet functions $\Psi_k(t_2)$ at the M collocation points of the extra layer, while $\boldsymbol{E}_1$ is a constant M-dimensional square matrix with $\Psi_k(t_2)$ at the M collocation points as its entries. All constant matrices are evaluated only once at the outset of the algorithm. $\hat{x}_{J_1M}(t_1)$ is an M-dimensional column vector of the unknown state-variables on layer J_1.

Substitution of (17) and (18) in (3) yields

$$\boldsymbol{E_1}\frac{d\bar{\boldsymbol{g}}}{dt_1} = -\boldsymbol{D_1}\bar{\boldsymbol{g}} - \boldsymbol{E_0}\frac{d\bar{\boldsymbol{c}}}{dt_1} - \boldsymbol{D_0}\bar{\boldsymbol{c}} + \boldsymbol{f}_M(\bar{\boldsymbol{c}}, \bar{\boldsymbol{g}}) + \boldsymbol{b}_M \tag{19}$$

where $\boldsymbol{D_0}$ is an $M \times N$ dimensional matrix whose columns are formed from the derivatives of the wavelet functions evaluated at each of the M collocation points in t_2, and $\boldsymbol{D}_1$ is an $M \times M$ dimensional matrix. Again, $\boldsymbol{D_0}$ and $\boldsymbol{D_1}$ are evaluated only once at the outset of the algorithm. $\boldsymbol{f_M}$ and $\boldsymbol{b_M}$ are column vectors comprising the values of f and b at the collocation points of level J_1.

Bearing in mind the notation introduced in (15) and (16), $d\bar{c}/dt_1$ may be expressed, using (6), as a function of $\bar{c}$

$$\frac{d\bar{\boldsymbol{c}}}{dt_1} = \boldsymbol{E}^{-1}[-\boldsymbol{D}\bar{\boldsymbol{c}} + \boldsymbol{f}_N(\bar{\boldsymbol{c}}) + \boldsymbol{b_N}]. \tag{20}$$

Substituting (20) in (19) yields the following equation:

$$\boldsymbol{E_1}\frac{d\bar{\boldsymbol{g}}}{dt_1} = -\boldsymbol{D_1}\bar{\boldsymbol{g}} + (\boldsymbol{E_0}\boldsymbol{E}^{-1}\boldsymbol{D} - \boldsymbol{D_0})\bar{\boldsymbol{c}} + \boldsymbol{f}_M(\bar{\boldsymbol{c}}, \bar{\boldsymbol{g}}) - \boldsymbol{E_0}\boldsymbol{E}^{-1}\boldsymbol{f}_N(\bar{\boldsymbol{c}}) + \boldsymbol{b}_M - \boldsymbol{E_0}\boldsymbol{E}^{-1}\boldsymbol{b}_N. \tag{21}$$

This may be written for convenience as

$$\boldsymbol{E_1}\frac{d\bar{\boldsymbol{g}}}{dt_1} = -\boldsymbol{D_1}\bar{\boldsymbol{g}} + \boldsymbol{F}_M(\bar{\boldsymbol{c}}, \bar{\boldsymbol{g}}) + \boldsymbol{B}_M \tag{22}$$

where

$$\boldsymbol{F}_M(\bar{\boldsymbol{c}}, \bar{\boldsymbol{g}}) = \big(\boldsymbol{E_0}\boldsymbol{E}^{-1}\boldsymbol{D} - \boldsymbol{D_0}\big)\bar{\boldsymbol{c}} + \boldsymbol{f}_M(\bar{\boldsymbol{c}}, \bar{\boldsymbol{g}}) - \boldsymbol{E_0}\boldsymbol{E}^{-1}\boldsymbol{f}_N(\bar{\boldsymbol{c}}) \tag{23}$$

and

$$\boldsymbol{B}_M = \boldsymbol{b}_M - \boldsymbol{E_0}\boldsymbol{E}^{-1}\boldsymbol{b}_N. \tag{24}$$

Equation (22) represents an $M \times M$ system of ODEs where the unknowns $\bar{\boldsymbol{g}}$ may be readily determined using a standard numerical technique for solving a system of ODEs [16]. A trapezoidal-rule integration scheme is recommended because of its stability qualities. The system in (22) is significantly smaller in dimension than that in (6) in that it involves M unknowns rather than $N + M$ unknowns when written for the same wavelet approximation level $J + 1$. Therefore, the computational cost for obtaining the circuit response is significantly reduced.

Furthermore, the structure of the equations in (6) and (22) is exactly the same. Therefore, the same model-order reduction technique, as presented in Section IV, may readily be applied to the system in (22). As a result, even more gains in computational efficiency may be achieved.

VI. Sample Systems

The above technique has been employed for two nonlinear systems: a diode rectifier circuit and a MESFET amplifier. The nonlinear diode rectifier circuit is deliberately selected, as it is strongly nonlinear in nature. The ability to efficiently simulate

the behavior of this circuit with good accuracy will provide a strong recommendation for employing the wavelet-based simulation technique presented here to simulate nonlinear circuits subjected to input signals that have widely separated rates of variation. The results for the MESFET amplifier response will further confirm the efficacy of the proposed method for structurally complex nonlinear circuits.

A. Nonlinear Diode Rectifier Circuit

The highly nonlinear diode rectifier circuit, as shown in Fig. 1, is excited with the following source:

$$b(t) = \sin\left(\frac{2\pi}{T_1}t\right)\sin\left(\frac{2\pi}{T_2}t\right) \tag{25}$$

where T_1 corresponds to the envelope period and T_2 corresponds to the carrier period.

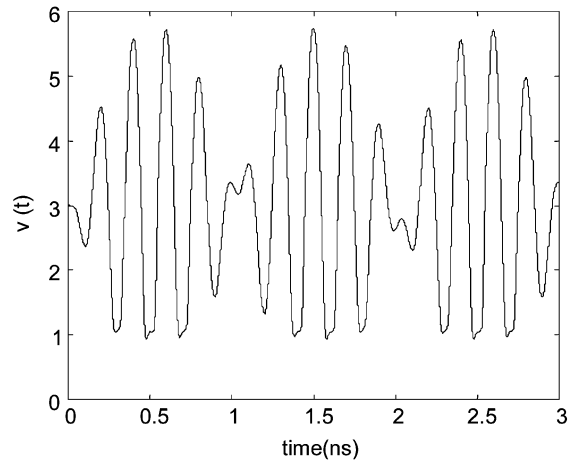
Fig. 2(a) shows the output from an ordinary differential-equation solver with a very short time step in order to obtain a highly accurate version of the output voltage to act as a benchmark for the purposes of confirming the accuracy of the proposed new simulation technique.

Fig. 2(b) presents the output from a full wavelet scheme with no model-order reduction applied, as was done in [9]. For the chosen wavelet parameters $J = 1$ and $L = 80$, the size of the ODE system is $N = 163$. An adaptive backward-Euler predictor corrector approach is employed for obtaining the solution. Good agreement is achieved when compared to the "accurate" result [see Fig. 2(a)]. However, the size of an ODE system that is solved is considerable and requires significant computer resources.

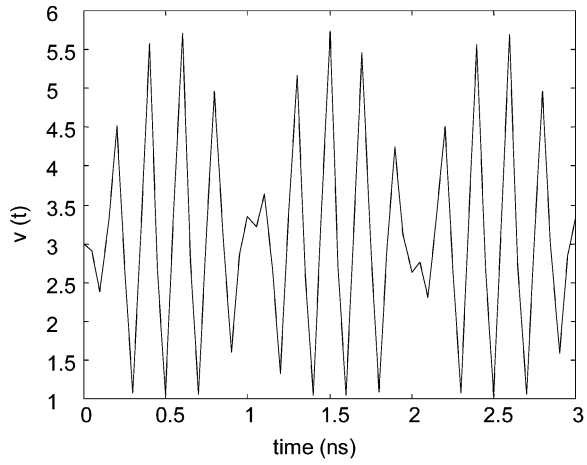
Fig. 2(c) shows the output when the model-order reduction technique proposed in Section IV is applied. For the same wavelet parameters $(J = 1, L = 80)$, the initial system of $N = 163$ unknown wavelet coefficients is reduced to $q = 5$ before obtaining the solution for the reduced-order system (11). In terms of accuracy, the relative difference between the result from the full wavelet scheme and the results obtained having applied the model reduction technique is negligible. However, in terms of computational time, the result obtained with the model reduction technique is computed in only 7% of the time necessary for the full wavelet scheme. This excellent gain in computational efficiency is due to the fact that, instead of solving an ODE system with 163 unknowns, a system with only five unknowns is solved.

Finally, Fig. 2(d) shows the result when a lower order full wavelet scheme is employed. In this case, $L = 5$ and $J = 0$ in (4). This results in an $N = 8$th-order system of equations, which has similar computational requirements to the reduced wavelet scheme with $q = 5$. As can be seen from Fig. 2(d), there is a significant loss in accuracy. This result clearly confirms that the approach presented in Section IV is significantly better than simply employing a full lower order wavelet scheme, especially when circumstances require high computational efficiency.

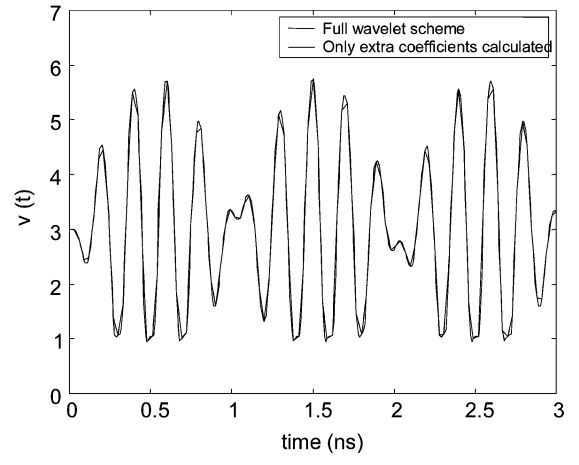
To emphasize the gains in accuracy achieved by the addition of an extra layer in the wavelet approximation series, Fig. 3(a) shows an example with wavelet layers $J = 1$ and $J = 2$. The collocation points range parameter L was deliberately chosen to be very low $(L = 10)$ so that gains in the accuracy due to adding an extra layer would be highlighted. The significant improvement in the accuracy of the circuit response, as evidenced from Fig. 3(a), confirms the rationale for employing extra layers.



(a)



(b)



(c)

Fig. 5. (a) Result from an ODE solver with a very short time step. (b) Result with full wavelet scheme $(J = 1, L = 20)$ with model-order reduction $(q = 20)$ applied. (c) Result with the proposed new higher order technique after adding an extra layer $(J = 2)$ in wavelet series approximation.

However, if the basic wavelet approach of Section IV for simulating a system is employed, the addition of extra layers increases the computational requirements greatly. However, with the novel technique proposed in Section V, this is no longer a barrier.

Fig. 3(b) shows the results for the diode rectifier circuit with a new layer added ($J_1 = J + 1 = 2$). The full line represents result obtained using a full wavelet scheme with model reduction. The dashed line is the circuit response calculated at the same wavelet level, but reusing results calculated from the lower order simulation. As can be seen, these two responses are practically indistinguishable.

However, it took only 14% of the computing time to obtain the higher degree of accuracy circuit response with the new method when compared to the time necessary to compute the circuit response by simply restarting the full wavelet simulation scheme with $J = 2$.

B. MESFET Amplifier

Fig. 4 presents a practical RF MESFET amplifier circuit whose behavior is described by a tenth-order ODE system. The source input has the same structure as in (25), i.e., it has widely separated rates of variation.

Fig. 5(a) shows the output from an ODE solver with a very short time step in order to obtain a highly accurate version of the output voltage to act as a benchmark.

Fig. 5(b) presents the MESFET response with the full wavelet scheme ($J = 1, L = 20$) employed. As can be seen, the general nature of the circuit response is obtained. However, the low-order wavelet approximation ($J = 1$) is not sufficient in this case to acquire the fine details of the output. Hence, there is a need to use a higher order wavelet approximation.

Fig. 5(c) presents the output obtained with the proposed new higher degree accuracy technique after adding an extra layer ($J = 2$) in the wavelet series approximation. It can be seen that the accuracy of the output voltage is considerably improved. However, it took only 21% of the computational time to obtain the circuit response with the new technique involving the calculation of an extra layer coefficients compared to the computational time required when the simulation is restarted from the beginning.

Therefore, the results presented here clearly confirm that, by employing the approach presented in Section V, the accuracy degree may be increased by adding an extra layer into the wavelet series approximation, but with considerably less computational costs than restarting a full wavelet scheme. This is possible since the coefficients calculated for a lower order approximation are reused to form the higher order approximation.

VII. Conclusion

A highly efficient wavelet-based simulation technique for high-frequency circuits has been presented.

The multitime partial differential-equation system describing the circuit has been solved using a pseudowavelet collocation method. A nonlinear model-reduction process has been applied, leading to significant gains in efficiency, but without a complementary loss in accuracy.

Furthermore, utilizing the multiresolution nature of wavelets, this paper has presented a further step toward a more accurate simulation technique with even greater efficiency within the overall design cycle. Rather than recalculating a complete set of new coefficients for higher order approximation of the unknown in the multitime partial differential-equation representation of the system, it utilizes the coefficients calculated from a previous simulation that involved a lower order approximation.

Therefore, the technique can be very useful for the IC designer since it enables the desired circuit response degree of accuracy to be achieved in steps rather than restarting simulations each time a higher degree of accuracy is sought. The results from highly nonlinear sample circuits indicate the efficiency and accuracy of the proposed approach.

As shown, the multilayer approach allows a controlled refinement and, for any practical usage, it would be helpful to derive the error measurement. This forms a basis of future work by the authors.

Acknowledgment

The authors wish to acknowledge the help of Prof. T. Brazil and the Microwave Engineering Research Group, University College Dublin, Dublin, Ireland.

References

[1] K. S. Kundert and A. Sangiovanni-Vincentelli, "Simulation of nonlinear circuits in the frequency domain," *IEEE Trans. Computer-Aided Design Integr. Circuits Syst.*, vol. CAD-5, no. 10, pp. 521–535, Oct. 1986.

[2] D. Long, R. Melville, K. Ashby, and B. Horton, "Full-chip harmonic balance," in *Proc. IEEE Custom Integrated Circuits Conf.*, Santa Clara, CA, 1997, pp. 379–382.

[3] M. Nakhla and J. Vlach, "A piecewise harmonic balance technique for determination of periodic response of nonlinear systems," *IEEE Trans. Circuits Syst.*, vol. CAS-23, no. 2, pp. 85–91, Feb. 1976.

[4] L. W. Nagel, "SPICE2: A computer program to simulate semiconductor circuits," Univ. California at Berkeley, Berkeley, CA, Tech. Rep. ERL-M520, 1975.

[5] E. Ngoya and R. Larcheveque, "Envelope transient analysis: A new method for the transient and steady-state analysis of microwave communications circuits and systems," in *IEEE MTT-S Int. Microwave Symp. Dig.*, San Francisco, CA, 1996, pp. 17–21.

[6] D. Sharrit, "New method of analysis of communication systems," presented at the IEEE MTT-S Nonlinear Computer-Aided Design Workshop, Jun. 1996.

[7] J. S. Roychowdhury, "Analyzing circuits with widely separated timescales using numerical PDE methods," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 48, no. 5, pp. 578–594, May 2001.

[8] J. C. Pedro and N. B. Carvalho, "Simulation of RF circuits driven by modulated signals without bandwidth constraints," in *IEEE MTT-S Int. Microwave Symp. Dig.*, Seattle, WA, 2002, pp. 2173–2176.

[9] M. Condon and E. Dautbegovic, "A novel envelope simulation technique for high-frequency nonlinear circuits," in *Proc. 33rd Eur. Microwave Week*, Munich, Germany, 2003, pp. 619–622.

[10] W. Cai and J. Z. Wang, "Adaptive multi-resolution collocation methods for initial boundary value problems of nonlinear PDEs," *SIAM J. Numer. Anal.*, vol. 33, pp. 937–970, Jun. 1996.

[11] C. E. Christoffersen and M. B. Steer, "State-variable-based transient circuit simulation using wavelets," *IEEE Microwave Wireless Compon. Lett.*, vol. 11, no. 4, pp. 161–163, Apr. 2001.

[12] P. K. Gunupudi and M. S. Nakhla, "Model reduction of nonlinear circuits using Krylov-subspace techniques," in *Proc. Design Automation Conf.*, New Orleans, LA, 1999, pp. 13–16.

[13] E. Dautbegovic, M. Condon, and C. Brennan, "An efficient nonlinear circuit simulation technique," in *IEEE RFIC Symp. Dig.*, Fort Worth, TX, 2004, pp. 623–626.

[14] C. S. Burrus, R. A. Gopinath, and H. Guo, *Introduction to Wavelets and Wavelet Transforms (A Primer)*. Englewood Cliffs, NJ: Prentice-Hall, 1998, pp. 10–19.

[15] J. M. Maron and R. J. Lopez, *Numerical Analysis: A Practical Approach*, 3rd ed. Belmont, CA: Wadsworth, 1991, ch. 6.

[16] ——, *Numerical Analysis: A Practical Approach*, 3rd ed. Belmont, CA: Wadsworth, 1991, ch. 8.

Emira Dautbegović (S'99) received the B.Sc. degree in control and electronic engineering from the University of Sarajevo, Sarajevo, Bosnia Herzegovina, in 2000.

She is currently a Ph.D. Researcher with the RF Modeling and Simulation Group, School of Electronic Engineering, Dublin City University, Dublin, Ireland. From 2000 to 2001, she was with Javno Preduzece Elektroprivreda Bosne i Hercegovine (JP EPBiH), the national electric power supplier in Bosnia Herzegovina. From 1999 to 2001, she was a Teaching Assistant with the Faculty of Electrical Engineering, University of Sarajevo. Her current research interests are in the area of simulation and modeling of high-speed linear and nonlinear circuits, development of computer-aided design (CAD) algorithms for high-frequency circuits and systems, simulation of interconnects, model-order reduction techniques, and the development of efficient numerical algorithms.

Ms. Dautbegović is member of the Institution of Engineers of Ireland (IEI), the Institution of Electrical Engineers (IEE), U.K., and the Association of Computing Machinery (ACM). She was co-founder and the vice-president of the IEEE Student Branch at the University of Sarajevo. She was a recipient of the 2002–2003 and 2003–2004 IBM Worldwide Fellowship Program. She was among the top 5% of students of the Faculty of Electrical Engineering, University of Sarajevo, for which she was the recipient of the Golden Medallion.

Marissa Condon (M'98) received the B.E. degree (with honors) and Ph.D. degree from the National University of Ireland, Galway, Ireland, in 1995 and 1998, respectively.

She then joined the National Grid of the Electricity Supply Board of Ireland for two years prior to joining the School of Electronic Engineering, Dublin City University, Dublin, Ireland. She is also joint leader of the RF Modeling and Simulation Group, Dublin City University. Her research interests are the analysis and simulation of high-frequency circuits, the development of algorithms for CAD packages, nonlinear model reduction, and the development of suitable macromodels for large-scale circuits and systems operating at high frequencies.

Dr. Condon is a reviewer for the European Conference on Circuit Theory and Design (ECCTD). She served on the Steering Committee of the Universities Power Engineering Conference from 1999 to 2000.

Conor Brennan (M'02) was born in Dublin, Ireland, in 1972. He received the B.A. (Mod.) degree in mathematics and Ph.D. degree from the University of Dublin Trinity College, Dubline, Ireland, in 1994 and 1998, respectively.

From 2000 to 2002, he was a Post-Doctoral Researcher with the Radio Propagation Group, Department of Electrical Engineering, University of Dublin Trinity College. In 2003, he joined the academic staff of the School of Electronic Engineering, Dublin City University (DCU), Dublin, Ireland, where he is joint leader of the RF Modeling and Simulation group. He is a member of the DCU-based Research Institute for Networks and Communications Engineering (RINCE).

Dr. Brennan was the recipient of a Forbairt Post-Doctoral Fellowship on the topic of efficient integral-equation methods.