

A novel envelope simulation technique for high-frequency nonlinear circuits

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Abstract – The paper proposes a new approach for the analysis and simulation of circuits subject to input signals with widely separated rates of variation. Such signals arise in communication circuits when an RF carrier is modulated by a low-frequency information signal. The approach will involve converting the ordinary differential equation system that describes the circuit to a partial differential equation system and subsequently solving the resultant system using a multiresolution collocation approach involving a cubic spline wavelet-based decomposition.

I. INTRODUCTION

The two most common circuit simulation techniques that are employed in RF and microwave circuit simulators are Harmonic Balance [1-3] and Time-Domain Integration [4]. Harmonic balance is employed for periodic or quasi-periodic steady-state analysis of mildly nonlinear circuits and hence can prove limited for the complex modulation formats encountered in today's high-speed systems or for systems involving strong nonlinearities. Time-domain integration on the otherhand is only suitable for baseband systems. For the simulation of circuits with digitally modulated high-frequency carriers with long bit sequences, time-domain integration is excessively slow. As a result, there is a need for some form of general purpose technique which can simulate over a long interval (to enable BER calculations for example) nonlinear systems with transient high-frequency signals or complex modulated RF carriers.

To this end, several envelope transient analysis approaches have been proposed whereby a mixed-mode technique is implemented [5-6]. The essence of these approaches is that the envelope of a signal is treated by time-domain integration and that the carrier is treated by Harmonic Balance. However, existing techniques have limitations, for example, restrictions in the bandwidth of the excitation signal [5] and the limitations of harmonic balance with respect to strong nonlinearities. Roychowdhury in [7] proposes converting the differential-algebraic equations which describe the circuit to multi-time partial differential equations and applying time-domain methods directly to solve the resultant equations. Pedro [8] also employs a multi-time partial differential equation approach but uses a combination of Harmonic Balance and Time-Domain Integration to solve the resultant system.

In this paper, the multi-time partial differential equation approach is again employed. However, a wavelet-based collocation approach is utilised for the purposes of solving the system. The particular solution technique employed is a variation or modification of that proposed by Cai and Wang [9]. The advantage of this approach is that it permits an adaptive multiresolution solution which is ideal for simulating responses of envelope-modulated circuits. Christoffersen and Steer [10] also employ a version of the cubic spline collocation method presented in [9] but they apply it directly to the original ordinary differential equation system rather than to a multi-time partial differential equation representation of the system as proposed in this contribution.

Thus, in summary, the paper combines a wavelet-based collocation technique with a multitime approach to result in a novel approach for the simulation of communications circuits subject to modulated signals.

II. MULTITIME PARTIAL DIFFERENTIAL EQUATION APPROACH

Consider a signal $x(t)$ which is composed of a carrier modulated by an envelope where the envelope signal is assumed to be uncorrelated with the carrier. In this case, the signal may be represented in two independent time variables as follows:

$$x(t) = \hat{x}(t_1, t_2) \quad (1)$$

t_1 relates to the low-frequency envelope and t_2 relates to the high-frequency carrier.

Now consider a general nonlinear circuit described by:

$$\dot{q}(x(t)) = f(x(t)) + b(t) \quad (2)$$

where $b(t)$ is the input to the circuit and f is a linear or nonlinear function modelling resistive elements. q models the memoryless linear or nonlinear charges or fluxes. $x(t)$ are the state variables.

The corresponding multitime partial differential equation system can be written as shown in [7] as:

$$\frac{\partial q(\hat{x}(t_1, t_2))}{\partial t_1} + \frac{\partial q(\hat{x}(t_1, t_2))}{\partial t_2} = f(\hat{x}(t_1, t_2)) + \hat{b}(t_1, t_2) \quad (3)$$

This multitime partial differential equation can be solved using entirely time-domain approaches as employed by Roychowdhury [7] or using a combination of time-step

integration for the envelope and Harmonic Balance for the carrier as in [8]. However, for strongly nonlinear circuits, the use of harmonic balance for the inner loop can prove limited as it leads to excessive computation. In this paper, the multitime partial differential equation system is solved using a pseudo-wavelet collocation method derived from that proposed by Cai and Wang [9].

III. WAVELET COLLOCATION METHOD FOR NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

The technique involves approximating the unknown function $\hat{x}(t_1, t_2)$ with a wavelet series in the t_2 dimension. I.e.

$$\hat{x}(t_1, t_2) = \bar{x}_{-1,-1}(t_1)\phi_b(t_2) + \sum_{k=0}^{T-4} \bar{x}_{-1,k}(t_1)\phi_k(t_2) + \bar{x}_{-1,T-3}(t_1)\phi_b(T-t_2) + \sum_{j=0}^{J-1} \sum_{k=-1}^{n_j-2} \bar{x}_{j,k}(t_1)\psi_{j,k}(t_2) \quad (4)$$

where J is an integer ≥ 0 and is the maximum subspace level being considered. $[0, T]$ is the interval where the unknown function, $\hat{x}(t_1, t_2)$, is defined. $T > 4$ and the interval may be scaled to obtain any required interval. $\phi(t)$ and $\psi(t)$ are basis functions known as scaling and wavelet functions respectively and are defined as in [9] as:

$$\varphi(t) = \frac{1}{6} \sum_{l=0}^4 \binom{4}{l} (-1)^l (t-l)_+^3 \quad t_+^n = \begin{cases} t^n & t \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

$$\psi(t) = -\frac{3}{7}\varphi(2t) + \frac{12}{7}\varphi(2t-1) - \frac{3}{7}\varphi(2t-2)$$

$$\varphi_b(t) = \frac{3}{2}t_+^2 - \frac{11}{12}t_+^3 + \frac{3}{2}(t-1)_+^3 - \frac{3}{4}(t-2)_+^3 + \frac{1}{6}(t-3)_+^3$$

$$\psi_{jk}(t) = \psi(2^j t - k)$$

$\bar{x}(t_1)$ are the unknown coefficients.

Eqn. 3 is then collocated on collocation points to result in a semidiscretised wavelet collocation method. The collocation points chosen are as given in [9] and are (subscript 2 refers to the t_2 variable):

$$\begin{aligned} t_{2,1}^{(-1)} &= 0, & t_{2,2}^{(-1)} &= 0.5, & t_{2,k}^{(-1)} &= k-2, & 3 \leq k \leq T+1 \\ t_{2,T+2}^{(-1)} &= T-0.5, & t_{2,T+3}^{(-1)} &= T \\ t_{2,-1}^{(j)} &= \frac{1}{2^{j+2}}, & t_{2,k}^{(j)} &= \frac{k+1.5}{2^j}, & 0 \leq k \leq n_j-3 \\ t_{2,n_j-2}^{(j)} &= T - \frac{1}{2^{j+2}} & n_j &= 2^j T \end{aligned} \quad (6)$$

To obtain a fully discretised wavelet collocation method, the time-derivative with respect to t_1 (representing slowly-varying envelope) is replaced by a suitable difference equation. An adaptive Backward-Euler predictor corrector approach is employed in the current work (A simple Forward Euler was suggested in [9]). This leads to significant gains in efficiency compared to fixed-step approaches. Consequently, the overall technique can be implemented in an extremely efficient manner. It obviates the need for solving nonlinear algebraic equations at each timestep thereby removing the potential difficulties that arise in other simulation approaches when large-scale nonlinear systems are present. Furthermore, for most systems, many of the wavelet coefficients may be neglected within a given tolerance ε . This permits the number of wavelet functions included to be adjusted dynamically thereby reducing the computing requirements while at the same time achieving a satisfactory level of accuracy. For example, if

$$|\bar{x}_{jk}(t_1)| < \varepsilon$$

then the wavelet function associated with this coefficient may be neglected. Furthermore, if the maximum coefficient in any level of resolution, J , is less than the error tolerance, then the level J of the wavelet expansion can be decreased to reduce computational requirements. I.e. $\max |\bar{x}_{jk}(t_1)| < \varepsilon$ would imply decreasing J to J' in eqn. 4 where $J' < J$.

IV. FIRST SAMPLE SYSTEM

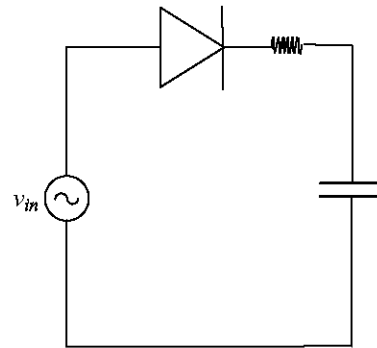


Fig. 1 Diode rectifier circuit

The first example taken consists of a diode rectifier circuit as shown in Fig. 1.

The source is:

$$b(t) = \sin\left(\frac{2\pi t}{T_1}\right) \sin\left(\frac{2\pi t}{T_2}\right) \quad (7)$$

where T_1 corresponds to the envelope period and T_2 corresponds to the carrier period. Fig. 2a shows the output voltage resulting from the new technique with a level of resolution chosen to obtain an acceptable simulation time. Fig. 2b shows the output from an ordinary differential equation solver with a very short time step in order to obtain a highly accurate version of the output voltage to act as a benchmark for the purposes of confirming the accuracy of the proposed new simulation technique. As evidenced by this result, the

new method achieves a satisfactory level of accuracy. Obviously, greater accuracy can be achieved by increasing the level of resolution in the wavelet scheme (or by setting a tighter tolerance value) but at the cost of increasing simulation time. Fig. 3 shows a result with a very coarse level of resolution which clearly captures the salient behaviour of the response.

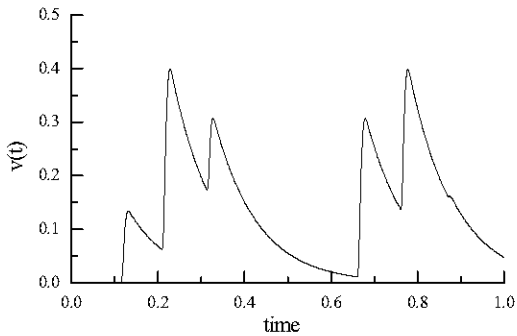


Fig. 2a Sample result from new method

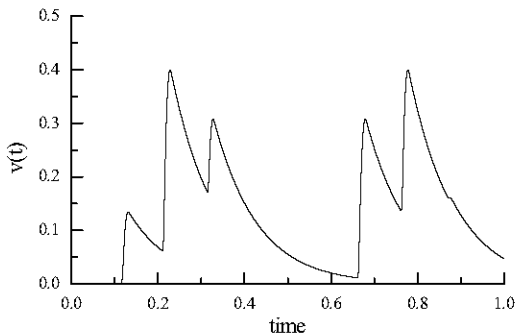


Fig. 2b Result from ODE solver with a very short timestep

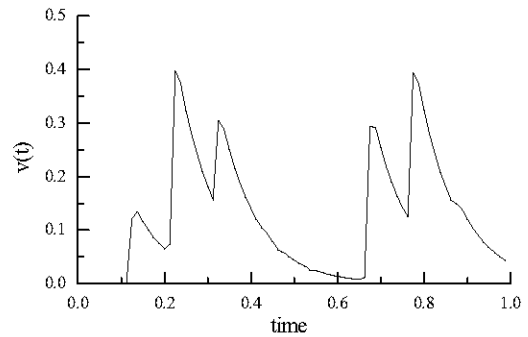


Fig. 3 Result with a very coarse level of resolution

V. SECOND SAMPLE SYSTEM

The second example taken is that of the single-ended MESFET amplifier shown in Fig. 4 and is deemed a suitable test circuit for the proposed technique. The amplifier is described by ten non-linear differential equations that are stiff in nature. The input to the circuit is a 2GHz wave modulated by a 0.2GHz wave.

Fig. 5 shows the output voltage obtained when a fourth-order Adams-Moulton predictor-corrector technique is employed with a time-step of ~ 0.12 ps. This is deemed an accurate representation of the output voltage for comparative purposes. Fig. 6 shows the corresponding voltage obtained with the new envelope wavelet technique. A level of resolution is chosen which achieves a high degree of accuracy. As evidenced by this result, the technique is highly effective in predicting the output voltage. Fig. 7 shows a result when a very coarse resolution is employed. As can be seen, even with this level of resolution, a very clear picture of the output voltage is obtained.

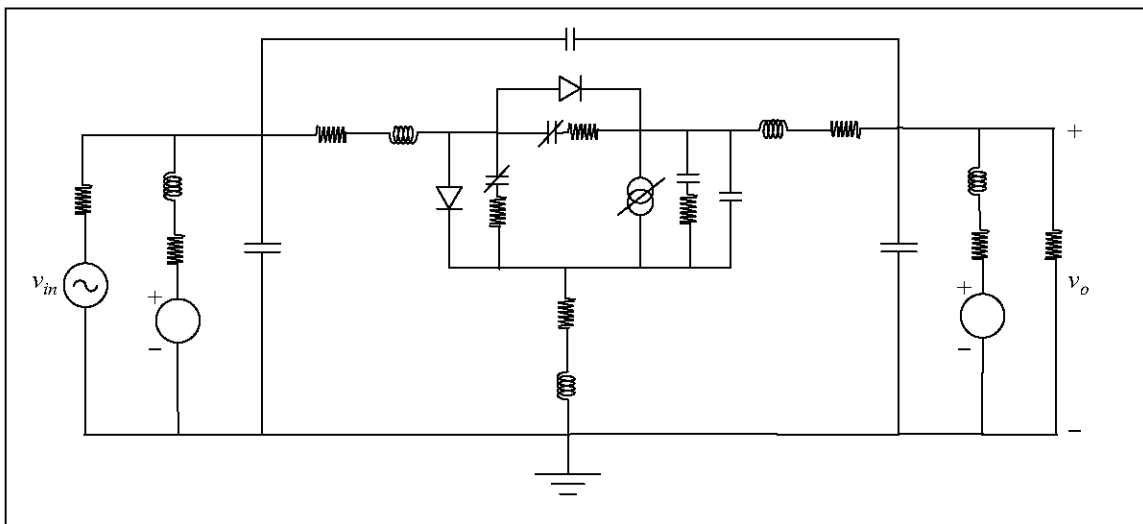


Fig. 4 Simple MESFET Amplifier

ACKNOWLEDGMENT

The authors wish to acknowledge the support of IBM for this work.

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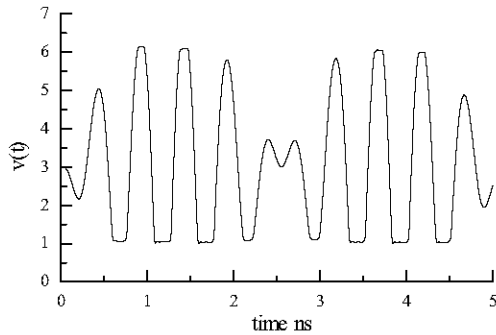


Fig. 5 Output voltage with Adams-Moulton technique

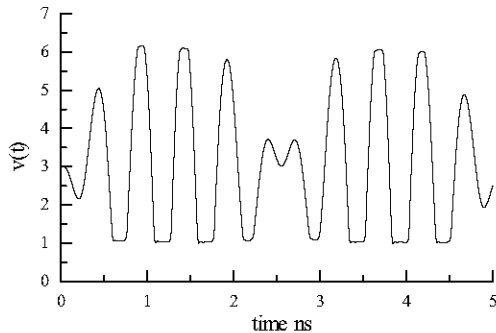


Fig. 6 Output voltage with wavelet envelope technique with a fine level of resolution

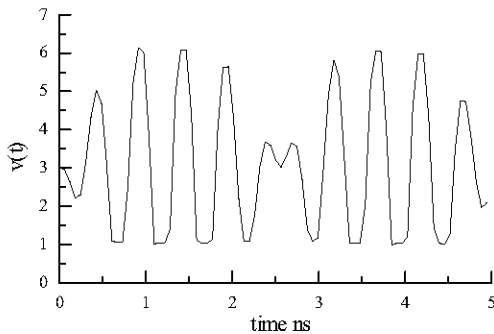


Fig. 7 Output voltage with wavelet envelope technique with a coarse level of resolution

VI. CONCLUSION

The paper has presented a new approach for the simulation of communications circuits with envelope-modulated signals. The approach involves the conversion of the ordinary differential equations that govern the system to multitime partial differential equations. The resultant partial differential equation system is solved using an adaptive pseudo-wavelet collocation scheme. Two sample systems have illustrated the efficacy of the proposed envelope simulation technique. Its principle advantages are that it may be applied in the case of strongly nonlinear circuits and that it permits an effective trade-off between accuracy and speed.