Compact Models for Wireless Systems

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***Abstract*—For the design and analysis of wireless systems, complex simulations are required and performed. Model order reduction techniques enable greater efficiencies to be achieved and concomitantly, a reduction in memory-resource usage. However, maintaining a certain level of accuracy is paramount. In this contribution, two techniques are combined to enable the formation of a compact model of a high-order system, structure or component. The first is a Krylov subspace method which reduces the original model to a moderate size and the second is a Fourier series expansion method that enables speed and ease of determination of the time-domain responses of the system to arbitrary inputs.**

I. Introduction

The technological advances in the circuitry involved in wireless systems are such that there is an ongoing need for efficient and effective model-order reduction techniques to counter the ever-increasing complexity of simulations. Many different techniques have been proposed, for example in reference [1 and all references therein]. In this contribution, two different methods are combined to form a compact model of a large high-order system thereby enabling fast and accurate repeated simulations.

The first method considered is the Krylov subspace method e.g. [2]. This method can handle very large systems and is numerically efficient. However, there is no error bound for the method and it can generate non-optimal models [3]. In general, the reduced models formed from Krylov methods contain information that is not necessary for a good approximation.

The second method addressed is based on a Fourier Series Expansion (FSE) [4]. The medium-size model obtained after application of the Krylov method is simulated and time-domain responses can be explicitly obtained in a simple form for an arbitrary input using only a compact set of FSE coefficients. Guaranteed stability and causality is assured with this model.

The proposed combined method is applied to a coplanar waveguide and results will highlight the efficacy of the proposed method.

II. Krylov Subspace Methods

Consider a linear system with a state-space representation as:

$$\begin{aligned} E\frac{\mathrm{d}x(t)}{\mathrm{d}t} &= Ax(t)+Bu(t) \\ y(t) &= Cx(t)+Du(t), \end{aligned} \qquad (1)$$

where $x(t)$ is the state-space of the system of dimension n, $u(t)$ is the input and $y(t)$ is the output of the system. Here A, B, C, D and E are the corresponding state-space matrices. Eqn. (1) may be transformed to the Laplace Domain and a transfer function $H(s)$ relating the input $U(s)$ to the output $Y(s)$ obtained:

$$H(s) = C(sE-A)^{-1}B + D. \qquad (2)$$

There are several variations of Krylov methods but the PRIMA [5] Krylov method is considered here. The Krylov space is then defined as:

$$K_q(\hat{A},\hat{B}) = [\hat{B}, \hat{A}\hat{B}, \dots, \hat{A}^{q-1}\hat{B}], \qquad (3)$$

where $\hat{A} = (A - s_0E)^{-1}E$ and $\hat{B} = (s_0E - A)^{-1}B$ and q is the dimension of the reduced system. An orthonormal basis, V, is formed for K_q. The reduced system is then formed as:

$$\begin{aligned} \tilde{E}\frac{\mathrm{d}\tilde{x}(t)}{\mathrm{d}t} &= \tilde{A}\tilde{x}(t)+\tilde{B}u(t) \\ \tilde{y}(t) &= \tilde{C}\tilde{x}(t)+\tilde{D}u(t), \end{aligned} \qquad (4)$$

with $\tilde{E} = V^TEV$, $\tilde{A} = V^TAV$, $\tilde{B} = V^TB$, $\tilde{C} = CV$ and $\tilde{D} = D$.

The reduced system obtained with PRIMA matches q moments of the original transfer function (2) and stability and passivity are preserved as proven in [5].

However, it is well-known that Krylov methods generate models which are, in general, larger than required. Several remedies for this redundancy have been proposed [3, 6-7]. In this contribution, the Fourier Series Expansion is proposed as an alternative post-processing step to cure the problem.

III. Fourier Series Expansion

The Fourier Series Expansion was first introduced in [4] and is summarized here for completeness. Let $\tilde{H}(\omega)$ be the transfer function of the reduced system obtained from application of the PRIMA algorithm of Section II. Suppose $\tilde{H}(\omega)$ is nonzero for $|\omega| \in [0, \omega_{\mathrm{m}}]$ where ω_{m} is assumed to be large, but finite. Also, assume $\tilde{H}(\omega) = \tilde{H}^*(-\omega)$. Then $\tilde{H}(\omega)$ may be expanded in a Fourier Series as follows, bearing in mind that it must be an even function of frequency.

$$\mathrm{Re}\,\tilde{H}(\omega) = \sum_{k=0}^{\infty} a_k \cos k\tilde{\omega}, \qquad (5)$$

where $\tilde{\omega} = \pi\omega/\omega_\mathrm{m}$. The expression in (5) describes an even function, defined for $\omega \in [-\omega_\mathrm{m}, \omega_\mathrm{m}]$ (i.e. $\tilde{\omega} \in [-\pi, \pi]$). To enforce causality, the expression for $\mathrm{Im}\,\tilde{H}(\omega)$ may be obtained from (5) via the Kramers-Kronig relations (Hilbert transform) [8]:

$$\mathrm{Im}\,\tilde{H}(\omega) = -\sum_{k=0}^{\infty} a_k \sin k\tilde{\omega}, \tag{6}$$

From (5) and (6), it follows that

$$\tilde{H}(\omega) = \sum_{k=0}^{\infty} a_k \mathrm{e}^{\mathrm{j}k\tilde{\omega}}, \tag{7}$$

for $\omega \in [-\omega_\mathrm{m}, \omega_\mathrm{m}]$.

The representation of the output in the time domain may be obtained by an Inverse Fourier Transform. The output caused by an (arbitrary) input $x(t)$ defined for $t > 0$, (i.e. input signal $x(t)\theta(t)$ with Fourier image $X(\omega)$, where $\theta(t)$ is the unit step-function), is:

$$\begin{aligned} y(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{j}\omega t} Y(\omega)\,\mathrm{d}\omega \\ &= \frac{1}{2\pi}\sum_{k=0}^{\infty} a_k \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{j}(t-\tilde{k})\omega} Y(\omega)\,\mathrm{d}\omega \qquad (8) \\ &= \sum_{k=0}^{\infty} a_k x(t-\tilde{k})\theta(t-\tilde{k}), \end{aligned}$$

where $\tilde{k} = \pi k/\omega_\mathrm{m}$.

Therefore, once the set of FSE coefficients is obtained from the frequency-domain simulations, then the response for an arbitrary input may be readily determined from (8).

To determine the set of FSE coefficients,let $\tilde{H}(\omega)$ be obtained at a number of points, ω_i:

$$\begin{aligned} F_i^{(1)} &= \mathrm{Re}\,\tilde{H}(\omega), \qquad i = 1, 2, \ldots, N_1; \qquad (9) \\ F_i^{(2)} &= \mathrm{Im}\,\tilde{H}(\omega), \qquad i = 1, 2, \ldots, N_2, \qquad (10) \end{aligned}$$

where N_1 is the number of real parts of the data points and N_2 is the corresponding number imaginary parts of the data points.

Then let a be the set of real coefficients $a = [a_0, a_1, \ldots, a_N]^T$. Introduce $M_{ik}^{(1)} = \cos k\tilde{\omega}_i$, $M_{ik}^{(2)} = -\sin k\tilde{\omega}_i$, and $\tilde{\omega}_i = \pi\omega_i/\omega_\mathrm{m}$, where $k = 1, \ldots, N$. Then from (9) and (10):

$$\begin{aligned} F^{(1)} &= M^{(1)}a + E^{(1)}; \qquad (11) \\ F^{(2)} &= M^{(2)}a + E^{(2)}. \qquad (12) \end{aligned}$$

Here $E^{(1,2)}$ represent the errors that arise due to limiting the summation in (5)-(8) to a finite number of terms, N; (11) and (12) may be merged to yield:

$$F = Ma + E \tag{13}$$

with

$$F = \begin{bmatrix} F^{(1)} \\ F^{(2)} \end{bmatrix}, \quad M = \begin{bmatrix} M^{(1)} \\ M^{(2)} \end{bmatrix}, \quad E = \begin{bmatrix} E^{(1)} \\ E^{(2)} \end{bmatrix}$$

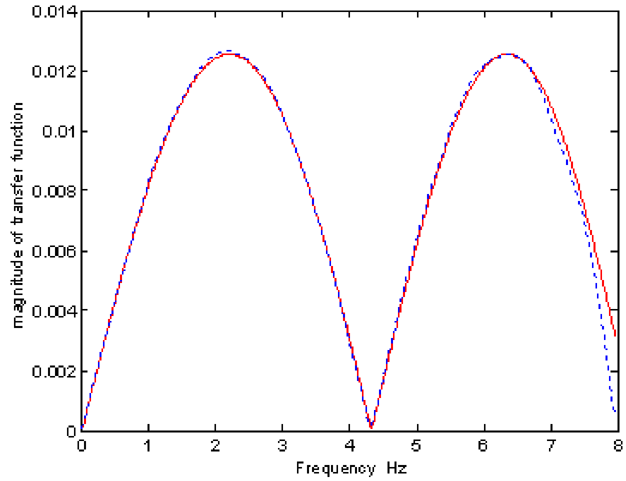

Fig. 1. Comparison of frequency responses for the coplanar waveguide – red solid line is full model, blue dashed line is reduced model.

and the minimal error, $E^T E$, for (13) is achieved with:

$$a = (M^T M)^{-1} M^T F. \tag{14}$$

The number of points N determines the size of this reduced system. By selecting N as small as possible to obtain the required error, any redundancy in the system in (4) can be removed.

IV. Example

The example considered is that of a coplanar waveguide. The original model is described by a state-space representation of the order of 300. The Krylov method is used to reduce this dimension to 200. The Fourier Series Method is then applied and the result obtained with $N = 20$ is shown in Fig. 1. It is superimposed on the frequency domain response of the original full model.

V. Conclusion

The paper has proposed a two-stage method for forming a reduced-order model of large-scale systems. The method combines two techniques, a Krylov method and a Fourier Series approach. The method achieves a high degree of accuracy and results in a compact model that enables ease of determination of the time-domain responses.

Acknowledgment

The author wishes to thank Dr. Dirk Deschrijver, Dept. of Information Technology, Ghent University, Ghent, Belgium for his provision of the dataset in the example.

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