A Parametric Macromodelling Technique

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***Abstract*—With the ever growing complexity of high-frequency systems in the electronic industry, formation of reduced-order models or compact macromodels of these systems is paramount. In this contribution, a Fourier series expansion technique is extended to form a modeling strategy to approximate the frequency-domain behaviour of a system based on several design variables. In particular, it is intended to provide a tool for the designer to identify the effect of manufacturer tolerances and process fluctuations or irregularities on system behaviour.**

I. Introduction

Parametric models of interconnect or packaging systems is of paramount importance for signal integrity analysis and system design and optimisation. Such models may be formed either from data obtained from time-consuming full-wave electromagnetic simulations or from measured data. These models characterize both the frequency behaviour of a system and its behaviour in relation to several physical properties such as length or substrate properties. The formation of parametric macromodels has increasingly become a focus of research e.g. [1], [2], [3], [4] and accurate formation of such models is not a trivial task.

In this paper, the formation of a passive, stable and causal parametric model from measured or simulated data is addressed by using a combination of two stages. The first stage is based on a Fourier Series Expansion [5]. The method is then extended for multivariate modelling by using spline interpolation.

The proposed method is most applicable for systems involving smooth responses. Consequently, it is useful for determining the effect of manufacturing tolerances or process fluctuations where parameter variability is relatively small. As an example, the method will be applied to an exponentially tapered transmission line and results will highlight the efficiency, simplicity and efficacy of the proposed method.

II. Fourier Series Expansion

The Fourier Series Expansion was first introduced in [5]. The method may be extended to a general multivariate case but for ease of understanding, the present paper will only address the bivariate case. Let the system be described by a transfer function $H(\omega, l)$, where ω is frequency and l is the second design variable or parameter. Let $H(\omega, l)$ be nonzero for $|\omega| \in [0, \omega_{\rm m}]$, where $\omega_{\rm m}$ is assumed to be large, but finite. Then $\operatorname{Re} H(\omega, l)$ may be expanded in a Fourier Series as follows, bearing in mind that it must be an even function of frequency:

$$\operatorname{Re} H(\omega, l) = \sum_{k=0}^{\infty} a_k(l) \cos k\tilde{\omega}, \qquad (1)$$

where $\tilde{\omega} = \pi\omega/\omega_{\rm m}$. The expression in (1) describes an even function, defined for $\omega \in [-\omega_{\rm m}, \omega_{\rm m}]$ (i.e. $\tilde{\omega} \in [-\pi, \pi]$). To enforce causality, the expression for $\operatorname{Im} H(\omega, l)$ may be obtained from (1) via the Kramers-Kronig relations (Hilbert transform) [6]:

$$\operatorname{Im} H(\omega, l) = -\sum_{k=0}^{\infty} a_k(l) \sin k\tilde{\omega}, \qquad (2)$$

From (1) and (2), it follows that

$$H(\omega, l) = \sum_{k=0}^{\infty} a_k(l) {\rm e}^{-{\rm j}k\tilde{\omega}}, \qquad (3)$$

for $\omega \in [-\omega_{\rm m}, \omega_{\rm m}]$. The Fourier coefficients, $a_k(l)$, are parameterised – they are a function of the additional parameter, l.

The representation of the output in the time domain may be obtained by an Inverse Fourier Transform. The output caused by an (arbitrary) input $x(t)$ defined for $t > 0$, (i.e. input signal $x(t)\theta(t)$ with Fourier image $X(\omega)$, where $\theta(t)$ is the unit step-function), is:

$$\begin{aligned} y(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} {\rm e}^{{\rm j}\omega t} Y(\omega)\, {\rm d}\omega \\ &= \sum_{k=0}^{\infty} a_k(l) \frac{1}{2\pi} \int_{-\infty}^{\infty} {\rm e}^{{\rm j}(t-\tilde{k})\omega} X(\omega)\, {\rm d}\omega \\ &= \sum_{k=0}^{\infty} a_k(l) x(t-\tilde{k}) \theta(t-\tilde{k}), \end{aligned} \qquad (4)$$

where $\tilde{k} = \pi k/\omega_{\rm m}$.

Therefore, once the set of FSE coefficients $\{a_k(l)\}$ are obtained from the frequency-domain simulations, then the response for an arbitrary input may be readily determined from (4).

To determine the set of FSE coefficients for a specific value of l, let $H(\omega, l)$ be obtained at a number of points, ω_i:

$$F_i^{(1)}(l) = \operatorname{Re} H(\omega_i, l), \qquad i = 1, 2, \ldots, N_1; \qquad (5)$$

$$F_i^{(2)}(l) = \operatorname{Im} H(\omega_i, l), \qquad i = 1, 2, \ldots, N_2, \qquad (6)$$

where N_1 is the number of real parts of the data points for each specific value of l and N_2 is the corresponding number imaginary parts of the data points.

Let $a(l)$ be the set of real coefficients $a(l) = [a_0(l), a_1(l), \ldots, a_N(l)]^T$. Let $M_{ik}^{(1)} = \cos k\tilde{\omega}_i$ and $M_{ik}^{(2)} = -\sin k\tilde{\omega}_i$, where $\tilde{\omega}_i = \pi\omega_i/\omega_{\mathrm{m}}$, $k = 1, \ldots, N$. Then from (5) and (6):

$$F^{(1)}(l) = M^{(1)}a(l) + E^{(1)}; \tag{7}$$

$$F^{(2)}(l) = M^{(2)}a(l) + E^{(2)}. \tag{8}$$

Here, $E^{(1,2)}$ represent the errors that arise due to limiting the summation in (1)-(4) to a finite number of terms, N. Eqns. (7) and (8) may be merged to yield:

$$F(l) = Ma(l) + E \tag{9}$$

with

$$F(l) = \begin{bmatrix} F^{(1)}(l) \\ F^{(2)}(l) \end{bmatrix}, \quad M = \begin{bmatrix} M^{(1)} \\ M^{(2)} \end{bmatrix}, \quad E(l) = \begin{bmatrix} E^{(1)} \\ E^{(2)} \end{bmatrix}$$

and the minimal error, $E^T E$, for (9) is achieved with:

$$a(l) = (M^T M)^{-1} M^T F(l). \tag{10}$$

III. Laguerre Basis

In [7] and [8], it is shown that the Fourier Series Expansion coefficients correspond to the Markov parameters of a discrete time system. This system has the form

$$H(z) = \sum_{k=0}^{N} \eta_k z^{-k} = C(zI - A)^{-1}B + D \tag{11}$$

where

$$A = [e_2, e_3, \ldots, e_N, 0], \quad B = [e_1],$$
$$C = [\eta_1, \eta_2, \eta_3, \ldots, \eta_N], \quad D = [\eta_0],$$

e_i denotes the i^{th} unit vector and 0 is a vector of zeros and N is the dimension of this discrete model. The sampling time of the discrete system is:

$$T = \frac{\pi}{\omega_{\mathrm{m}}}. \tag{12}$$

However, in [9], it is shown that the orthonormal expansion of a continuous transfer function $G(s)$ in terms of a Laguerre basis is equivalent to a Taylor expansion inside the unit disk:

$$G(s) = \sum_{k=0}^{N} G_k \Phi_k^{\alpha}(s) = \frac{1}{\sqrt{2\alpha}} \sum_{k=0}^{N} G_k \bar{z}^k, \tag{13}$$

where

$$\Phi_{\alpha}^{k}(s) = \frac{\sqrt{2\alpha}}{s+\alpha}\left(\frac{s-\alpha}{s+\alpha}\right)^k, \tag{14}$$

$$\bar{z} = \frac{1}{z}, \tag{15}$$

$$\bar{z} = \frac{s-\alpha}{s+\alpha}. \tag{16}$$

The relationship in (16) is a variant of the familiar bilinear transform. $\Phi_{\alpha}^{k}(s)$ is the Laplace Transform of the scaled Laguerre function $\phi_{\alpha}^{k}(s) = \sqrt{2\alpha}\mathrm{e}^{-\alpha t} l_k(2\alpha t)$ and $l_k(t)$ is the Laguerre polynomial. Therefore, comparing (11) and (13),

$$\eta_k = \frac{G_k}{\sqrt{2\alpha}}, \quad \text{and} \quad \alpha = \frac{2}{T}. \tag{17}$$

Therefore, the Fourier Series Expansion is equivalent to identification of the coefficients of a Laguerre basis for the complete system. A continuous version of the system representation in (11) may be formed by using the inverse bilinear transform.

IV. Stability and Passivity

A truncated Fourier Series preserves stability [7]. However, a function represented by a truncated Fourier Series is not guaranteed to be positive real and hence, the corresponding system is not guaranteed to be passive [10]. To be passive, the transfer function of a system must be positive real [11]. However, it is possible to guarantee positive realness if the Cesàro average is taken into account [10]. The Cesàro average defined as:

$$K_N(x) = \frac{1}{N+1} \sum_{k=0}^{N} \phi_n(x), \tag{18}$$

where

$$\phi_n(x) = \sum_{k=-n}^{n} c_k \mathrm{e}^{\mathrm{j}kx}$$

is a Fourier series of a function $\xi(x)$. When $\xi(x)$ is periodic, the Cesàro average will converge to $\xi(x)$ as $N \to \infty$. To guarantee passivity, the function

$$H(z) = \sum_{k=0}^{N} \eta_k \mathrm{e}^{-\mathrm{j}k\tilde{\omega}}$$

is replaced by:

$$H(z) = \sum_{k=0}^{N} \left(1 - \frac{k}{N+1}\right) \eta_k \mathrm{e}^{-\mathrm{j}k\tilde{\omega}}. \tag{19}$$

V. Spline Interpolation

To determine an efficient bivariate macromodel, cubic spline interpolation is employed to determine the coefficients $a(l)$ between the known values. In general, the variation of the measured data $H(\omega, l)$ with the design variables is smooth and small. Hence, cubic spline interpolation is adequate for identifying the Fourier Series coefficients, $a(l)$. Cubic spline interpolation with not-a-knot end conditions is employed.

VI. Example

The example selected is taken from [4] and is an exponential tapered transmission line. It is shown in Fig. 1. $Z_0 = 50\,\Omega$ and $Z_L = 300\,\Omega$. The relative dielectric constant is $\varepsilon_r = 2$. The length of the line is varied as $L \in [1\,\mathrm{cm}, 2\,\mathrm{cm}]$ over the frequency range $[1\,\mathrm{kHz}, 50\,\mathrm{MHz}]$.

Fig. 2 shows the magnitude of the reflection coefficient, S_{11}, evaluated using a dense set of $80{\times}200$ data samples.

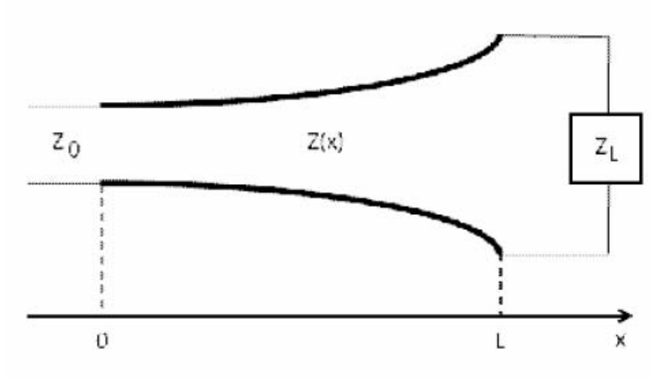

Fig. 1. Exponential tapered transmission line

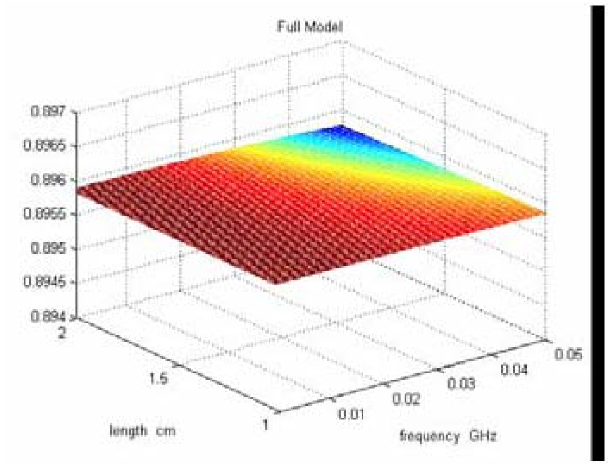

Fig. 2. Reflection coefficient S_{11} from full model

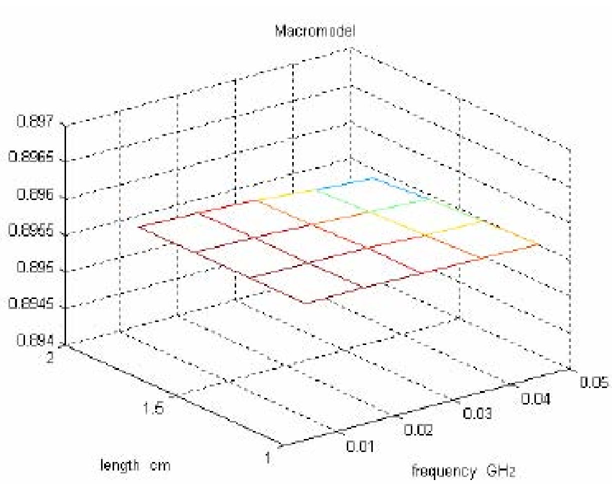

Fig. 3. Reflection coefficient S_{11} of macromodel

Fig. 3 shows the result achieved using 4×5 samples using the technique described in Sections II to IV. The error is less than $-30\,\text{dB}$.

So with a very simple model formed with only a few sample points, time-domain responses to arbitrary inputs can be readily and rapidly be obtained from (10). Thus the effect of parameter variations can be efficiently assessed. In addition, the problem of unstable poles that arises and must be addressed with vector fitting is avoided [12].

VII. Conclusion

The paper has proposed a straightforward method for forming a compact macromodel of high-frequency linear systems subject to parameter variations. The method combines two techniques, a Fourier Series approach and a spline interpolation method to extend the technique to multivariate modelling. The resultant model is passive, stable and causal. The method is accurate and its simplicity renders it a suitable tool for rapidly assessing the effect of manufacturing tolerances or process fluctuations.

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