Cross-Correlation Dynamics in Financial Time Series

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Abstract

The dynamics of the equal-time cross-correlation matrix of multivariate financial time series is explored by examination of the *eigenvalue spectrum* over sliding time windows. Empirical results for the S&P 500 and the Dow Jones Euro Stoxx 50 indices reveal that the dynamics of the small eigenvalues of the cross-correlation matrix, over these time windows, oppose those of the largest eigenvalue. This behaviour is shown to be independent of the size of the time window and the number of stocks examined. A basic one-factor model is proposed, which captures the main dynamical features of the eigenvalue spectrum of the empirical data. Through the addition of perturbations to the one-factor model, (leading to a market plus sectors model), additional sectoral features are added, resulting in an Inverse Participation Ratio comparable to that found for empirical data.

Key words: Correlation Matrix, Eigenspectrum Analysis, Econophysics

1. Introduction

In recent years, the analysis of the equal-time cross-correlation matrix for a variety of multivariate data sets such as financial data [1–8], electroencephalographic (EEG) recordings [18,19], magnetoencephalographic (MEG) recordings [9] and others, has been studied extensively. In particular, Random Matrix Theory (RMT) has been applied to filter the relevant information from the statistical fluctuations inherent in empirical cross-correlation matrices, constructed for various types of financial time series [1–8]. By comparing the eigenvalue spectrum of the correlation matrix to the analytical results obtained for random matrix ensembles, significant deviations from the RMT eigenvalue predictions are said to contain genuine information about the correlation structure of the system.

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This genuine information has been used to reduce the difference between the predicted and realised risk of different portfolios.

Several authors have suggested recently that there may, in fact, be some real correlation information hidden in the RMT defined random part of the eigenvalue spectrum. A technique, involving the use of power mapping to identify and estimate the noise in financial correlation matrices, has been described [10]. This power mapping allows the suppression of those eigenvalues, associated with the noise, to reveal different correlation structures buried underneath. The relationship, between the eigenvalue density c of the true correlation matrix, and that of the empirical correlation matrix C, was derived to show that correlations can be measured in the random part of the spectrum [11,12]. A Kolmogorov test was applied to demonstrate that the bulk of the spectrum is not in the Wishart RMT class [13]. In this paper, the authors demonstrate that the existence of factors such as an overall market effect, firm size and industry type is due to collective influence of the assets. More evidence that the RMT fit is not perfect was provided, [14], where it was shown that the dispersion of "noise" eigenvalues is inflated, indicating that the bulk of the eigenvalue spectrum contains correlations masked by measurement noise.

The behaviour of the largest eigenvalue of a cross-correlation matrix for small windows of time, has been studied [15] for the DAX and Dow Jones Industrial average Indices (DJIA). Evidence of a time-dependence between 'drawdowns' ('draw-ups') and an increase (decrease) in the largest eigenvalue was obtained, resulting in an increase of the *information entropy* [1] of the system. Similar techniques were used, [16], to investigate the dynamics between the stocks of two different markets (DAX and DJIA). In this case, two distinct eigenvalues of the cross-correlation matrix emerged, corresponding to each of the markets. By adjusting for time-zone delays, the two eigenvalues were then shown to coincide, implying that one market leads the dynamics in the other.

Equal-time cross-correlation matrices have been used, [17], to characterise dynamical changes in nonstationary multivariate time-series. It was shown that, as the synchronisation of k time series within an $M-$dimensional multivariate time series increases, this causes a repulsion between eigenstates of the correlation matrix, in which k levels participate. Through the use of artifically created time series with pre-defined correlation dynamics, it was demonstrated that there exist situations, where the relative change in eigenvalues from the lower edge of the spectrum is greater than that for the large eigenvalues, implying that information drawn from the smaller eigenvalues is highly relevant.

The technique in [17] was applied to the dynamic analysis of the eigenvalue spectrum of the equal time cross-correlation matrix of multivariate Epileptic Seizure time series, using sliding windows. The authors demonstrated that information about the correlation dynamics is visible in *both* the lower and upper eigenstates. The equal-time correlation matrix of EEG signals was further studied, [18], with a view to investigating temporal dynamics of focal onset epileptic seizures [2]. It was shown that the zero-lag correlations between multichannel EEG signals tend to decrease during the first half of a seizure and increase gradually before the seizure ends. This work was further extended to the case

[1] In information theory, the Shannon entropy or information entropy is a measure of the uncertainty associated with a random variable.

[2] A partial or focal onset seizure occurs when the discharge starts in one area of the brain and then spreads over other areas.

of *Status Epilepticus* [19], where the equal-time correlation matrix was used to assess neuronal synchronisation prior to seizure termination.

It was shown [20], for particular examples, that information about cross correlations can be found in the RMT bulk of eigenvalues and that the information extracted at the *lower* edge is statistically *more significant* than that extracted from the larger eigenvalues. The authors introduced a method of unfolding the eigenvalue level density, through the normalisation of each of the level distances by its ensemble average, and used this to calculate the corresponding individual nearest-neighbour distance. Through this unfolding those parts of the spectrum, dominated by noise, could be distinquished from those containing information about correlations. Application of this technique to multichannel EEG data showed the smallest eigenvalues to be more sensitive to detection of subtle changes in the brain dynamics than the largest.

In this paper, we examine the eigenvalue dynamics of the cross-correlation matrix from multivariate financial data. The methods used are reviewed in Section 2. In Section 3 we describe the data studied, while in Section 4 we look at the results obtained both for the empirical correlation matrix and the model correlation matrices described.

2. Methods

2.1. *Empirical Dynamics*

The equal-time cross-correlation matrix, between time series of equity returns, is calculated using a sliding window where the number of assets, N, is smaller than the window size T. Given returns $G_i(t)$, $i = 1, \ldots, N$, of a collection of equities, we define a normalised return, within each window, in order to standardise the different equity volatilities. We normalise G_i with respect to its standard deviation σ_i as follows:

$$g_i(t) = \frac{G_i(t) - \widehat{G_i(t)}}{\sigma_i} \tag{1}$$

Where σ_i is the standard deviation of G_i for assets $i = 1, \ldots, N$ and $\widehat{G_i}$ is the time average of G_i over a time window of size T.

Then the equal-time cross-correlation matrix is expressed in terms of $g_i(t)$

$$C_{ij} \equiv \langle g_i(t)\, g_j(t) \rangle \tag{2}$$

The elements of C_{ij} are limited to the domain $-1 \leq C_{ij} \leq 1$, where $C_{ij} = 1$ defines perfect positive correlation, $C_{ij} = -1$ corresponds to perfect negative correlation and $C_{ij} = 0$ corresponds to no correlation. In matrix notation, the correlation matrix can be expressed as

$$\mathbf{C} = \frac{1}{T}\mathbf{G}\mathbf{G}^{\tau} \tag{3}$$

Where $\mathbf{G}$ is an $N \times T$ matrix with elements g_{it}.

The eigenvalues λ_i and eigenvectors $\hat{\mathbf{v}}_i$ of the correlation matrix $\mathbf{C}$ are found from the following

$$\mathbf{C}\hat{\mathbf{v}}_i = \lambda_i \hat{\mathbf{v}}_i \tag{4}$$

The eigenvalues are then ordered according to size, such that $\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_N$. The sum of the diagonal elements of a matrix, (the Trace), must always remain constant under linear transformation. Thus, the sum of the eigenvalues must always equal the Trace of the original correlation matrix. Hence, if some eigenvalues increase then others must decrease, to compensate, and vice versa (*Level Repulsion*).

There are two limiting cases for the distribution of the eigenvalues [17,18]. When all of the time series are perfectly correlated, $C_i \approx 1$, the largest eigenvalue is maximised with a value equal to N, while for time series consisting of random numbers with average correlation $C_i \approx 0$, the corresponding eigenvalues are distributed around 1, (where any deviation is due to spurious random correlations).

For cases between these two extremes, the eigenvalues at the lower end of the spectrum can be much smaller than λ_{max}. To study the dynamics of each of the eigenvalues using a sliding window, we normalise each eigenvalue in time using

$$\tilde{\lambda}_i(t) = \frac{(\lambda_i - \bar{\lambda})}{\sigma^{\lambda}} \tag{5}$$

where $\bar{\lambda}$ and σ^{λ} are the mean and standard deviation of the eigenvalues over a particular reference period. This normalisation allows us to visually compare eigenvalues at both ends of the spectrum, even if their magnitudes are significantly different. The reference period, used to calculate mean and standard deviation of the eigenvalue spectrum, can be chosen to be a low volatility sub-period, (which helps to enhance the visibility of high volatility periods), or the full time period studied.

2.2. *One-factor Model*

In the *one-factor model* of stock returns, only *correlations with the market*, ρ_0, are taken into account. The spectrum of the associated correlation matrix consists of only two values, a large eigenvalue of order $(N-1)\rho_0 + 1$, associated with the market, and an $(N-1)-$fold degenerate eigenvalue of size $1 - \rho_0 < 1$. Any deviation from these values is due to the finite length of time series used to calculate the correlations. In the limit $N \to \infty$ (even for *small correlation*,i.e. $\rho \to 0$) a large eigenvalue appears, which is associated with the eigenvector $v_1 = \left(\frac{1}{\sqrt{N}}\right)(1, 1, 1 \ldots 1)$, and which dominates the correlation structure of the system.

2.3. *Market plus sectors model*

To expand the above to a "market plus sectors" model, we perturb a number of pairs N of the correlations $\rho_0 + \rho_n$, where $-1 - \rho_0 \leq \rho_n \leq 1 - \rho_0$. Additionally, we impose a constraint $\sum\limits_N \rho_n = 0$, ensuring that the average correlation of the system remains equal to ρ_0. These perturbations allow us to introduce *groups of stocks* with similar correlations, (corresponding to Market Sectors).

Using the correlation matrix from the "one-factor model" and the "market plus sectors model", we can construct correlated time series using the Cholesky decomposition A of a correlation matrix $C = AA^{\tau}$. We can then generate finite correlated time series of length T,

$$x_{it} = \sum_j A_{ij} y_{jt} \qquad t = 1, \ldots, T \tag{6}$$

where y_{jt} is a random Gaussian variable with mean zero and variance 1 at time t. Using Eqn. 2 we can then construct a correlation matrix using the simulated time series. The finite size of the time series introduces 'noise' into the system and so the empirical correlations will vary from sample to sample. This 'noise' could be reduced through the use of longer simulated time series or through averaging over a large number of time series.

In order to compare the eigenvectors from each of the Model Correlation matrices and that constructed from the equity returns time series, we use the Inverse Participation Ratio (IPR) [4,22]. The IPR allows quantification of the number of components that participate significantly in each eigenvector and tells us more about the level and nature of deviation from RMT. The IPR of the eigenvector u^k is given by $I^k \equiv \sum_{l=1}^{N} \left(u_l^k\right)^4$ and allows us to compute the inverse of the number of eigenvector components that contribute significantly to each eigenvector.

3. Data

In order to study the dynamics of the empirical correlation matrix over time, we analyse two different data sets. The first data set comprises the 384 equities of the Standard & Poors (S&P) 500 where full price data is available from January 1996 to August 2007 resulting in 2938 daily returns. The S&P 500 is an index consisting of 500 large capitalisation equities, which are predominantly from the US. In order to demonstrate that our results are not market specific, however, we examine a second data set, made up of the 49 equities of the Dow Jones Euro Stoxx 50 where full price data is available from January 2001 to August 2007 resulting in 1619 daily returns. The Dow Jones Euro Stoxx 50 is a stock index of Eurozone equities designed to provide a blue-chip representation of supersector leaders in the Eurozone.

4. Results

We analyse the eigenvalue dynamics of the correlation matrix of a subset of 100 S&P equities, chosen randomly, using a sliding window of 200 days. This subsector was chosen such that $Q = \frac{T}{N} = 2$, thus ensuring that the data would be close to non-stationary in each sliding window. Figure 1(a) shows broadly similar sample dynamics from the 5^{th}, 15^{th} and 25^{th} largest eigenvalues over each of these sliding windows. The sum of the 80 smallest eigenvalues are shown in Figure 1(b), while the dynamics of the largest eigenvalue is displayed in Figure 1(c). The level repulsion between the largest eigenvalue and the small eigenvalues is evident here, (comparing 1(b) and 1(c)), with the dynamics of the small eigenvalues contrary to those of the largest eigenvalue. As noted earlier, this is a consequence of the fact that the trace of the correlation matrix must remain constant under transformations and any change in the largest eigenvalue must be reflected by a change in one or more of the other eigenvalues. Similar results were obtained for different subsets of the S&P and also for the members of the Dow Jones Euro Stoxx 50.

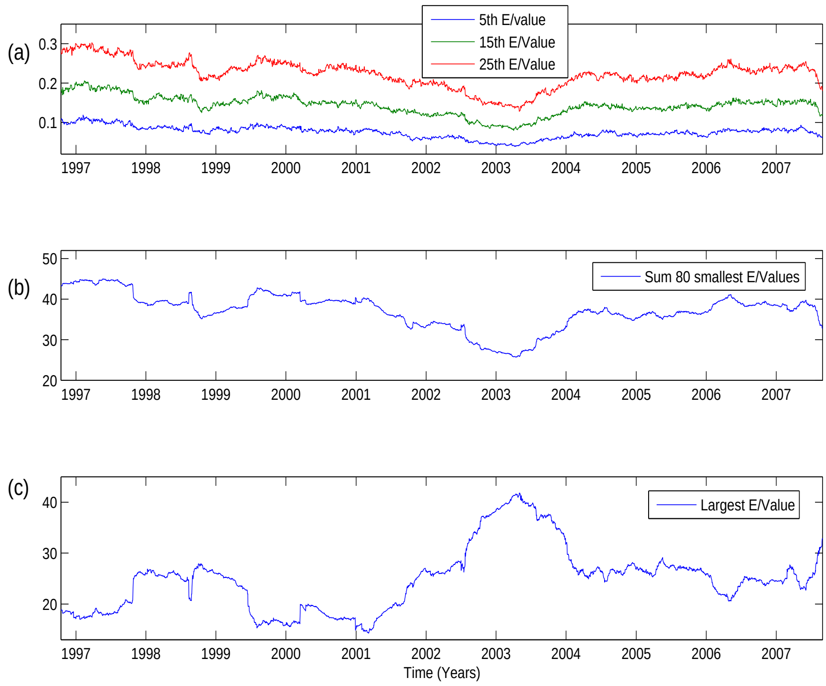

Fig. 1. Time Evolution of (a) Three small eigenvalues (b) Sum of the 80 smallest eigenvalues (c) The largest eigenvalue

4.1. *Normalised Eigenvalue Dynamics*

Using normalised eigenvalues as described above, (Eqn. 5), we performed a number of experiments to investigate the dynamics of a set of small eigenvalues versus the largest eigenvalue. The various experiments are described below:

(i) As in Section 4, the dynamics for the same subset of 100 equities are analysed using a sliding window of 200 days. The normalisation is carried out using the mean and standard deviation of each of the eigenvalues over the entire time-period. Figure 2(a) shows the value of the S&P index from 1997 to mid$-$2007.

The normalised largest eigenvalue is shown in Figure 2(b) along with the average of the 80 normalised small eigenvalues. The compensatory dynamics mentioned earlier are shown more clearly here, with the largest and average of the smallest 80 eigenvalues having opposite movements. The normalised eigenvalues for the entire eigenvalue spectrum are shown in Figure 2(c), where the colour indicates the number of standard deviations from the time average for each of the eigenvalues over time. As shown, there is very little to differentiate the dynamics of the $80-90$ or so smallest eigenvalues. In contrast, the behaviour of the largest eigenvalue is clearly opposite to that of the smaller eigenvalues. However, from the 90^{th} and subsequent eigenvalue there is a marked change in the behaviour, (Figure 2(d)), and the eigenvalue dynamics are distinctly different. This may correspond to the area outside the "Random Bulk" in RMT. Similar to [15,16], we also find evidence of an increase/decrease in the largest eigenvalue with respect to 'drawdowns'/'draw-ups'. Additionally, we find the highlighted *compensatory dynamics* of the small eigenvalues. These results were tested for various time windows and normalisation periods, and found to be more pronounced since additional features are captured and emphasised.

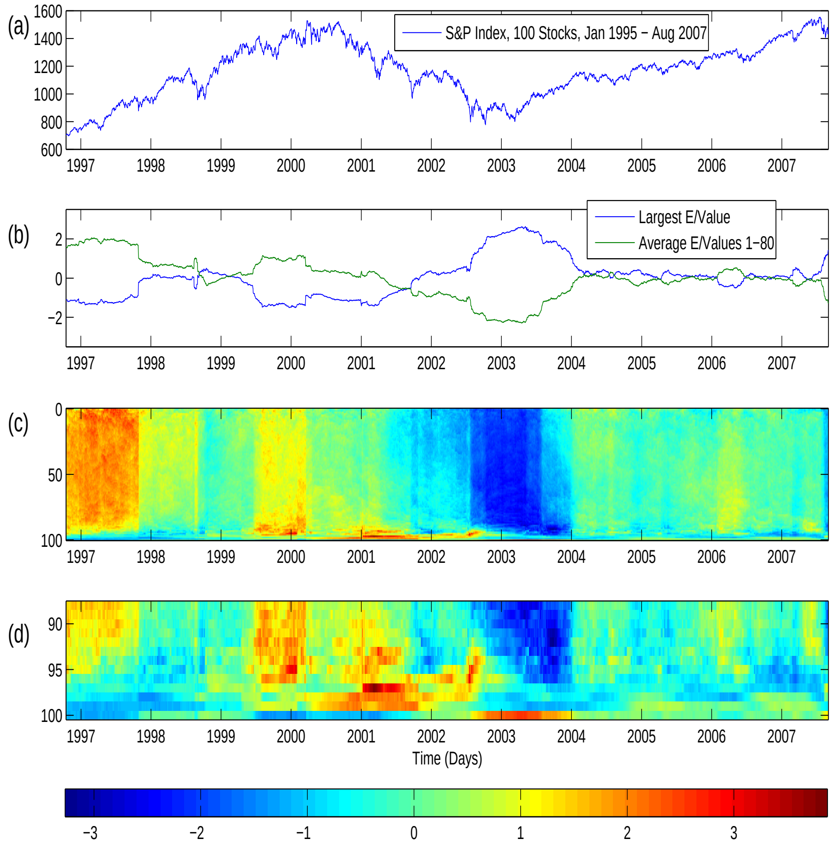

Fig. 2. (a) S&P Index (b) Normalised Largest Eigenvalue vs. Average of 80 smallest normalised eigenvalues (c) All Normalised Eigenvalues (d) Largest 12 Normalised Eigenvalues

(ii) To demonstrate the above result for a *different level of granularity*, we chose 50 equities randomly with a time window of 500 days, giving $Q = \frac{T}{N} = 10$. The results obtained, (Figure 3), are in keeping with those for $Q = 2$ earlier, with a broad-band increase (decrease) of the 40 smallest eigenvalues concurrent to a decrease (increase) of the largest eigenvalue, as required by level repulsion.

(iii) The previous examples used random subsets of the S&P universe in order to keep $Q = \frac{T}{N}$ as large as possible. To demonstrate that the above results were not sampling artifacts, we also looked at the full sample of 384 equities, (that survived the

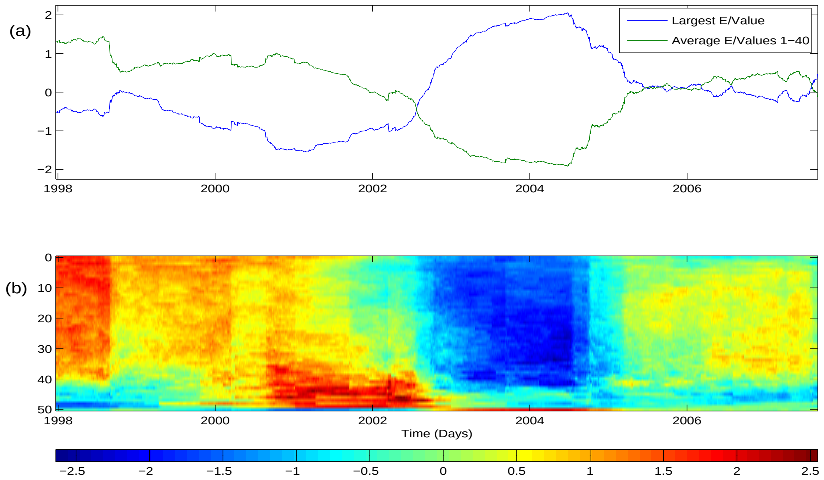

Fig. 3. (a) Normalised Largest Eigenvalue vs. Average of 40 smallest normalised eigenvalues (b) All Normalised Eigenvalues

entire 11 year period), with a time window of 500 days ($Q = 1.30$). The results, as shown in Figure 4, are similar to those above, with the majority of the small eigenvalues compensating for changes in the large eigenvalue. As indicated previously, however, there is a small band of large eigenvalues, where the behaviour is different to that of both the band of small eigenvalues and the largest eigenvalue.

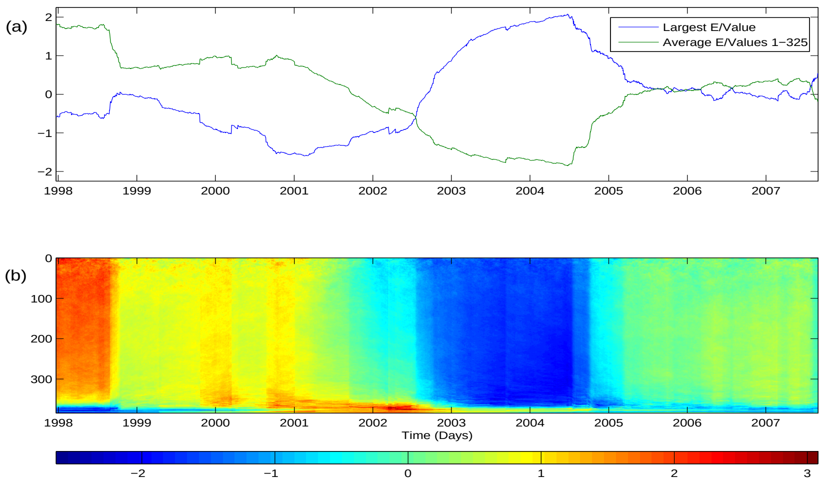

Fig. 4. (a) Normalised Largest Eigenvalue vs. Average of 325 smallest normalised eigenvalues (b) All Normalised Eigenvalues

(iv) All examples discussed so far have focused on the universe of equities from the S&P 500 that have survived since 1997. To ensure that the results obtained were not exclusive to the S&P 500, we also applied the same technique to the 49 equities of the EuroStoxx 50 index that survived from January 2001 to August 2007. The sliding window used was 200 days, such that $Q = 4.082$. The results found were again similar (Figure 5) to those found before, with a wide band of small eigenvalues “responding to” movements in the largest eigenvalues. In this case, the band of deviating large eigenvalues (ie. those which correspond to the area outside the “Random Bulk” in RMT), (Figure 5(d)), is not as marked as in the previous example. This effectively implies that equities in this index are dominated by the “Market”.

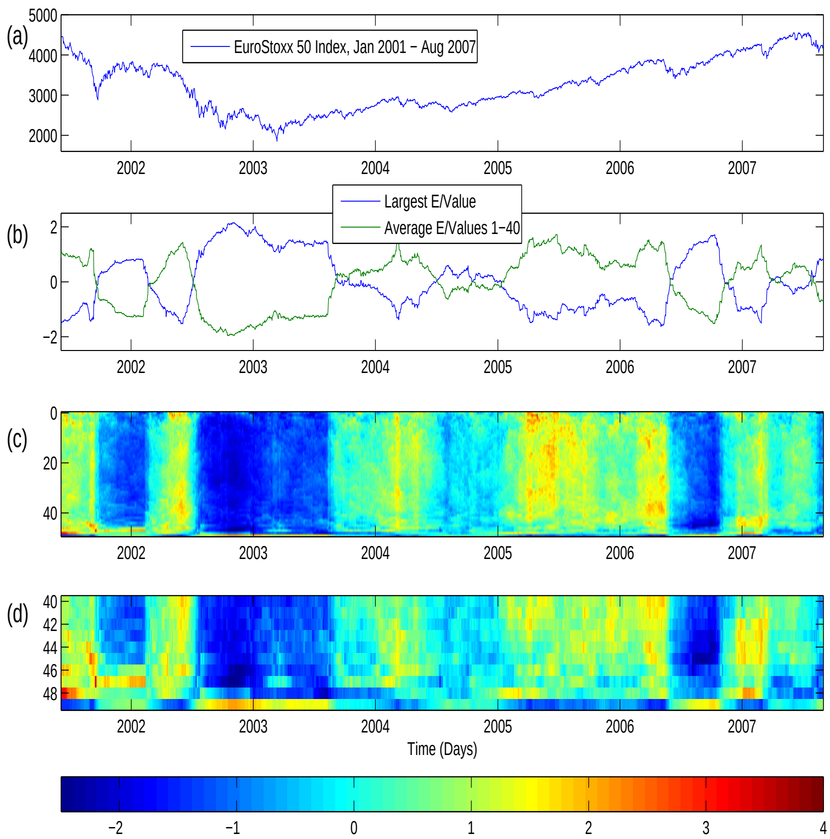

Fig. 5. (a) EuroStoxx 50 Index, Jan 2001 - Aug 2007 (b) Normalised Largest Eigenvalue vs. Average of 40 smallest normalised eigenvalues for EuroStoxx 50 (c) All Normalised Eigenvalues (d) The 9 Largest Normalised Eigenvalues

4.2. *Model Correlation Matrix*

The results, described, demonstrate that the time dependent dynamics of the small eigenvalues of the correlation matrix of stock returns move counter to those of the largest eigenvalue. Here, we show how a simple one-factor model, Section 2.2, of the correlation structure reproduces much of this behaviour. Furthermore, we show how additional features can be captured by including perturbations in this model, essentially a *"market plus sectors"* model, Section 2.3, [13,22,23].

In order to compare the empirical results, Section 4, to those of the single factor model, we first constructed a correlation matrix where each non-diagonal element was equal to the average correlation of the empirical matrix in each sliding window. We then calculated the eigenvalues of this matrix over each sliding window and normalised these as before, (Section 2). The results of the single-factor model are displayed in Figure 6 for the EuroStoxx 50 index with a sliding window of 200 days. As can be seen, the main features of the dynamics are in agreement with those of Figure 5 for the empirical data. The large eigenvalue has equal and opposite value to the average of the 40 smallest eigenvalues. As expected, there is no fluctuation across the lower eigenvalues, suggesting that the empirical features missing from the single-factor model are explained by the perturbations due to group dynamics.

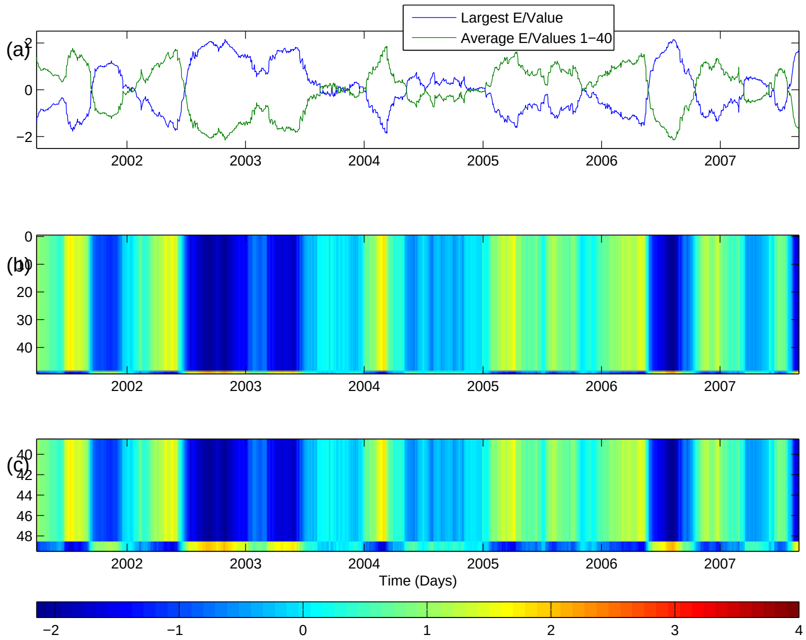

Fig. 6. (a) Normalised Largest Eigenvalue vs. Average of 40 smallest normalised eigenvalues (b) All Normalised Eigenvalues using one-factor model correlation structure

To examine the properties of the eigenvector components, we use the Inverse Participation Ratio. For the single factor model, we created a synthetic correlation matrix using

Eqn. (6), with average correlation (0.204) equal to that of the Euro Stoxx 50 over the time period studied. As shown in Figure (7), the IPR retains some of the features found for empirical data [4,22] with the IPR corresponding to the largest eigenvector having a much smaller value than the mean. This corresponds to an eigenvector to which many stocks contribute, (effectively the market eigenvector), [4,22].

In an attempt to include additional empirical features, such as the band of deviating large eigenvalues between the bulk and the largest eigenvalue, we performed one further experiment. We considered a perturbation, with two groups of stocks having correlation $\rho_0 - 0.15$ and $\rho_0 + 0.15$, and kept the average correlation at each time window the same. In this case (Figure (7)), additional features of the IPR are found. The extra group structure results in a larger IPR for the smallest eigenvalue and for the second largest eigenvalue. This is in keeping with [4] where, for empirical data, the group structure resulted in a number of small and large eigenvalues with a larger IPR than that of the bulk of eigenvalues. The large eigenvalues were shown, [4,8], to be associated with correlation information related to the group structure.

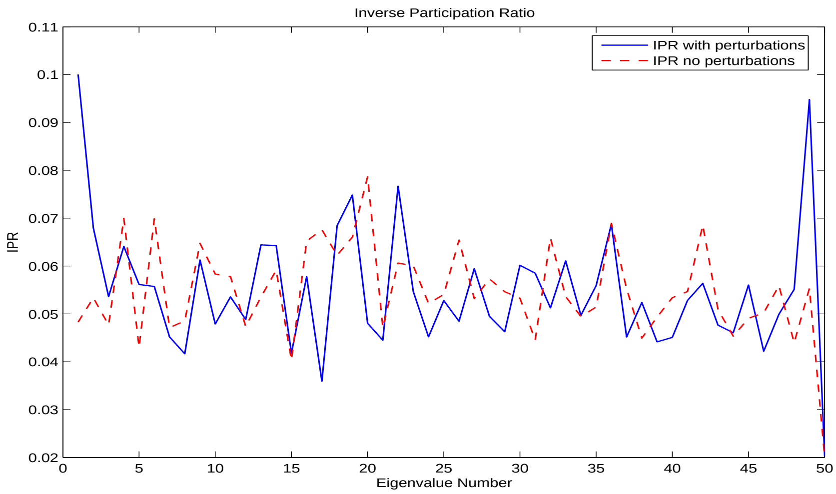

Fig. 7. (a) Normalised Largest Eigenvalue vs. Average of 325 smallest normalised eigenvalues (b) All Normalised Eigenvalues

5. Conclusions

The correlation structure of multivariate financial time series was studied by investigation of the eigenvalue spectrum of the equal-time cross-correlation matrix. By filtering the correlation matrix through the use of a sliding window we have been able to examine the behaviour of the largest eigenvalue over time. As shown graphically, Figures 2 - 5, the largest eigenvalue moves counter to that of a band of small eigenvalues, due to *level repulsion*. A decrease in the largest eigenvalue, with a corresponding increase in the small eigenvalues, corresponds to a redistribution of the correlation structure across more dimensions of the vector space spanned by the correlation matrix. Hence, additional eigenvalues are needed to explain the correlation structure in the data. Conversely, when the correlation structure is dominated by a smaller number of factors (eg. the "single-factor model" of equity returns), the number of eigenvalues needed to describe the correlation structure in the data is less. In the context of the previous work, [15,16], this means that fewer eigenvalues are needed to describe the correlation structure of 'drawdowns' than that of 'draw-ups'.

By introducing a simple one-factor model of the correlation in the system (Section 4.2) we were able to reproduce the main results of the empirical study. The compensatory dynamics, described, were clearly seen for a correlation matrix with all elements equal to the average of the empirical correlation matrix. The model was then adapted, by the addition of pertubations to the correlations, with the average correlation remaining unchanged. This "markets plus sectors" type model was then able to reproduce additional features of the empirical correlation matrix, demonstrated by the Inverse Participation Ratio (IPR). The IPR of the "markets plus sectors" model was shown to have group characteristics typically associated with Industrial Sectors, with a larger than average value for the smallest eigenvalue and for the second largest eigenvalue.

Future work includes a more detailed study of the relationship between the direction of the market and magnitude of the eigenvalues of the correlation matrix. Studying the multiscaled correlation dynamics over *different granularities* may shed some light on the different collective behaviour of traders with different strategies and time horizons. Additional analysis of high frequency data may also be useful in the characterisation of correlation dynamics, especially prior to market crashes. It would also be worthwhile to study the possible relationship between the dynamics of the small eigenvalues and additional correlation information which, according to some authors [10–14,17,20], may be hidden in the part of the eigenvalue spectrum normally classifed as noise. Similar to [17,20], this could be acheived through analysis of the relative dynamics of the small and large eigenvalues at times of extreme volatility (such as during market crashes).

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