An effective method for the determination of the locking range of an Injection-Locked Frequency Divider

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***Abstract*-The paper proposes a methodology for the determination of the locking range of an Injection-Locked Frequency Divider. The technique involves the use of the Warped Multi-time scale model and is applicable to oscillators in general. The ability to determine, in an efficient manner, the locking ranges of Injection Locked Frequency Dividers is of great importance to design engineers as ILFDs are suitable for lower-power wireless applications.**

I. INTRODUCTION

In many branches of science, the phenomenon of injection locking is of importance. This phenomenon occurs when the natural frequency of an oscillator changes to become identical to an external perturbating frequency. In wireless communications, the phenomenon has been exploited for very beneficial purposes in applications such as frequency synthesis. In the feedback loop of a frequency synthesiser, a frequency pre-scaler is employed to divide the frequency by a fixed number. Injection Locked Frequency Dividers (ILFD) consume less power than static dividers [1] and hence are preferable for low-power wireless applications. Unfortunately, the bandwidth over which locking occurs for ILFDs is limited. However, they are usually employed in LC-VCO-based Phase Locked Loops (PLL) which have a limited tuning range. Consequently, the restriction on the bandwidth of the ILFDs is not an impediment to their usefulness. However, the ability to determine it to an adequate level of accuracy is an important requirement for the electronic design industry [1-2].

The present contribution proposes a novel simulation technique for the determination of the locking range of an Injection-Locked Frequency Divider (ILFD) or any general oscillator circuit. The approach involves two options. The first involves determining an estimate of the locking based on experimental knowledge of ILFDs. The second provides a more accurate prediction if a high degree of accuracy is required.

The simulations involved in determining the locking range employ the warped multi-time scale model which is a variant of the standard multi-time scale model [3]. However, unlike the standard multi-time scale model, it is capable of handling variations in the input and local frequency of the oscillator circuit [3]. For the determination of the locking range, three time scales are considered in the warped multi-time scale model. The first time scale is for the oscillator autonomous solution. The second time-scale is for the input signal to which the oscillator circuit synchronises when locking occurs. The third time scale is for the transient evolution of the system. The use of the warped multi-time scale model enables identification of the natural frequency of the ILFD which may then be compared with the input frequency.

Section 2 will briefly describe the ILFD and give experimental results for the locking range. Section 3 will describe the warped multi-time scale model. Section 4 will present the method for the determination of an estimation of the locking range. Section 5 will then detail the accurate method for the determination of the locking range. Finally, results and a brief conclusion will be given.

II. INJECTION-LOCKED FREQUENCY DIVIDER

The ILFD under consideration is the popular topology shown in Fig. 1. A simplified circuit model is shown in Fig. 2. Equation (1) gives the governing equations for the circuit in Fig. 2:

$$\begin{aligned} C\frac{dV_C}{dt} &= I_L - f(V_C) \\ L\frac{dI_L}{dt} &= -I_L R - V_C \end{aligned} \qquad (1)$$

$f(V_C)$ is the driving point characteristic of the non-linear resistor and for the present work takes the form:

$$\begin{aligned} f(V_C) &= aV_C - bV_C^3 \\ b &= \frac{a}{V_{DD}^2} \end{aligned} \qquad (2)$$

When a signal is injected into the circuit in Fig. 1, the parameter *a* varies and this is modelled by expanding *a* about V_{GS} as follows:

$$a = \hat{a}(V_{GS} + v_{GS}) = \hat{a}(V_{GS}) + \left.\frac{d\hat{a}}{dV_{GS}}\right|_{VGS} v_{GS} \qquad (3)$$

When the expansion in (3) is inserted into (1), the result is:

$$C\frac{dV_C}{dt} = I_L - (A + daV_{in})V_C + \frac{A + daV_{in}}{V_{DD}^2}V_C^3$$
$$L\frac{dI_L}{dt} = -I_L R - V_C \qquad (4)$$

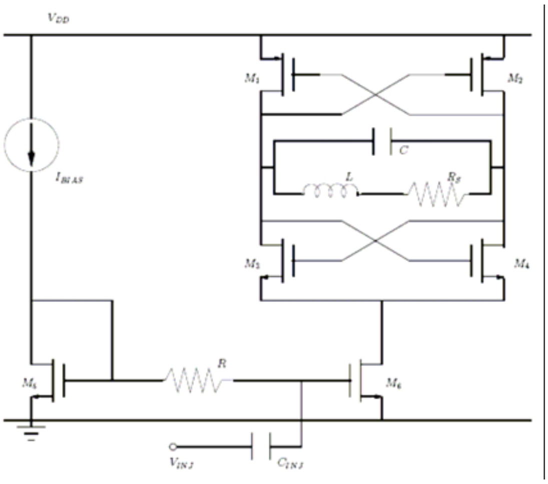

Fig. 1 *Circuit schematic of ILFD*

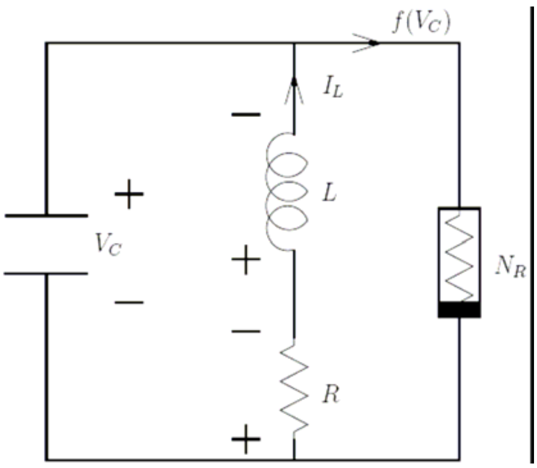

Fig. 2 *Simplified circuit model*

III. WARPED MULTI-TIME SCALE MODEL

Transient simulation of oscillators using standard numerical integration techniques requires a very long simulation time and thus, the Warped Multi-time scale model (WaMPDE) was proposed in [3] in order to speed up the computation process. In this method, the system of ordinary differential equations (ODEs) governing the oscillator behaviour is converted to a system of partial differential equations with different time axes to account for the fast and slow time scales involved in the response of the oscillator to an excitation. Furthermore, the WaMPDE explicitly involves the natural/autonomous frequency of the oscillator and this enables subsequent determination of the locking range of the oscillator.

Consider the following general ODE system:

$$\frac{dx(t)}{dt} + f(x(t)) = b(t) \qquad (5)$$

Eqn. 1 is an example of this type of system. The p+1 dimensional WaMPDE corresponding to eqn. 5 is:

$$\sum_{i=1}^{p}\left(\omega_i(\tau_{p+1})\frac{\partial\hat{x}}{\partial\tau_i}\right) + \frac{\partial\hat{x}}{\partial\tau_{p+1}} + f(\hat{x}) = b(\tau_1,\cdots\tau_{p+1}) \qquad (6)$$

$\tau_1\cdots\tau_p$ correspond to the warped time scales and τ_{p+1} is the time-scale of the original system in (5). $\hat{x}$ and $\hat{b}$ are multivariate functions of the p+1 time variables. The relationship between (5) and (6) stems from the fact that:

$$x(t) = \hat{x}(\phi_1(t),\cdots\phi_p(t),t) \qquad (7)$$

where:

$$\phi_i(t) = \int_0^t \omega_i(\tau)d\tau \qquad (8)$$

Thus, if a solution to eqn. 6 is found, a solution to eqn. 5 is automatically found.

For the determination of the locking range of the ILFD, three time-scales are considered. The autonomous solution is considered in the τ_1 time scale. The input signal is considered in the τ_2 time scale and the third time scale corresponds to real-time. Thus, for the simulations in this paper, the WaMPDE is:

$$\omega_0(\tau_3)\frac{\partial\hat{x}}{\partial\tau_1} + \omega_{inj}\frac{\partial\hat{x}}{\partial\tau_2} + \frac{\partial\hat{x}}{\partial\tau_3} + f(\hat{x}) = b(\tau_3) \qquad (9)$$

ω_0 is the oscillator natural frequency and ω_{inj} is the input frequency. The WaMPDE may be solved with time-domain methods or with a mixture of time-domain and frequency-domain methods. For this contribution, time-domain methods were employed as in [4]. The derivatives with respect to the warped variables are calculated using the five-point centred difference formula in [4]. The Backward Euler Method is used for the transient evolution in the τ_3 time-scale. The resultant nonlinear algebraic equations are solved using Newton's Method [6].

IV. ESTIMATION OF THE LOCKING RANGE

To determine an estimate of the locking range of the ILFD, some prior knowledge of the behaviour of the ILFD with respect to a varying input frequency is employed. This can be obtained from experimentation. Repeated performance of such experiments is time-consuming. However, for this estimation technique, all that is required is a general knowledge of the type of behaviour of the ILFD and consequently, an experimental setup is not required each time that the method is applied. From the experimental results on an ILFD performed in [5] and as shown in Fig. 3, it is noted that the relationship between ω_{inj}/ω_0 and ω_{inj} is approximately linear between locking intervals (the ILFD locks at multiples of its natural frequency – the *n*th locking range is when the ILFD locks with $\omega_{inj}/\omega_0 = n$). During the locking intervals, the slope is obviously zero. Consequently, two simulations are performed with ω_{inj} at values known *not* to lock the ILFD and to be below the lower limit of the particular *n*th locking range. From this, an estimate of the start of the *n*th locking range can be obtained. For example, the start of the divide-by-one locking range is when $\omega_{inj}/\omega_0 = 1$. Thus:

$$\omega_{start} = \omega_{inj\_1} + \frac{1 - \left(\frac{\omega_{inj}}{\omega_0}\right)_1}{m_{below}} \qquad (10)$$

where *m* is the slope of the line connecting the two points determined from simulations. $(\omega_{inj}/\omega_0)_1$ is the value of one of the simulations and ω_{inj_1} is the corresponding input frequency.

Similarly, an estimate for the upper limit of the locking range can be determined by performing two simulations above the upper limit in regions known not to lock the ILFD.

$$\omega_{end} = \omega_{inj\_1} + \frac{1 - \left(\frac{\omega_{inj}}{\omega_0}\right)_1}{m_{above}} \qquad (11)$$

For each divide-by-*n* locking zone, a similar procedure would be performed to obtain an estimate of the corresponding locking range.

V. Accurate Determination of the Locking Range

In [4], a procedure was detailed for the determination of the Locking Range of an oscillator. The procedure is summarised as follows: An input frequency known NOT to lock the ILFD but close to the locking range limit is injected into the ILFD. Eqn. 9 is then solved until steady-state conditions are obtained. At each time step, the Jacobian matrix employed in Newton's method is noted. If the column of the Jacobian that corresponds to the local frequency is near–zero, then the implication is that the system has become almost independent of the local frequency and only forced oscillations are present. Hence, locking is deemed to have occurred.

VI. Results

Fig. 3 shows the experimental results obtained in [5] for the relationship between ω_{inj}/ω_0 and ω_{inj}. With the techniques described in the previous sections, the locking range for $\omega_{inj}/\omega_0 = 2$ is obtained as [0.87MHz 1.03MHz]. Similarly, the locking range for $\omega_{inj}/\omega_0 = 4$ is [1.69MHz 1.89MHz]. This shows the efficacy of the approach.

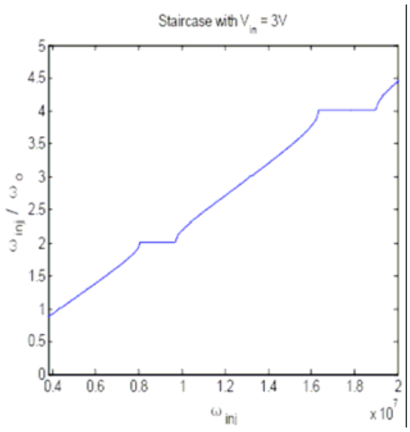

Fig. 3 *Experimental results*

VII. Conclusions

The paper has presented a simulation strategy for the determination of the locking range of an ILFD. The strategy involves the use of the WaMPDE. Firstly, an estimate of the locking ranges may be obtained using linear extrapolation. Secondly, a more accurate technique for fine-tuning the estimate is proposed. Results confirm the efficacy of the approach.

Computer simulation for the determination of the locking range is advantageous in avoiding the need for time-consuming experiments. It also greatly aids in design work involving ILFDs and their use as lower power frequency dividers in PLLs for wireless systems.

Acknowledgment

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