

DETECTION THRESHOLDING USING MUTUAL INFORMATION

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Abstract: In this paper, we introduce a novel non-parametric thresholding method that we term *Mutual-Information Thresholding*. In our approach, we choose the two detection thresholds for two input signals such that the mutual information between the thresholded signals is maximised. Two efficient algorithms implementing our idea are presented: one using dynamic programming to fully explore the quantised search space and the other method using the Simplex algorithm to perform gradient ascent to significantly speed up the search, under the assumption of surface convexity. We demonstrate the effectiveness of our approach in foreground detection (using multi-modal data) and as a component in a person detection system.

1 INTRODUCTION

The selection of thresholds is an important task in computer vision and detection systems. A threshold set too high will result in many missed detections; set too low, there will be many false positives. A fixed threshold may not perform well if the properties of the scene or environment change. For example, the same threshold is unlikely to be optimised for both daytime and night time scenes. By dynamically adapting the threshold to cater for different scenarios, these limitations can be addressed.

Research on dynamic (or adaptive) thresholding is extensive. The most common approach is to observe the signal's properties and to determine the best threshold to suit these properties. Signal histogram based methods have generated much interest (Otsu, 1979) (Kapur et al., 1985) (Rosin, 2001). The spatial distribution of the signal and noise has also been used (Rosin, 1998). Another similar approach is to perform a clustering of signal values, for example, using K-means (Duda et al., 2001), and to choose a threshold to separate some of the clusters. Our approach is different in that we do not observe the properties of a single signal, but observe how the choice of threshold will affect its relationship with another signal.

Mutual information has been used in computer vision and machine learning for various applications, including data alignment (Viola, 1995), particularly

in medical imaging (Pluim et al., 2003). The fusion of object detector outputs (Kruppa and Schiele, 2001) and feature selection for classifier training (Peng et al., 2005) are also applications where mutual information has proven useful.

In this paper, we introduce a novel non-parametric thresholding method that we term *Mutual-Information Thresholding*. In our approach, the two detection thresholds for two input signals are selected so that the mutual information between the thresholded signals is maximised. This encourages high agreement between detectors, as well as high information content. We describe two efficient implementations of this approach: one using dynamic programming to perform a full-search on the threshold-pair-space, and another more efficient approach using the Simplex (Nelder and Mead, 1965) gradient ascent to find the optimum solution, with the assumption of surface convexity.

The paper is organised as follows. We introduce our thresholding algorithm in section 2 and provide two efficient implementations in section 3. Section 4 shows the results of using our approach for foreground detection in multi-modal video sequences and on pedestrian detection in thermal infrared images. We conclude in section 5 with a summary of the paper and note some potential areas for future research.

2 PROPOSED ALGORITHM

There are generally two ways in which different data sources are combined. One approach is to create a new data representation, providing a better platform from which to perform analysis. Examples of this include linear combinations of the data, fusion using the max or min operator, or other non-linear combinations. The other common approach is that the analysis (such as thresholding) is performed separately on both sources of data and results are subsequently combined (using a binary operator, such as AND or OR, for example). Our novel method is to perform the analysis on both sources of data simultaneously and to use information from each source to assist the analysis of the other. In this way, we obtain results from two separate sources, but enhanced by each other. We threshold two signals, choosing the thresholds so that the mutual information between the two thresholded signals is maximised.

Formally, we describe our algorithm as follows. We define a *detection score* as a confidence measure that indicates the presence or absence of an event when it has a high or low value respectively. Given two sets of detection scores, X and Y , with $X = \{x_1, x_2, \dots, x_N\}$ and $Y = \{y_1, y_2, \dots, y_N\}$, that are aligned (spatially and temporally), we can choose thresholds, T_X and T_Y , to decide whether the event was present at a particular point, according to each set. By thresholding each set, we obtain the event detection sets, X' and Y' , with $X' = \{x'_1, x'_2, \dots, x'_N\}$ and $Y' = \{y'_1, y'_2, \dots, y'_N\}$.

$$x'_i = \begin{cases} 1 & \text{if } x_i \geq T_X \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$y'_i = \begin{cases} 1 & \text{if } y_i \geq T_Y \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

These thresholds, T_X and T_Y , are chosen so as to maximise the mutual information between the distributions of X' and Y' , expressed as

$$I(X; Y) = \sum_{u \in \{0,1\}} \sum_{v \in \{0,1\}} p_{xy}(u, v) \log \frac{p_{xy}(u, v)}{p_x(u)p_y(v)} \quad (3)$$

where $p_{xy}(u, v)$ is the probability that $x'_i = u$ and $y'_i = v$, $p_x(u)$ is probability that $x'_i = u$ and $p_y(v)$ is the probability that $y'_i = v$. In most applications, these probabilities are easily computed by counting occurrences and dividing by N .

Choosing the thresholds in this way leads to two desirable benefits. Firstly, it encourages agreement between the two detection sets, so that they often agree on whether the event has been detected or not. Secondly, it leads to high information content (or entropy). Without this constraint, agreement could

be maximised by setting the thresholds very high (or very low) but the detectors would always return the same answer, regardless of the data they are analysing.

2.1 Fusion

After thresholding, one is left with two binary maps. If a single map is required, these results need to be fused in some way to obtain the final decision for each event. One method is to use a binary operator, such as AND or OR, to combine the maps. An approach which is more robust against noise is to use the spatial information to determine the local support of each event. Support can be defined, for example, as the number of neighbouring events that have the same value as the central event. If the maps disagree on a detection result, the result with the greater support can be used. This is very effective at removing isolated noise. If the support values are equal, this could be an example of an object which is undetectable in one modality, such as a room-temperature bag using thermal infrared. Depending on the application, this disagreement could provide additional semantic knowledge.

3 EFFICIENT IMPLEMENTATION

Every pair of thresholds used on two signals will provide a corresponding mutual information (MI) value. By computing the MI value for every pair of thresholds, a *MI surface* is obtained. In this section, we present two methods to maximise the MI value. The first method is to use dynamic programming to compute the entire MI surface using all pairs of thresholds (chosen from two discrete sets). The second method is to use the simplex algorithm and perform gradient ascent to find the maximum MI value, under the assumption of surface convexity.

3.1 Full Surface Mapping

A brute-force approach to computing the MI surface involves iterating over all pairs of thresholds (chosen from two discrete sets), using them to threshold both signals, then computing the MI between the thresholded signals. If T_c thresholds are tried for each signal, this results in T_c^2 pairs and a computation in the order of $O(T_c^2 N)$, where N is signal size (e.g. the number of pixels in an image). The dynamic programming algorithm we describe achieves the same results in time $O(T_c^2 + N)$.

Firstly, we denote $A = \{a_1, a_2, \dots, a_P\}$ as the set of thresholds we wish to evaluate for the first signal and $B = \{b_1, b_2, \dots, b_Q\}$ as the set of thresholds we wish to evaluate for the second signal. Next, we note that equation (3) requires the four values for $p_{xy}(u, v)$, with $u, v \in \{0, 1\}$. $p_x(u)$ and $p_y(v)$ can be obtained from these values (e.g. $p_x(1) = p_{xy}(1, 0) + p_{xy}(1, 1)$). Each of these four values are computed by counting the number of occurrences where $x'_i = u$ and $y'_i = v$, then dividing by the total number of values, N . Therefore, we wish to compute these four counts for each pair of thresholds we wish to evaluate. We denote the counts as $C_{u,v}(a_i, b_j)$, which equals the number of occurrences where $x'_i = u$ and $y'_i = v$, when the thresholds are set at $T_X = a_i$ and $T_Y = b_j$. Initially the counts are all set to zero. For each data point we have the values x_k and y_k . From these values, we can deduce that $C_{0,0}(a_i, b_j)$ will be increased by one when both $a_i > x_k$ and $b_j > y_k$. Similarly, $C_{0,1}(a_i, b_j)$ will be increased by one when both $a_i > x_k$ and $b_j \leq y_k$. Count maps $C_{1,0}$ and $C_{1,1}$ have similar rules. For each data point, we could increase the counters in each map by iterating over all thresholds that should be increased. A faster method is to store markers at the positions in the map where the count increases and integrate afterwards. This is a similar, complementary technique to the standard dynamic programming method used in (Viola et al., 2003) to quickly find the sum of all pixels in a rectangular area of an image. The pseudo-code describing how to update the count maps for a data-point is shown in figure 1. Finally, we integrate the counts horizontally, as follows:

$C_{u,v}(a_i, b_j) \leftarrow C_{u,v}(a_i, b_j) + C_{u,v}(a_{i-1}, b_j)$
and then vertically,

$C_{u,v}(a_i, b_j) \leftarrow C_{u,v}(a_i, b_j) + C_{u,v}(a_i, b_{j-1})$
This array now stores, at location $C_{u,v}(a_i, b_j)$, the number of occurrences where $x'_k = u$ and $y'_k = v$, when the thresholds are set at $T_X = a_i$ and $T_Y = b_j$. Using the obtained values, this approach can be used to compute the entire MI surface using equation (3).

3.2 Simplex Maximum Search

Although the MI surface is not guaranteed to be convex, strong convexity was present in the vast majority of types of data we have investigated. Any gradient ascent method will be very computationally efficient, compared to a full search, even using the above dynamic programming strategy. Using a gradient ascent approach (such as the Simplex algorithm) also has the advantage that the thresholds do not need to be quantised into discrete values. Any full-search approach will require a finite set of pairs of thresholds, therefore demanding a quantisation of the values. This means that the Simplex search finds a more precise optimum

<p>Given: data point (x_k, y_k) Find largest threshold a_i such that $a_i \leq x_k$ Find largest threshold b_j such that $b_j \leq y_k$ $C_{1,1}(a_1, b_1) ++$ $C_{1,1}(a_{i+1}, b_1) --$ $C_{1,1}(a_1, b_{j+1}) --$ $C_{1,1}(a_{i+1}, b_{j+1}) ++$ $C_{1,0}(a_1, b_{j+1}) ++$ $C_{1,0}(a_{i+1}, b_{j+1}) --$ $C_{0,1}(a_{i+1}, b_1) ++$ $C_{0,1}(a_{i+1}, b_{j+1}) --$ $C_{0,0}(a_{i+1}, b_{j+1}) ++$</p>

Figure 1: Pseudocode for algorithm in subsection 3.1

solution. Simplex (or another gradient ascent method) can also be used efficiently for higher dimensional thresholding. For example, if we wished to choose P thresholds that would maximise the mutual information between P thresholded signals, a full-search would usually be unfeasible for $P > 2$.

3.2.1 Initialisation and Scale

In order to use Simplex, the initial position and simplex size needs to be specified. The choice of these parameters may depend on the application. We propose to initialise Simplex in the following manner, as it was deemed suitable for our target application of video processing. In the first two video frames, a full search is performed, using as fine a quantisation as is possible within the time constraints. The thresholds found using the full search can be used to initialise the Simplex search in subsequent frames (i.e. The thresholds found in the previous frame are used as the starting position for the current frame). The simplex size can be determined by setting it to be a fraction (e.g. 10%) of the change in thresholds between the first two frames. This size can be left fixed or adapted to minimise convergence time. Alternatively, multiple initialisation positions and scales can be evaluated to choose the one that provides the greatest MI value.

3.2.2 Convexity Assumption

If there are multiple peaks in the MI surface, simplex will not be guaranteed to find the global maximum. However, by initialising the simplex using the thresholds of the previous frame, the temporal coherence of the thresholds is enforced, rather than tolerating the thresholds jumping between two similarly MI valued peaks. We also found that multiple peaks were only likely to occur in two scenarios: either there

was a correlation between the detectors false positives/negatives or the signals did not share much mutual information, in which case the peaks were caused by noise.

3.2.3 Efficiency Analysis

In order to gauge how efficient the gradient ascent approach is compared to the full-search, we calculated the number of iterations required to converge to the correct foreground-detection thresholds for each of 200 frames in a multimodal (thermal infrared and visible spectrum) video sequence. We used a median background image for both the visible and infrared sequences. We initialised our simplex at 10 different scales. In only two tests (out of 2000) did it converge to a sub-optimum solution. This occurred at the two smallest scales. We found that larger scales, in general, required more iterations to converge, but were more likely to converge to a more precise solution. The average number of iterations to convergence was 26.72. When compared to a full-search, using 256 thresholds for each signal, the Simplex method is over 2400 times faster.

4 EXPERIMENTAL RESULTS

4.1 Foreground Detection

To test our algorithm, we used it to choose thresholds for foreground detection for multi-modal (thermal infrared and visible spectrum) video data. The surveillance-type video was captured using the a joint IR-Visible camera rig (Ó Conaire et al., 2005). We used the non-parametric background model described in (Elgammal et al., 2000) to separately model the colour and thermal background of the scene. For each pixel, the models each return the probability that the pixel belongs to the background. Since we used a linear quantisation of the threshold space, we got better resolution by using the negative logarithm of the probability. Specifically, we used $\min(-\log(p), 255)$ in the foreground detection map for each pixel, where p is the background probability. This spread out the detection values (similar to histogram equalisation), so that they were not all clumped into one bin.

Our tests were run on three multi-modal sequences of approximately 850 frames each. Two were daytime scenes and one was captured at night. In order to evaluate our approach to thresholding, we compare the thresholds produced by our method to those produced by Kapur thresholding (Kapur et al., 1985). Kapur et al. also used an information theoretic approach to thresholding. Using the signal’s histogram, their approach was to explain positive and negative detections

as two different signals and choose the threshold that would maximise the sum of the two-class entropies. In a comparison of thresholding methods (Rosin and Ioannidis, 2003), Kapur thresholding was determined to have the best all-round performance. The results of our experiments are shown in figure 2.

In the daytime scenes, there is strong mutual information and the results are good. The Kapur thresholds behave in exactly the opposite way to our approach. While the Kapur threshold is very stable in the visible spectrum, the MI threshold varies significantly. On the other hand, the Kapur threshold is very unstable in the infrared spectrum, the MI threshold is very stable. Our method seems to perform counter intuitively, since the thermal infrared images are far noisier than the visible spectrum. However, if one imagines two well separated distributions, as is the case when there is a high signal-to-noise ratio, then there is a wide range of thresholds that would give very good performance. In a noisy signal, the noise and signal are not as well separated, so there is only a very narrow band of thresholds that give the correct separation. This is why our method has a very stable threshold for the infrared images, as there is only a very narrow range of values where the infrared agrees with the visible spectrum. The visible spectrum threshold, on the other hand, can vary a lot without causing any performance degradation, since the noise is so low.

In the night time scene, there is very little mutual information between the visible and infrared foreground maps. Pedestrians are practically undetectable in visible spectrum images. This leads to a low value at the MI surface peak and poor thresholds for both modalities. The MI value itself can be used as a quality measure to determine the reliability of the thresholds returned. However, the mutual *information* is dependant on how much foreground is present, so we therefore considered a more robust quality measure that takes the foreground size into account. If we compute f , defined as the fraction of all pixels that both maps agree is foreground, then the highest possible MI value is $M_{max} = -f \log(f) - (1-f) \log(1-f)$. By dividing the obtained MI score by M_{max} , we obtain a quality (or reliability) measure of the returned thresholds. This quality score was computed for all sequences and is shown in figure 2(d).

Future work will involve determining how to cater for scenarios where the threshold quality score is low. This scenario could mean that one or both signals are performing very poorly (such as the visible spectrum in nighttime scenes), or that there is no mutual information to utilise (such as when there are no objects or people in the scene). One approach could be to revert to using a single-band thresholding method for each signal (such as Kapur). Another approach might be to use the motion information in each of the modalities.

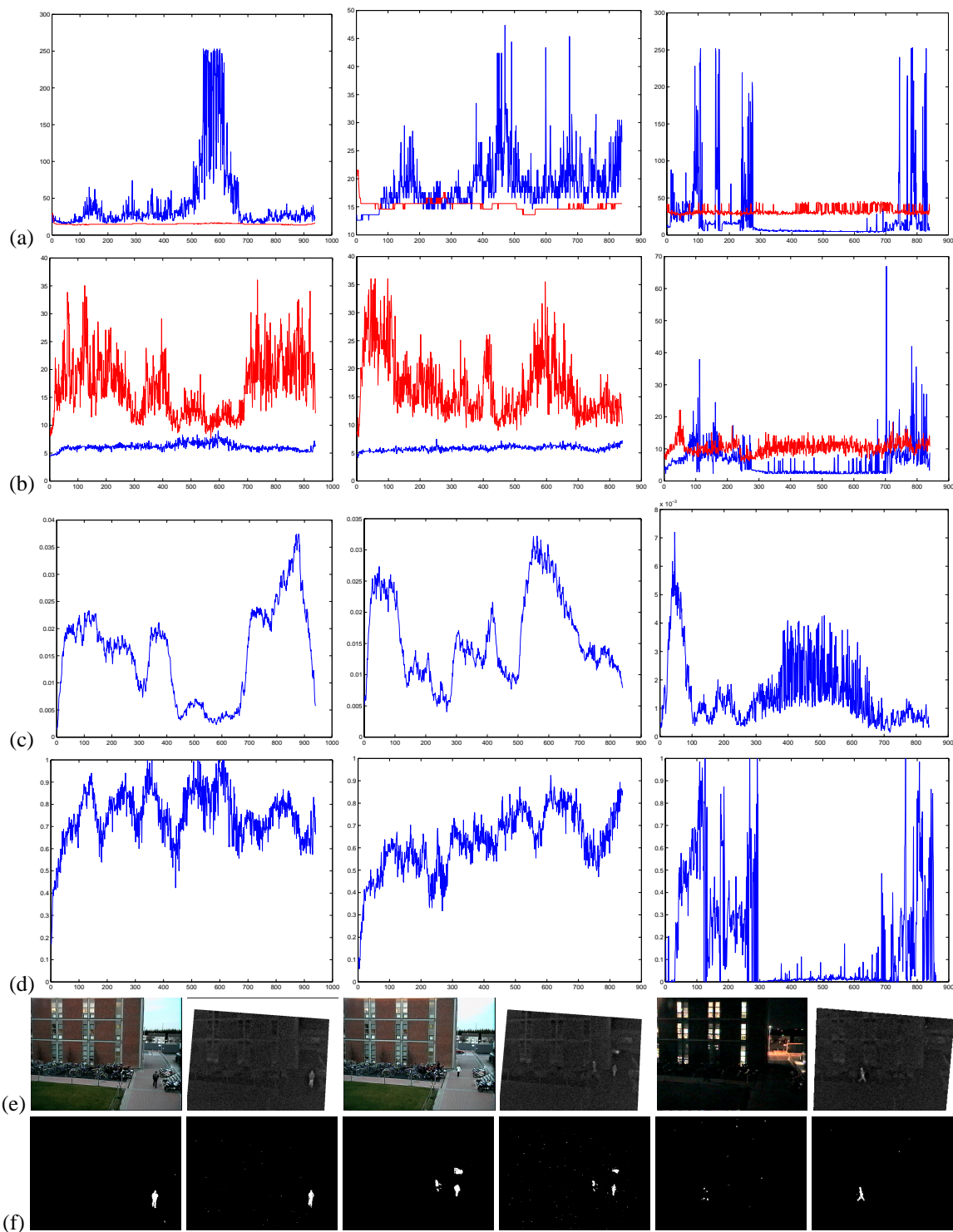


Figure 2: Comparison of our method to Kapur thresholding. Left and Centre columns are from daytime sequences. Right column is from a night-time sequence. Rows correspond to: thresholds for (a)Visible Spectrum and (b)Infrared, (c)Mutual Information, (d)Threshold Quality Measure, (e)Example frames from each sequence, (f)Example thresholded images using our method. In rows (a) and (b), the Kapur thresholds are shown in red, our method's thresholds are in blue.

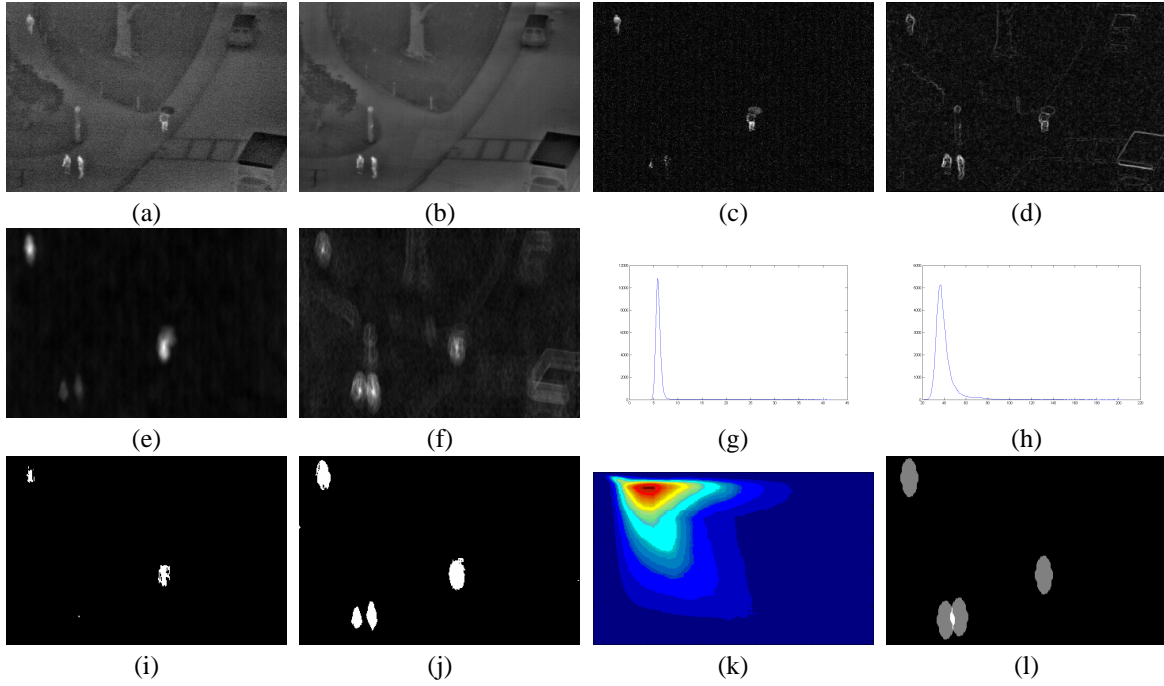


Figure 3: Person-detection example: (a)Current image, (b)Background image, (c)Background difference, (d)Image edges, (e)Silhouette detection map, (f)Contour detection map, (g)Histogram of (e), (h)Histogram of (f), (i)Kapur thresholded result, (j)Our method, (k)Mutual information surface, (l)Detected People

4.2 Person Detection

To further test our algorithm, we incorporated it into a person detection system and used the OSU Thermal Pedestrian Database from the OTCBVS Benchmark Dataset (Davis and Keck, 2005) to evaluate performance. The database contains images of pedestrians taken with a thermal infrared camera in a wide variety of environments. Since our goal was to evaluate the thresholding component of the system, the other components were chosen to be as simplistic as possible.

The system worked as follows. First, the median background image was computed. Then for each image, two detectors were used: one based on pedestrian contour and the other based on silhouette. The contour detection map was obtained by convolving the pedestrian contour template with the Sobel edges of the image. The silhouette detection map was obtained by convolving the pedestrian silhouette template with the absolute difference image between the current image and the background image. Thresholds for these maps were obtained using our mutual information thresholding algorithm (subsection 3.1). Pedestrian regions were determined as all pixels that had above threshold values in both maps. Next, each local maxima in the contour detection map within these regions was paired with the closest local maxima in the silhouette detection map within these regions. Maxima

in the silhouette detection map were then paired with the closest maxima in the contour detection map. Person candidates corresponded to each pair of maxima, from the two separate maps, that were both paired to each other (i.e. they were both closest to each other). Candidates were then evaluated according to the minimum description length principle, in respect to how much of the pedestrian regions they could explain. We use a pedestrian candidate template to evaluate the fitness of each candidate by calculating the maximum number of pedestrian-region pixels it overlaps with, when centred on either maxima of the candidate. The best candidate is considered a ‘true’ person and the pedestrian region pixels it overlaps are removed. This process continues until there are no remaining candidates, or no candidate can explain more than a predefined number of pixels (which was set at one tenth of the template size).

We used the dynamic-programming full-search thresholding algorithm and it performed well for almost all images. For some images, there were two peaks in the MI surface. We speculate that this extra peak was due to the correlation between the noise in both detection maps, since they were both derived from the same image. This peak was usually smaller than the correct peak but it was occasionally greater. We catered for this scenario by evaluating all local maxima in the surface and evaluating them in order

Table 1: The results of our pedestrian detection system on the OTCBVS database are shown below

Sequence	People	Precision	Recall
1	91	0.95	0.98
2	100	0.95	0.98
3	101	0.87	1.00
4	109	0.94	1.00
5	101	0.92	0.96
6	97	0.98	1.00
7	94	0.93	0.99
8	99	0.97	0.99
9	95	1.00	1.00
10	97	0.92	0.98
Total	984	0.95	0.99

of descending MI score. We discarded peaks whose thresholds produced binary maps with very high Euler numbers (an Euler number of a binary image is the number of regions minus the number of holes and can be calculated quickly using local pixel information. A high value indicates high noise). An example of person detection is shown in figure 3. In this difficult example, the two people, in the bottom left of the image, have been standing in the same spot for the entire sequence, so have been included in the background image. However, the motion of the people leaves an impression on the difference image and hence, on the silhouette based detector map. Our method causes the silhouette threshold to drop so that it agrees with the strong detection in the contour-based detection map. Kapur, on the other hand, sets the two thresholds independently and therefore fails to detect all the people. The results of our system are shown in table 1. They are comparable to those obtained in (Davis and Keck, 2005).

5 CONCLUSION AND FUTURE WORK

In this paper, we introduced a novel non-parametric thresholding method that chooses two detection thresholds for two input signals so that the mutual information between the thresholded signals is maximised. We described two efficient implementations of our algorithm using dynamic programming for a full-search and Simplex gradient ascent for a faster search with the assumption of surface convexity. We evaluated our method by comparing it to a standard non-parametric thresholding algorithm using multi-modal video sequences. We also incorporated our method into a person detection system and achieved good results using the publicly available OTCBVS pedestrian database.

Our thresholding method works on aligned data so can be used for local, as well as global thresholding. It can also be used to threshold space-time slices, such as groups of video frames. In these scenarios, the window size is an important parameter: too small and it may be sensitive to noise, too large and there is a chance the signal properties have changed and a global threshold would not be appropriate. Investigating how the window sizes should be set automatically is an interesting area of further work.

Determining the types of data that can be used with our method is another area for future studies. Sources that are completely independent do not share any mutual information and therefore are not suitable. On the other hand, data sources that are linearly dependent will produce thresholds equal to the median data values, as this maximises their mutual information. Independence in the noise of both sources would seem an important factor to ensure that good thresholds are produced. The use of derivatives, such as edges, as a second data source to select thresholds, has proven useful, although it may violate the noise independence criterion. Similarly, using two sources of data that come from the same sensor (the red and green colour bands, for example), may also violate this criterion and produce multiple peaks in the MI surface. In small-scale experiments for foreground detection, using a combination of two colour bands, or using the edges of the absolute difference map as a second source, our method produced good thresholds, so further testing is required to evaluate when these sources might fail.

The results of our pedestrian detection system were encouraging. However, the OTCBVS pedestrian database does not contain much clutter, so a future system will be tested on more difficult pedestrian data, using multi-modal data we have captured.

Currently, our method does not consider spatial information or the proximity of pixels when choosing the thresholds. Incorporating this information into our method is another avenue of research to consider. For example, the two parameters (low and high thresholds) for hysteresis segmentation could be selected by maximising the MI between the resulting segmentation and another source of data.

Finally, using this method on three or more sources of data is another area for future investigation. The quality measure we developed gives an estimate of the reliability of the results and hence, this might be used to make a system more robust against the failure of one or more components, if it can quickly detect unreliability in the data sources. The combination of three or more sources provides many interesting challenges, such as whether they should all be combined simultaneously, or whether a pair-wise combination, using the quality values returned, provides better performance.

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