

# **Overcoming Laser Diode Nonlinearity Issues in Multi-Channel Radio over Fiber Systems**

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## **Abstract**

The authors demonstrate how external light injection into a directly modulated laser diode may be used to enhance the performance of a multi-channel radio over fiber system operating at a frequency of 6 GHz. Performance improvements of up to 2dB were achieved by linearisation of the lasers modulation response. To verify the experimental work a simulation of the complete system was carried out using Matlab. Good correlation was observed between experimental and simulated results.

## **Introduction**

Broadband wireless networks of the future will probably use a radio over fiber architecture to provide wireless connectivity to their users. In this architecture, data signals are generated at a central station (CS), modulated onto an optical carrier, sent over optical fiber to many base stations (BSs), and then transmitted over the air to users. This type of system is extremely cost efficient because it reduces the complexity of the many BSs which must be deployed by having most of the complex processing equipment at the CS [1-3]. It is expected these broadband hybrid radio/fiber systems will divide the available radio spectrum into a number of channels for broadcast, and that the RF bands to be used will be higher than the 2.4 GHz RF band to be used for 3G systems. Taking these two points into account it is clear that a major problem that may be encountered in these networks will be nonlinearity problems in the various devices used. In particular, in systems that use direct modulation of the laser transmitter with the multi-channel RF signal, the dynamic non-linearity of the laser diode may impose serious limitations on the system performance due to inter-modulation distortion (IMD) effects [4,5]. One method of reducing the non-linearity of laser diodes is the use of external injection into the directly modulated laser from a second laser source. Under external injection

conditions the relaxation frequency of the laser may be increased significantly [5-12], and the modulation response at lower frequencies can be made significantly more linear than that without external injection [5,13]. This helps to greatly reduce IMD problems. The exact alteration in the modulation response of the laser under external injection conditions is dependent on both the injected power and the detuning between the master and slave laser, and by varying these two parameters it is feasible to optimise the lasers modulation response for different applications [14].

In terms of work that has been carried out on complete systems, it has been shown how the performance of a hybrid radio/fiber system based on a directly modulated laser, and operating in an RF band beyond the bandwidth of the free running laser, can be greatly improved by using external injection [9,10]. In this case the greatly improved system performance is due simply to the enhanced modulation response of the laser at the RF band employed in the hybrid system. However the non-linear response of the laser with external injection may still cause problems for multi-carrier radio-over-fiber systems. To our knowledge there has been no work undertaken to examine how external light injection may be used to improve the performance of a practical multi-carrier radio-over-fiber system by linearising the laser's modulation response around the operating RF band.

In this paper we examine how the external light injection into a directly modulated laser transmitter may be used to improve the performance of a multi-carrier hybrid radio/fiber system by making the modulation response of the device more linear around the RF band of operation. It is important to note that the experimental work is set-up such that the relative modulation response of the laser with and without external injection is the same, thus the measured improvement in system performance

is simply due to the improved linearity of the directly modulated laser transmitter. The experimental set-up employs a multi-carrier RF signal (with each channel having a data rate of 10 Mbit/s) operating around 6GHz, which is a possible operating band for next generation wireless systems. Our results show an improvement in system performance of greater than 2 dB by linearization of the laser's modulation response using external light injection. To verify our results we have also developed a complete working model of the system using MATLAB, and good correlation has been achieved between the experimental and theoretical results.

### **Experimental Set-up**

The laser used in this work was an NEL 1550nm DFB laser diode. The device has a threshold current of 27mA and an intrinsic modulation bandwidth of around 8GHz when biased at 40 mA. As explained in the introduction, external light injection into the laser diode may be employed to significantly alter the modulation response of the device such that modulation bandwidth is increased, and the response at lower frequencies becomes more linear. Figure 1 shows the change in modulation bandwidth that can be achieved with an external injection level of 3 mW from a tuneable external cavity laser (ECL). In this case the detuning between the modulated DFB laser and the ECL was approximately -4GHz, and the external injection is achieved with by using the ECL in conjunction with an optical coupler and a polarization controller.

The aim of this work is then to show how the linearisation of the modulation response due to the external light injection (around the relaxation frequency of the free running device), may greatly improve the performance of a multi-carrier radio-over-fiber communication system. Figure 2 shows our complete experimental set-up for

characterising the system performance. We first combined a 30Mhz carrier and a 60Mhz carrier in an RF power coupler. This composite signal was then mixed with a local oscillator at 6Ghz to give 5 carriers spaced by 30Mhz and centred at 6Ghz. A 10Mb/s NRZ data stream from an Anritsu pattern generator was then low pass filtered using a filter with a bandwidth of 10 MHz (to minimize the bandwidth occupied by the data signal), and subsequently mixed with the 5 carriers to give the multi-carrier RF data signal. A channel spacing of 30Mhz between the RF channels was chosen in order to ensure that the cross channel interference between them was negligible. The multi-carrier RF signal was then amplified before being added to a bias current and used to modulate either the free running laser, or the laser with external light injection from the ECL. The resulting optical microwave carrier was then passed through 10 km of dispersion shifted fiber (DSF), amplified in an erbium doped fiber amplifier, and detected using a 25Ghz photodetector from New Focus. The use of 10 km of DSF in our work ensured any frequency chirping, due to the direct modulation of the laser diode, has little or no effect on system performance. The received multi-carrier RF signal was electrically amplified and then down-converted by mixing it again with the 6Ghz local oscillator. Note that only the central channel was down-converted to a digital base-band signal. The down-converted signal was subsequently passed through a low-pass electrical filter with a bandwidth of 10 MHz to select out the one required base-band signal and then electrically amplified before being examined using a high-speed oscilloscope or an error detector.

There are two characteristics of the laser diode response that will affect the performance of a radio over fiber system, the magnitude of the response and the linearity of the response. Firstly consider the magnitude of the modulation response of the laser diode. As the magnitude of the laser response increases, then for a given

power in the RF signal driving the laser, the detected electrical signal power will increase. Thus by using the external injection to enhance the response of the laser at frequency bands beyond the bandwidth of the free running device, it will clearly result in a major improvement in the performance of the hybrid radio/fiber system [9,10]. The second important characteristic is the linearity of the response at the frequency of interest. If we are operating in a linear portion of the modulation response, then IMD is kept to a minimum. However as the operating RF band approaches the relaxation frequency of the laser, the response becomes quite nonlinear, which will thus affect the performance of a multi-carrier radio-over-fiber system. To overcome this non-linearity problem it is possible to employ external light injection to make the laser's response more linear around the relaxation frequency of the free running device [5,13]. In our work we will show for the first time how the linearisation of the modulation response improves the performance of a multi-carrier radio-over-fiber system.

### **Experimental Results**

Figure 3(A) shows the electrical spectrum of our 5-channel RF data signal that is used to directly modulate the DFB laser diode, and Figure 3(B) displays the received electrical spectrum after passing through the optical link and being detected with the 25 GHz pin diode. As we can see from this figure, the dynamic non-linearity of the laser diode around the operating frequency of 6 GHz results in the generation of sidebands around the received signal due to IMD. In addition to the sidebands that are visible in the received spectrum, there are also IMD products sitting on the received data channels that will affect the performance of these channels. When using a multi-channel RF data signal as in this work, there are two types of inter-modulation products generated that cause interference and degrade the overall performance of the

SCM system. These are the inter-modulation products of the type  $2f_x - f_y$ , and also those due to beating between three frequencies  $f_x + f_y - f_z$ . To overcome the problems caused by these inter-modulation products we subsequently used the external injection technique as described above. In this case the received electrical spectrum is as shown in Figure 3(C). It is clear from this result that the improved linearity of the device, around the operating frequency band, greatly reduces the level of IMD.

These spectra clearly show us that the nonlinearity in the free running laser is introducing signal distortion that can be significantly reduced using external injection. To determine how the improvement in linearity affects the performance of the overall system it was necessary to measure the Bit Error Rates of the received signal. It is important to note that relative response of the laser around the operating frequency band, with and without external light injection, was the same, thus the improvement in system performance was solely due to the reduction in non-linearity of the device having decreased the IMD.

Figure 4 shows the received eye diagrams of the down-converted signal having passed through the optical microwave link with and without external injection into the directly modulated DFB laser. From this figure it can be seen that the relative response is identical as the amplitude of the received signal is the same in both cases. It can also be seen that the opening of the eye for the free running DFB laser (Figure 4A) is degraded when compared to the system employing external injection (Figure 4B).

To determine the improvement in system performance under external injection conditions, an optical attenuator was used to vary the received power and the bit error

rate was measured as a function of the received power. Figure 5 shows the plot of BER against received powers for the overall system with and without external injection into the laser, and shows that a 2dB improvement was achieved.

### **Matlab Model**

A Matlab model was the designed to simulate the 5-channel system described above and to verify the results obtained. It used the laser rate equations to describe the operation of the laser.

There are many different forms of the laser rate equations and almost every work undertaken uses a slightly different form. The form we used for our free running laser was very similar to those used before normalisation by Le Bihan and Yabre in [15]. In [16] Yabre neglects the gain compression factor,  $\epsilon$ , by assuming that the optical power is moderate enough to allow the approximation ( $\epsilon S \ll 1$ ) and hence  $(1 - \epsilon S) \approx 1$ . For simplification purposes we used this approximation in our model as the optical power levels in our work are moderate, and also the modulation frequency employed is less than the relaxation frequency of the injection locked laser. For the injection locked case we added the final term in Equation (2) and the final term in Equation (3) [12]. These terms describe the level and the phase of the injected light.

The single mode rate equations for injection locked lasers with photon density  $N(t)$ , corresponding phase  $S(t)$  and carrier density  $\phi(t)$  used in the model are as follows:

$$\frac{dN(t)}{dt} = \frac{I(t)}{qV} - \frac{N(t)}{\tau_n} - g_0(N(t) - N_{om})S(t) \quad (1)$$



$$\frac{dS(t)}{dt} = \Gamma g_0 (N(t) - N_{om}) S(t) - \frac{S(t)}{\tau_p} + \Gamma \beta \frac{N(t)}{\tau_n} + 2K_c \sqrt{S_{inj} S(t)} \cos(\phi(t)) \quad (2)$$

$$\frac{d\phi(t)}{dt} = \frac{\alpha}{2} \left( \Gamma g_0 (N(t) - N_{om}) - \frac{1}{\tau_p} \right) - \Delta\omega - K_c \sqrt{\frac{S_{inj}}{S(t)}} \sin(\phi(t)) \quad (3)$$

where  $g_0$  is the gain coefficient,  $\tau_p$  is the photon lifetime,  $k_c$  is the coupling coefficient for the injected light,  $\alpha$  is the linewidth enhancement factor,  $I(t)$  is the injection current,  $\tau_n$  is the carrier lifetime,  $N_{om}$  is the transparency carrier density,  $q$  is the electron charge,  $\Delta\omega$  is the detuning frequency,  $S_{inj}$  is the photon density of the injected light,  $V$  is the volume of the active region,  $\Gamma$  is the optical confinement factor and  $\beta$  is the spontaneous emission factor.

The values used in our work are given in table 1. Some of these parameters were obtained from device specifications given by the supplier of the laser chip used in the experimental work, while others were taken from parameters used in reference 12.

The first step in the design of the full system model was the characterisation of the intensity modulation response of the laser diode. Small signal analysis was used. In small signal analysis, time varying components are considered to have a dc part and an ac part. The following were substituted into (1), (2) and (3) above.

$$\begin{aligned} I(t) &= I_0 + \delta I \\ S(t) &= S_0 + \delta S \\ N(t) &= N_0 + \delta N \\ \phi(t) &= \phi_0 + \delta \phi \end{aligned} \quad (4)$$

where  $I_0, S_0, N_0, \phi_0$  are the dc parts, and  $\delta I, \delta S, \delta N, \delta \phi$  are the ac parts of the current, photon density, carrier density and phase, respectively. Ignoring the steady state

solution and higher order terms yields a set of linearized equations for the ac components of  $S(t)$ ,  $N(t)$  and,  $\phi(t)$ ;

$$\begin{pmatrix} j\omega + a_{11} & a_{12} & a_{13} \\ a_{21} & j\omega + a_{22} & a_{23} \\ a_{31} & a_{32} & j\omega + a_{33} \end{pmatrix} \begin{pmatrix} \delta N \\ \delta S \\ \delta \phi \end{pmatrix} = \begin{pmatrix} \delta I / qV \\ 0 \\ 0 \end{pmatrix} \quad (5)$$

where

$$\begin{aligned} a_{11} &= \frac{1}{\tau_n} + g_0 S_0 & a_{12} &= \frac{1}{\tau_p} - \frac{X}{\Gamma S_0} & a_{13} &= 0 \\ a_{21} &= -\Gamma g_0 S_0 & a_{22} &= \frac{X}{2S_0} & a_{23} &= 2S_0 Y \\ a_{31} &= -\frac{\alpha}{2} \Gamma g_0 & a_{32} &= \frac{-Y}{2S_0} & a_{33} &= \frac{X}{2S_0} \end{aligned}$$

and

$$X = 2K_c \sqrt{S_{inj} S_0} \text{Cos}(\phi_0)$$

$$Y = K_c \sqrt{\frac{S_{inj}}{S_0}} \text{Sin}(\phi_0)$$

The modulation response was taken as  $\frac{\delta S}{\delta I}$ .

$I_0$  is a known constant which represents the bias current. The dc values of photon density, current density and phase can be obtained by letting the left hand side of equations (1),(2),(3) above equal zero. Using the relationship,  $\text{Cos}^2 A + \text{Sin}^2 A = 1$ , and adding manipulated versions of 2 and 3, yields the following:

$$K_c^2 \frac{S_{inj}}{S_0} = \frac{1}{4} \left[ \left( \Gamma g_0 (N_0 - N_{om}) - \frac{1}{\tau_p} \right) + \frac{\Gamma \beta N_0}{S_0 \tau_n} \right]^2 + \left[ \frac{\alpha}{2} \left( \Gamma g_0 (N_0 - N_{om}) - \frac{1}{\tau_p} \right) - \Delta \omega \right]^2 \quad (6)$$

Letting the left hand side of Equation (1) equal zero and rearranging gives:

$$\frac{\frac{I_0}{qV} - \frac{N_0}{\tau_n}}{g_0(N_0 - N_{om})} = S_0 \quad (7)$$

Substituting for  $S_0$  in Equation (6) yields a quartic equation in  $N_0$  that Matlab can easily solve, giving the DC value for carrier density. From this  $S_0$  can then be obtained using Equation (7) and  $\phi_0$  can be obtained from equation (2) or (3). Once these are obtained, then every value in the equation is known and hence the modulation response can be plotted.

There has been much work reported on the stability and locking range of lasers subject to external injection [17,18,19]. We found that a detuning parameter of  $-11\text{GHz}$  gave us stability in our system over the range of injection ratios that we used. Figure 6 displays the locking range over which the light injection significantly alters and improves the modulation response of the laser diode.

Figure 7 shows the modulation response of the laser under free running conditions and under three different levels of external light injection. The injection ratio is defined as the ratio of injected photon density to the photon density of the slave laser. The injection ratios in Figure 7 were 0.017, 0.025 and 0.045. Careful examination of the response at 6GHz shows that the free running laser response is rather nonlinear. Injection ratio 0.017 gives a higher magnitude of response but its linearity is not much improved than from the free running case. 0.025 is the closest match to the experimental results used above because although the linearity has improved with injection, the relative response level is the same as the free running laser. Again, this will allow us to obtain a measure of how much performance improvement is due to

the linearity of the response alone. As we continued to increase the injection ratio the response became more linear but also had a lower response.

As was mentioned previously, most of the work in this area to date has been on the laser alone [5-8,11,12]. There has been no work reported which simulates the full communication system in which external injection is used to linearise the modulation response of the directly modulated laser. Once the laser response had been determined, the rest of the system was built around it. The entire system comprised of a data source, rise time filter, local oscillator, mixer, power combiner, laser, detector, receiver filter, demodulator, bit error rate tester and oscilloscope. Simulation allows a much more ideal filter to be employed and hence we used channel spacing of 20Mhz rather than the 30Mhz of the experimental system. Only thermal noise at the receiver was taken into consideration.

Figure 8 shows the simulated eye diagrams of the received 10 Mbit/s data signals from the hybrid radio/fiber system with and without external injection into the directly modulated laser transmitter. The external injection ratio is set to 0.025 in the simulation to ensure that the relative response is the same for both cases and the difference in performance is this solely attributed to the different linearity of the response around the operating frequency. We can clearly see the degraded performance due to IMD for the free running laser transmitter (Figure 8A).

In order to quantitatively determine the improvement in system performance we have used the simulation model to determine the BER vs. received power curves for the overall system. Figure 9 shows quite clearly the different effects that the linearity and the magnitude of the laser's modulation response, have on the system performance.

By turning on injection, performance was improved by almost 3.5dB for injection ratio of 0.025. This 3.5dB improvement is solely due to the improved linearity as the relative response around the operating frequency is the same for both the free running laser, and the device with external injection. However, as more light (injection ratio of 0.045) is injected into the laser the response becomes flatter, but the relative response decreases, thus reducing the improvement in system performance to only 2.5 dB. For injection levels less than 0.025 (i.e. 0.017) the modulation response becomes more linear and the magnitude of the response is also increased. Both of these changes in the modulation response add together to give an overall improvement in system response of greater than 4 dB.

Our simulation may also be used to determine the effects of having a larger number of RF data channels in the SCM system. When the channel number is increased to 9, the improvement in system performance for the externally injected laser (injection ratio of 0.025), compared with the free running device is found to be in the order of 5.3 dB. This increase in improvement in system performance, under external injection, for a larger channel number, is as expected. The reason comes from the fact that for an increased number of channels the number of IMD products generated increases, and so does the interference on the various RF data channels.

### **Conclusions and Discussion**

In this paper we have shown the improvement in system performance that can be achieved in a multi-carrier radio-over-fiber communication system, by using external injection into the laser transmitter. The improvement in system performance is solely due to the enhanced linearity of the device around the operating RF band, as the relative response of the laser is kept equal for the free running and externally injected

case. In the experiment we have used a 5-channel system with each channel carrying 10Mb/s of data and examined the effect that the nonlinearities had on the performance of these channels. Improvement in system performance of greater than 2dB was achieved. The complete system was then simulated using Matlab, with the laser rate equations used to mimic the nonlinear response of the laser. Good correlation was observed between experimental and simulated results.

Hybrid fiber radio (HFR) architectures will most probably be used for the transmission of RF data signals to the users in a future broadband wireless networks. The simplest method of generating an optical RF carrier is to use direct modulation of a laser diode. However the dynamic non-linearity of laser transmitter may cause signal distortion due to IMD and degrade the overall system performance of multi-carrier radio-over-fiber systems. Although in our work we have concentrated on an SCM system with 5 data channels, practical SCM systems may have a much larger number of RF carriers. In this case the number of IMD products will greatly increase and the affect on system performance due to the laser non-linearity will become even more severe. Under these circumstances (large number of RF channels) the improvement in system performance as a result of reduced non-linearity (due to external light injection) will be far more pronounced. Clearly laser transmitters based on external light injection may thus be very suitable sources for such SCM systems as they have improved linearity. For practical systems applications it will be more suitable to integrate the slave and master laser into a single chip that exhibits the type of linearity required for next generation multi-carrier RF systems [20].

Figure 1 - Modulation Response of the single mode laser measured under free running conditions and with external injection of 3mW from the external cavity laser. In both cases the dc bias current was 40 mA.

Figure 2 - The experimental setup for HFR system using a directly modulated laser with external light injection.

Figure 3 - Electrical Spectra showing (A) the signal which modulated the laser, (B) the received signal without injection locking and showing the IMD effects of laser nonlinearity and (C) the received signal with injection, showing the reduction in IMD due to increased linearity.

Figure 4 - Received eye diagrams of the two 10Mb/s data signals from the optically fed microwave system using (A) free running laser diode and (B) laser diode with the external injection level of 3mW. Received optical power (before photodiode) was -18.7dBm in both cases.

Figure 5 - BER versus received optical power for the central RF data channel using directly modulated laser with (●) and without (▲) external injection.

Figure 6 – Detuning versus injection ratio showing the locking range over which the external injection significantly improves the lasers modulation response.

Figure 7 - Modulation Response of the simulated laser under free running conditions and with injection ratios of 0.017, 0.025, 0.045. In all cases the bias current was 70mA.

Figure 8 - Simulated eye diagrams of the two 10Mb/s data signals from the optically fed microwave system using (A) free running laser diode and (B) laser diode with the external injection ratio of 0.025. Received optical power (before photodiode) was -18.23dB in both cases.

Figure 9 - BER versus received optical power for the simulated central RF data channel using directly modulated laser with injection ratio 0.017 (▲), injection ratio 0.025 (■), injection ratio 0.045(◆) and without any external injection(◆).

Table 1 - Parameters used in the Matlab simulation.



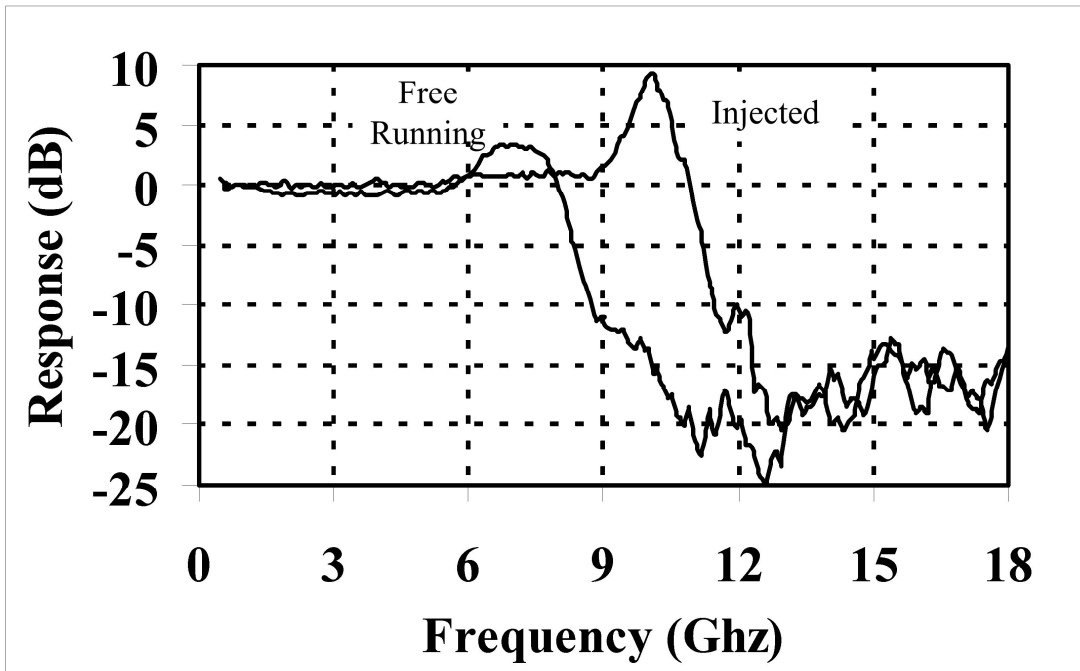


Figure 1

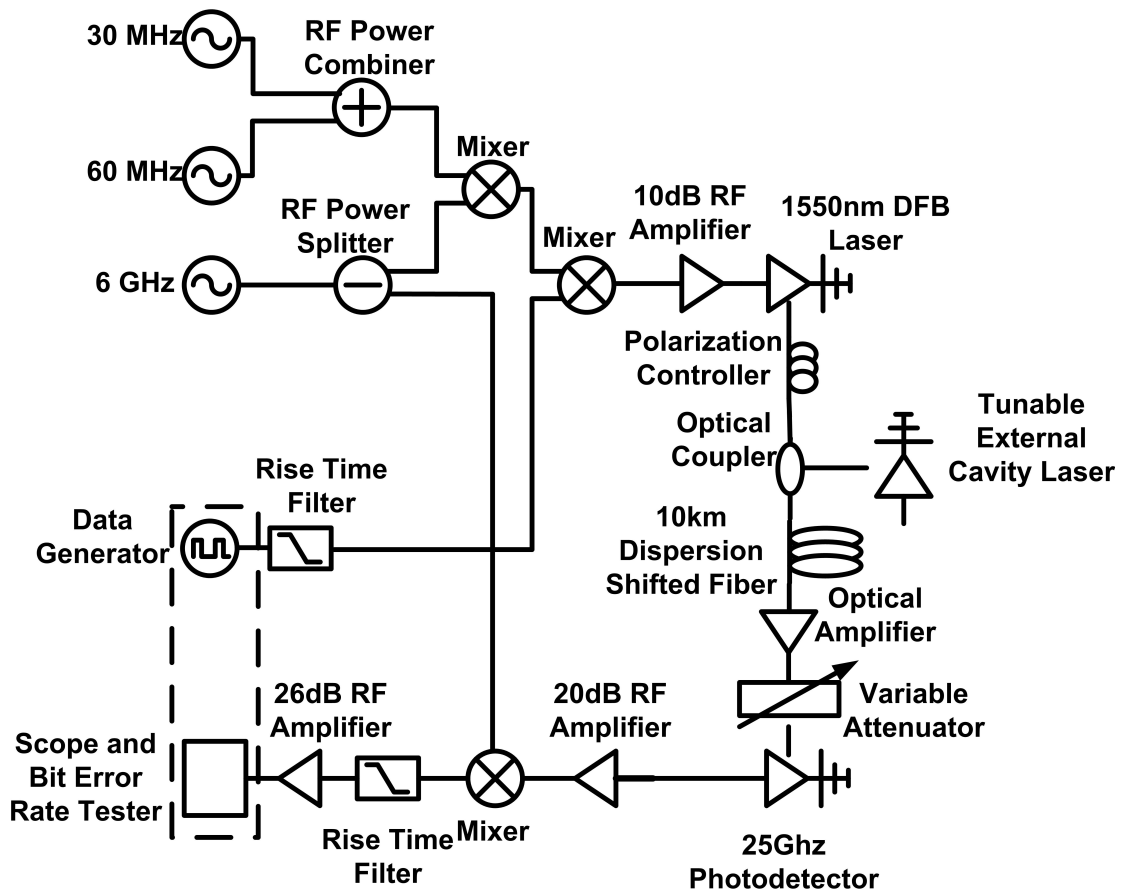


Figure 2

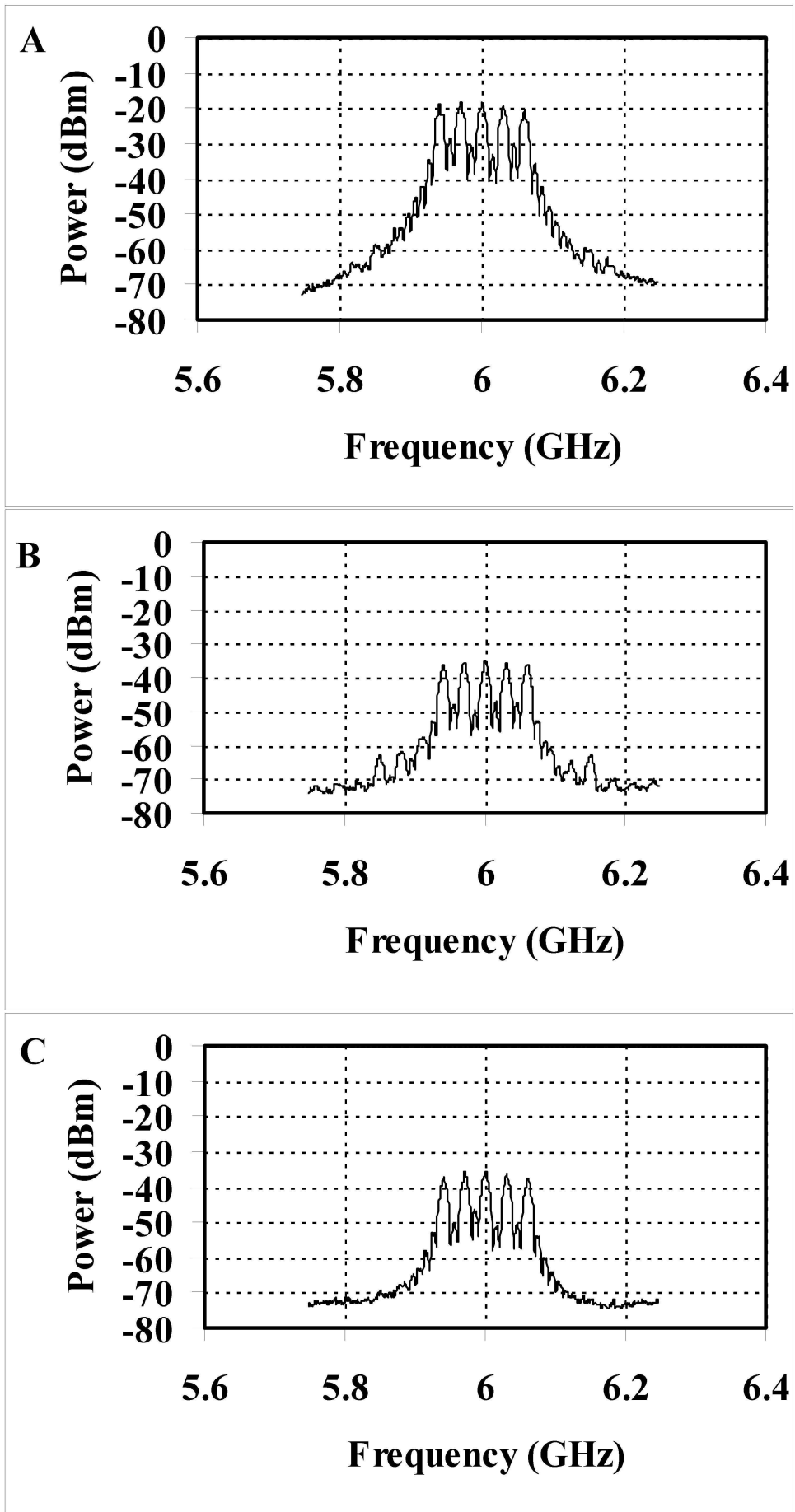


Figure 3

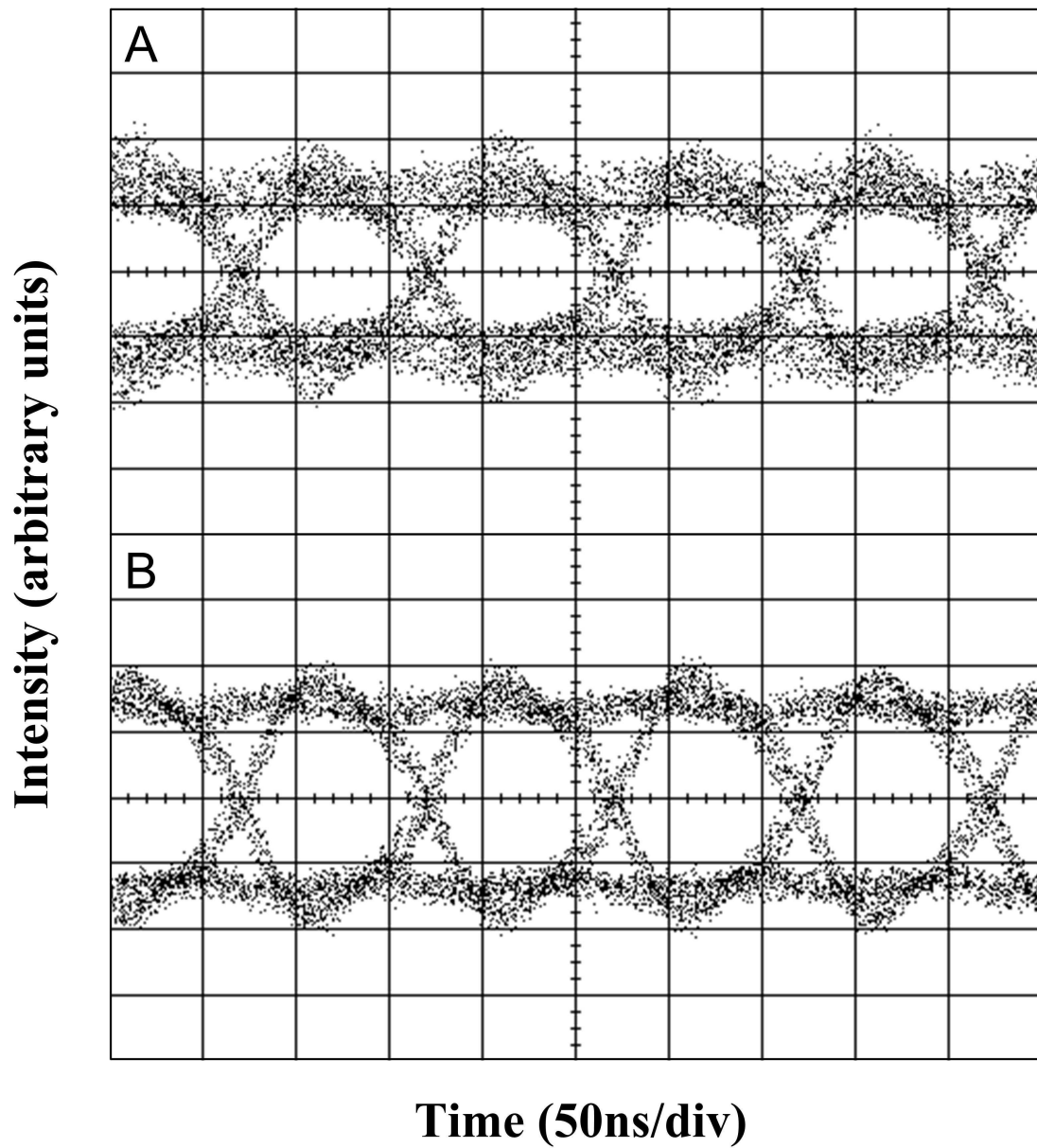


Figure 4

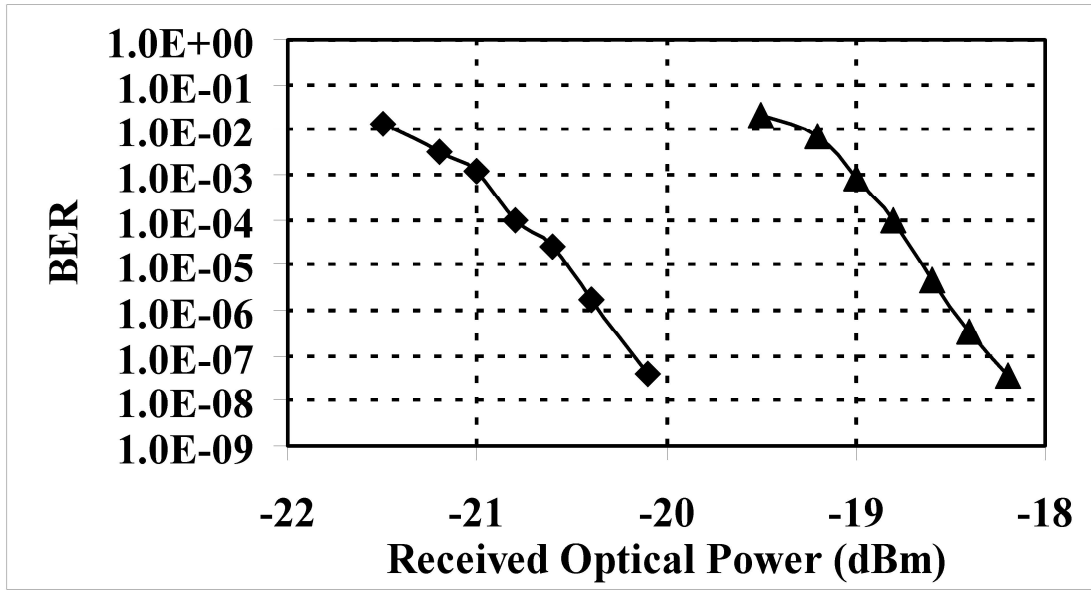


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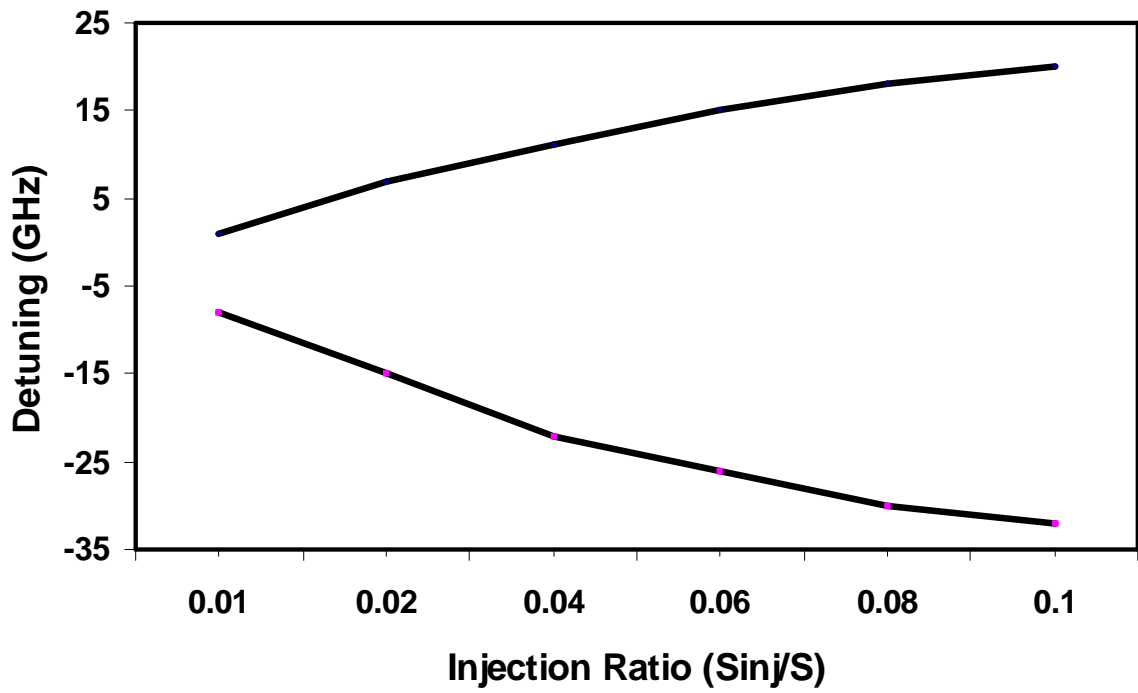


Figure 6

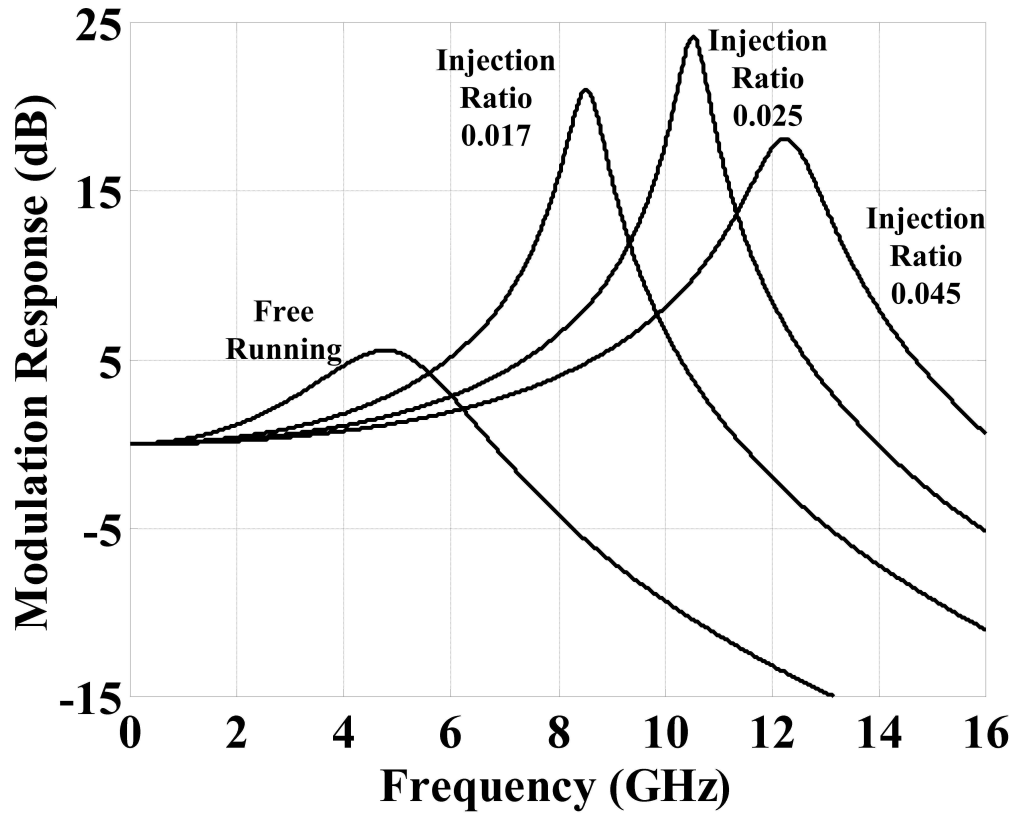
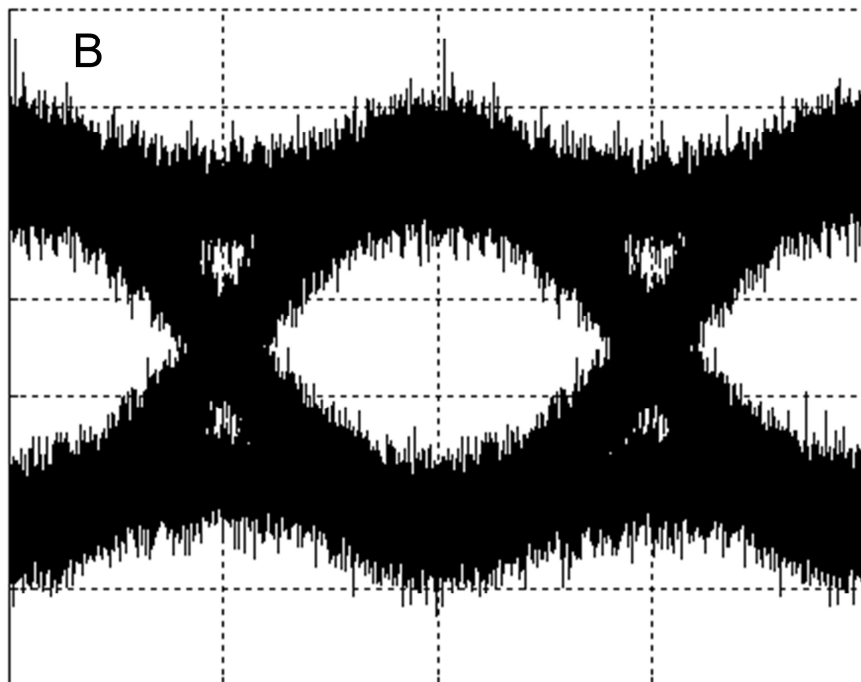
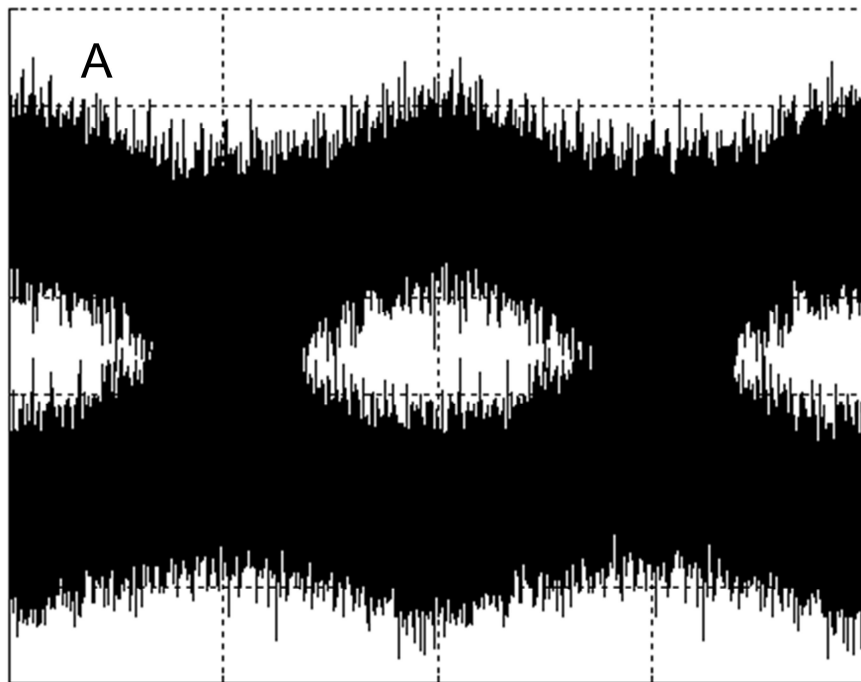


Figure 7

Intensity (arbitrary units)



Time (50ns/div)

Figure 8

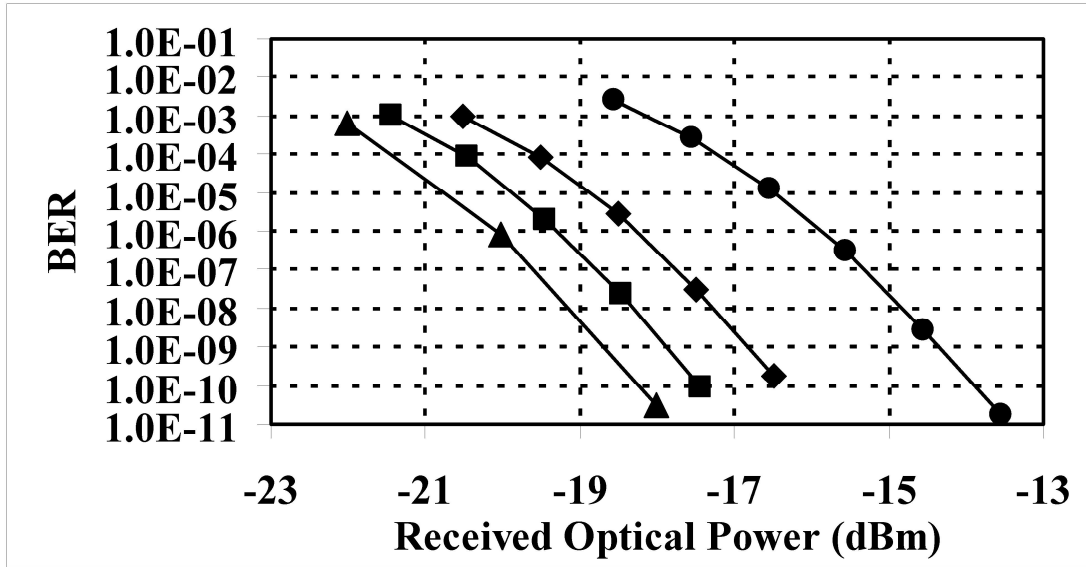


Figure 9

$g_0$	$1e-12 \text{ m}^3 \text{ s}^{-1}$
$N_{om}$	$1.4 \times 10^{23} \text{ m}^{-3}$
$V$	$11e-17 \text{ m}^3$
$\tau_p$	$2 \times 10^{-11} \text{ s}$
$\tau_n$	$0.3 \times 10^{-9} \text{ s}$
$\Gamma$	0.35
$\beta$	0.0
$q$	$1.6e-19 \text{ C}$
$\alpha$	6.8
$I(t)$	70mA
$\Delta f$	-11Ghz
$\Delta \omega$	$2\pi \Delta f$
$S_{inj}$	$35e20 \text{ m}^{-3}$
$k_c$	$2.5 \times 10^{11} \text{ s}^{-1}$

Table 1

## References

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