

## Few-body problems in nuclear astrophysics

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**Abstract.** Few-body methods provide very useful tools to solve different problems important for nuclear astrophysics. Some of them are discussed below.

### TRIPLE COLLISIONS

Binary collisions are dominant in stellar environments. But since nuclear reactions occur in a stellar plasma, it is important to estimate the impact of the medium on the elementary processes. The dynamic effect of the medium is to produce triple nonradiative collisions. To calculate the reaction rates for such processes, we use a genuine three-body Coulomb scattering wave function, which is exact in the asymptotic region where two particles are close to each other and far away from the third (spectator) particle. We have estimated reaction rates of  ${}^7\text{Be}(ep, e){}^8\text{B}$  and  ${}^7\text{Be}(pp, p){}^8\text{B}$  triple collisions leading to the nonradiative formation of  ${}^8\text{B}$ . For solar core conditions we find that the triple collision rates for  ${}^7\text{Be}(ep, e){}^8\text{B}$  and  ${}^7\text{Be}(pp, p){}^8\text{B}$  are approximately  $10^{-5}$  and  $10^{-12}$  of that for the binary one  ${}^7\text{Be}(p, \gamma){}^8\text{B}$ . Triple collisions play a minor role in stellar matter unless temperatures and densities are high.

### COULOMB BREAKUP

It is very difficult, or often impossible, to measure under lab conditions nuclear cross sections at stellar energies. That is why different indirect techniques, such as Coulomb breakup reactions, are useful for getting at the desired astrophysical information. This information can be distorted by the final-state three-body Coulomb interaction (post-decay Coulomb acceleration). A correct treatment of the final-state rescattering requires the use of a genuine scattering wave function for three charged particles in the contin-

**TABLE 1.** The second column is the binary reaction rate of  ${}^7\text{Be}(p, \gamma){}^8\text{B}$ , in the third and fourth columns are shown triple reaction rates for reactions with electron and proton spectators  ${}^7\text{Be}(ep, e){}^8\text{B}$  and  ${}^7\text{Be}(pp, p){}^8\text{B}$ , respectively.

$T_7$ ( $10^7 K$ )	$N_A \langle \sigma v \rangle$ ( $cm^3 mol^{-1} s^{-1}$ )	$N_A^2 \langle \Sigma(T) \rangle_e$ ( $cm^6 mol^{-2} s^{-1}$ )	$N_A^2 \langle \Sigma(T) \rangle_p$ ( $cm^6 mol^{-2} s^{-1}$ )
1.4	$1.7 \times 10^{-12}$	$1.7 \times 10^{-19}$	$1.2 \times 10^{-26}$
4	$3.2 \times 10^{-7}$	$1.7 \times 10^{-14}$	$7.8 \times 10^{-20}$
8	$1.4 \times 10^{-4}$	$3.4 \times 10^{-12}$	$2.3 \times 10^{-16}$
20	$7.7 \times 10^{-2}$	$4.6 \times 10^{-10}$	$1.6 \times 10^{-12}$
40	3.6	$5.0 \times 10^{-9}$	$2.1 \times 10^{-10}$
60	25.4	$1.3 \times 10^{-8}$	$2.0 \times 10^{-9}$
80	90.9	$2.1 \times 10^{-8}$	$7.1 \times 10^{-9}$
100	229.2	$2.8 \times 10^{-8}$	$1.7 \times 10^{-8}$

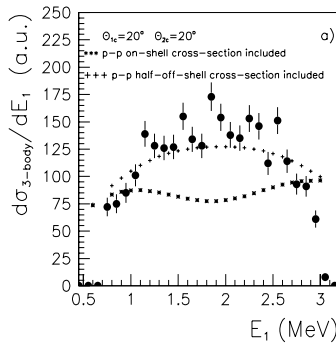
**TABLE 2.** The ratio for the single differential cross sections calculated with and without the final-state three-body Coulomb rescattering at the incident energies  $E_i = 46.5$  and  $83$  MeV/A as a function of the  ${}^7\text{Be} - p$  relative kinetic energy  $E_{rel}$ .

$E_{rel}$ [MeV]	0.1	0.15	0.2	0.4	0.6	0.8	1.0
Ratio ( $E_i = 46.5$ MeV/A)	1.90	1.18	1.03	0.97	0.98	1.00	1.00
Ratio ( $E_i = 83$ MeV/A)	1.13	1.02	1.00	1.00	1.00	1.00	1.00

uum. As an example we consider the  ${}^{208}\text{Pb}({}^8\text{B}, {}^7\text{Be}, p){}^{208}\text{Pb}$  reaction (Coulomb breakup of  ${}^8\text{B}$ ). Results presented in Table 2 show the renormalization of the astrophysical factor  $S_{17}(E_{rel})$  extracted from the Coulomb breakup reaction due to the three-body final-state interactions.

## ASYMPTOTIC NORMALIZATION COEFFICIENTS

The asymptotic normalization coefficient (ANC) method has proven to be another important indirect technique to get astrophysical  $S$  factors. Often the use of the weak radioactive beams is required to determine the ANCs for light proton-rich nuclei of importance to nuclear astrophysics. Alternatively, one can extract the mirror neutron ANC using stable-particle beams and find the required proton ANC assuming charge symmetry. We point out a few cases where the mirror symmetry can be used. The direct capture  ${}^{14}\text{N}(p, \gamma){}^{15}\text{O}(\frac{3}{2}_1^+)$  controls the energy production in the CNO cycle and its astrophysical factor is determined by the ANC of the overlap  $\langle {}^{14}\text{O} | {}^{15}\text{O}(\frac{3}{2}_1^+) \rangle$ . The  ${}^{15}\text{O}(\frac{3}{2}_1^+)$  state is separated from the neighbouring  ${}^{15}\text{O}(\frac{5}{2}_2^+)$  state by only 70 KeV, which influences the precision of measurements involving this state. The spacing between the mirror states in  ${}^{15}\text{N}$  is larger and therefore the ANC for the  $\langle {}^{14}\text{N} | {}^{15}\text{N}(\frac{3}{2}_1^+) \rangle$  overlap integral can be determined using neutron transfer reactions to higher accuracy than the  ${}^{15}\text{O}(\frac{3}{2}_1^+)$  ANC. Also, direct contributions to the cross sections of the  ${}^{22}\text{Mg}(p, \gamma){}^{23}\text{Al}$  and  ${}^{26}\text{Si}(p, \gamma){}^{27}\text{P}$  reactions, involving proton-rich radioactive nuclei, could be calculated through the mirror neutron ANC's which can be determined using stable targets  ${}^{22}\text{Ne}$  and  ${}^{26}\text{Mg}$ . These



**FIGURE 1.** Single differential cross section for  $p(6\text{MeV}) + d \rightarrow p + p + n$  as a function of the energy of one of the protons at fixed scattering angles of both protons  $\theta_{1c} = \theta_{2c} = 20^\circ$ .

reactions are relevant to the nucleosynthesis in novae and are being intensively investigated. An unsolved problem is the estimation in a few-body approach of the charge symmetry breaking for proton and neutron mirror ANCs for bound and resonant states. Another unsolved problem is the charge symmetry breaking for mirror  $\alpha$  ANCs for light nuclei.

## TROJAN HORSE

"Trojan Horse" is another very promising and powerful indirect method for extracting astrophysical factors for various nuclear reactions at astrophysically relevant energies. Using a few-body approach, one can derive a relationship between the measured  $2 \rightarrow 3$  cross section and that of the  $2 \rightarrow 2$  sub-reaction, and can estimate the influence of the "off-shellity" on the extracted  $2 \rightarrow 2$  cross section. In Fig. 1 we demonstrate the impact of the off-shell effects for the simplest three-body breakup  $p + d \rightarrow p + p + n$ . The single differential cross section has been calculated in the impulse approximation usually used to reproduce the energy dependence in the quasielastic kinematics assuming that the  $p + p$  scattering amplitude is on-shell. We can see from Fig. 1 that for the symmetric kinematics the on-shell approximation fails.

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