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Letters

On Abolishing Symmetry Requirements in the Formulation of a Five-Level Selective Harmonic Elimination Pulse-Width Modulation Technique

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***Abstract*—Selective harmonic elimination pulse width modulation (SHE–PWM) techniques offer a tight control of the harmonic spectrum of a given voltage waveform generated by a power electronic converter along with a low number of switching transitions. These optimal switching transitions can be calculated through Fourier theory, and for a number of years quarter-wave and half-wave symmetries have been assumed when formulating the problem. It was shown recently that symmetry requirements can be relaxed as a constraint. This changes the way the problem is formulated, and different solutions can be found without a compromise. This letter reports solutions to the switching transitions of a five-level SHE–PWM when both the quarter- and half-wave symmetry are abolished. Only the region of high-modulation indices is reported since the low-modulation indices region requires a unipolar waveform to be realized. Selected simulation and experimental results are reported to show the effectiveness of the proposed method.**

***Index Terms*—Five-level inverter, inverter control, non-symmetrical pulsewidth modulation (PWM), selective harmonic elimination (SHE).**

I. Introduction

SELECTIVE harmonic elimination pulsewidth modulation techniques (SHE–PWM) have been mainly developed for two- and three-level converter schemes (i.e., bipolar and unipolar waveforms) [1]. The main challenge associated with SHE–PWM techniques is to obtain the analytical solution of the resultant system of non-linear transcendental equations that contain trigonometric terms which in turn provide multiple sets of solutions. This has been reported in numerous technical articles [2]–[5]. Several algorithms have been reported in the technical literature concerning methods of solving the resultant nonlinear transcendental equations which describe the SHE–PWM problem. These algorithms include the well-known iterative approach [6], [7], elimination theory [5], Walsh functions [8], optimization techniques [2], [3] and genetic algorithms [4], [9], and [10].

On the other hand, multilevel converters have drawn tremendous attention in recent years and have been studied for several high-voltage and high-power applications [11]. Switching losses in these high-power, high-voltage converters represent an issue. Any switching transitions that can be eliminated without compromising the harmonic content of the final waveform are considered advantageous. Numerous topologies have been investigated and widely studied [11]. The desired output of a multilevel converter is synthesized with several methods including staircase modulation, sinusoidal PWM with multiple triangular carriers, and multilevel space-vector modulation. SHE–PWM methods have been introduced to multilevel converters in several technical articles. Initially the switching frequency was restricted to line frequency and therefore, the staircase multilevel waveform was arranged in such a way as to control the fundamental and eliminate the low-order harmonic from the waveform [4]. A new active harmonic elimination technique was recently introduced to the line-frequency method aiming to eliminate higher orders of harmonics by simply generating the opposite of the harmonics to cancel them [5]. Another variation of the multilevel SHE–PWM method is the unipolar case waveform where the waveform takes a positive, zero and negative value, and a phase-shifted technique is used between the converters to build the multilevel system control [12], [13]. Other approaches have also been reported including one where harmonic elimination is combined with a programmed method [4] and another where a criterion based on power equalization between various cascaded connected H-bridge multilevel converters is used to obtain the angles of the harmonic elimination method [14]. More recently, multilevel SHE–PWM defined by the well-known multicarrier phase-shifted PWM (MPS–PWM) was proposed in [15] where the modulation index defines the distribution of the switching angles. The problem of SHE–PWM is then applied to a particular operating point aiming to obtain the optimum position of these switching transitions that offers elimination to a selected order of harmonics. The main advantages of the method included an increased bandwidth for the same switching transitions and higher dc bus utilization.

However, in the previously mentioned approaches, the problem was formulated with quarter-wave symmetry constraints where the solutions to the switching transitions are sought in the region between 0 and $\pi/2$. This restriction could result in sub-optimal solutions with regards to the uncontrolled harmonic distribution. Furthermore, the harmonics can only be in phase or out of phase. References [16]–[18] suggested that the constraint of quarter-wave symmetry can be relaxed to half-wave symmetry where all the even harmonics are zero but the harmonic phasing is free to vary. The advantages of generalizing the problem definition to not require quarter-wave symmetry include finding several solutions not obtainable using the classic symmetrical approach and the flexibility to select uncontrolled harmonic content.

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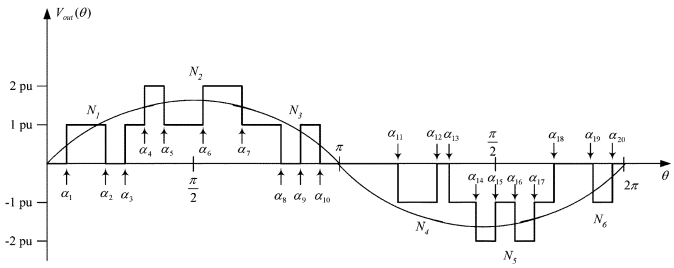

Fig. 1. Five-level defined non-symmetrical SHE–PWM waveform generation for 1< m_a <2.

This may in return result in further overall improvements associated with the PWM method in question. References [16] and [17] also presented solutions for both single- and three-phase cases (i.e., triplen harmonics are controlled and uncontrolled, respectively) and studied the effect of varying the phase angle of the fundamental to the harmonic profile. However, only two- and three-level waveforms were reported. Although the technique was extended to a generalized multilevel case (i.e., m-level, n-harmonic) where the problem becomes more complicated, especially when the switching angles are relaxed between 0 and 2π, solutions for higher than the three-level waveform were not presented. In [18], several multilevel waveforms are published demonstrating the generalization of [16] and [17] in the m-level n-harmonic case. However, in [18] a continuum of solutions across a variety of modulation indices is not provided.

The main contribution of this letter is to report solutions and results for a novel five-level PWM technique when both the quarter- and the half-wave symmetries are abolished. Although only the five-level case is presented here, the proposed method can be equally applied to any number of levels and for any number of switching transitions provided the algorithm converges. The switching transitions of the proposed method and for the region of high modulation indices are presented. Selected experimental results are provided to support the theoretical considerations.

The letter is organized in the following way. Section II describes in detail the proposed five-level SHE–PWM. The case when a five-level converter is implemented with 20 switching transitions within an entire period is discussed in Section III. The resulting voltage waveform with its spectrum for a given operating point of the inverter is plotted using a software program and is experimentally verified. Conclusions are summarized in Section IV.

II. Proposed Five-Level SHE–PWM

The proposed five-level SHE–PWM technique is defined according to the example waveform shown in Fig. 1. The multilevel converter considered in the implementation of the technique is composed of two cascaded H-bridge converters with equal dc sources and therefore the number of levels of the waveform is assumed to be five, (i.e., 2 p.u., 1 p.u., 0 p.u., -1 p.u., and -2 p.u.). However, it should be noted that other multilevel converter topologies could be made to operate with the proposed technique. Let N be the number of total switching transitions (angles) of the waveform sought within the full period of the waveform (i.e., N is relaxed between 0 and 2π). However, N is chosen to be an even number so as to have the same number of switching transitions (i.e., notches) in both the positive and negative half-cycle, thereby guaranteeing that the resultant waveform is realizable and physically correct. A closer inspection of the waveform shown in Fig. 1 reveals that there are six different regions of operation described as follows.

Let N_1 be the number of switching transitions placed between the 0 p.u. level and 1 p.u. This number can only be odd, since the first switching transition is chosen to be from the 0 p.u. to 1 p.u. level. Let N_2 be the number of switching transitions placed between the 1 p.u. level and 2 p.u. This number can only be even, since we have only a five-level waveform. Let N_3 be the number of switching transitions placed between the 1 p.u. level and the 0 p.u. This number again can only be odd. Let N_4 be the number of switching transitions placed between the 0 p.u. level and -1 p.u. This number also can only be odd, since the first switching transition is chosen to be from the 0 p.u. to -1 p.u. level. Let N_5 be the number of switching transitions placed between the -1 p.u. level and -2 p.u. This number can only be an even number, so as to have a five-level waveform. Finally, let N_6 be the number of switching transitions placed between the -1 p.u. level and 0 p.u. This number can only be odd, since the first switching transition is chosen to be from the 0 p.u. to -1 p.u. level. It is worth noting that N should be equal to the sum of all these transitions (i.e., $N = \sum_{i=1}^{6} N_i$)

The Fourier series expansion of the waveform presented in Fig. 1 is given by

$$V_{\mathrm{out}}(\theta) = a_0 + \sum_{n=1}^{\infty} C_n \cos(n\theta + \phi_n) \tag{1}$$

where

$$C_n = \sqrt{A_n^2 + B_n^2}, \quad \text{and} \tag{2}$$

$$\phi_n = \tan^{-1}\frac{B_n}{A_n}. \tag{3}$$

Assuming a unity dc level (1 p.u.) for each inverter cell, the generalized expressions of A_n, B_n, and a_0 for the proposed five-level converter are given by (4)–(6), shown at the bottom of the page, where

$$\begin{aligned} &n = 1, \ldots, \frac{N}{2} - 1 \\ &P_1 = N_1, P_2 = P_1 + N_2, P_3 = P_2 + N_3, \\ &P_4 = P_3 + N_4, P_5 = P_4 + N_5 \quad \text{and} \\ &P_6 = P_5 + N_6 \end{aligned}$$

and α_k is the kth switching transition (angle).

$$a_0 = \frac{1}{2\pi}\left[\begin{array}{l}\sum_{k=1}^{P_1}(-1)^k\alpha_k + \sum_{k=P_1+1}^{P_2}(-1)^{k-1}\alpha_k + \sum_{k=P_2+1}^{P_3}(-1)^k\alpha_k + \sum_{k=P_3+1}^{P_4}(-1)^{k-1}\alpha_k + \sum_{k=P_4+1}^{P_5}(-1)^k\alpha_k \\ +\sum_{k=P_5+1}^{P_6}(-1)^{k-1}\alpha_k\end{array}\right] \tag{4}$$

$$A_n = !\frac{1}{n\pi}\left[\begin{array}{l}\sum_{k=1}^{P_1}(-1)^k \sin n\alpha_k + \sum_{k=P_1+1}^{P_2}(-1)^{k-1}\sin n\alpha_k + \sum_{k=P_2+1}^{P_3}(-1)^k \sin n\alpha_k \\ +\sum_{k=P_3+1}^{P_4}(-1)^{k-1}\sin n\alpha_k + \sum_{k=P_4+1}^{P_5}(-1)^k \sin n\alpha_k + \sum_{k=P_5+1}^{P_6}(-1)^{k-1}\sin n\alpha_k\end{array}\right] \tag{5}$$

$$B_n = \frac{1}{n\pi}\left[\begin{array}{l}\sum_{k=1}^{P_1}(-1)^{k-1}\cos n\alpha_k + \sum_{k=P_1+1}^{P_2}(-1)^k \cos n\alpha_k + \sum_{k=P_2+1}^{P_3}(-1)^{k-1}\cos n\alpha_k \\ +\sum_{k=P_3+1}^{P_4}(-1)^k \cos n\alpha_k + \sum_{k=P_4+1}^{P_5}(-1)^{k-1}\cos n\alpha_k + \sum_{k=P_5+1}^{P_6}(-1)^k \cos n\alpha_k\end{array}\right] \tag{6}$$

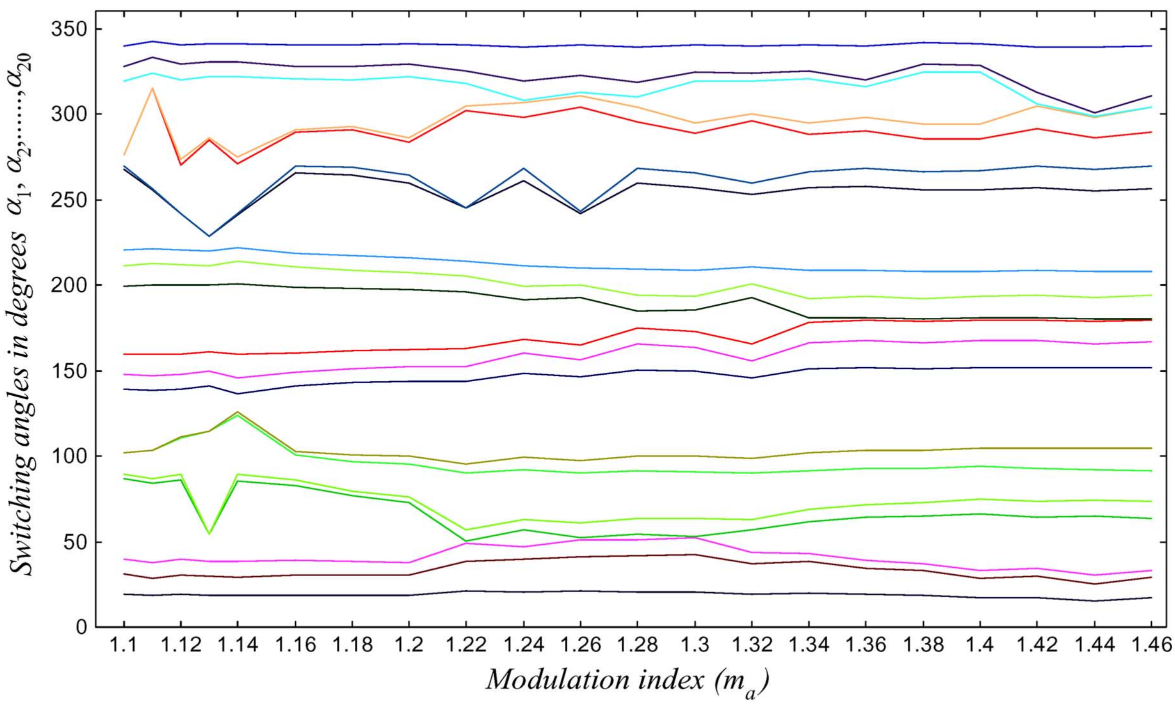

Fig. 2. Switching angles solution versus modulation index m_a. (Color version of Fig. 2 is available online at http://ieeexplore.org.)

Unlike the case of a waveform having quarter- and half-wave symmetry, the proposed formulation results in a dc component and all even harmonics. These are no longer equal to zero and therefore need to be controlled. Hence, for N switching transitions (angles), $N/2-1$ harmonics can be controlled/eliminated, including the fundamental, provided that solutions exist. One of the remaining two degrees of freedom is used to control the dc component at zero level; utilization of the second one remains an open question [17].

The problem defined by (4)–(6) can be solved using any of the techniques that have been previously applied to the case of the more restrictive quarter-wave symmetry problem, including iterative approaches [6], [7], resultant theory [5], minimization techniques [2], [3], and [15] or genetic algorithms [4], [9], and [10]. Any sets of solutions resulting from the case where the quarter- or half-wave symmetry are followed will also be a solution to the more generalized formulation proposed in this letter. The minimization technique, assisted with a real-coded hybrid genetic algorithm in order to reduce the computational burden associated with the nonlinear transcendental equations of the SHE–PWM method proposed in [9] and [10], has been applied to investigate the proposed five-level converter. Details regarding the proposed hybrid genetic algorithm can be found in [10].

In order to proceed with the optimization/minimization technique, an objective function describing a measure of effectiveness of the proposed method is specified as

$$F(\alpha_1, \ldots, \alpha_N) = a_0^2 + (C_1 - m_a)^2 + C_2^2 + \cdots + C_n^2 \quad (7)$$

where

$$m_a = \frac{V_F}{V_{\mathrm{dc}}} \quad (8)$$

where V_F is the fundamental component to be generated and ($0< m_a <2$). It is worth noting that the proposed method is investigated at high modulation indices only; it is implied that the low-modulation indices are satisfied with a three-level SHE–PWM technique which has been addressed in [15]. Therefore, the modulation indices in this letter will be searched from 1 to 2 (i.e., $1< m_a <2$), although there is no guarantee that solutions will be found for the entire interval.

The optimal switching angles are generated by minimizing (7) when it is subject to the constraints of (9) and (10). Consequently, selected harmonics are minimized. These switching angles are generated for different values of m_a and then stored in look-up tables to be used to control the inverter for a given operating point

$$(0 < \alpha_1 < \alpha_2 < \ldots < \alpha_{\frac{N}{2}} < \pi) \quad (9)$$

$$(\pi < \alpha_{\frac{N}{2}+1} < \ldots < \alpha_N < 2\pi) \quad (10)$$

where the constraints ensure that the solution is physically realizable and the waveform can be implemented for the given five-level converter. It should be noted that the above constraints are imposed to ensure that the switching transitions are equal between the positive and negative half cycles of the waveform, recognizing at the same time that the space of the overall solutions is reduced.

III. Simulation and Experimental Results

In this letter, 20-switching transitions (angles) (i.e., $N = 20$) were chosen as an example to implement the proposed five-level converter. They are distributed (Fig. 1) as follows $N_1 = 3$, $N_2 = 4$, $N_3 = 3$, $N_4 = 3$, $N_5 = 4$, $N_6 = 3$. As the theory of SHE–PWM suggests, 18-switching angles are used to control the first nine harmonics, including the fundamental component (i.e., two angles for each harmonic), and one of the switching angles is used to control the dc component. The results reported in this letter include only the case when both the symmetries are abolished. The effect of the harmonic phasing varying is not

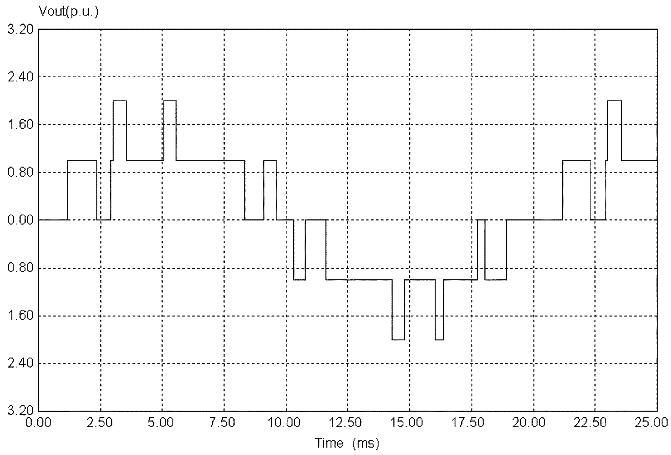

Fig. 3. Output voltage waveform (simulation, $m_a = 1.3$).

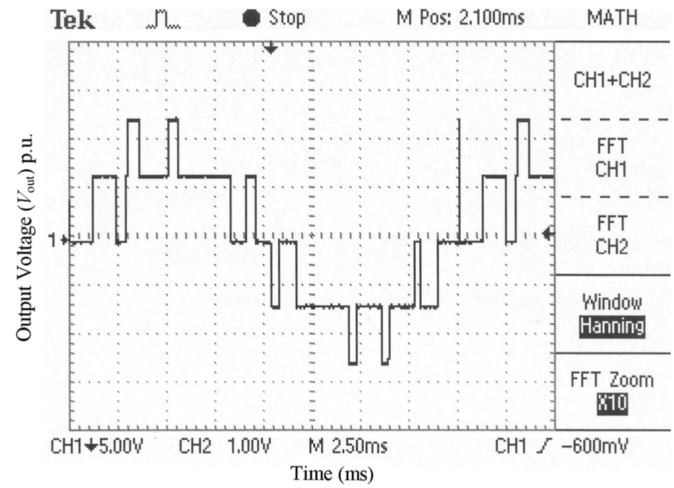

Fig. 5. Output voltage waveform (experimental, $m_a = 1.3$).

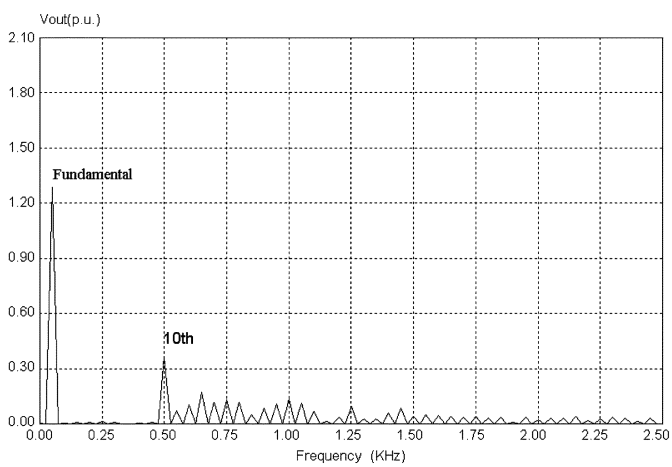

Fig. 4. Spectrum of the output voltage (simulation, $m_a = 1.3$).

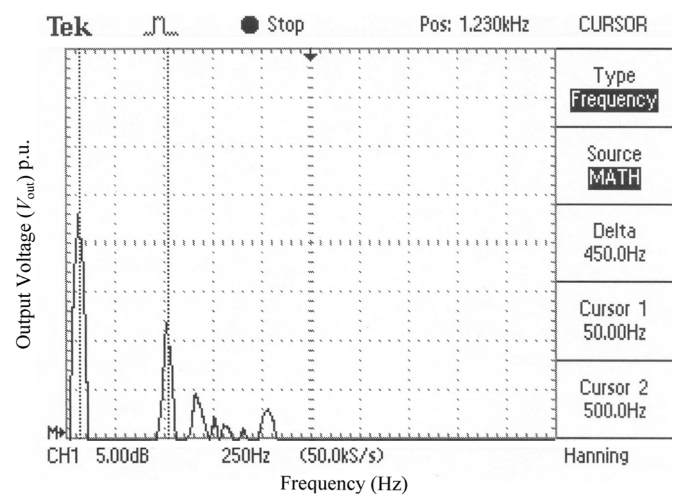

Fig. 6. Spectrum of the output voltage (experimental, $m_a = 1.3$). (Cursor 1 is on the fundamental component (50 Hz) and cursor 2 is on the next significant harmonic i.e., 10 ×50 Hz).

investigated. It is also confirmed that multiple sets of solutions exist. However, the evaluation to identify the optimal set that offers a better performance is once again beyond the scope of this letter. Therefore, only one continuous set is reported. Fig. 2 shows the solution of the switching angles (i.e., $\alpha_1, \ldots, \alpha_{20}$) for a certain range of $m_a(1.10 \geqslant m_a \geqslant 1.46)$.

An operating point of $m_a = 1.3$ was chosen and first simulated using the PSIM software package [19] to illustrate the effectiveness of the proposed method. The output voltage waveform is shown in Fig. 3, where it is clearly illustrated that the switching transitions are distributed throughout the period without any restriction of meeting half- and/or quarter-wave symmetries. Fig. 4 depicts the spectrum of the output voltage where the absence of the selected harmonics (even and odd) is evident and the fundamental is maintained at a pre-defined value (1.3 p.u.). On the other hand, the next significant harmonic that appears in the output waveform is the tenth, confirming the theoretical considerations of the proposed technique.

A low-power laboratory five-level inverter prototype based on the two insulated-gate bipolar-transistor H-bridges configuration was developed and tested to verify the feasibility and validity of theoretical and simulation findings. The pre-calculated PWM signals are implemented using a low-cost, high-speed Texas Instruments TMS320F2812 digital signal processor (DSP) board with an accuracy of 20 μs. A digital real-time oscilloscope (Tektronix TDS210) was used to display and capture the output waveforms. The spectrum of the output voltage was obtained with a feature of the fast Fourier transformer.

In order to draw a sensible comparison between experimental and simulation results, the same operating point is once again chosen. The output voltage waveform is shown in Fig. 5 and its spectrum illustrated in Fig. 6. As it can be seen, a very good correlation between experimental and simulation results is achieved. The spectrum of the output voltage clearly shows the absence of the targeted harmonics while controlling the fundamental at a predefined value. Although the proposed technique is investigated and discussed with only a single-phase, five-level converter, this does not restrict its application for the three-phase system. However, in the three-phase case, only the location of the controlled harmonics varies as the input to the formulated problem and does not include the triplen harmonics since the load can cancel their effect.

IV. Conclusion

A novel five-level SHE–PWM technique for voltage-source converters has been proposed in this letter. The problem of finding the switching transitions is reformulated without requiring quarter- and half-wave symmetry for the output waveform. An efficient optimization/minimization technique assisted with a hybrid genetic algorithm is applied to find the switching transitions (i.e., angles) for a valid modulation index value of the fundamental component. Although multiple sets of solutions have been found, only a continuous one is reported due to space limitations. Selected simulation and experimentally verified results are presented to confirm the validity of the theoretical analysis.

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