A Seven-Level Defined Selective Harmonic Elimination PWM Strategy

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Abstract—Selective harmonic elimination pulse-width modulation (SHE-PWM) techniques offer an optimized control approach for a given converter and are therefore suitable for the low switching frequency high-power applications. Optimization techniques can be successfully used to obtain the solutions of the equations defining the SHE-PWM waveform. In this paper, a seven-level multilevel strategy (MSHE-PWM) defined on the line-to-neutral basis and based on a ratio of a variable number of angles distributed over three levels to be able to calculate the transition points is reported. The technique provides eighteen switching transitions for every quarter period in the standard modulation index range. In the overmodulation region, this can be changed in order to increase the gain of the modulator which in turn results in a compromised bandwidth. The switching angles as a function of the modulation index are reported for the standard as well as the overmodulation range. Selected simulation results are presented to verify the effectiveness and feasibility of the proposed method.

I. INTRODUCTION

Selective harmonic elimination pulse-width modulation (SHE-PWM) techniques have been extensively studied for a two-level, three-level and recently for multilevel converters [1]-[20]. There have been many approaches to the SHE-PWM problem reported in the technical literature including: sequential homotopy-based computation [6], resultants theory [7], optimization search [8], Walsh functions [9]-[10], optimal methods [11]. The bipolar waveform has been treated in detail in [12] where a minimization technique is employed along with a biased optimization search method to get the multiple sets as predicted in [6].

These methods were extended in multilevel systems where the staircase waveform was used to find the angles and per unit values in order to minimize a number of harmonics and synthesize multilevel waveforms [13]. Such a waveform is shown in Fig. 1. This was further investigated and reported in [14], [15], [16] introducing an extra switching angle to address the limitations of the previous methods [14] in areas where optimized switching angles cannot be found. Specifically, the theory of resultants and its performance for a multilevel staircase waveform was reported in [14]. A unified approach was presented in [15]. More recently, the use of symmetric polynomials is combined with the resultant theory for a multilevel converter [16].

Multilevel SHE-PWM (MSHE-PWM) systems have been controlled using the unipolar (three-level) waveform and phase-shifted techniques [17]. Other approaches have also been reported including one where the harmonic elimination

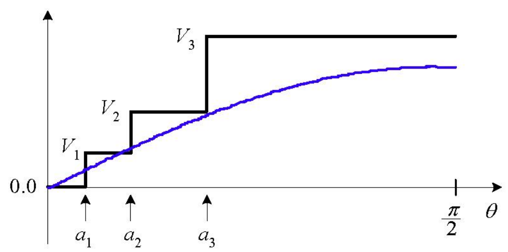


Fig. 1. The generalized staircase waveform suitable for multilevel systems and related angles of transition between the various voltage levels.

is combined with a programmed method [18] and another where a criterion based on power equalization between various cascaded connected H-bridge converters is used to obtain the angles of the harmonic elimination method [19].

A recent paper [20] reported a MSHE-PWM strategy defined for five-levels as shown in Fig. 2. This method did not only seek single transitions (Fig. 1) but rather found multiple switching angles in order to establish a PWM waveform (Fig. 2). The switching angles were reported and the comparison of the MSHE-PWM technique against the well-know sinusoidal PWM employing phase-shifted carriers confirmed the superiority of the former.

The objective of this paper is to report switching angles for a seven-level waveform. This extends our knowledge of MSHE-PWM techniques for the first time into a seven-level case (Fig. 3). Solutions for the switching angles for the entire range of modulation indices, i.e. standard and overmodulation are reported in the paper and verified to confirm the effectiveness of the proposed strategy.

The paper is organized as follows. Section II presents in detail the proposed seven-level MSHE-PWM. Switching angles as a function of the modulation indices are reported. The standard modulation index range and the overmodulation are investigated and results are presented in Section III and conclusions are summarized in Section IV.

II. PROPOSED SEVEN-LEVEL MSHE-PWM

The proposed MSHE-PWM strategy is defined according to the waveform shown in Fig. 3 and represents the line-to-neutral waveform of the converter. The number of levels of the waveform is assumed to be seven, i.e., 1p.u., 2p.u., 3p.u., 0p.u., -1p.u., -2p.u. and -3p.u.

The problem is formulated in a generalized form as follows. Let P be the number of levels of the waveform and the case where this number is odd is considered although an even number can be equally studied. In this paper we

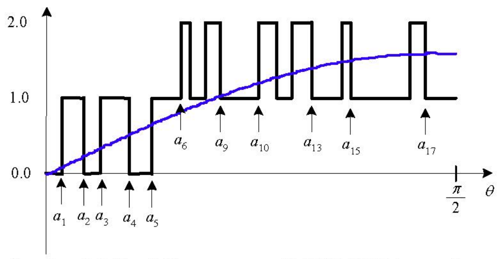


Fig. 2. A five-level defined (line-to-neutral) SHE-PWM waveform shown for a distribution ratio of 5/12 (z_1=5, z_2=12, N=z_1+z_2=17).

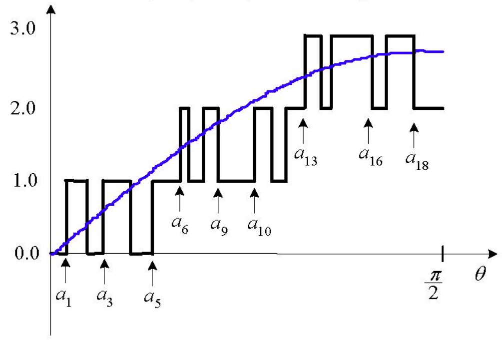


Fig. 3. A seven-level defined (line-to-neutral) SHE-PWM waveform shown for a distribution ratio of 5/7/6 (z_1=5, z_2=7, z_3=6, N=z_1+z_2+z_3=18).

consider P=7. Let N be the number of switching transitions (angles) of the waveform sought within the quarter of the period of the waveform. Let z_i be the switching angles (transitions) in every level and

$$i = 1,2,3.....(\frac{P-1}{2}) \quad (1)$$

The equation that describes the Fourier analysis of the multilevel waveform is then:

$$f_h = \sum_{k=1}^{z_1} (-1)^{k-1} \cos(h \cdot \alpha_k) + \sum_{n=2}^{(P-1)/2} \sum_{k=1+\sum_{q=1}^{n-1} z_q}^{\sum_{q=1}^{n} z_q} (-1)^{k-1-\sum_{q=1}^{n-1} z_q} \cos(h \cdot \alpha_k) \quad (2)$$

where

$$\sum_{q=1}^{(P-1)/2} z_q = N \quad (3)$$

The problem is solved through constraint minimization [8] and the following function when N=even is considered:

$$\mathrm{MIN}\left\{(f_1 - M)^2 + f_5^{\,2} + f_7^{\,2} + ... + f_{3N-1}^2\right\} \quad (4)$$

with the constraints:

$$0 < \alpha_1 < \alpha_2 < ...... < \alpha_N < \frac{\pi}{2} \quad (5)$$

The idea here is the classic SHE method that tries to find switching angles in order to eliminate a number of harmonics and control simultaneously the fundamental component. The challenge of the proposed method is that for the first time such method is applied to a seven-level multilevel PWM waveform (Fig. 3). The typical half-wave and quarter wave symmetries are followed for the waveform, i.e., simply when the switching angles for all modulation indices are obtained for the angles between zero and $\pi/2$, the usual reflection occurs to find the rest of the angles. Since there are N switching angles (i.e., $\alpha_1, \alpha_2, \alpha_3, ... \alpha_N$), N-1 harmonics can be eliminated if solutions can be found. For a three-phase inverter, the harmonics to be eliminated from the waveform are assumed to be non-triplen odd harmonics (i.e., 5th, 7th, 11th, 13th, n^{th} where n=3N-1 when N=even and n=3N-2 when N=odd). The strategy then relies on the structure of the power circuit in order to remove the triplen ones from the line-to-line voltage waveforms. For the seven-level case we have:

$$0 \le M \le 3 \quad (6)$$

If V_1 is the amplitude of the fundamental component to be generated, then

$$V_1 = \frac{4M}{\pi} \quad (7)$$

whereas the square-waveform of 3p.u. amplitude can generate $(12/\pi)$ p.u. maximum value at fundamental frequency.

III. Results and Discussion

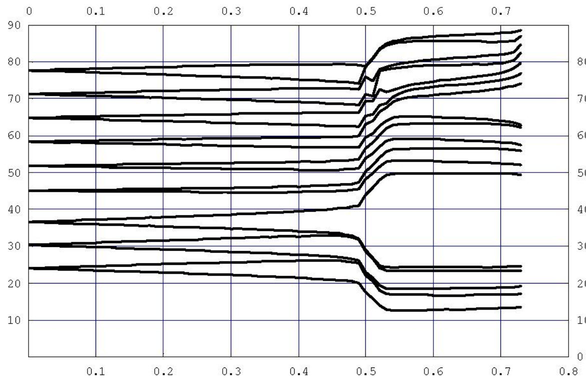
The minimization technique proposed in [8] has been applied and software is used to investigate the method [21]. For this paper N=18. However, this number and the various ratios can be changed as desired and the proposed method would provide the respective solutions provided they exist.

A. *Standard Modulation Range*

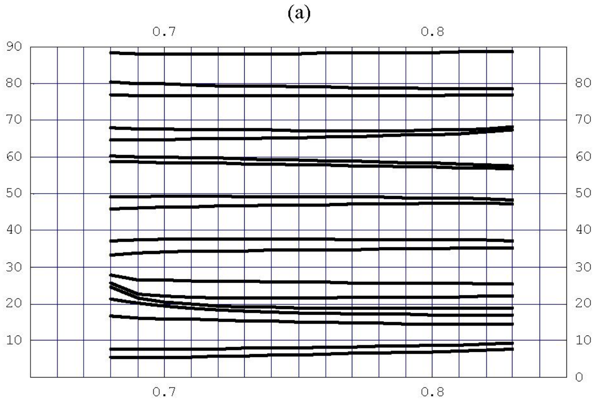
The standard modulation rage includes areas of M where solutions can be found. The maximum value found is M=2.58 for the case of 1/1/16 (Fig. 4(p)). It should be noted that the result obtained from the proposed method when the ratio is chosen to be 18/0/0 (Fig. 4(a)) provides solutions for all modulation indices up to 0.73 (continuous solutions) and other sets can increase the maximum obtainable M to 0.83 (Fig. 4(b)). Further results for all combinations of ratios are plotted in Fig. 4. The results are summarized in Table 1. Scanning the results presented in Fig. 4, one can see that there is an overlap between the regions, ensuring the method can be implemented for the entire range of modulation indices. The last harmonic that can be eliminated according to equation (4) is the 53p.u. and this is confirmed in Fig. 6. Specifically, Fig. 6(a) shows this waveform and its spectrum confirms that it does have only multiple of triplen harmonics (Fig. 6(a)). These triplen harmonics are cancelled out in a three-phase system from the line-to-line waveforms (Fig. 6(c)). This is confirmed through the associated spectrum of the line-to-line voltage waveform shown in Fig. 6(d).

B. *Overmodulation*

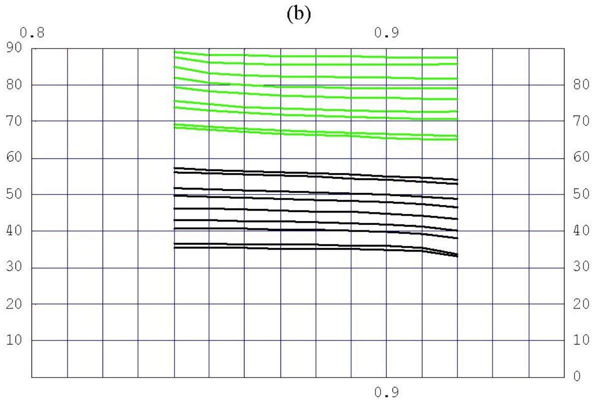
In order to increase the modulation index further, a pulse-dropping approach can be adopted and the same method [8] can be used to obtain the switching angles. In this paper, a number of variations are investigated and reported. The respective switching angles are presented in Fig. 5. The results are summarized in Table 2. The number of harmonics controlled in the overmodulation region is dropped and N


0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
90 80 70 60 50 40 30 20 10
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
80 70 60 50 40 30 20 10 0

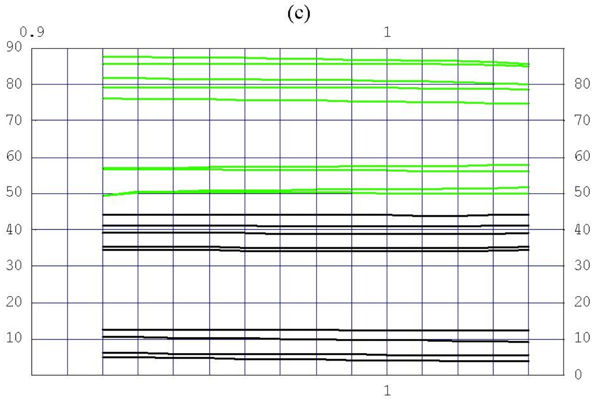
(a)


0.7 0.8
90 80 70 60 50 40 30 20 10
0.7 0.8
80 70 60 50 40 30 20 10 0

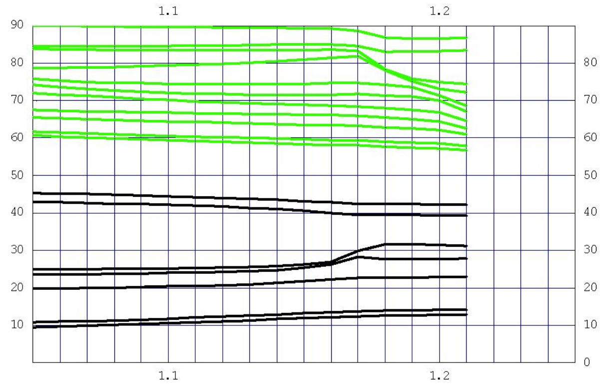
(b)


0.8 0.9
90 80 70 60 50 40 30 20 10
0.9
80 70 60 50 40 30 20 10 0

(c)


0.9 1
90 80 70 60 50 40 30 20 10
1
80 70 60 50 40 30 20 10 0

(d)


1.1 1.2
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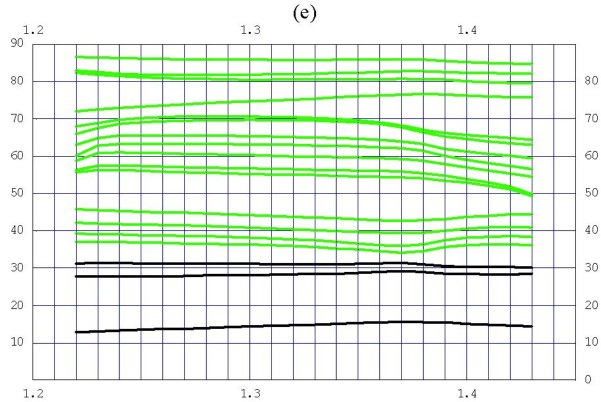
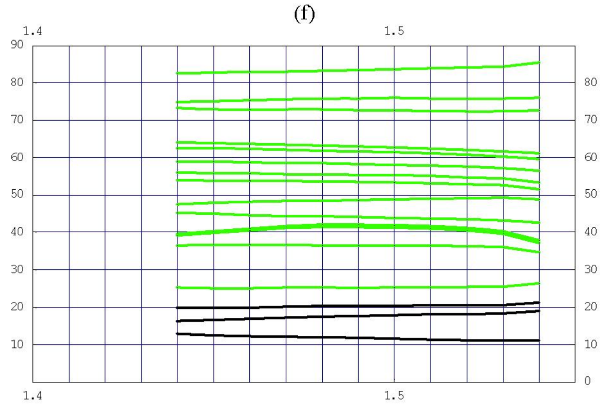
1.1 1.2
80 70 60 50 40 30 20 10 0

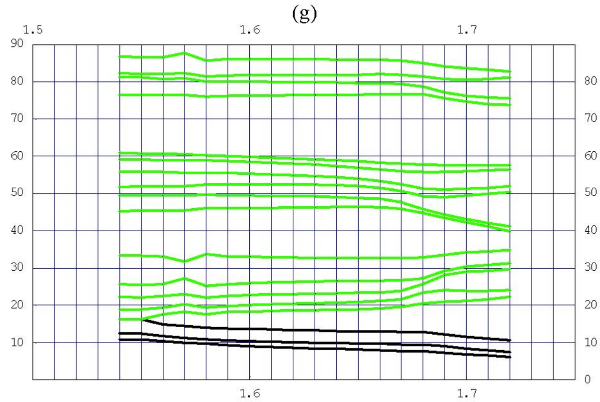
(e)


1.2 1.3 1.4
90 80 70 60 50 40 30 20 10 0
1.2 1.3 1.4
80 70 60 50 40 30 20 10 0

(f)


1.4 1.5
90 80 70 60 50 40 30 20 10
1.4 1.5
80 70 60 50 40 30 20 10 0

(g)


1.5 1.6 1.7
90 80 70 60 50 40 30 20 10
1.6 1.7
80 70 60 50 40 30 20 10 0

(h)

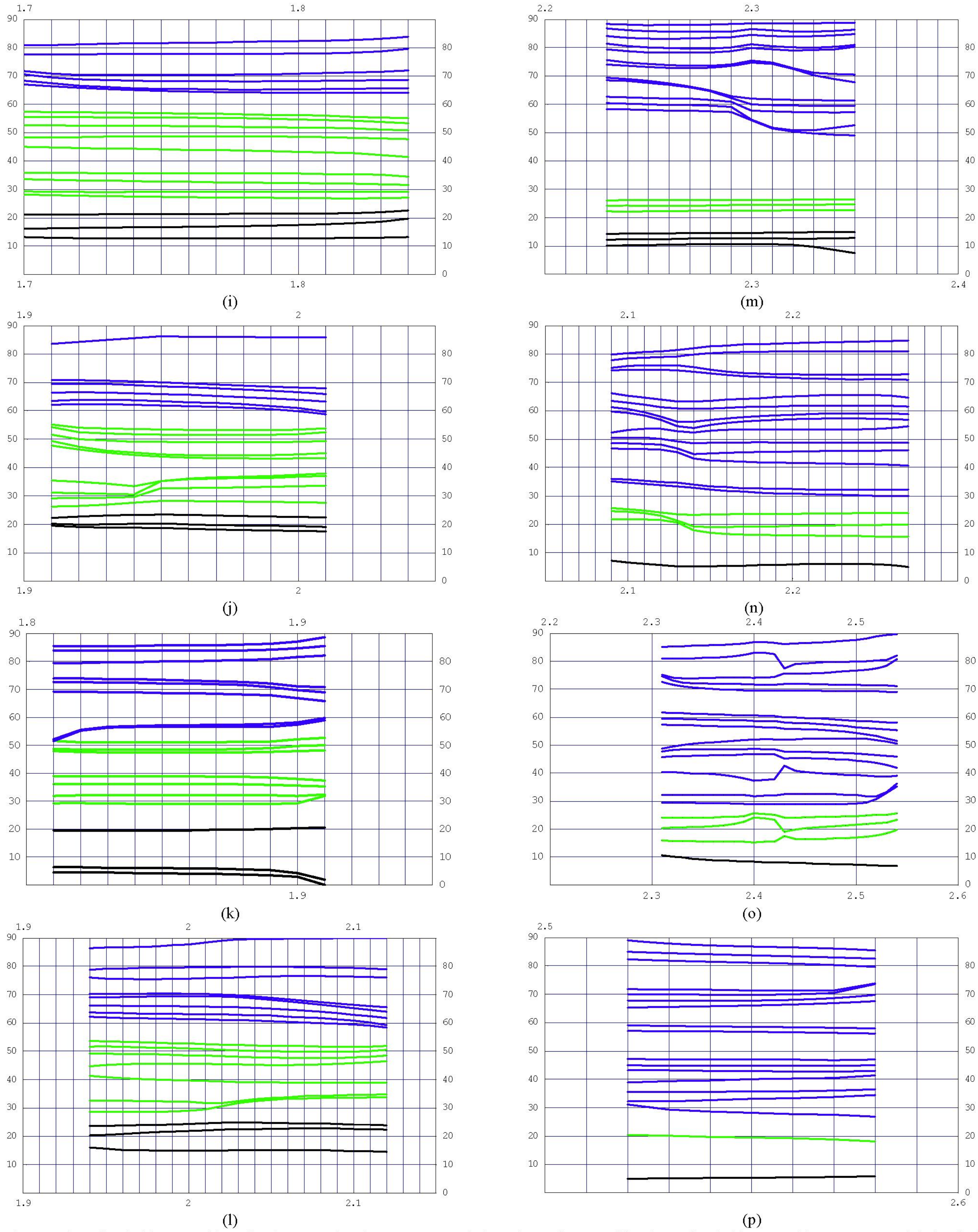


Fig. 4. Selected switching transitions for the seven-level SHE-PWM technique for various combinations of switching transitions versus modulation index. The results are summarized in Table 1. (a) Case 18/0/0, $0 \leq M \leq 0.73$ (b) Case 18/0/0, $0.68 \leq M \leq 0.83$ (c) Case 9/9/0, $0.84 \leq M \leq 0.92$ (d) Case 9/9/0, $0.92 \leq M \leq 1.05$ (e) Case 7/11/0, $1.05 \leq M \leq 1.21$ (f) Case 3/15/0 $1.22 \leq M \leq 1.43$ (g) Case 3/15/0, $1.44 \leq M \leq 1.54$ (h) Case 3/15/0, $1.54 \leq M \leq 1.72$ (i) Case 3/9/6, $1.91 \leq M \leq 2.01$ (j) Case 3/9/6 $1.7 \leq M \leq 1.84$ (k) Case 3/7/8 $1.81 \leq M \leq 1.91$ (l) Case 3/7/8, $1.94 \leq M \leq 2.12$ (m) Case 3/3/12, $2.23 \leq M \leq 2.35$ (n) Case 1/3/14, $2.09 \leq M \leq 2.27$ (o) Case 1/3/14, $2.31 \leq M \leq 2.54$ (p) Case 1/1/16, $2.52 \leq M \leq 2.58$.

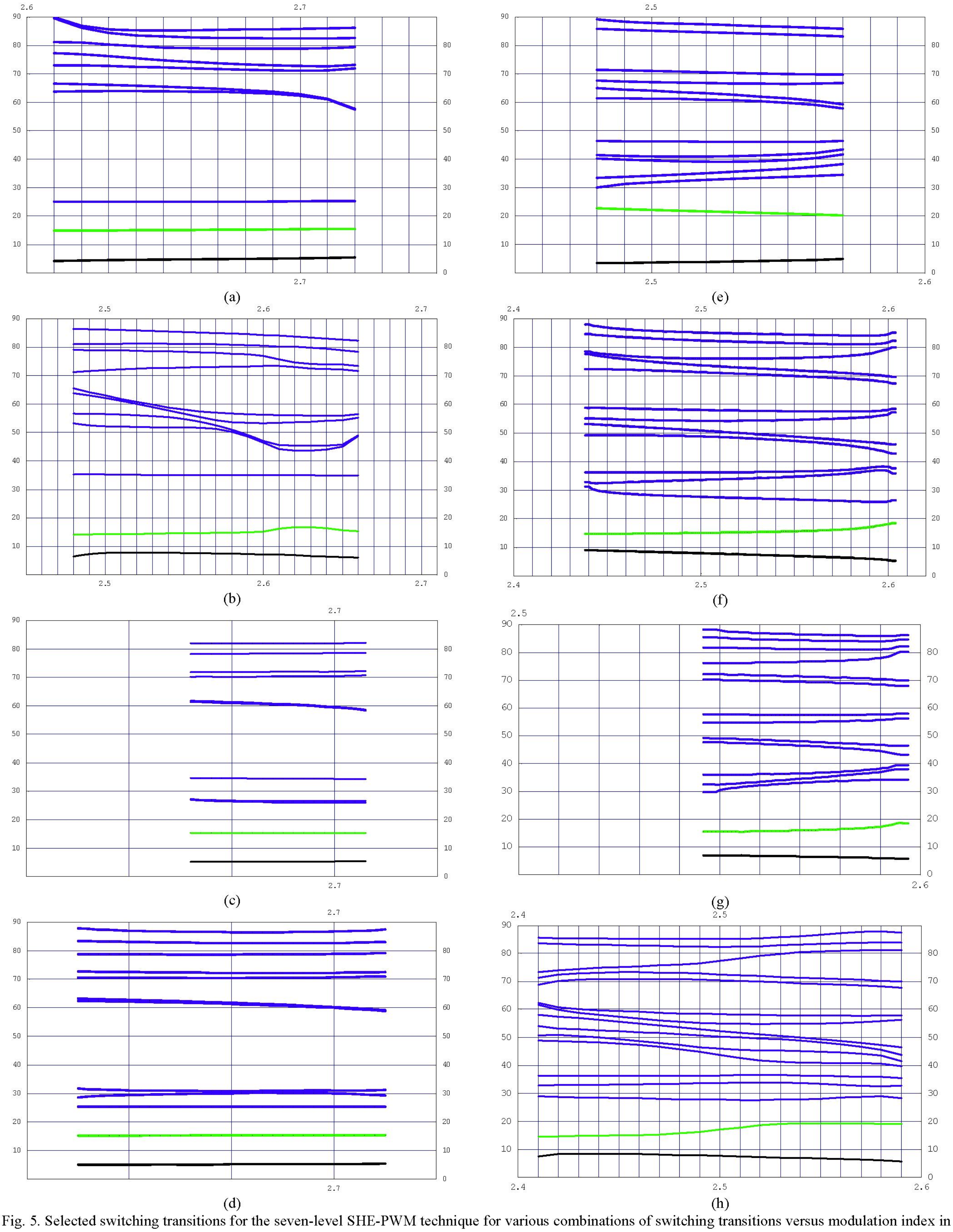


Fig. 5. Selected switching transitions for the seven-level SHE-PWM technique for various combinations of switching transitions versus modulation index in the overmodulation region. The results are summarized in Table 2. (a) Case 1/1/8, 2.61≤M≤ 2.72 (b) Case 1/1/9, 2.48≤M≤ 2.66 (c) Case 1/1/9, 2.66≤M≤ 2.72 (d) Case 1/1/10, 2.58≤M≤2.71 (e) Case 1/1/11, 2.48≤ M≤ 2.57 (f) Case 1/1/12 2.445≤M≤ 2.605 (g) Case 1/1/13, 2.545≤M≤ 2.59 (h) Case 1/1/14, 2.41≤M≤ 2.59.

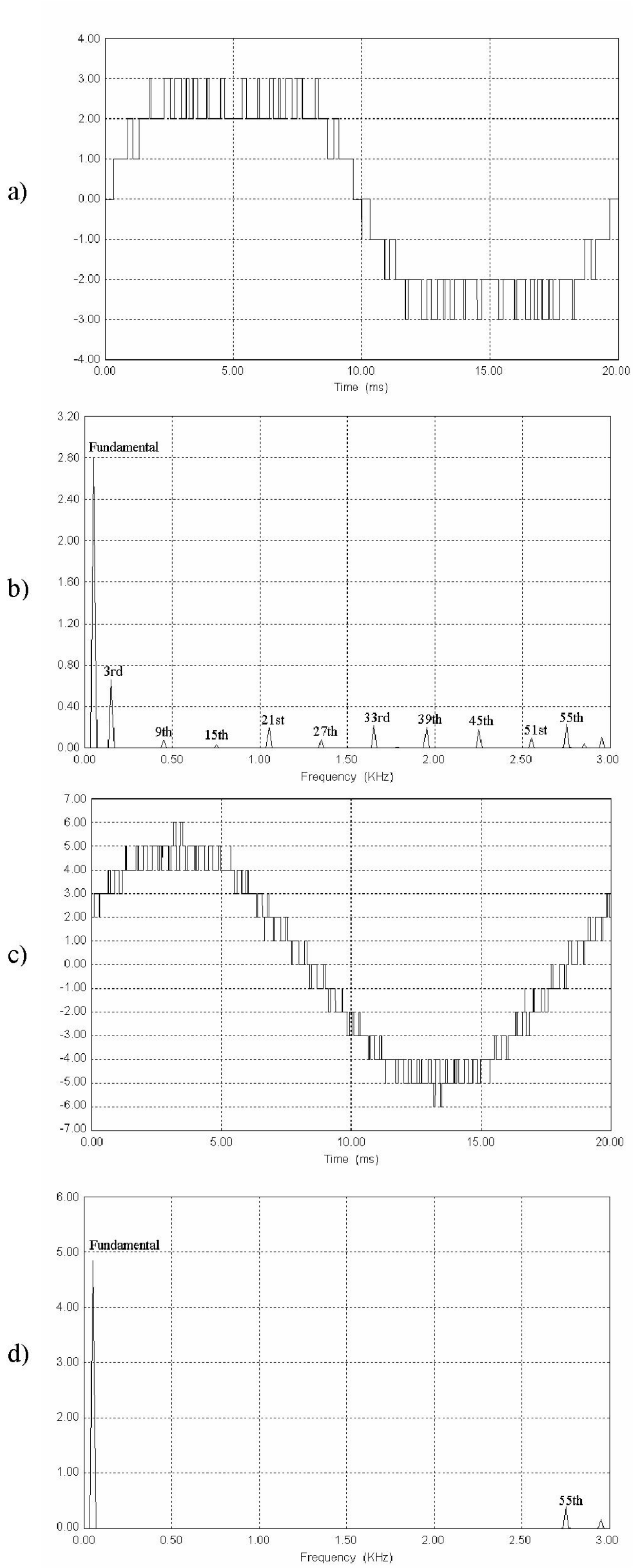


Fig. 6: Implementation of the proposed technique for the following case within the standard modulation index range: 1/3/14, M=2.2. (a) Line to neutral voltage waveform. (b) Spectrum of the line-to-neutral voltage waveform. (c) Line-to-line voltage waveform. (d) Spectrum of the line-to-line voltage waveform.

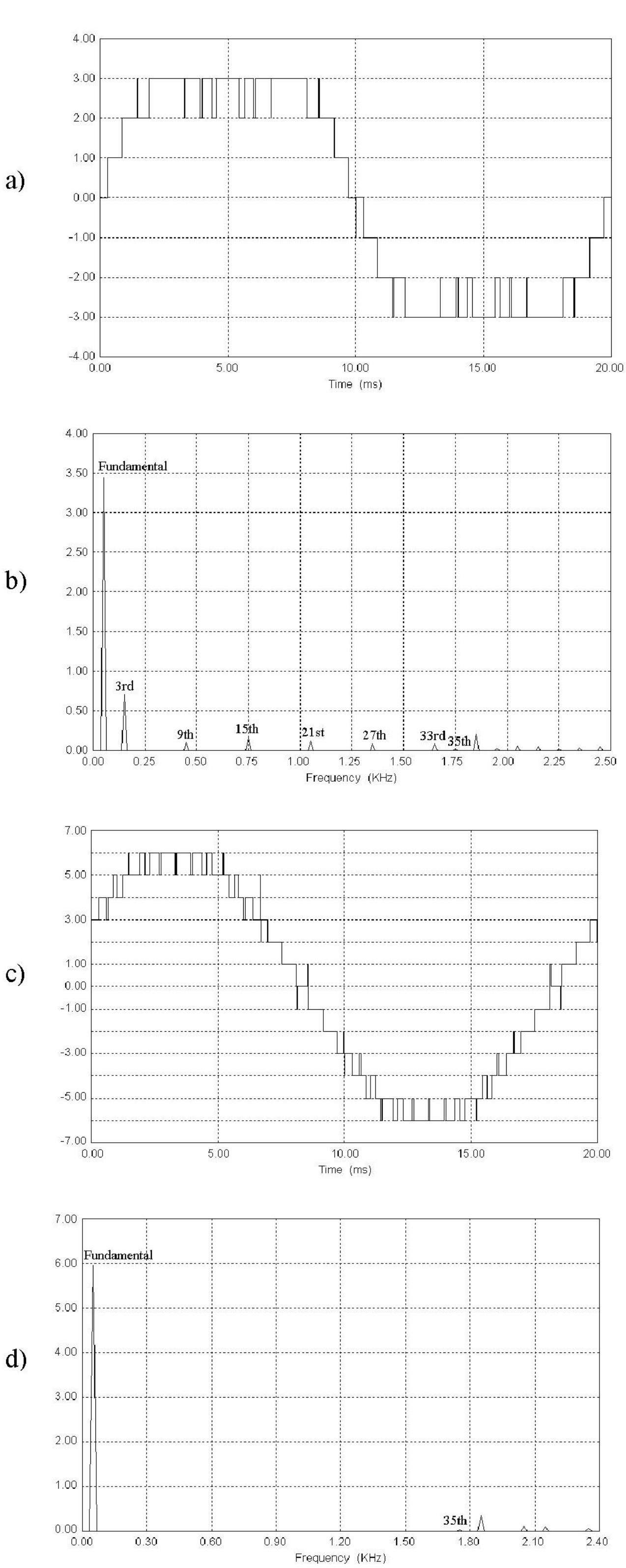


Fig. 7: Implementation of the proposed technique for the following case within the overmodulation index range: 1/1/9, M=2.7 controlling 10 non-triplen harmonics. (a) Line to neutral voltage waveform. (b) Spectrum of the line-to-neutral voltage waveform (c) Line-to-line voltage waveform. (d) Spectrum of the line-to-line voltage waveform.

Table 1: Summary of solutions for switching angles in regions when the variable ratio of distribution of the switching angles changes between the multiple levels ($N = 18$).

$z_1/z_2/z_3$	Standard Modulation Region
18/0/0	$0 \le M \le 0.73$ & $0.68 \le M \le 0.83$
9/9/0	$0.84 \le M \le 0.92$ & $0.92 \le M \le 1.05$
7/11/0	$1.05 \le M \le 1.21$
3/15/0	$1.22 \le M \le 1.43$ & $1.44 \le M \le 1.54$ $1.54 \le M \le 1.72$
3/9/6	$1.91 \le M \le 2.01$ & $1.7 \le M \le 1.84$
3/7/8	$1.81 \le M \le 1.91$ & $1.94 \le M \le 2.12$
3/3/12	$2.23 \le M \le 2.35$
1/3/14	$2.09 \le M \le 2.27$ & $2.31 \le M \le 2.54$
1/1/16	$2.52 \le M \le 2.58$

Table 2: Summary of solutions for switching angles in the overmodulation regions when the variable ratio of distribution of the switching angles changes between the multiple levels (N=varies).

$z_1/z_2/z_3$	Overmodulation Region
1/1/8, N=10	$2.61 \le M \le 2.72$
1/1/9, N=11	$2.48 \le M \le 2.66$ & $2.66 \le M \le 2.72$
1/1/10, N=12	$2.58 \le M \le 2.71$
1/1/11, N=13	$2.48 \le M \le 2.57$
1/1/12, N=14	$2.445 \le M \le 2.605$
1/1/13, N=15	$2.545 \le M \le 2.59$
1/1/14, N=16	$2.41 \le M \le 2.59$

varies. To confirm the validity of the method, the line-to-neutral and the line-to-line voltage waveforms are plotted in Fig. 7(a) and (c) respectively for M=2.7 and the case of 1/1/9. The spectrum of both waveforms presented in Fig. 7(b) and 7(d) respectively confirm that the bandwidth is compromised but the harmonics are still tightly controlled confirming the superiority of the proposed method. Since N=11, 10 non-triplen harmonics are controlled. By investigating other ratios of switching angles a further increase of the modulation index can be obtained but it compromises the harmonic bandwidth even further.

IV. CONCLUSIONS

A seven-level MSHE-PWM strategy suitable for multilevel converters has been documented in this paper. The method is mathematically defined in order to eliminate a number of harmonics from the waveform. It is shown that by distributing a number of switching angles over multiple levels, solutions can be found offering a degree of overlapping and thus redundancy, allowing implementation of the proposed method over the entire range of modulation indices. Switching angles have been reported for both the standard and the overmodulation range. The spectrum of both the line-to-neutral and therefore the line-to-line waveforms are well controlled. This is confirmed through selected results presented to validate the theory.

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