Generalized Formulation of Multilevel Selective Harmonic Elimination PWM: Case I -Non-Equal DC Sources

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Abstract— The paper presents optimal solutions for eliminating harmonics from the output waveform of a multilevel staircase pulse-width modulation (PWM) method with non-equal dc sources. Therefore, the degrees of freedom for specifying the cost function increased without physical changes as compared to the conventional stepped waveform. The paper discusses an efficient hybrid real coded genetic algorithm (HRCGA) that reduces significantly the computational burden resulting in fast convergence. An objective function describing a measure of effectiveness of eliminating selected order of harmonics while controlling the fundamental for any number of levels and for any number of switching angels is derived. It is confirmed that multiple independent sets of solutions exist and the ones that offer better harmonic performance are identified. Different operating points including five- and seven-level inverters are investigated and simulated. Selected experimental results are reported to verify and validate the effectiveness of the proposed method.

I. INTRODUCTION

Multilevel inverters have drawn tremendous interest in the field of high-voltage and high-power applications such as laminators, mills, conveyors, pumps, fans, blowers, compressors, and so on. The term multilevel starts with the introduction of the three-level inverter [1]. By increasing the number of levels in a given topology, the output voltages have more steps generating a staircase waveform, which approaches closely the desired sinusoidal waveform and also has reduced harmonic distortion [1]-[6]. One promising technology to interface battery packs in electric and hybrid electric vehicles are multilevel inverters because of the possibility of high VA rating and low harmonic distortion without the use of transformer [3].

Various multilevel inverters structures are reported in the literature, and the cascaded multi-cell inverter appears to be superior to other multilevel inverters in application at high power rating [3]. It is worth noting that in most of the works reported in the literature, the level of the dc sources was assumed to be equal and constant, which will probably not be the case in applications even if the sources are nominally equal [3].

Selective harmonic elimination (SHE-PWM) has been mainly developed for two- and three-level inverters in order to achieve lower total harmonic distortion (THD) in the voltage output waveform [2], [9], [10] and [12]-[14]. The common characteristic of the selective harmonic elimination method is that the waveform analysis is performed using Fourier theory [9]-[10]. Sets of non-linear transcendental equations are then derived, and the solution is obtained using an iterative procedure, mostly by Newton-Raphson method [2]. This method is derivative-dependent and may end in local optima; further, a judicious choice of the initial values alone will guarantee convergence [6]-[10]. Another approach uses Walsh functions [7] where solving linear equations, instead of solving non-linear transcendental equations, optimizes the switching angles. More recently, a technique based on a combination of an interval-search procedure and Newton's method is proposed to find all solutions to the nonlinear transcendental equations [12]. However, the method was only applied to the two-level inverters. References [13] and [14] generalized the problem formulation where constraint of quarter-wave symmetry is relaxed to half-wave symmetry where all the even harmonics are zero but the harmonic phasing is free to vary. However, only the two- and three-level waveforms were reported and the technique has not been extended to multilevel case where the problem becomes more challenging.

On the other hand, SHE-PWM methods have been introduced to multilevel converters in several technical articles. Initially the switching frequency was restricted to line frequency and therefore, the stair case multilevel waveform was arranged in such a way as to control the fundamental and eliminate the low order harmonic from the waveform [3], [4], [6] and [8]. In references [4], [5] and [16], these transcendental equations are converted into polynomial equations where the resultant theory is applied to determine the switching angles to eliminate specific harmonics. However, as the number of DC sources increases, the degrees of the polynomials in these equations are large which increase the computation difficulty and furthermore if one wanted to apply this method to multilevel converter with non-equal DC source which is always the case in most applications, the set of transcendental equations to be solved are no longer symmetry and require the solution of a set of high-degree equations which is beyond the capability of contemporary computer algebra. A new active harmonic elimination technique was recently introduced to the line frequency method aiming to eliminate higher order of harmonics by simply generating the opposite of the harmonics to cancel them [15]. However, the disadvantage in that is that it uses a high switching frequency to eliminate higher order harmonics. Other approaches have also been reported including one where the harmonic elimination is combined with a programmed method [5] and another where multilevel SHE-PWM defined by the well-know multicarrier phase-shifted PWM (MPS-PWM) was proposed in [11] where the modulation index defines the distribution of the switching angles and then the problem of SHE-PWM is applied to a particular operating point aiming to obtain the optimum position of these switching transitions that offer elimination to a selected order of harmonics. In this paper a minimization technique which has been reported in [9], [10] is once again applied to find the solution for the resultant non-linear equations. However, the SHE-PWM technique was only reported for five-level and with equal DC sources.

More recently, a general genetic algorithm using MATLAB GA Optimization Toolbox was applied to solve the same problem of SHE [6]. However, the paper shows only the solutions to the equal DC sources and with the fundamental frequency switching method which has lower degrees of freedom for specifying the cost function compared with the proposed PWM method for the same physical structure. Moreover, the authors have not identified the best solution sets among those found.

In this paper, a cascade multilevel PWM inverter with non-equal dc sources is investigated. The main objectives of this paper are: first to reformulate the problem of SHE for multilevel inverters based on PWM waveform using the staircase method. Where individual cells are operated with a frequency that is close to the fundamental frequency (two or three times) so that the switching losses are relatively low and higher degrees of freedom for specifying the cost function and therefore higher bandwidth when compared to existing family of techniques such as conventional stepped waveform is gained. Secondly, to introduce a minimization technique assisted with a hybrid genetic algorithm in order to greatly reduce the computational burden associated with the nonlinear transcendental equations of the selective harmonic elimination method [8].

The paper is organized as follows. Section II presents the formulation along with analysis for the generalized staircase PWM voltage waveform. Section III discusses the implementation of the hybrid genetic algorithm. Results for a number of selected cases are provided in Section IV and finally conclusions are summarized in Section V.

II. PROBLEM FORMULATION AND ANALYSIS

Fourier series expansion of the generalized staircase PWM output waveform of the single-phase multilevel inverter shown in Fig. 1(b) can be easily expressed as follows:

$$V_{out} = \sum_{n=1}^{\infty} A_n \cos n\theta + B_n \sin n\theta \qquad (1)$$

Owing to the PWM waveform characteristics of odd function symmetry and half-wave symmetry, the output voltage can be reduced to:

$$V_{out} = \sum_{n=1,3,5,\ldots}^{\infty} B_n \sin n\theta \qquad (2)$$

where, B_n is the Fourier coefficient.

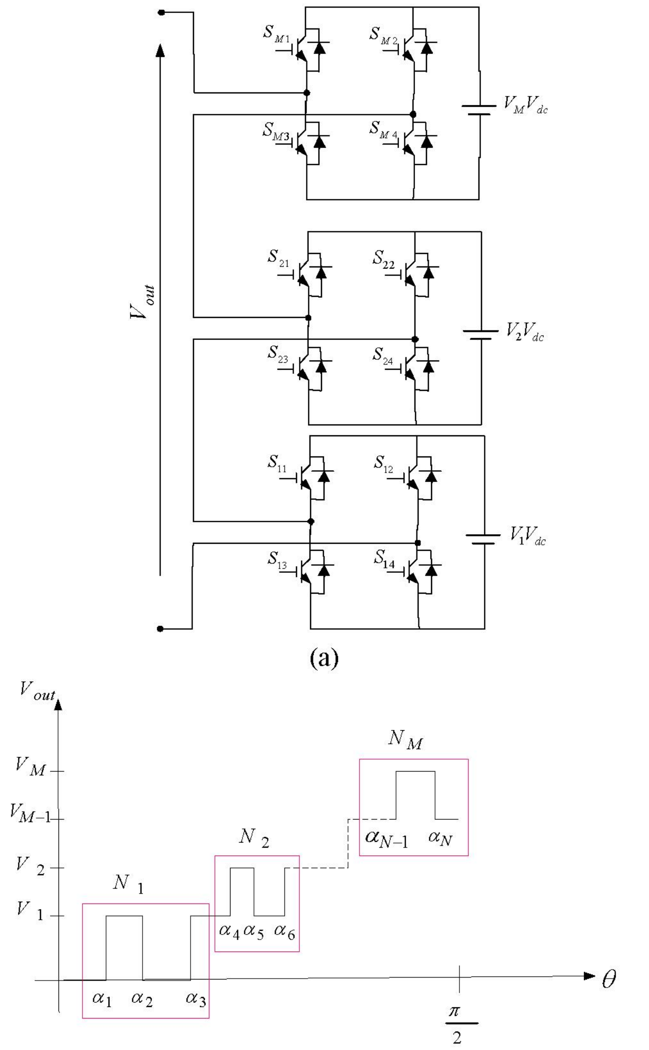


Fig.1. Phase circuit of a multilevel inverter system. (a) Cascaded H-bridge based inverter. (b) Generalized staircase PWM waveform.

With the assumption of non-equal dc level of each inverter cell, a generalized expression of B_n (i.e. for the single-phase case) and for any number of switching angles (even or odd, provided that the waveform is physically correct and can be implemented) is given by

$$B_n = \frac{4V_{dc}}{n\pi}\Bigg[\sum_{n=1,3,5,\ldots}^{2N-1} \Bigg(V_1 \sum_{k=1}^{P_1} (-1)^{k+1} \cos n\alpha_k \pm V_2 \sum_{k=P_1+1}^{P_2} (-1)^{k} \cos n\alpha_k \pm \ldots \pm V_M \sum_{k=P_{M-1}+1}^{P_M} (-1)^{k} \cos n\alpha_k\Bigg)\Bigg] \qquad (3)$$

where,

$n = 1, 3, 5, \ldots, 2N-1$, for single-phase system

$n = 1, 5, 7, \ldots, 3N-2$, for three-phase system and N is odd

$n = 1, 5, 7, \ldots, 3N-1$, for three-phase system and N is even

M = the number of dc sources (i.e. inverter cells) and the product $V_M V_{dc}$ is the value of the M^{th} dc source

$P_1 = N_1$, $P_2 = N_1 + N_2$,….., $P_M = N_1 + N_2 + \cdots + N_M$,

$N_1, N_2, \ldots, N_M$ are the number of pulses per-quarter cycle at inverter $1, 2, \ldots, M$, respectively.

$N = N_1 + N_2 + \ldots + N_M$ (Number of pulses per-quarter cycle of the multilevel inverter output voltage),

α_k is the k^{th} switching angle, and

In eqn. (3), the polarity $\pm$ is positive if P_{M-1} is an odd number otherwise it is negative.

Eqn. (3) has N unknown variables ($\alpha_1, \alpha_2, \ldots, \alpha_N$) and a set of solutions is obtainable by equating $N-1$ harmonics to zero and assigning a specific value to the fundamental.

Solutions to eqn. (3) can be obtained through any of those methods highlighted in section I.

However, these methods suffer from various drawbacks, such as prolonged and tedious computational steps [2], [12] convergence to local optima, increase the degrees of the polynomials if the technique in [3], [4] is considered, etc. Further, the number of harmonics eliminated by these techniques is linked to the number of switching angles; hence, the more the number of harmonics to be eliminated, the larger the computational complexity and time.

To alleviate such problems, a hybrid genetic algorithm approach is proposed, which solves the same problem with a simpler formulation, with any number of levels or switching angles and without extensive derivation of analytical expressions.

An objective function describing a measure of effectiveness of eliminating selected order of harmonics while controlling the fundamental must be defined. Two predominate methods in choosing the switching angles, $\alpha_1, \alpha_2, \ldots, \alpha_N$ can be considered: 1) eliminate the lower frequency dominant harmonics or 2) minimize the total harmonic distortion (THD). The first method is the most popular and straightforward of the two techniques, that is to eliminate the lower dominant harmonics and the filter or the nature of the load will take care of the higher residual frequencies. In this paper the first method is also chosen. Hence, the objective function (for the single-phase case) is defined as

$$F(\alpha_1, \alpha_2, \ldots, \alpha_N) = (B_1 - A_0)^2 + B_3^2 + B_5^2 + \ldots + B_n^2 \quad (4)$$

where $A_0 = \dfrac{m_a \pi}{4}$, and (5)

$m_a = \dfrac{V_F}{V_{dc}}$, is the modulation index

Noting that V_F is the fundamental component,

and $0 < m_a < M$.

The optimal switching angles are obtained by minimizing eqn. (4) when it is subject to the constraint of eqn. (6). Consequently selected harmonics are eliminated. These switching angles are generated for different operating points and then stored in look-up tables to be used to control the inverter for certain operating point.

$$(0 \le \alpha_1 \le \alpha_2 \le \ldots \le \alpha_N \le \frac{\pi}{2}) \quad (6)$$

III. Hybrid Genetic Algorithm Implementation

Genetic algorithms (GAs) are highly suited to search spaces which are not well defined or have a high number of local minima, which plague more traditional calculus-based search methods. By removing the need for auxiliary information regarding the optimization surface the computational requirements are greatly reduced.

The GA necessitates the need for the optimization variables ($\alpha_1, \alpha_2, \ldots, \alpha_N$) to be coded as population of strings transformed by three Genetic operators: selection, crossover, and mutation.

The presented hybrid genetic algorithm combines a standard real coded GA and the phase-2 of conventional search technique.

A. Phase-1 (Real Coded) Algorithm

Real coded genetic algorithm is implemented as follows,

1- A population of N_P trail solution is initialized. Each solution is taken as a real valued vector with their dimensions corresponding to the number of variables ($\alpha_1, \alpha_2, \ldots, \alpha_N$). The initial components of α_i are selected in accordance with a uniform distribution ranging between 0 and 1.

2- The fitness score for each solution vector α_i is evaluated, after converting each solution into corresponding switching instants α_a using upper and lower bounds.

3- Roulette wheel based selection method is used to produce N_P offspring from parents.

4- Arithmetic crossover and non-uniform mutation operators are applied to offspring to generate next generation parents.

The algorithm proceeds to step 2, unless the best solution does not change for a pre-specified interval of generations.

B. Phase-2 (Direct Search Optimization Method) Algorithm

After the phase-1 is halted, satisfying the halting condition described in the previous section, optimization by direct search and systematic reduction of the size of search region method is employed in the phase-2. In the light of the solution accuracy, the success rate, and the computation time the best vector obtained form the phase-1 is used as an initial point for the phase-2.

The optimization procedure based on direct search and systematic reduction in search region is found effective in solving various problems in the field of nonlinear programming [16]. This direct search optimization procedure is implemented as follows

1- The best solution vector obtained from the first phase of the hybrid algorithm is used as an initial point $\alpha(0)$ for phase-2 and an initial range vector is defined as

$$R(0) = RMF \times Range \quad (7)$$

Where, $Range$ is defined as the difference between the upper and lower bound (the upper and lower bound for each switching angle here are $\pi/2$ and 0, respectively) and RMF is a range multiplication factor. The value of RMF varies between 0.0 and 1.0.

2- N_S trail solution vectors around $\alpha(0)$ are generated using following relationship,

$$\alpha_i = \alpha(0) + \alpha(0) .* rand(1, n) \tag{8}$$

where, α_i is the i^{th} trail solution vector, $(.^*)$ represents element-by-element multiplication operation, and $rand(1, n)$ is a random vector, whose element value varies from – 0.5 to 0.5.

3- For each feasible trail solution vector find objective function value and find the trail solution set, which minimizes $F(\alpha)$ and equate it to $\alpha(0)$ as follows,

$$\alpha(0) = \alpha_{best} \tag{9}$$

where, α_{best} is the trail solution set with minimum $F(\alpha)$.

4- Reduce the range by an amount given by $R(0) = R(0) * (1 - \beta)$.
where, β is the range reduction factor, whose typical value is 0.05.

5- The algorithm proceeds to step 2, unless the best solution does not change for a pre-specified interval of generations.

IV. Results and Discussion

In order to verify the validity of the proposed algorithm, a program was developed using MATLAB 6.0 software package [17]. The program is run for a number of independent trials. The proposed technique has been applied successfully to different cases in order to indicate its ruggedness. The simulation results are obtained accordingly using PSIM software package [18].

A low-power laboratory five-level inverter prototype based on the two IGBT H-bridges configuration was developed and tested to verify the feasibility and the validity of the theoretical and the simulation findings. The pre-calculated PWM signals are implemented using low-cost high-speed Texas Instruments TMS320F2812 digital signal processor (DSP) board with an accuracy of 20µseconds. A digital real-time oscilloscope (Tektronix TDS210) was used to display and capture the output waveforms and with the feature of the fast Fourier transform (FFT), the spectrum of each of the output voltage was obtained.

A discussion on the results is presented in the following sections. It is worth noting that in the following discussion, the levels of the dc sources are non-equal and can be measured; furthermore, each dc source has a nominal value of 1 p.u.

A. *Case I: Five-level inverter with V_1 =1, V_2 = 0.80 p.u.*

In this case, two-cell inverter with different DC source levels (i.e. $V_1 = 1, V_2 = 0.80$ p.u.) was considered. It is assumed here that each individual inverter cell is operating at frequency of three times of the fundamental one. As a result, there are three switching angles per-quarter cycle at each level of the output waveform. Hence, there are six switching angles per-quarter cycle of the output waveform of the multilevel inverter, which offer elimination of five low order of harmonics and controlling the fundamental at a certain value. The switching angles variation against m_a for the said case is plotted in Fig. 2. It is confirmed that more than one set of solutions are exist and for a certain range of m_a (i.e. set 1, $1.1 \le m_a \le 1.94$, set 2, $1.1 \le m_a \le 2.0$ and set 3, $1.94 \le m_a \le 2.0$). As an example, an operating point when $m_a = 1.48$ was chosen to illustrate the effectiveness of the proposed method and the results (i.e. simulation and experimental) are shown in Fig. 3 (a)-(d). It is evident that the targeted harmonics (3rd, 5th, 7th, 9th, and 11th) are eliminated and the next significant harmonic appearing in the output voltage is the 13th. Good agreement between the simulation and the experimental one is obtained.

On the other hand, the next significant harmonic could be considerably shifted further (i.e. 19th) if three-phase system is considered where all triplen harmonics are eliminated by the phase shift between the two phases.

B. *Case II: Seven-level inverter with V_1 =1, V_2 = 0.95 and V_3 = 0.85 p.u.*

Three inverter cells with unequal dc sources are cascaded in this case, once again each inverter cell is operated at a three times of the fundamental frequency. Therefore, there are three switching angles per-quarter cycle at output waveform of each inverter cell. As a result there are nine degrees of freedom offering the elimination of eight low order of harmonics and controlling the fundamental component. The solution sets for the switching angles for a certain range of m_a (set 1, $2.1 \le m_a \le 2.9$, set 2, $2.1 \le m_a \le 2.9$ and set 3, $2.92 \le m_a \le 3.0$) is illustrated in Fig. 4. Fig. 5 (a) depicts the output voltage waveform and its spectrum in Fig. 5 (b) where the absence of selected harmonics (3rd, 5th,….,17th) is clearly evident for the given fundamental value which is 2.90 p.u. It is worth noting that the most significant order of harmonic appears in the output waveform is the 19th and once again it could be further shifted to the 31st if the three-phase system was considered.

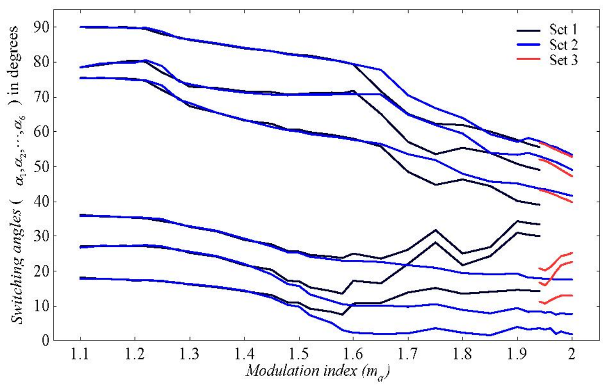
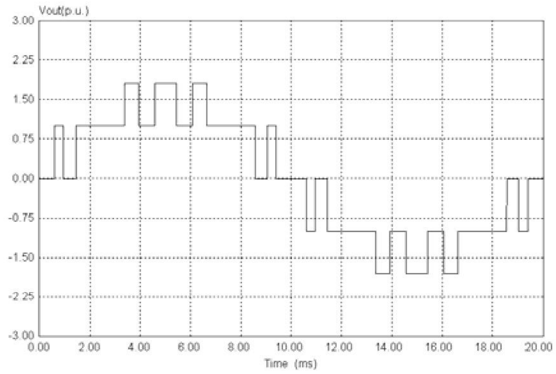
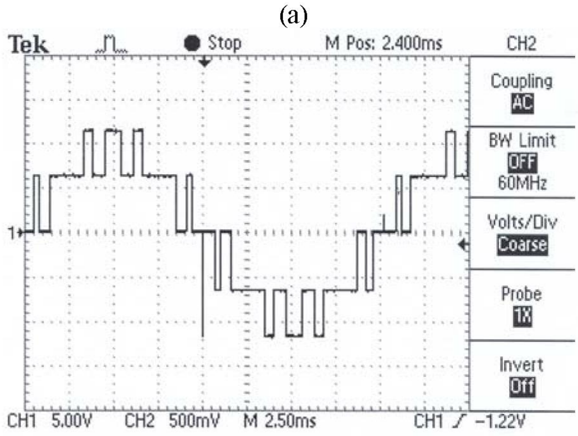


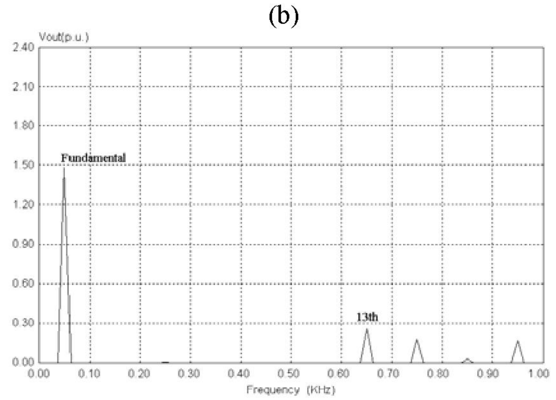
Fig. 2: Switching angles vs. m_a (case *I*).



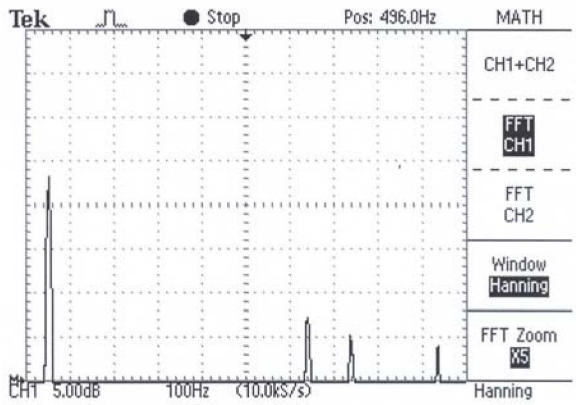
(a)



(b)



(c)



(d)

Fig. 3: Case I: Five-level inverter with $m_a = 1.48$. (a) Output voltage waveform (simulation). (b) Spectrum of output voltage (simulation). (c) Output voltage waveform (experimental). (d) Spectrum of output voltage (experimental)

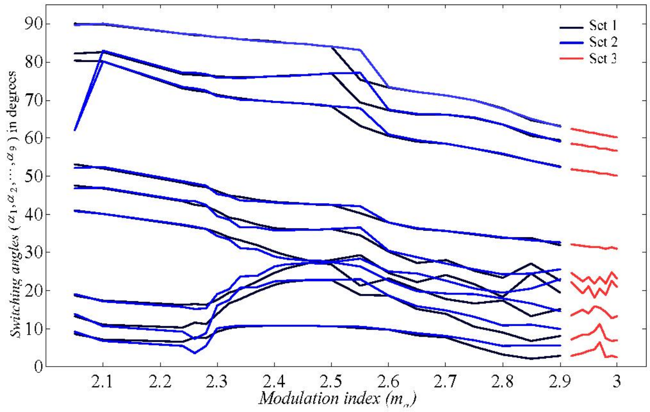
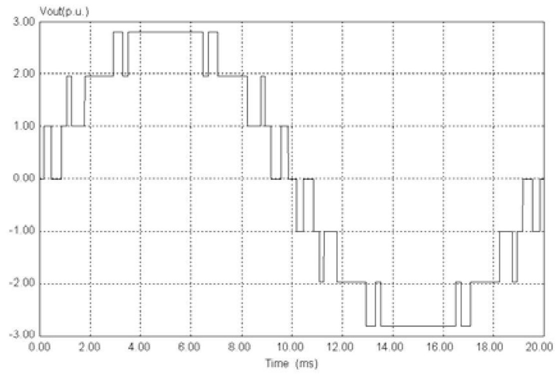
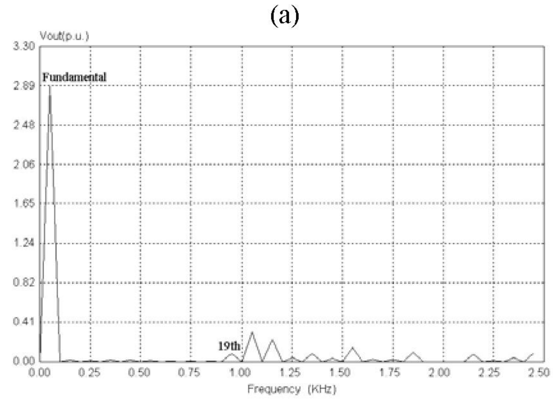


Fig. 4: Switching angles vs. m_a (case *II*)



(a)



(b)

Fig. 5: Case II: Seven-level inverter with $m_a = 2.90$. (a) Output voltage waveform (simulation). (b) Spectrum of output voltage (simulation).

C. *Performance index of optimality*

It has been clearly shown that there are more than one set of solutions with a different range of m_a. It is therefore, important that the optimum set be identified with respect to harmonic performance. For this purpose, the total harmonic distortion (THD) is plotted in Fig. 6 for the two wide continuous sets (set 1 and set 2).

For practical reasons, the THD is calculated using eqn. (10) and up to 39th order of harmonics is taken into account. The low pass filter and the nature of the highly inductive load will take care of the higher order of harmonics.

$$THD = \frac{\sqrt{\sum_{n=2}^{39} V_n^2}}{V_1} \text{ (p.u.)} \tag{10}$$

It is obviously found that set 1 performs better than set 2 for both *case I* and *case II*. Furthermore, it is noticed that the higher the value of m_a and the number of levels the lower the THD.

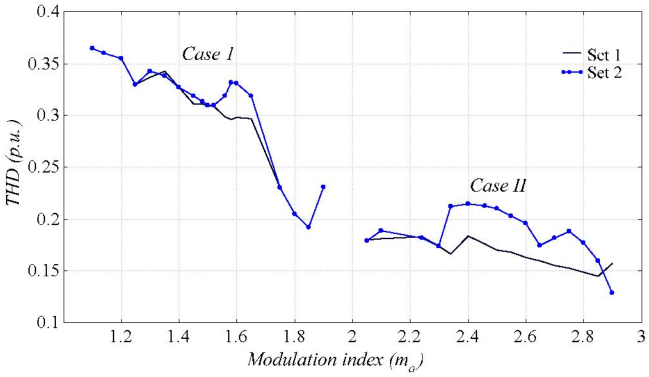


Fig. 6: Total harmonic distortion (THD) vs. m_a

V. Conclusions

A technique to generate optimal switching angles to eliminate a certain order of harmonics from the output waveform of multilevel PWM inverter waveform with non-equal dc sources is reported in this paper. The number of eliminated harmonics is increased by more than two times compared to the conventional stepped waveform for the same physical structure (i.e. number of cells). Two-phase hybrid genetic algorithm namely real-coded and direct search is proposed to overcome the computational burden and to ensure the accuracy of the calculated angles [8]. The algorithm was developed using MATLAB software and is run for a number of times independently to ensure the feasibility and the quality of the solution. The algorithm finds the complete set of solutions and confirms that more than one exist. The merit of superiority of this method is that neither a close initial point to the exact one that is for Newton-Raphson nor conversion of the problem to a set of polynomial equations are needed, further ensuring an optimal solution and reducing the computational effort. As a figure of optimality, THD is chosen to evaluate the performance of the proposed method and to identify the optimal solution set. The simulation results presented for two different cases including five- and seven-level inverters and selected experimental results are presented to verify the theoretical and simulation findings.

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