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Graphics calculators in upper secondary courses

prepared for the
Secondary Education Authority

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Purpose

This paper has been produced on request of the Secondary Education Authority, as part of a process of considering the potential impact of graphics calculators on upper secondary school courses. The paper provides background on this matter for the Authority and for committee members, including syllabus committees that may be affected by the decision to permit the use of graphics calculators in Tertiary Entrance Examinations.

Introduction

Graphics calculators are small, hand-held portable calculators, with significant mathematical capabilities. Although relatively novel and recent in Western Australia, they have been available to the general public in the United States and Europe since 1985, and have enjoyed increasing popularity in secondary schools and undergraduate colleges abroad over the last decade. Most graphics calculators are a little larger and a little heavier than typical scientific calculators of the kinds that are popular in secondary schools, but are still small enough and light enough to be carried with ease from class to class or from school to home in a schoolbag. (A typical weight is around 200 g.) All graphics calculators are battery powered; none are yet solar powered, due presumably to their need for substantial power. Most tend to draw a bit more heavily on batteries than do scientific calculators. All graphics calculators have a constant memory facility, retaining the contents of the memory when the calculator is turned off, and an automatic shutdown facility, turning the device off after several minutes of inactivity. The calculators are designed on the assumption that owners will use them to store information (such as statistical data or programs) for long periods. All graphics calculators can be operated quietly. (The few that have a 'beep' function allow for it to be turned off.)

Graphics calculators are generally readily recognisable because they have a considerably larger display screen than do typical scientific calculators. The screen allows for several lines of information to be displayed or for graphical information to be presented. This graphical capability has led to the use of adjectives such as 'graphics', 'graphing' and 'graphical' to describe these calculators, and the resulting over-emphasis on the graphing capabilities of the device. While graphics calculators typically enjoy substantial graphical capabilities, they also have many mathematical capabilities that do not depend on graphical representations. Several commentators have suggested in fact that the term, 'graphics calculator' is something of a misnomer, since the devices are actually pocket computers with significant inbuilt mathematical software, only some of which is graphical in orientation. Indeed, several currently available models have around 32K of available memory, which was regarded as appropriate for a fairly large microcomputer in schools in the late 1970's and early 1980's.

Calculator capabilities

The suite of mathematical capabilities on a graphics calculator varies from machine to machine, depending in part on manufacturer preferences and impressions of the needs of the market. However, since most graphics calculators have been constructed with the educational market in mind, there are many capabilities shared by most machines, especially newer ones. The most common capabilities found on graphics calculators are:

- Standard scientific calculator functions are available, generally using conventional algebraic syntax. Thus it is quite unnecessary for anyone to own *both* a scientific calculator and a graphics calculator, unless conflicting demands are made by courses in schools. A number of memories, generally at least one for each letter of the alphabet, is available for storage and retrieval of single numbers.
- Elementary functions can be graphed with arbitrary ranges on the coordinate plane. Several functions can be graphed simultaneously, and graphs manipulated to investigate more closely aspects of interest, such as points of intersection, intercepts with the axes and extreme values of functions. On later machines, commands to find *automatically* intercepts of graphs with axes, points of intersection of pairs of graphs and relative extrema of functions are also available, reducing the tedium of scrolling and tracing graphs.
- Most calculators allow for at least one other kind of graph to be drawn, and many allow for at least two others. Polar coordinates and parametrically defined functions are the two likely choices in addition to coordinate graphs; at least two calculators also accommodate recursively defined functions with corresponding graphs (such as 'cobweb' graphs).
- Some calculators include a facility to graph inequalities, by shading the area under or over a graph. In some (but not all) cases, this facility can be used to graph compound inequalities; in some other cases, the facility is limited to two functions, which is less than the requirements of typical linear programming contexts prevalent in secondary schools.
- Data can be stored and analysed. Typically, at least two variables can be dealt with simultaneously, and the analysis is at least comparable with that of so-called bivariate scientific calculators (i.e., means, standard deviations, regression coefficients and product-moment correlation). Graphical displays of data are also available, including usually histograms, scatter plots and least-squares regression lines (both linear and non-linear). Stored data can be reviewed, edited, augmented or deleted, unlike the case of scientific calculators, for which only summary data are stored.
- Matrices can be stored and manipulated. Minimally, matrix addition, multiplication and inversion are available, so that most systems of linear equations can be numerically solved by matrix methods. Other matrix operations are available, such as finding determinants and transposes of matrices, and performing elementary row operations.
- Numerical calculus facilities are generally available (although some machines produced for junior high school in the US do not have these). Typically, numerical values of derivatives of elementary functions at a point as well as definite integrals are available to reasonable accuracy. Some calculators also provide visual representations of definite integrals as areas under curves, although these tend to be less accurate.
- Most calculators have some mechanisms for providing approximate numerical solutions to elementary equations. In some cases, these are restricted to graphical approximations, but most rely on iterative procedures unrelated to graphs.
- More recent machines have tabular facilities that allow for a numerical representation of a function as an alternative to a graphical representation. (It seems unlikely that future machines will be produced without such a capability, in fact.) Such facilities allow for tables of values to be scrutinised with a view to finding directly approximate solutions to equations or relative extrema of functions. On some calculators, it is possible to split the screen so that a table of values and a graph are shown simultaneously.
- Complex numbers are dealt with by some calculators as a separate mode of operation, while other calculators display complex numbers whenever they occur naturally, such as in the solution of some equations. Some calculators, designed for younger students, do not have facilities for handling complex numbers at all, however.
- Most graphics calculators have some facility to communicate with other calculators or with peripheral devices, including microcomputers and overhead projectors. The mechanisms for this communication vary somewhat, including for example, optional purchase of a suitable cable, a cable included routinely with each calculator, special models with communication ports, infra-red facilities built into

each machine and special overhead projector models. As yet, the industry has not decided which is the optimal arrangement. Communication usually allows any kinds of transfer of information, with the most likely use being the transference of calculator programs, data matrices, graphical environments or complete memory contents.
- Some graphics calculators have facilities for dealing with sequences and series, in some cases through the use of list structures. Typically, sequences can be generated automatically using either an explicit or recursive rule, and successive terms summed automatically on request.

Potentially troublesome capabilities

Some graphics calculator capabilities may be a source of unease to syllabus committees, principally because of concerns regarding public examinations. One kind of concern may be that some students with more powerful calculators may have an advantage over those with less powerful calculators. A second kind of concern is the possibility of students bringing personal notes of some kind with them into an examination, despite directions not to do so.

- Programmability is built into all graphics calculators, on the assumption that the user will want to customise the device to their own needs to an extent. Programming languages vary from being rather primitive (like those of programmable scientific calculators of the 1980's) to being rather better structured (including *while*, *repeat*, and *do* kinds of statements). Machines vary somewhat in terms of their available memory for programs. As noted later in this paper, there may be some advantage taken of programmability to reduce differences betwen calculator models.
- Most graphics calculators could be used to store a limited amount of alphabetic information, such as notes, although it is generally rather troublesome to do so. In some cases, a facility for brief memos is available, while in other cases the programmable facility would have to be used for such a purpose (by writing a 'program' consisting of notes.) At present graphics calculators do not have QWERTY keyboards, so that storage of notes is tedious. The structure of the calculator memories would generally mean that retrieval of notes is tedious and difficult, too.
- Some graphics calculators (those manufactured by Hewlett Packard, and shortly also one by Texas Instruments) have limited symbolic manipulation facilities, about which some concern has already been expressed by examination authorities in other places (including Victoria, which prohibits calculators with symbolic manipulation capabilities). These facilities allow for elementary algebraic operations to be performed, such as polynomial expansion, equation solution, differentiation and indefinite integration. In many cases, the operations seem to be cumbersome and produce inelegant results that still require further simplification. It seems doubtful that the facilities would provide students with much of an advantage over what is already in the SEA Mathematical Tables book, (which includes, for example, indefinite integrals) but this question needs more thorough examination.

Current models

There are essentially four manufacturers of graphics calculators for the secondary school market. Each produces a range of machines for a range of prices. It is inappropriate to describe all of these in great detail. However, they tend to fall roughly into three categories, with prices to match:

> *High-end* machines, which contain most of the features mentioned above, and possibly some others not mentioned above. These are intended for serious users of mathematics, who are likely to keep their calculator from secondary school into the undergraduate years. They usually have a lot of memory available.
>
> *Mid-range* machines, which contain many of the above features, with the exception of some of those pertaining to more advanced mathematics. In most cases, mid-range calculators are similar in operation and design to high-end calculators by the same manufacturer, so that users can readily transfer their skills from one to the other. These tend to have less memory available than the high-end machines.

Introductory machines, which contain a limited subset of capabilities from above, probably intended at first for junior high school use in the USA. These, too, are designed so that upward mobility to more sophisticated machines is fairly easy. Such machines have only appeared quite recently.

Each company produces a range of peripheral devices as well, including overhead projection facilities, communications equipment and printing facilities. One company also produces an attachment that allows for real-time data collection to be interfaced to one of their calculators.

In alphabetical order, the four companies are:

Casio
This Japanese-based company produced the first range of graphics calculators popular in high schools in the USA in 1985, and has continued to produce new models ever since. They have produced at least 10 different models, which have become successively easier to use and more feature-laden. In 1995, for example, they made the first graphics calculator with colour capabilities. At present, the company has all three levels of calculator available, including high-end, mid-range and introductory models. Casio has a distinguished record of production of scientific calculators for schools and industry, with many innovative features. Similarly, their graphics calculators contain some features not elsewhere seen. The company is interested in developing better machines to suit schools, and is keen to get advice from schools to do so.

Hewlett Packard
This US company produced the first mathematically sophisticated graphics calculators (in 1986), mainly for use by scientists and engineers. Over time, these have been redesigned and improved, but are mainly used by tertiary workers and students, rather than high school personnel. In 1995, the company produced its first graphics calculator aimed specifically at a high school market, a high-end machine with impressive new capabilities to rival its competitors. This machine has not yet been officially released in Australia, but is apparently enjoying considerable success in the USA. It was produced following consultation with school personnel, unlike previous HP equipment. The company has an extremely good record of producing very high quality reliable equipment, generally at a slightly higher price than its competitors, and was one of the first manufacturers of scientific calculators.

Sharp
This Japanese company produced its first graphics calculator in 1987, and followed this with a better model in the early 1990's. Both a mid-range and a high-end model are available, and each of these is in use in Western Australian schools, where they are popular with both teachers and students. They seem to be especially easy to use, and have very good graphics capabilities. The Sharp calculators do not appear to be as successful in the USA, possibly because they are not as competitively priced there. I am largely unaware of Sharp's activities amongst the educational community, but suspect that they have received good advice from school teachers regarding their calculator design. For many years, Sharp have produced good quality scientific calculators and recently very good elementary calculators for primary schools.

Texas Instruments
This US company also has a distinguished record of producing well-designed and well-accepted calculators for primary and high school, and was one of the original entrants into the school calculator market in the mid 1970's. Their first graphics calculator was not produced until 1990, but was very thoroughly researched among school personnel, and was an immediate success. It was regarded as a very user-friendly machine. This mid-range machine was complemented by two high-end machines in 1991 and 1993, and these too have been very popular in the USA and elsewhere. My impression is that the Texas Instruments calculators still have the lion's share of the high school market in the USA, and that the company works hard at supporting teachers to maintain this position. There are many contented Texas Instruments users in Western Australian schools, too.

Each company distributes its calculators through third parties, some of whom have a national focus. A Western Australian firm, Abacus Pty Ltd, is quite prominent in this field and well-known nationally. They sell both to schools and to the public graphics calculators from all four firms, and have developed a good reputation for doing so. Other outlets, such as educational book stores and retail outlets sell a more limited range of machines, and tend to be less knowledgeable about new calculator developments.

Close inspection of the available models of graphics calculators makes it clear that each of these four companies presently produces good quality devices for secondary schools. There are also clear signs that each of the companies is keen to increase their market share in Australian schools, and quite prepared to offer schools various kinds of support (such as sponsorships, newsletters, advice, published information, special deals). My impression is that the companies and their Australian agents are aware of the significance of price as one element of their marketing strategies, but are also aware that there are educational issues to be addressed as well. Each company is foreign to Australia, and it less likely (although certainly not impossible) that specific machines will be produced with Australian schools in mind.

This is a rapidly changing field, making it very difficult to predict even five years ahead. For example, there is already available in the USA a very sophisticated graphics calculator produced by Texas Instruments (the TI-92) that includes significant symbolic manipulation software, similar to *Derive* and a version of the dynamic geometry programme (*Cabri Geometry*); together with other capabilities, these render the calculator almost as powerful as a good microcomputer of a few years ago, yet priced and sized very much less.

Current prices

In a competitive market, prices are bound to fluctuate somewhat, depending on suppliers, manufacturers and even foreign exchange rates. Schools are often able to obtain a better price through bulk purchasing (such as a class set of calculators). At present (mid-1995), approximate tax-exempt prices to schools seem to be:

High-end about $135

Mid-range about $120

Introductory about $100

Sales tax of 22% is added for purchasers unable to claim tax exemption; this would usually include private individuals, such as students. Recommended retail prices are considerably higher than the above prices, but it is not clear that these would ever apply.

It is perhaps significant that current taxation law allows schools to purchase calculators tax exempt and then sell them secondhand to students after a period of twelve months. I assume that this means that students who were paying a hire fee for use of a graphics calculator could legally have the fees contribute to the purchase of a calculator at the tax-exempt rate after a year, thus legally obviating the need to pay sales tax. However, this assumption should be checked with the relevant authorities.

It is my impression that many schools and individuals would find a hire purchase arrangement the most suitable way for students to acquire a graphics calculator for personal use. If this began in, say, Year 9, and continued over the remaining years of secondary school, an annual fee of about $30 at today's prices would allow students to own the calculator on leaving school. Presumably, calculators could be sold secondhand by students to recover some of the costs, if they did not intend using it beyond school. It is assumed that students would not also purchase a scientific calculator under such an arrangement, and that they would be responsible for their own batteries.

Many schools have commented that a class set of graphics calculators is a better practical investment than a class microcomputer for mathematics, especially when the costs of software for the computer are taken into account, since it allows wide individual access.

Although very many students now have access to a microcomputer at home, it is much less likely that they have access to suitable mathematical software as well. A distinct advantage of graphics calculators is that the software, while limited, comes with the calculator.

It is exceedingly difficult to predict the future pricing of graphics calculators with any measure of confidence. Nonetheless, it would be surprising if the prices rose from the present values in the competitive environment described, and more likely that they will stay the same or drop a little. Inflationary effects in addition will reduce the prices in time. Manufacturers and distributors may see some psychological comparative advantages in pricing some of their calculators below $100, but I have no evidence on which to base this speculation. To some extent, the Australian prices are controlled by corporate decisions in boardrooms in the US and Japan, and I am not privy to their thinking. However, graphics calculators generally cost a bit more in Australia than they do in the USA, suggesting that there is some room for downward movement. In real dollar terms, graphics calculators are already less expensive than were scientific calculators at the time of their introduction into the tertiary entrance examinations in the late 1970's.

Curriculum impact

There are a number of ways in which upper school curricula may be affected by the ready availability to students of graphics calculators. Some of these effects might have been observed in the case of microcomputers, had it been possible for microcomputers to be as readily accessible and as portable as graphics calculators. However, in most cases, it has not been possible to assume that each student in a class has ready access to a microcomputer at all times, so there has been a tendency for such technology to be regarded as an optional extra to enrich or enliven the curriculum, rather than a fundamental part of it. Courses designed on this assumption have paid remarkably little attention to uses of technology to support quantitative thinking. In contrast, and as an example, in the USA, the National Council of Teachers of Mathematics in 1989 constructed its *Curriculum and Evaluation Standards* for school mathematics on the assumption that all students have access to a scientific calculator with graphing capabilities at all times from the ninth grade onwards. In what follows, it is assumed that students have a personal graphics calculator at their disposal, and can use it when they wish, including during assessment.

Some changes may need to be made to curricula in mathematics, physics and accounting to acknowledge the ways in which students may profitably use graphics calculators. Such changes are likely to be rather small, in my estimation, and represent mainly shifts in emphasis away from more procedural activities and towards more interpretive kinds of activities. Such changes would seem to be consistent with a discernible trend over the last few decades away from memorisation of facts and routine procedures as a highly valued activity in many curriculum domains. The clearest evidence for this is the availability to students of table books containing various mathematical formulae, chemical data sheets, physics formula sheets and similar forms of acknowledgment that what students can do is much more important than what they can remember.

It would seem important for different syllabus committees to consider carefully their particular courses with a view to understanding in detail the impact of graphics calculators on student activity and learning, including student assessment. Some of the following dimensions of impact may need to be considered:

Some tasks become much easier

Some mechanical aspects of mathematics are rendered much easier with a graphics calculator. Numerical solution of equations, definite integration, matrix inversions and matrix multiplication, arithmetic with complex numbers, curve sketching, solution of linear systems and the solution of inequalities are examples of this. Graphics calculators do not provide exact solutions to mathematical questions, of course, but they will provide very good numerical approximations. Outside mathematics courses, some of these capabilities may be regarded as providing students with easier access to important learning, since the practical problem of dealing with mathematical aspects (e.g., solving an equation or evaluating an integral) is diminished. In many cases, it would be unwise to rely entirely on the calculator to perform routine tasks, but in most cases the calculator will allow for some freeing up of student time for more important interpretive activity.

New opportunities for learning appear

Students can undertake explorations of significant ideas in new ways. Some examples include comparisons of the graphs of families of functions, the study of convergence by graphical or numerical means, the effects of data transformations on statistics, the simulation of random processes, the exploration of asymptotic behaviour and comparisons of the graph of a function with the graph of its derivative. The Calculator Based Laboratory (CBL) produced by Texas Instruments is an example of a development specifically directed at the physical sciences; the CBL allows for data to be collected and then down-loaded into a calculator for processing. It is likely that teachers will need professional development support to take advantage of the variousopportunities afforded by graphics calculator technologies.

Alternative methods become available

Some tasks can be performed a number of different ways, so that the task of students is to develop enough proficiency with these to make good choices of solution method. A good example is the solution of equations. Students can choose between various graphical methods, numerical refinement methods (using tables of values) or direct methods (using solver facilities on a calculator). Similarly, the data analysis capabilities of calculators will usually present students with a choice of representations of data (e.g., a box plot or a histogram, linear or non-linear regression), and they will need to learn ways of making good choices. Students in the physical sciences, who often deal with mathematical formulas, can solve the resulting equations readily using solver facilities, where some may find more formal procedures less accessible. Rather than being restricted to arithmetic and geometric sequences, students can directly sum the terms of other sequences for which formulas are not as readily available,

Curriculum sequences may be affected

Having a graphics calculator allows students to do some things that were formally only available after many years of study. A good example is that of finding a relative maximum of a function on an interval. Previously, this was not generally possible until calculus was studied. Graphically, it can be handled quite well by a lower school student with a graphics calculator. The same is true for curve sketching. A consequence of such changes is that calculus may be seen as a way of formalising capabilities that students have already developed earlier, emphasising the exact solutions rather than approximations. In a similar way, a graphics calculator provides students with a mechanism for finding approximate solutions to elementary nonlinear equations, well beyond their algebraic capabilities.

Calculator use needs to be learned

Modern graphics calculators are sophisticated pieces of technology, and require some student time to learn how to use them efficiently. Because of their complexity, and the range of possible calculator activities, more time will be needed by students than has generally been needed to learn to use a scientific calculator. Generally speaking, graphics calculators are becoming more user-friendly, and few teachers report much difficulty with students learning how to use them for the specific range of purposes relevant to a particular course. Most students would need to learn to use no more than a small proportion of the range of calculator capabilities for any particular course. However, it seems unwise to delay the introduction of graphics calculators into school curricula until senior secondary school. The optimal time for students to begin using graphics calculators is probably around Year 9, when they begin to study mathematics courses for which many of the calculator capabilities would be useful. Teachers will probably find it inconvenient for students in a classroom to use a variety of calculator models, as happens at present with scientific calculators, but individual students are less likely to find this a problem.

At present, there appear to be many teachers of mathematics who have had little personal experience with graphics calculators, and who will need appropriate professional development support. Outside mathematics, there are even less teachers with significant experience of this technology to date. Clearly, the phasing-in period of graphics calculators will need to be used to provide adequate help for such teachers. Two kinds of help are needed: learning to use particular calculators for particular kinds of purposes, and then making use of the calculators within particular curriculum areas. It is especially important that examining panels are provided with access to appropriate support of these kinds,. given the perceived significance of the external examinations.

Calculators can be customised

It is likely that most of the mathematical needs of an upper school course are already covered by existing models. However, it is possible to use some features of graphics calculators to produce machines that suit better particular courses. The principal mechanism for doing this is through programming. Programs may be written and given to students to allow them access to particular mathematical tasks germane to an area of inquiry. For example, students in Physics or Accounting may find it helpful to have a suite of programs concerned with particular tasks, perhaps focussing on formulas of special significance. Some calculators allow for formulas to be stored for use when appropriate, rather than requiring specific programs to be written for such a purpose. Similarly, students with less powerful calculators may be provided with programs that augment calculator functions (e.g., to evaluate a definite integral or solve a nonlinear equation.)

Access to appropriate technology will be increased

At the present time, many students in secondary schools have a measure of access to graphics calculators, since their schools have seen fit to purchase class sets for use in mathematics, even though their use is not permitted in external examinations. However, there are many other students who are denied such access, and any advantages associated with it, because their schools are unprepared or unable to commit resources to what is regarded as technically unnecessary equipment. Although I have no data to support the claim, it seems likely that students in the former group attend better resourced schools in more affluent areas nearer to the city, while students in the latter group are likely to attend schools with smaller resources in less affluent areas and are further from the city. The official sanctioning of graphics calculators in examinations is likely to ensure that the latter groups of students gain significantly better access to technology for mathematics and other courses involving quantitative reasoning than would otherwise have been the case.

Consequences for examinations

It is widely believed that Tertiary Entrance Examinations exert powerful effects on school curricula over the secondary years, and not only on the courses subject to external examination. An important consequence of permitting the use of graphics calculators in examinations is that the examination may better reflect the nature of the course, and increase the likelihood that there is a coherence between teaching and learning and assessment in courses. Thus, students will recognise that it is valuable to develop expertise with calculators to complete various kinds of mathematical tasks, since they are not artificially prevented from using this expertise in examinations.

Examination options

There are at least three possible options for examinations to accommodate the use of graphics calculators.

Examinations might be constructed so as to be effectively calculator-neutral, so that no significant advantage is given to students attempting questions with the aid of a graphics calculator over students who merely have a scientific calculator available to them. (The Victorain examinations authority has signalled this intention, to ease initial concerns about equity.) One way of doing this is to remove from examinations direct questions that can be readily completed quickly on most graphics calculators with a high degree of success and without much evidence of understanding. Examples of such tasks include numerical solution of an equation, matrix arithmetic and sketching curves. Another way of doing so is to remove numerical questions and replace them with symbolic questions. There are some difficulties in each of these strategies, which may lead to unwanted distortion in the course and may also make examinations harder than intended, eroding student confidence. However, an even more significant problem is that a continuing reliance on calculator-neutral examinations sends a clear signal that calculators are *not* useful items of equipment, and it is not really necessary to learn how and when to use them well. So, even if examining panels find it easier in the first few years to set papers that are effectively calculator-neutral, perhaps because of equity concerns, the deliberate construction of calculator-neutral examinations would be an unfortunate long-term strategy.

Examinations might be divided into two separately-timed parts, in the first of which no calculators at all are used, while their use is freely permitted in the second half. During the first part, candidates could be instructed to place their calculators on the floor. There are a number of formal testing programs in which separate parts of examinations are separately timed, under different conditions, and papers collected accordingly. Such a strategy was used in Leaving Mathematics Examinations in WA in the early 1960's for example, when a multiple-choice component was involved, and is presently used in the Advanced Placement examinations of the College Board in the USA. While administratively awkward, but nonetheless manageable, this kind of approach might ease concerns of some panels that students are likely to become deskilled by an over-reliance on technology. There are significant difficulties with this approach too, however. In the first place, it may mean in effect that students are expected to know how to do many things both with and without a calculator, which generally means that the same amounts of course time have to be spent developing skills that calculators handle better as well as learning how to use calculators to do them. Rather than reduce the computational burden, the graphics calculator would effectively increase the burden on students, which seems counter-productive and counter-intuitive. In the second place, there would be no incentive for students to learn the most critical skills of calculator use, concerned with deciding whether or not to use a calculator in a particular situation. Ultimately, a strategy of separate examinations would serve neither students nor courses well, in my opinion.

Should a strategy of separate examinations or parts of examinations be used, it is imperative that students are not permitted to use *any* calculator in the parts of an examination in which graphics calculators are not permitted, since to do otherwise would demand that they have *two* calculators, with obvious difficulties created for students and their families, The examination authorities in Victoria have made a grave error in this regard, in my opinion.

Finally, examinations might be set precisely as they are at present, to try to reflect the most important ideas in courses, mindful of the fact that students will have encountered those ideas and developed their understanding of them with access to graphics calculators. In my opinion, this is the preferred option. It should be expected that students will have learned both when and how to use their graphics calculators for the kinds of activities germane to a particular course. In many cases, the examinations might be seen as providing (at least in part) an opportunity for students to demonstrate that they can make efficient use of modern technology when it is sensible to do so, and some questions should be set that need some use of graphics calculators for successful completion to allow them to do so. In other words, students might be *required* to use graphics calculators in examinations, rather than merely being *allowed* to do so. In this way, it is likely that the graphics calculators will be effectively integrated into the fabric of a course, rather than being seen as optional extras or as a means of avoiding thinking. Setting such examinations may require careful analysis of courses by examiners, as already happens. It is clearly imperative that examiners are thoroughly familiar with typical graphics calculator capabilities of the kinds that students are likely to have available to them.

Question style

In some cases, changes of phraseology may be needed to ensure that students understand what is being asked of them. A good example in the case of mathematics is the command 'solve'. In some cases, it may mean 'solve exactly', in other cases, it may mean 'provide a numerical approximation to a solution', while in still other cases, it may mean 'provide an argument to prove that all solutions have been found'. If students are expected to provide more than a numerical answer to a question, it is important that phraseologies be developed to communicate this unambiguously. A principle that might be considered in examinations is that, if students are required to show their working, it should be worth showing in its own right, and not only as a means of awarding partial credit. In a classroom, students might be instructed to show their working so that the teacher can make diagnostic use of it, but this argument does not hold in examinations which are not returned to students. A consequence of this line of thought is that a numerical answer to a complex question should be accepted as appropriate, as at present, unless students are explicitly told to provide something else. It is assumed that examining panels are aware of ways in which such numerical answers are accessible to users of graphics calculators.

Restrictions on calculators

In the past, it has been customary to specify particular calculator models as appropriate for examination use. This is a very difficult strategy to conduct in an area of technology undergoing very rapid change, such as graphics calculators, and I recommend that it *not* be used. It is necessary however to specify some restrictions on allowable calculators. In this regard, it is interesting to note the generic restrictions used by the College Board in the USA for their Advanced Placement examinations in calculus:

> Unacceptable machines include the following:
>
> Powerbooks and portable computers
> Pocket Organizers (Wizard, etc.)
> Electronic Writing Pads or Pen Input Devices (Newton, etc.)
> Palm top computers with QWERTY keyboards (HP-95, etc.)

In Victoria, the examinations board also suggested that minicomputers, models with paper tapes, models that make a noise or 'talk' and models that require an electrical outlet be excluded. To ease concerns that students with more powerful calculators will be advantaged in examinations, the College Board announced that particular care will be taken to ensure that this is not the case, which presumably involves examining panels working through draft questions with various calculators at their disposal. The Board further specified that graphics calculators should have a *minimal* set of capabilities, and that the examinations would be set on the assumption that students had access to these and had some experience with using them. The four minimal properties suggested for the calculus courses were:

> produce the graph of a function within an arbitrary viewing window;
> find the zeros of a function;
> compute the derivative of a function numerically; and
> compute definite integrals numerically

For calculators constructed without all of these capabilities, the College Board suggested that students should use the programming facility of their machines to ensure that they had access to all four of these; it also provided suitable programs for students to use for this purpose, and suggested that these be inserted into the calculators before coming to the examination.

Such strategies seem appropriate to the Western Australian situation, and deserve some consideration by syllabus committees. The particular suite of properties assumed by particular courses is likely to differ from that above, which focuses on the calculus. For example, some calculators have an inbuilt facility for complex number arithmetic, while others do not, of some significance to students in TEE Calculus.

Symbolic calculators

The availability of symbolic manipulation software on graphics calculators needs special attention. At present, some aspects of elementary algebra and calculus are available on some calculators, with the consequence that either symbolic or numerical answers to examination questions are accessible to a knowledgable student with such a calculator. To date, only Hewlett Packard has produced calculators with such capabilities, although (as noted above) a very sophisticated calculator with a computer algebra system has recently been released by Texas Instruments (on a machine that has a QWERTY keyboard and hence which might be regarded as too powerful anyway by another criterion, however).

The examinations authority in Victoria has already ruled out calculators with any symbolic manipulation capabilities. In contrast, the College Board seems untroubled by potential advantages students with such calculators might have over other students, since it mentions that both the HP-28 and HP-48 series are acceptable calculators within the rules specified above. Each of these calculators includes symbolic manipulation capabilities.

Rather than a blanket ban, it may be more appropriate to study the actual symbolic capabilities available on such calculators with a view to determining their actual effects in examination settings, and ways of minimising any advantages by careful question design. Although I have not studied this issue in depth, my initial impressions are that the symbolic capabilities are very limited, often inelegant, cumbersome to use, and unlikely to make much difference to student performance in present examinations. My understanding is that the Examination Boards in the UK do not yet prevent students from using calculators with these sorts of capabilities, although there are concerns about the next generation of technology, including the TI-92 in particular.

Apart from responding to real, rather than imagined, problems, there is another reason to give careful thought to the issue of symbolic manipulation. The Hewlett Packard Company has recently produced a new calculator (the HP-38G) specifically for the high school market, that happens to include a few symbolic manipulation commands. My understanding is that the calculator is selling very well in the USA. Given the nature of graphics calculator competition, it is conceivable that other manufacturers may follow suit, in order to capture parts of the huge US market. Should this happen, it is possible that most new graphics calculators will have symbolic manipulation capabilities, in which case a blanket ban on any symbolic manipulation capabilities may become in effect a ban on all new graphics calculators. It is also optimistic to expect that graphics calculator manufacturers will be happy to produce separate models for Australian conditions, given the relatively small size of the Australian market in comparison with those of the USA, Europe and South East Asia. This argument is entirely speculative, since I am unaware of any intentions of these kinds by other manufacturers at present.

Calculator memories

As already noted, it is possible to store a limited amount of information in some parts of calculator memories, including notes and formulas. Such storage is cumbersome and tedious, but nonetheless possible. Retrieval of stored information is likewise cumbersome and tedious, since the calculators are not organised as data bases, designed to allow for fast storage and retrieval of information.

It is difficult to believe that there is much advantage to students in using a graphics calculator in this kind of way in Tertiary Entrance Examinations, since there seems to be fairly little premium placed on recall of information in the examinations themselves. The mathematics courses allow students to use the SEA table book, which contains a number of formulas as well as statistical tables and indefinite integrals, acknowledging that choosing and using these is more important than remembering them. The Chemistry and Physical Science courses similarly provide students with a data sheet including the periodic table, while the Physics course provides students with various formulas and constants. A significant counter-example to these is the Accounting course, which appears to expect students to memorise a number of standard formulas expressed in words rather than symbols. It is not clear to me whether (or why) students are expected to memorise these, but it would not be difficult for them to store them in a graphics calculator for retrieval during an examination.

A strategy of asking students to clear memories or remove batteries before entering an examination is unlikely to be a good idea, since it would prevent students from using program facilities to upgrade their calculators, as described above. In addition, it would be very difficult for invigilators or teachers to check that each calculator was cleared completely, since most calculators have backup battery systems, which are difficult to access, and it is an easy matter for a knowledgable student to thwart checking mechanisms by writing a small program that displays an incorrect message such as "Memory cleared".

Once again, a more careful analysis of the possible ways in which students might use memory facilities, together with careful question design, may be a better alternative than demanding that memories be cleared or calculators be banned in case they are used improperly.

Current examination papers

An important process to undertake is for current examination papers to be analysed with a view to determining ways in which graphics calculators might have been used to advantage by students, and whether or not there are significant differences in capabilities of different calculator models that would lead to comparative advantages. The appropriate persons to do this are syllabus committees who have acquired some knowledge of graphics calculator capabilities. The results may be surprising.

In my estimation, excluding mathematics, there are few places in most of the current examinations where a graphics calculator would have been more advantageous to students than a typical scientific calculator. This is a reflection on the nature of the examinations, which place considerable emphasis on arguments, reasoning and interpretation and less emphasis on routine calculation or manipulation, as well as a reflection on the nature of the subjects themselves.

As far as mathematics courses are concerned, the impact is more evident, especially in Calculus and Applicable Mathematics; there is less evident impact on Discrete Mathematics. The consequential impact on Year 11 courses should also be noted. In each case, however, careful attention to the details of questions would allow knowledgable examiner panels to adjust papers to incorporate graphics calculators appropriately.

Conclusion

The use of graphics calculators in mathematics courses in upper school seems to be an important and positive step towards improving the courses to take advantage of the possibilities provided by new technologies. Some modifications to courses are needed to do this. The effects of graphics calculators in other courses are likely to be fairly small, but generally positive. Syllabus Committees need to analyse carefully their courses and the associated examinations to understand well the interaction between these newer technologies and the existing courses.