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Geometry meets the computer

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Introduction

In a rather short space of time, computers have changed in character from being large numerical devices that could only be communicated with obliquely to small visual devices that allow for much more direct forms of person-machine communication. We have gone from the roomfull to the pocketfull, from paper tape and punched cards to keyboards, mice and touch screens and from strings of binary digits to visual images. All of this has taken not much more than one (human) generation. The IBM Corporation confidently predicted in 1945 that there would never be a market for more than two or three computers in the *world*, and yet in affluent countries like Australia, there are already many *households* with more computers than that, depending a bit on how one defines 'computer'.

Such dramatic technological changes have many consequences, and one of them is the possibility that computers may be of value to children studying geometry with some access to technology. The main purpose of this paper is to describe one development of this kind, the so-called 'dynamic geometry' software that has recently begun to appear in educational settings. The first early experiments in this field involved the *Geometric Supposer* series of software, which allowed secondary school students to explore geometric situations efficiently. However, recent refinements of this idea are much more powerful, and include *Cabri Géomètre* and *The Geometer's Sketchpad*. A version of *Cabri* is now also available on personal technology, the Texas Instruments' astonishingly powerful TI-92 graphics calculator, reinforcing its status as an important new mathematical tool.

Geometry has certainly fallen from its position of prominence in western countries over the past generation. Indeed, even the word 'geometry' is in danger of disappearing from the vocabulary of many students who may undergo a secondary education without ever reading a book with 'geometry' in the title or taking a course described as a course in 'geometry'. While changes of language are natural and not necessarily problematic, and there is much of great value in the space strands of various Australian curricula, there have also been voices of concern raised about the rapid demise of what used to be a significant part of the curriculum for all students.

All too often, in fact, the 'geometry' of years gone by referred to the rather formal synthetic geometry typical of school geometry until recently, and quite similar to the original formulation in Euclid's *Elements* (the most successful textbook in history, with a print run at least two millennia long!). The focus was on the development of formal geometric results concerned with the plane, particularly congruence and similarity, as well as some of the geometry of circles. We have now recognised that the emphasis on formal proof was misplaced for most young students, and that many important spatial ideas were neglected while students were trying to come to terms with proofs of geometric results. Consequently, many changes have occurred to the geometry curriculum in the past two decades, and most of them have been for the better.

In this vein, in a recent superb publication associated with the NCTM Curriculum and Evaluation Standards for School Mathematics, Art Coxford (1991) suggested that a continuation of the broadening of geometry is to be welcomed:

> Geometry, today and tomorrow, must be approached from multiple perspectives to permit the user to make the most of the content as its uses broaden and expand into heretofore unknown regions of science and nature. Fractals, which are founded on the concept of similarity, which are represented graphically (visually), and which are a creation at least partially dependent on powerful computers for their existence, are the new geometric tool of the near future. But what of the more distant future? What will the new tool be? What geometric content will it build on? No one knows, but you can be sure that it will demand an awareness of geometry from multiple perspectives for its comprehension. (p. 4)

This indicates a significant change in direction for the US mathematics geometry curriculum , which has long fixed solely on the geometry of Euclid.

The development of computers and calculators has affected our views of what is important in arithmetic, algebra and statistics, and have affected what sorts of realistic applications of mathematics we regard as appropriate for schools. So it is perhaps surprising that there has been almost no simultaneous impact of the computer on geometry in schools so far, especially in view of the earlier observation that computers have become more visual in nature.

Despite the broadening and the enriching of geometry into 'space', there have persisted some lingering doubts that some important babies may well have been inadvertently discarded along with the Euclidean bath water. It is perhaps ironic that it may be the computer that will generate fresh interest in Euclidean geometry. Indeed, in a rich recent text, Heinz Schumann & David Green (1994) even describe *Cabri* as 'Euclid's revenge':

> Geometry used to have pride of place in mathematical education but in the last fifty years its role has diminished and formal geometry has disappeared. There are three major reasons for this: firstly the difficulty of actually performing the necessary constructions accurately, secondly the considerable time consumed in repeating drawings, and thirdly the realisation that the proof concept fundamental to traditional Euclidean geometry is inherently difficult for most students and that parrot learning of proofs has no merit. However, new IT tools are now arriving, one of which is called *Cabri-géomètre*, which can address these issues and make traditional geometry live again. the cry of the modern mathematics movement of the early Sixties was "Euclid must go!"; the cry of the Nineties could be "Come back Euclid!" (p 9)

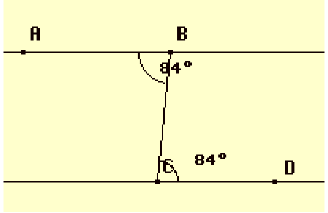
Whether dynamic software realises this ambitious description remains to be seen. However, it is timely now that we turn attention to what the computer may have to offer the students and the teacher as far as geometry is concerned.

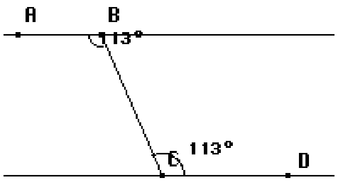
Description

The *Geometric Supposer* software allowed students to construct geometric objects and make measurements on them. For example, they could construct a triangle, measure the angles and add them. What the computer added that normal geometric construction tools could not was efficiency: once a construction was made, a single command allowed it to be repeated. In this way, having constructed a triangle and found that the sum of its angle sizes was 180°, students could quickly construct some other triangles and check that the property was common to all of them, rather tan being unique to the first one.

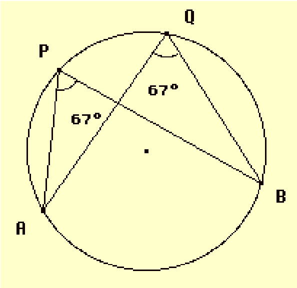
What dynamic geometry software adds to this process, as its name suggests, is *dynamism*. Rather than repeat the construction, the object constructed can be *moved* in some sense, so that what is general can be distinguished from what is particular. In the case of the angles of a triangle, a dynamic geometry program such as *Cabri* allows the user to move the vertices of a triangle at will, all the while repeating the measurement of the angles and the determination of their sum. (It is easier to do this than it is to describe it in words.)

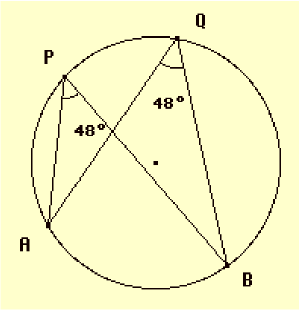
So, an important aspect of *Cabri* is that it allows users to construct a geometric object that can be readily manipulated to observe what changes and what stays the same. The example below shows an elementary instance of this, allowing students to see that alternate angles formed by a transversal across a pair of parallel lines are congruent. As point B, constrained to stay on the line AB, is dragged left and right with the mouse, the two angle measurements at B and C are continually updated and are always the same.





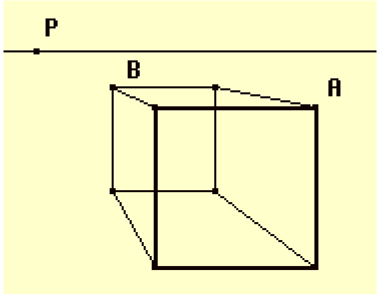
As another example concerning angles, the two angles subtended by the arc below are the same, readily seen by moving point Q around the circle to different places. Furthermore, if the arc is changed (by moving point B, for example), the same congruence is evident. Together, these movements suggest the generalisation that the angles subtended by the same arc are congruent.



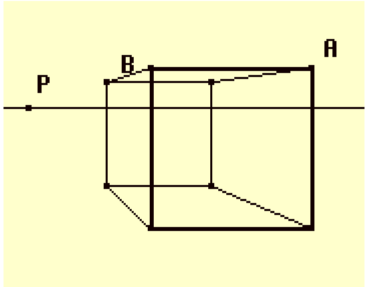


As a third example of dynamic geometry , the series of perspective drawings below start with a rectangular box with a square end located in the plane of this page. One vertex of the square is A, and there is a vertex

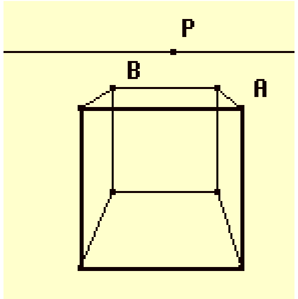
B at the back. The horizon and the vanishing point P are also shown.



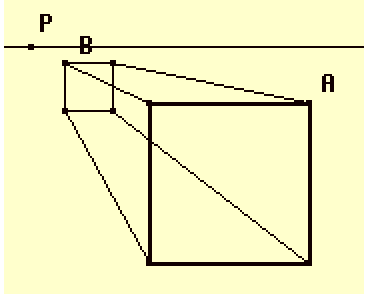
A change of perspective is accomplished by dragging suitable parts of the object, using the mouse. The object looks different if the horizon is 'lowered':



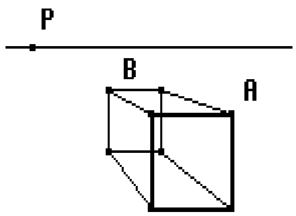
If the vanishing point is moved to the right, a new view appears automatically:



If the box is changed - made 'longer' by dragging point B - its perspective drawing changes:



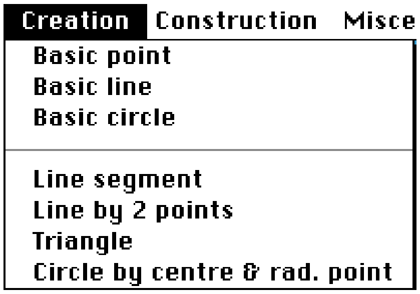
Similarly, dragging point A to make a smaller box also changes the perspective view.

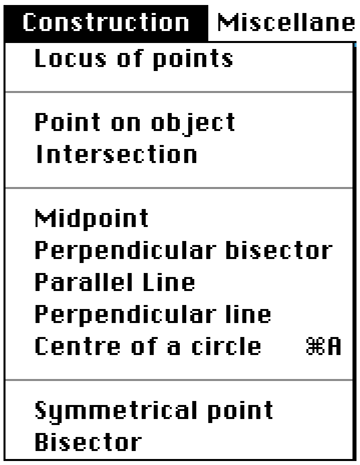


The static page does not do justice to the dynamic aspect of *Cabri* and similar software. An impression of movement is readily created by actions such as those represented in the snapshots above. It is interesting to speculate what the effect on the history of art would have been if the pre-Renaissance painters (who did not understand perspective) had had access to a device permitting these kinds of explorations!

Capabilities

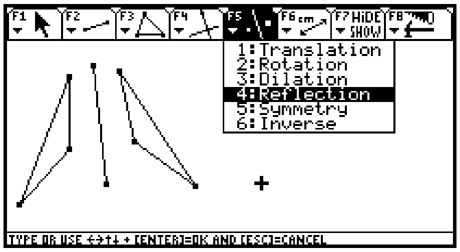
So what can be constructed in *Cabri*? Essentially anything that can be constructed with the traditional Euclidean tools of compass and straightedge. The difference is that manual dexterity with the instruments is not necessary, and arguably of course was never really as important as knowing which constructions to make and in which particular order.

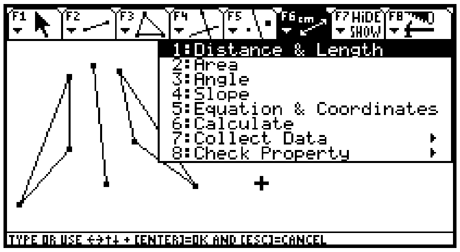




The two screens above show some of the menu choices available to a *Cabri* user. Most of these are familiar and collectively they comprise a powerful new kind of mathematical tool. Later versions of *Cabri* (all the screen shots in this paper are from version 1) and the *Geometer's Sketchpad* include even more constructions, including the elementary transformations of reflection, rotation and translation.

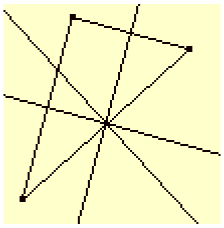
For example, the screens below, taken from a Texas Instruments TI-92 graphics calculator, show some of these transformational constructions as well as some of the measurements that can be automated with software of this kind. In the first screen, for example, a triangle has been reflected about a line segment. It is possible to change the triangle (by dragging vertices of the triangle) and seeing the effect on the image, or changing the line segment (again by dragging) and seeing the effect on the reflection.

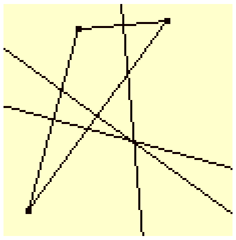




It is indeed interesting that the owner and distributors of microcomputer versions of more recent versions of *Cabri* in the USA is the Texas Instruments Company, well known for its development of fine graphics calculators. This new partnership reflects both the significance of dynamic geometry software for mathematics and the significance of personal technology for education.

Other powerful features can be built into software of this kind. For instance, locus can be studied directly, by instructing the computer to trace the path of a point as it moves in a certain way. Measurements of length, angle and area allow for quantitative geometric relationships to be explored. Macros, particular sequences of constructions, can be constructed by sophisticated users or used by others; an example is the construction of the circumcircle of a triangle, using a single menu entry. *Cabri* incorporates a 'property-checker', which allows the user to ask the software whether or not a particular (apparent) property, such as the concurrency of the perpendicular bisectors of the sides of a triangle, holds generally. The screens below, snapshots from a dynamic range, suggest quite persuasively that the property holds in general.





Reviewing the constructions might help students to see *why* this extraordinary property should hold, but this will not always be the case. Of course, providing a formal proof of properties that hold generally involves a great deal more than recognizing the generality, and dynamic geometry software will not do this for us. But the computer at least implicitly raises the issue of *why* a particular relationship holds generally or doesn't.

Implications

Although experience is being accumulated, it is still a little early to tell what are the best ways for this sort of software to be used in schools, and it seems likely that its importance will depend quite critically on students having enough experience with it to begin to use it independently - which makes the idea of building it into a calculator especially interesting, of course.

As with spreadsheets, a range of levels of student involvement is possible. Teachers may draw an object (such as those shown above) and provide students with the chance to explore some of its geometric features by manipulating the object. Students might be given instructions, at various levels of detail and specificity, regarding how to draw particular kinds of objects to examine. Students may even be given license to explore geometric objects in whatever way they wish, once they have learned how to get started with the software. There have already been a number of instances of young students discovering new geometric generalisations using software of these kinds! Euclid and his colleagues did not record all the possibilities, and did not have the benefits of access to computers to help their thinking.

Chris Little and Rosamund Sutherland (1995, pp 5-6) describe a generic approach in a set of student activities for *Cabri*:

> Geometry has in the past often been taught as a set of facts to learn. *Cabri* provides an ideal medium for pupils to construct elementary theorems for themselves. In *Cabri* pupils can work on a variety of examples of the same geometrical figure. They do this by dragging the basic points of their constructed figure. Pupils can then use the evidence from their constructed figure to make conjectures, and check under what conditions their conjectures hold for other drawings.

> *Cabri* helps pupils understand what changes and what stays the same in a geometric figure. In this sense pupils are learning about geometric invariants.

Doing geometry requires pupils to use perception, experimentation but also logic. An equally important aspect of geometry is to seek *reasons* for results and theorems, using previous knowledge and results.

All the computer work in this unit follows the same basic pattern.

Construct
First construct a figure in Cabri.

Question and conjecture

By moving the basic points and observing the size of the angles, make a simple *conjecture*. For example:

the angles add up to 360 degrees

the two angles ... are equal

Cabri check

Once the conjecture is written down clearly, more drawings can be produced by dragging basic objects. Is the conjecture *always* true? Can you find a counter-example?

Communicate and explain

Can you *explain why*?

How do you *know* that the angles of a triangle add up to *exactly* 180 degrees?

How do you *know* the sum isn't 179.9 degrees, or 180.0000001 degrees?

Can you *use* what you know already?

Communicate and explain on paper

Make a note of what you have found. This should include copies of diagrams or printouts.

The merits of these kinds of activities are not yet clear, although they appear on the surface to offer important new ways of coming to grips with geometry. Thorough exploration of the classroom and curriculum implications of this kind of software is still to be completed. The development in the long term of new partnerships between geometry and computers will take a good deal of work, especially within classrooms. But it certainly has a great intuitive appeal for mathematics education.

The question of visualisation is important in many parts of mathematics, not only in geometry, of course. In their introduction to a stunning recent publication, Walter Zimmermann and Steve Cunningham, referred to the words of David Hilbert, arguably the greatest mathematician of the twentieth century:

> "In mathematics we find two tendencies present. On the one hand, the tendency toward abstraction seeks to crystallize the *logical* relations inherent in the maze of material that is being studied, and to correlate the material in a systematic and orderly manner. On the other hand, the tendency toward *intuitive understanding* fosters a more immediate grasp of the objects one studies, a live rapport with them, so to speak, which stresses the concrete meanings of their relations. ... With the aid of visual imagination [*Anschauung*] we can illuminate the manifold facts and problems of geometry, and beyond this, it is possible in many cases to depict the geometric outline of the methods of investigation and proof."

> Paraphrasing Hilbert, it is our goal to explore how "with the aid of visual imagination" one can "illuminate the manifold facts and problems of mathematics." We take the term *visualization* to describe the process of producing or using geometrical or graphical representations of mathematical concepts, principles or problems, whether hand drawn or computer generated. (p 1)

It certainly seems as if dynamic graphing software such as Cabri has the potential to enhance visualisation.

The possibilities opened by the development of dynamic geometry software reflect the new liaison between the computer and geometry, and it is hard not to believe that the consequences for mathematics education are positive. Exploring these consequences is made relatively accessible, since the AAMT offers each of the excellent references below for sale in its catalogue of publications, and also distributes the *Cabri Géomètre* software, all at member discounts. Much more is to be gained from manipulating objects and constructing new ones then from reading about others doing so, of course. Schools, students and teachers need to have at least access to some software for this to be possible.

After the hand calculator was invented, arithmetic could never be the same again. Following the invention of data analysis software, statistics could never be the same again. Now that algebra is available not only on large computer systems, but also on graphics calculators and personal technologies like the TI-92, algebra and calculus can never be the same again. It now seems, too, that geometry can never be the same again.

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