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# GRAPHICS CALCULATORS AND ASSESSMENT\*

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**ABSTRACT:** Graphics calculators are powerful tools for learning mathematics and we want our students to learn to use them effectively. The use of these hand held personal computers provides opportunities for learning in interactive and dynamic ways. However, it is not until their use is totally integrated into all aspects of the curriculum that students regard them with due importance. This includes their use in all kinds of assessment tasks such as assignments, tests and examinations as well as in activities and explorations aimed at developing students' understanding. The incorporation of graphics calculators into assessment tasks requires careful construction of these tasks. In this paper, discuss issues of equity relating to calculator models, levels of calculator use and the purpose and design of appropriate tasks. We also describe a typology we have developed to assist in the design and wording of assessment tasks which encourage appropriate use of graphics calculators, but which do not compromise important course objectives.

## INTRODUCTION

The educational significance of graphics calculators is closely related to the likelihood that they will be accessible to students in many situations: in classrooms, studying at home and, critically for this paper, in assessment situations. The size, ready portability and rapidly affordable cost of graphics calculators together make it necessary to reconsider assessment of students' mathematical capabilities under the assumption that they will be permitted to make use of a graphics calculator. This paper provides an overview and analysis of the relationships between graphics calculators and assessment, and identifies a number of important issues.

Over recent years, we have worked as a team to gradually incorporate graphics calculators into an undergraduate mathematics course at Murdoch University in Western Australia. (Bradley, Kemp & Kissane 1994). Our principal motivation for including graphics calculators into the fabric of the course was (and still is) the improvement of both teaching and learning mathematics. However, as others in similar circumstances have found, issues of assessment arose naturally and needed attention. Although some of the issues are amenable to analysis, most of them have required practical work in teaching and curriculum development. The course concerned mainly comprises work in pre-calculus, algebra and trigonometry, although it also includes some work with the idea of a derivative function. However, our curriculum development work in this course has allowed us to see the wider implications of graphics calculators for assessment in other educational contexts.

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There seems to have been surprisingly little written about assessment issues related to graphics calculators yet. (For example, none of the papers in Andrews & Kissane (1994) dealt with the matter.) It is also interesting to note that, despite the apparent availability of microcomputers for mathematics education for a considerable time now, issues of assessment seem not to have been carefully analysed in the literature and have not been prominent in thinking about the relationships between technology and mathematics education. The explanation for these phenomena is that it has not been necessary to grapple with the issues, since in most practical situations, students have not had enough access to the technology to demand that we contemplate its use in assessment. The development of the personal technology of graphics calculators has changed this, and thus provides the imperative for this paper.

## WHAT IS ASSESSMENT?

Assessment in mathematics education is concerned with finding out what students know, understand and can do, with a view to making use of this information in some way. Assessment takes many forms. Recognisable around the world are formal examinations, which are sometimes external to the school, and less formal versions such as class tests and quizzes, usually of shorter duration and generally local to a particular classroom. Much formal assessment is timed and supervised. While there are many kinds of questions asked, they can often be classified into two broad groups; short answer questions, such as multiple choice questions, which require students to think about and analyse some situation, and then select the best of the responses offered; extended response questions, which require student to structure a response by themselves, and generally demand that students give some evidence of their thinking, not only their conclusion. Our experience and interest is mainly with the latter category, although much of this paper has implications for short answer questions as well.

Formal assessment sometimes has a 'high stakes' character, such as an external examination used to determine in part whether a student will be admitted to a particular institution or programme, or whether a student will be permitted to graduate. In nationally competitive situations, the stakes can become very high, and issues of assessment and graphics calculators can become issues in the public domain, not only practical issues for the classroom teacher of mathematics.

As well as formal assessment, much useful information can be gleaned from students work in less formal settings, including weekly assignments, projects and investigations, which may be completed by students working in collaboration, or with substantial help from resources such as textbooks, and within much less tightly defined time constraints. In our experience, the use of graphics calculators in such situations is very important for both learning and assessment. In addition, there is much informal assessment in classrooms, relying on observation, interview and conversation. Although such formative assessment might arguably even be more important, in fact, than formal assessment, it is not the subject of this paper.

## WHY IS THE USE OF GRAPHICS CALCULATORS IN ASSESSMENT IMPORTANT?

The central reasons for coming to terms with assessment issues related to the use of graphics calculators are related to the integration of technology into the curriculum. (Kemp, Kissane & Bradley 1995). The coherence of assessment and learning environments is critical. Graphics calculators enjoy a significant advantage over other computers in terms of accessibility; students are potentially able to use calculators in many settings, such as in the classroom, at home when working

on homework or assignments, and in formal assessment situations. Although graphics calculators provide important new learning opportunities, and access to new ways of dealing with mathematical situations, in our experience some students are inclined to undervalue both of these if the calculators are not integrated into the assessment program as well as the rest of the curriculum. (Bradley, Kissane & Kemp 1996) This is especially important in the case of ‘high-stakes’ assessment.

A different kind of change is that an important new course outcome might be expected, concerned with the extent to which students can use technology well; this involves deciding when to use a graphics calculator and when not to do so, using it efficiently, interpreting the results obtained and describing them in appropriate mathematical language. Such an important course outcome needs to be assessed, which involves changing the traditional style of assessment.

Regarding student opinion on the use of graphics calculators, we have elsewhere reported data suggesting that there is a discernible positive shift associated with use in all areas of assessment. (Kissane, Kemp & Bradley, 1995). In the table below, the mean attitudes of successive groups of our undergraduate students to various aspects of calculator use are summarised, augmented by 1996 results. The 1994 group used calculators in class and one class test, but not in assignments or the final exam, while the latter two groups used graphics calculators in all phases of assessment. The mean scores are constrained to fall between 1 and 4, with a higher mean score associated with a more positive attitude, and a score of 2.5 representing ambivalence.

	<b>1994</b>	<b>1995</b>	<b>1996</b>
Using the graphics calculators helped me to understand graphs of polynomial and rational functions.	3.15	3.19	3.33
Using the graphics calculators helped me to understand graphs of trigonometric functions.	3.10	3.13	3.23
It was a good idea to be able to use the graphics calculators in the test.	3.01	3.22	3.39
Using the graphics calculator helped me to understand the relationship between graphs and solutions to equations and inequalities.	2.96	3.20	3.35
Using the graphics calculators helped me to understand matrices and their uses to solve systems of equations.	2.88	3.09	3.33
Overall I enjoyed using the graphics calculators.	2.83	3.03	3.30
Some assignment questions should require the use of graphics calculators.	2.76	3.13	3.23
I think that we should be allowed to use graphics calculators in the final examination.	2.71	3.35	3.53

**Table 1:** Mean scores on selected Likert items

This table illustrates clear shifts in successive cohorts of student opinion regarding graphics calculators. While the shift in view on items regarding assessment is quite predictable, the shifts *not* directly related to assessment support the argument for integration of graphics calculators into assessment, to take advantage of the consequences of wider student use. Further details of our experiences, the course concerned and other aspects of student reactions are given in Kissane, Kemp & Bradley (1995).

## GRAPHICS CALCULATOR CAPABILITIES

In considering the relationship between assessment and graphics calculators, the mathematical capabilities of the calculators themselves need to be taken into account. Two issues quickly emerge. In the first place, despite the way in which they are described, graphics calculators are quite powerful computers and provide access to many mathematical capabilities that are not essentially graphical in nature. Secondly, there are considerable differences between the capabilities of different models. The pairs of calculator screens in Figures 1, 2 and 3 illustrate these two points.

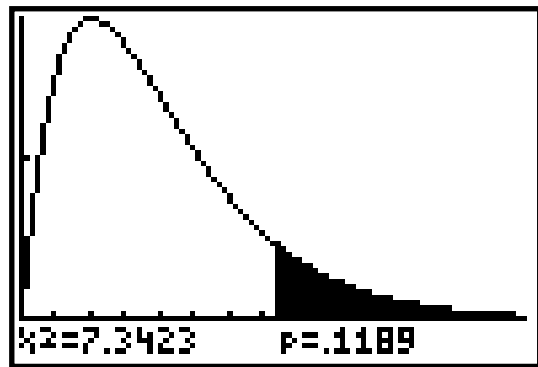
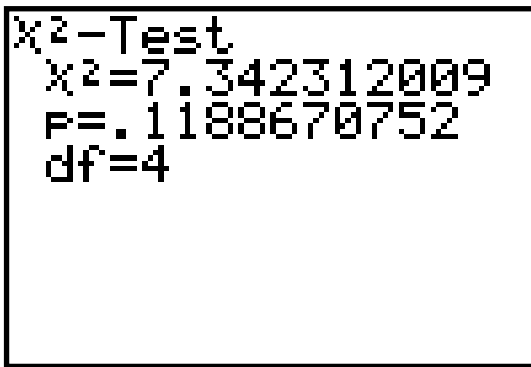


Figure 1: A chi-squared hypothesis test on a Texas Instruments TI-83

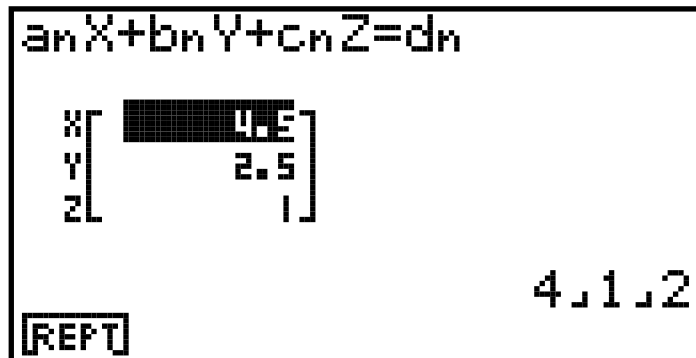
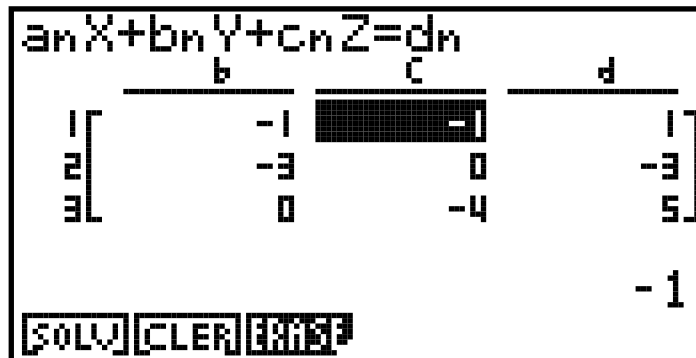


Figure 2: The solution of a system of linear equations on a Casio fx-9700



$$x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7$$

**Figure 3:** A Taylor polynomial on a Hewlett Packard HP-38G

Other examples might readily have been chosen for which the main mathematical support provided by the calculator is not essentially graphical (although it may have a graphical element, such as in the example of the hypothesis test above). Graphics calculators can be used directly to find successive terms of a recursively defined sequence, construct confidence intervals perform matrix arithmetic, approximate definite integrals, operate with complex numbers, and so on. Some recent calculators have very powerful symbolic capabilities, raising significant concerns for both curriculum and assessment. Some illustrative examples are shown in Figure 4, containing screens from a Texas Instruments TI-92 graphics calculator, the best available example of a hand-held computer algebra system.

Top screen:

- factor( $2 \cdot x^4 + x^3 - x^2 + x - 3$ )  
 $(x - 1) \cdot (2 \cdot x + 3) \cdot (x^2 + 1)$
- expand( $((2 \cdot a + 3 \cdot b)^3)$ )  
 $8 \cdot a^3 + 36 \cdot a^2 \cdot b + 54 \cdot a \cdot b^2 + 27 \cdot b^3$
- $\int (x \cdot e^x) dx$   
 $x \cdot e^x - e^x$
- $\int (x e^x, x)$

Bottom screen:

- $\int_{-1}^1 \sqrt{1 - x^2} dx$   
 $\frac{\pi}{2}$
- $\frac{d}{dx} ((2 \cdot x + 1)^5)$   
 $10 \cdot (2 \cdot x + 1)^4$
- $\sum_{n=1}^k (n^2)$   
 $\frac{k \cdot (k + 1) \cdot (2 \cdot k + 1)}{6}$
- $\Sigma (n^2, n, 1, k)$

**Figure 4:** Symbolic manipulation on a Texas Instruments TI-92

A complication when considering assessment with graphics calculators, especially assessment beyond the local level, where there may be a reasonable expectation that most students have the same model calculator, is that not all graphics calculators have precisely the same capabilities. The more sophisticated capabilities of some calculators are not always available on other models. In some cases, this is a consequence of variations among capabilities of the same brands of calculators (such as the differences between Texas Instruments TI-81 and TI-83 calculators), whilst in other cases, the differences reflect different manufacturer's views of what is important at different levels.

In many cases, differences can be minimised by either skilful calculator use or through the use of short programs. For example, neither the Texas Instruments TI-81 nor the Casio fx-7700 has an automatic root-finding routine, unlike more powerful models. The short programs in Figure 5 allow students to upgrade their calculators to achieve the same end, albeit a little less conveniently, once they have drawn a graph and decided approximately where the roots are located. Students can be provided with such a program by their teacher, and do not need to know about programming or the Newton-Raphson procedure to use the program, any more than do students who use the inbuilt versions of routines like these on a more sophisticated calculator.

<pre> :Lbl 0 :X→D :X-(Y1/NDeriv(Y1 ,X)→X :If abs (X-D)&gt;0. 0000001 :Goto 0 :Disp X </pre>	<pre> Lbl 0e X→De X-(Y1÷d/dx(Y1,X))→Xe Abs (X-D)&gt;0.00000001⇒ Goto 0e X </pre>
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**Figure 5:** Short programs for a Texas Instruments TI-81 and a Casio fx-7700

Similar programs can be written to reduce other differences, such as to generate successive terms of a sequence for a calculator without recursive function capabilities or to evaluate definite integrals numerically.

Unlike the preceding examples, some calculator capabilities are more difficult to replace, and are thus potentially of greater concern when assessment is taken into account. Some examples of these include graphing in polar coordinates, arithmetic with complex numbers, automatic solution of systems of linear equations, generation of terms of Taylor series, statistical hypothesis testing and symbolic manipulation generally.

### EQUITY ISSUES ASSOCIATED WITH DIFFERENT MODELS

Understandable concerns have been raised about the possibility that more affluent students might be advantaged during formal assessment merely because they can afford more sophisticated graphics calculators, which generally cost more to purchase than less sophisticated models. While it may be possible (to a limited extent) to resolve such problems at the local level by a process of encouraging, or even insisting, that all students use the same graphics calculator, such a strategy in itself is iniquitous and ultimately unmanageable. In any event, such ‘solutions’ are not possible for the wider level, outside the domain of a single class, school, district or state.

Experience has already begun to accumulate indicating that most students in practice are likely to use the least sophisticated parts of their calculator most well, and to be less confident of the more sophisticated capabilities. Frequently, more sophisticated calculator operations are only valuable to a more sophisticated and experienced user. Our informal observations at Murdoch University are consistent with this claim. It needs also to be acknowledged that someone who has spent a lot of time using a more powerful calculator will most likely be better able to handle many mathematical situations than someone else; in fact, they almost certainly know more about mathematics as a result of the time spent using their powerful calculator! In other words, it is not only the calculator that has more capabilities; it is also the person operating the calculator.

Nonetheless, the equity issue is a real one, and needs to be considered in the context of assessment, particularly high-stakes assessment such as Advanced Placement examinations in the

USA, A-level examinations in the UK, and tertiary entrance examinations in Australia and elsewhere. (Kissane, Bradley & Kemp, 1994).

One partial solution to the problem is the specification and publication of minimum capabilities needed for courses or exams (as happens for the Advanced Placement Examinations in Calculus in USA). This places the onus on those responsible for the examinations to ensure that questions are not rendered easier to those with more sophisticated calculators, and at the same time makes clear to students, their parents and their teachers what capabilities they should ensure are accessible. In the case of students with less expensive, or merely older, calculator models, small programs, such as those shown in the previous section, should be made available long enough in advance for students to be comfortable with their fluent operation.

The use of programs in this way means that students need to have access to calculator memories, and suggests that it would be quite inequitable to insist that calculator batteries are removed or memories cleared before an examination. In turn, this position raises a fresh issue related to text storage in some graphics calculator models. Although limited forms of text storage are possible in any graphics calculator model (by merely writing a program consisting of words, for example), some calculators actually have a facility to write, store and retrieve notes. Students with such a calculator, in an assessment situation in which memories are not required to be cleared in advance, might be able to take advantage of this. While on the surface this might seem to be of concern, in fact it is not conceptually different from allowing students access to sheets of notes, table books containing mathematical formulas and even open-book exams. At Murdoch University, we have routinely allowed students to take a page or two of notes into examinations with them for some time now, to reinforce the emphasis that mathematical thinking is not mainly a question of *remembering* things, but of choosing suitable mathematical formulations of problems, interpreting solutions sensibly, expressing mathematical arguments, and so on. It may be, however, that the continued use of graphics calculators may be a vehicle to encourage mathematics educators to consider more carefully what are the really important features of mathematical work.

As well as minimum capabilities, graphics calculators have already progressed to the point where it is necessary to acknowledge that some capabilities are so powerful that no amount of skilful use or programming of much less sophisticated machines will overcome the equity problems. The most obvious example of this is the extraordinary Texas Instruments TI-92, some screens from which were shown previously. At present, although local inequity may be reduced in the obvious way (by ensuring that all students have similar access to the same calculator), there seems no alternative in wider assessment contexts to restricting access to such calculators, especially those capable of *extensive* symbolic manipulation. An essential problem is that it is too hard to tell the difference between the responses in an examination of someone who knows a lot about mathematics from someone who knows only how to use their calculator. This is even a problem when all students have access to the same calculator, incidentally. Of course, the 'solution' of banning the use of the technology in assessment is not an appropriate long-term solution, and we have much to think about in adjusting our curricula and their assessment to the next generation of graphics calculators. This issue is discussed in more depth by Bradley (1995).

Some problems of inequity can be reduced or even eliminated by careful design of assessment tasks, as a later section of this paper shows. For example, if students are asked to find roots of a function accurate to only one decimal place, rather than to several decimal places, the advantage normally afforded by automatic root finders over a process of tracing and zooming disappears. In fact, in many practical situations, neither exact answers nor approximate answers to many decimal places are important; what matters are the number and nature of solutions and their approximate size. In any event, even if more accuracy is warranted in practice, it doesn't mean that we need to ask for it in a formal assessment situation.



## LEVELS OF CALCULATOR USE IN ASSESSMENT

There are essentially three options for dealing with graphics calculators in assessment situations. Calculators may be used without any restriction except for limitations on which models and capabilities will be permitted. They may be completely barred from assessment, which is already the case in many mathematics education settings around the world, or there may be some combination of these two (such as when a section of an examination is required to be done without access to calculators).

### *Unrestricted calculator access*

The most persuasive argument for allowing unrestricted graphics calculator access is that this is most like the normal classroom teaching and learning situations. If we are to integrate technology into our curricula, it is prudent to reduce the differences between assessment conditions and typical classroom conditions. Hence, we have a strong preference for this strategy of calculator use. With unrestricted access to a graphics calculator, students are generally expected to choose for themselves when and how to use a calculator, although some advice may be offered for particular questions. It is also possible to devise questions that allow us to find out directly how well students have learned to use their graphics calculator capabilities in the service of dealing with mathematical situations.

### *Calculator-neutral assessment*

When graphics calculators are permitted in assessment, it has been suggested by some that emphasis should be placed on ‘calculator-neutral’ questions, for which there is no advantage to students with a graphics calculator over those who do not have access to one. This seems to us to be a most unwise strategy, and not a sensible approach to the equity issues it is attempting to resolve. In the first place, questions that are regarded as calculator-neutral very often are not, particularly for skilled users of graphics calculators. Such students can often check their answers using the graphics calculator and actually obtain some comparative advantage over those without a graphics calculator. As another example, students using a calculator that can graph integral functions would be advantaged over others without access to such help, when trying to determine an indefinite integral.

One way of rendering assessment tasks calculator-neutral, perhaps the main way in fact, is to use symbolic expressions rather than numbers. Although this probably is effective in neutralising the influence of the graphics calculator (in many, but certainly not all, situations at least, as indicated by the integration example), it can easily have the quite undesirable consequence of making the test questions a good deal harder than intended, or really necessary. (See Bradley(1995) for a discussion of this).

However, of even more significance is the fact that this kind of assessment practice runs perilously close to sending the wrong message that calculators and their intelligent use are not really useful. Graphics calculators have the potential to prove a powerful ally to students learning mathematics, to teachers teaching mathematics and to curriculum developers trying to bring about curriculum change that acknowledges, and makes mathematical advantage of, the invention of the microprocessor. It would be a tragedy if we sent the community the message that graphics calculators are not really useful devices after all.

### *Calculator-free assessment*

The final way to address the issues of calculator use is to refuse to allow students to use them at all. This certainly solves one problem, but at the expense of other problems. When graphics calculators are removed from use in even one part of assessment, we lose our capacity to determine whether or not students have learned to make intelligent use of the technology. When teachers,

textbook authors and test constructors control when and how students will make use of their graphics calculators, any opportunity or incentive for students to learn to make such decisions by themselves disappear. Calculator-free assessment prevents students from developing such discrimination skills, a crucial aspect of intelligent use of any kinds of machine. Rather than encouraging students to use graphics calculators to help them think about mathematics, this kind of practice encourages them to only do so when they are told to.

An additional problem comes with tests that permit graphics calculators to be used for part of the time, but not used for other parts of the time, since this may inadvertently make a course harder too. It is possible that one consequence of this practice is that calculators add to the content of a course, since students have to know both the old ways and the new ways of doing a particular piece of mathematics. Thus, while inverting a  $2 \times 2$  matrix is easy enough to be completed by hand, possibly using a scientific calculator, the same is not true for matrices with dimensions  $3 \times 3$  or higher. So, rather than regarding numerical algorithms for the inversion of a  $3 \times 3$  matrix as a curious historical anachronism, if students are expected to do this by hand, as well as sometimes use their calculator to do such things (arguably a much more sensible use of their time), the net effect is that the calculator has actually created more work for the students, which would indeed be an ironic consequence of making use of a labour-saving device. The argument that students will 'understand' better what they do if they develop fluency with long-hand ways of doing things like this is very difficult to defend, although sometimes attempts are made. To argue using an analogy, very few students, if any, developed a better feel for the nature of a square root by finding them by using the tedious, complicated and ancient algorithm that was still taught in schools only 30 years ago, than by using a table book or a scientific calculator.

Some important mathematical tasks are already calculator-free, of course. These include notions of proof, translation from words into symbols, analysis of real situations, symbolic manipulation, mathematical modelling, and most aspects of advanced mathematics; some examples appear in the next section of this paper. Such tasks can be readily included on tests that permit graphics calculators.

## DESIGNING ASSESSMENT TASKS TO INTEGRATE GRAPHICS CALCULATORS

When students are permitted to use graphics calculators in assessment situations, it is essential that the assessment tasks are carefully designed with this in mind. This requires a clear view of exactly what we intend to find out with a particular assessment task, and a good understanding of what graphics calculators might have to do with what we are trying to find out. Integration of graphics calculators into assessment involves more attention to detail than merely sanctioning their use on existing assessment instruments, and in this section of the paper we propose and exemplify an analysis of the relationships between assessment tasks and graphics calculators.

The first aspect of design occurs before any assessment tasks are planned. As the preceding section of this paper indicates, different levels of calculator use can be adopted, and a decision needs to be made about this and communicated adequately to students. In practical terms, assuming that assessment is not to be calculator-free, the essential choice to be made is between 'allowing' and 'requiring' graphics calculators to be used. These two choices conjure up different images for the task of the designer. In the case of 'allowing' graphics calculator use, there is an implicit tone of reluctance and forbearance, rather than encouragement. There is also an implicit suggestion that some students might prefer to not take up the offer to use a graphics calculator, or might not be able to take up the offer, because they do not have access to appropriate equipment. Consequently, attention may well focus on how to ensure that no disadvantage is associated with this.

The messages associated with 'requiring' calculator use are quite different from this. The graphics calculator is assumed to be part of the 'tools of the trade' for students, and they are expected to demonstrate that they are capable of using it efficiently and autonomously in

appropriate circumstances, when dealing with mathematical tasks. They are also expected to realise that sometimes it is not appropriate to use a graphics calculator, or that the use must be supported and augmented in various ways by careful mathematical analysis. In short, when graphics calculators are integrated into the fabric of a course, they will be required for use in assessment, rather than merely tolerated.

As a result of our experience with integrating graphics calculators into courses, we have developed a typology, shown in Table 2, of the possible relationships between the tasks given to students in examinations and our intentions regarding graphics calculator use.

<b><i>Graphics calculators are expected to be used</i></b>	
1	Students are explicitly advised or even told to use graphics calculators
2	Alternatives to graphics calculator use are very inefficient
3	Graphics calculators are used as scientific calculators only
<b><i>Graphics calculators are expected to be used by some students but not by others</i></b>	
4	Use and non-use of graphics calculators are both suitable
<b><i>Graphics calculators are not expected to be used</i></b>	
5	Exact answers are required
6	Symbolic answers are required
7	Written explanations of reasoning are required
8	Task involves <del>ing</del> <i>in</i> the mathematics from a situation or rep situation mathematically
9	Graphics calculator use is inefficient
10	Task requires that a representation of a graphics calculator screen will be interpreted

**Table 2:** Expected usage of graphics calculators and examinations  
(From Kemp, Kissane & Bradley 1996)

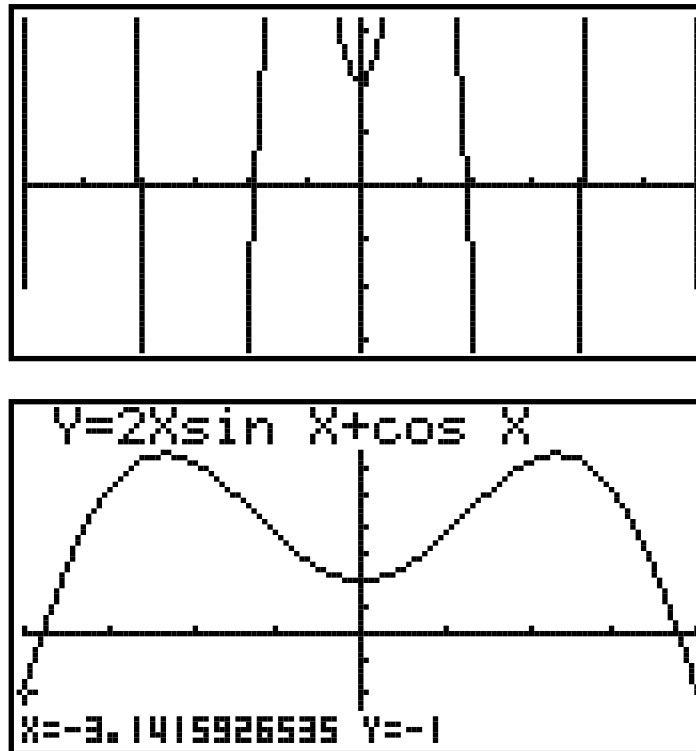
This typology is described at some length in Kemp, Kissane & Bradley (1996). There is space here only to describe and illustrate briefly the various types of intended relationships. The next section of this paper also provides a number of examples.

### *Graphics calculator use is expected*

There are a number of reasons why we might design assessment task for which we expect graphics calculators to be used. The most compelling of these is that sometimes there is no alternative *for the student* to using a graphics calculator to solve a problem. At other times, there may be alternatives, but they are inefficient because they take too long, or are too complicated. In our typology, we have identified separately the situation in which students are explicitly advised to use a graphics calculator for a particular task, rather than being expected to decide this for themselves. For example, consider the following examination questions:

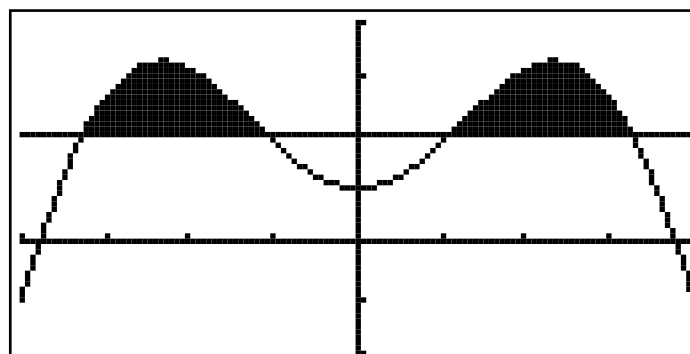
- (a) Sketch a graph of the function  
 $f(x) = 2x \sin x + \cos x$  for  $-\pi \leq x \leq \pi$ ,  
 showing any critical features.
- (b) Use the graph to solve  $2x \sin x + \cos x \geq 2$  on this interval.

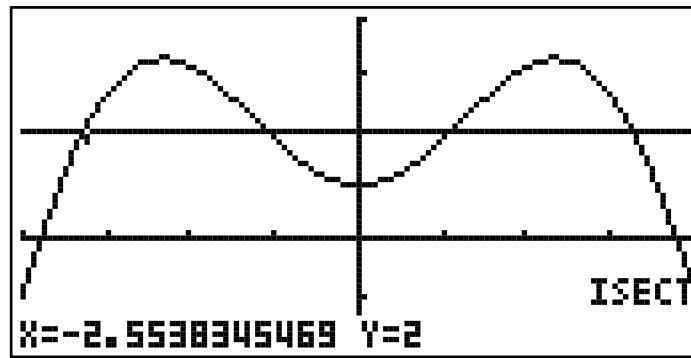
There are time-honoured procedures, using the calculus, to identify the important features of a function and hence to sketch a graph of the function on an interval. However, students who have not yet studied the calculus (or even those who have studied calculus, but want only an approximate solution rather than an exact one) will find a graphics calculator a useful aid in situations like this. But it is often not a trivial matter to make intelligent use of graphics calculators for sketching curves. The calculator screens shown in Figure 6 give examples of some of the ways in which students might use the calculator to aid their thinking.



**Figure 6:** Using a Casio fx-9700 to graph a function

The first screen shows what will happen if a student uses the default axis settings, even if the calculator is set to radians (which, of course is not always the case). Producing a graph like that shown in the second screen requires the students to undertake some mathematical analysis, including thinking about what features of a graph are ‘critical’.





**Figure 7:** Using a Casio fx-9700 to examine an inequality

As suggested by Figure 7, the solution of the inequality in part (b) might involve students in shading (in fact, or in their mind) above  $y = 2$ , and it may involve tracing or use of automatic procedures for finding points of intersection. In each case, substantial mathematical thinking is involved. For less experienced or sophisticated students, we may prefer to advise them explicitly to use their graphics calculator (a Type 1 question, in terms of our typology) and we may even suggest how they should do so. However, addressing this question without the use of a graphics calculator would not be appropriate for most students, so that we can expect that it will allow us to assess well some important aspects of mathematics achievement.

As noted earlier, the case for graphics calculator use in mathematics is not restricted to drawing and analysing graphs of functions. The example below illustrates another context in which a graphics calculator would be expected to be used

A study into adult health compared the average number of hours a week spent exercising ( $x$ ) with a measure of how fit people were ( $y$ ) for a small sample. The data are shown below.

$x$	5.3	19.2	9.1	11.3	2.1	8.1	4.5	9.0	6.2	5.8	19.2	14.0	15.5	16.2	10.9	14.1
$y$	2.6	12.9	6.1	5.9	2.4	5.8	5.3	8.0	3.7	14.8	12.9	9.8	10.3	13.1	8.0	12.3

(To check that you have entered the data correctly, note that the means of  $x$  and  $y$  are 10.65625 and 8.36875 respectively.)

- Find the line of best fit for these data. In a sentence or two, describe how well the data can be modelled with a line.
- One of these observations appears to be an outlier. Which one? Explain how you can use the scatterplot to detect outliers.
- Remove the most obvious outlier, identified in part (b), and find the line of best fit for the reduced data set.
- Describe two ways, one graphical and one numerical, in which you can tell that the line in (c) is a better fit to the data than that in (a).
- Use the line in (c) to predict the fitness level of someone who exercises for 12 hours per week on average.

A significant advantage of graphics calculators over scientific calculators is that they provide an opportunity for data analysis of the kind suggested here. Once data are stored in a calculator, they can be analysed in different ways, outliers can be removed to determine their effects directly, and students expected to use some initiative in deciding how to address important questions. Assessment of these kinds of activities is more likely to be congruent with the objectives of the

course concerned, with the classroom teaching used and with good statistical practice than is more conventional assessment without graphics calculator use.

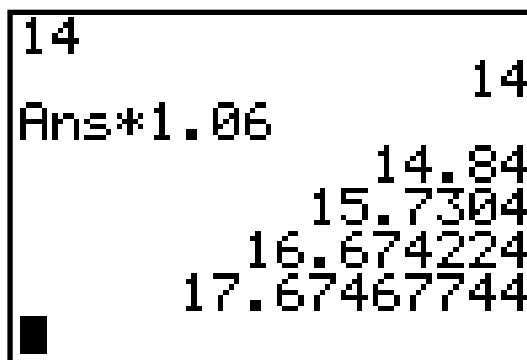
*Graphics calculator use is expected by some students only*

Whether or not a particular student chooses to make use of a graphics calculator for a particular task will often depend on both the student and the task. We think that it is appropriate for some variation to occur, and expect that students will need some help in learning to make good decisions about whether, and how, to use their graphics calculator. As an illustration, consider the example below.

A microbiologist is studying the growth of a virus, which grows very rapidly. She estimates that a specimen contains 14 thousand virus cells with the number of cells increasing by 6% every hour.

- (a) How many cells will the virus have after 15 hours?
- (b) How long will it take before the specimen has 20 thousand cells?

There are many ways in which students might respond to part (a). For example, a sophisticated student may simply calculate  $14(1.06)^{15}$  on a scientific calculator. A less sophisticated student may construct a recursive sequence like that shown in Figure 8, where a new term is generated each time the command *Ans\*1.06* is entered by pressing the 'Enter' key.



**Figure 8:** Using a Texas Instruments TI-81 to generate a recursively-defined sequence

Other students may use recursive function capabilities of their graphics calculator and construct a table of values, or a graph of discrete values which can be traced. Similarly, when responding to part (b) of the question, some students will make use of various graphics calculator capabilities (such as finding points of intersection of  $y = 14(1.06)^x$  and  $y = 20$ , using a 'solve' command, scrolling a table of values, etc.). Some students who know about logarithms may prefer to solve the equation  $14(1.06)^x = 20$  by first taking the logarithm of each side. Regardless of the method chosen, students will need to deal with issues of accuracy and interpretation of their answer.

*Graphics calculator use is not expected*

There are a number of assessment situations for which graphics calculator use would not be appropriate, and the typology outlines what these are. The following question provides an example from an elementary calculus course:

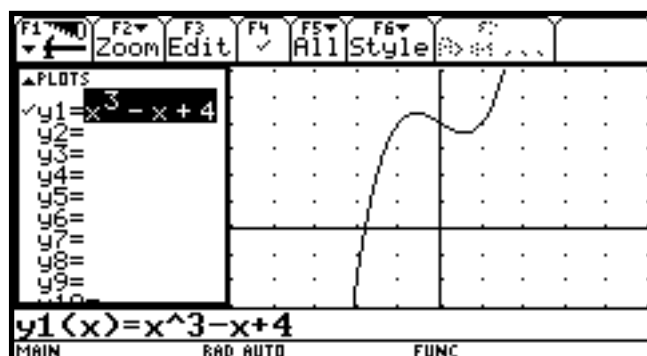
Find the exact value of  $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos x} dx$ .

Asking for the exact value requires that the answer be expressed as  $\ln(4/3)$  after the integral has been performed symbolically. If a graphics calculator (without considerable symbolic manipulation capabilities) were used by students, a numerical integral could be obtained, which may be a useful check on the symbolic manipulation involved. However, this would not be regarded as an appropriate response to a question in its present form.

### DESIGNED USE OF GRAPHICS CALCULATORS

In this section, we use the typology to illustrate how assessment tasks might be designed and adapted for presentation to students to reflect various appropriate uses of graphics calculators. We have chosen to take a particular context, concerned with understanding the relationships between functions and their graphs, in order to show how the typology might help to consider the design of suitable assessment tasks. Other contexts can be considered in a similar way. For the context we have chosen, each of the various types is relevant, while this might not always be the case for all contexts.

Each of the illustrative questions is related to a particular cubic function  $f(x) = x^3 - x + 4$ . For the convenience of the reader, a graph of this function is shown in Figure 9, drawn on a Texas Instruments TI-92 screen with  $-5 \leq x \leq 5$  and  $-4 \leq y \leq 6$ . The graph is the same shape as a graph of  $g(x) = x^3 - x = x(x - 1)(x + 1)$  moved up 4 units.



**Figure 9:** Graph of  $f(x) = x^3 - x + 4$  on a Texas Instruments TI-92

The ten assessment tasks given here have been chosen to show how different aspects of student thinking and mathematical behaviour can be addressed by asking questions in different ways. For convenience, we refer below to the typology by number. The questions should not all be regarded as mere alternatives to each other; they deal with a number of different aspects of this function and its graph. Similarly, it is unlikely that students would be asked to respond to several of these questions in the same test or assignment. The purpose of providing these examples is to illustrate the kinds of thinking needed when designing assessment tasks with the possible use of graphics calculators in mind.

#### *Type 1*

Use your graphics calculator to find solutions to  $x^3 - x + 4 = 0$ , correct to two decimal places.

Inexperienced students may need advice about when and even how to use their graphics calculator. A task like this can be used to determine whether or not students can use a process of

numerical refinement and understand the concept of a solution to an equation. Figure 10 shows some possible successive iterations on a Texas Instruments TI-82. The first screen may help students realise that there is likely to be only one solution, located somewhere between  $x = -2$  and  $x = -1$ ; the second screen verifies that there is in fact only one solution on this interval; the third screen allows an approximation to the solution to the specified accuracy to be obtained.

X	Y1
-3	-20
-2	-2
-1	10
0	28

X = -3

X	Y1
-2	-2
-1.9	-.959
-1.8	-.032
-1.7	.787
-1.6	1.504
-1.5	2.125
-1.4	2.656

X = -2

X	Y1
-1.8	-.032
-1.799	-.0253
-1.798	-.0146
-1.797	-.0059
-1.796	.00279
-1.795	.01147
-1.794	.02013

X = -1.796

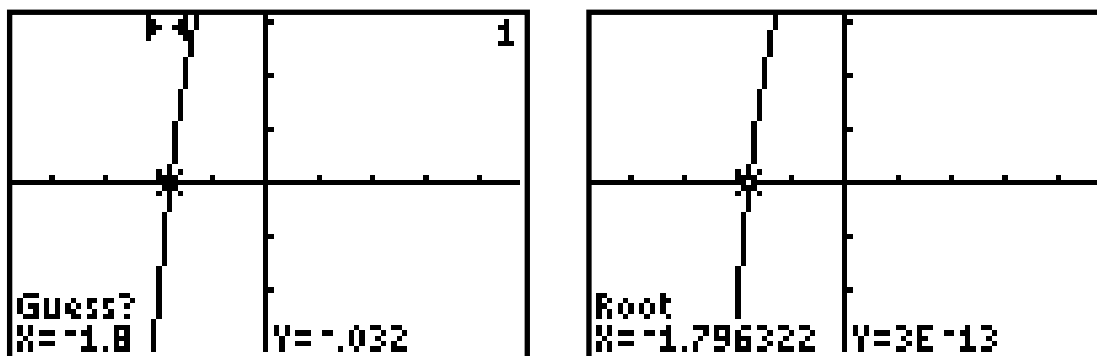
**Figure 10:** Using tables on a Texas Instruments TI-82 to solve an equation

This task also assesses whether or not students understand the meaning of ‘correct to three decimal places’. More sophisticated or experienced students who have found out how to use the automatic solve features of their calculator (if they have them) may prefer to use those instead of the tabular iteration.

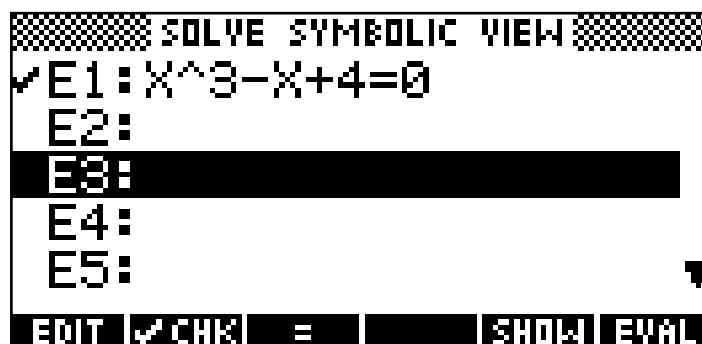
*Type 2*

Solve correct to two decimal places  $x^3 - x + 4 = 0$ .

With no direction at all, students must here decide for themselves what procedure is appropriate. On many graphics calculators, there may be several choices. For example, Figure 11 and Figure 12 show a graphical method on a Texas Instruments TI-82 and the use of an automatic solver on a HP-38G.



**Figure 11:** Using an automatic root finder on a Texas Instruments TI-82







**Figure 12:** Using the Solve aplet on a Hewlett Packard HP-38G

Whichever machine and method is chosen, the student needs to know how to set up the calculator to solve the equation, and is still expected to provide a solution to the specified level of accuracy. Students need to be able to navigate their way around graphics calculator modes, menus and syntax, and know how to interpret the resulting screens. For example, the TI-82 procedure for automatic root-finding is nicely implemented, but the student still needs to realise that a ‘root’ is needed (even though the assessment task does not use this term), know what the significance of the ‘guess’ is, and be able to recognise  $3E-13$  as a number very close to zero. An assessment task like that shown is providing evidence that students can make use of their graphics calculator as well as providing evidence of their mathematical thinking.

*Type 3*

Given  $f(x) = x^3 - x + 4$ ,

(a) Evaluate  $f(-1.795)$  and  $f(-1.799)$

(b) Hence give an estimate (to 3 decimal places) for the value of  $c$  if  $f(c) = 0$ .

Although this question requires no more computation than is available with a scientific calculator, and also requires some understanding of linear interpolation, a skilful user of a graphics calculator may well find efficient ways of completing part (a) and may use an alternative method to help them with part (b).

*Type 4*

Given  $f(x) = x^3 - x + 4$ , solve  $f(x) = 12 - x$ .

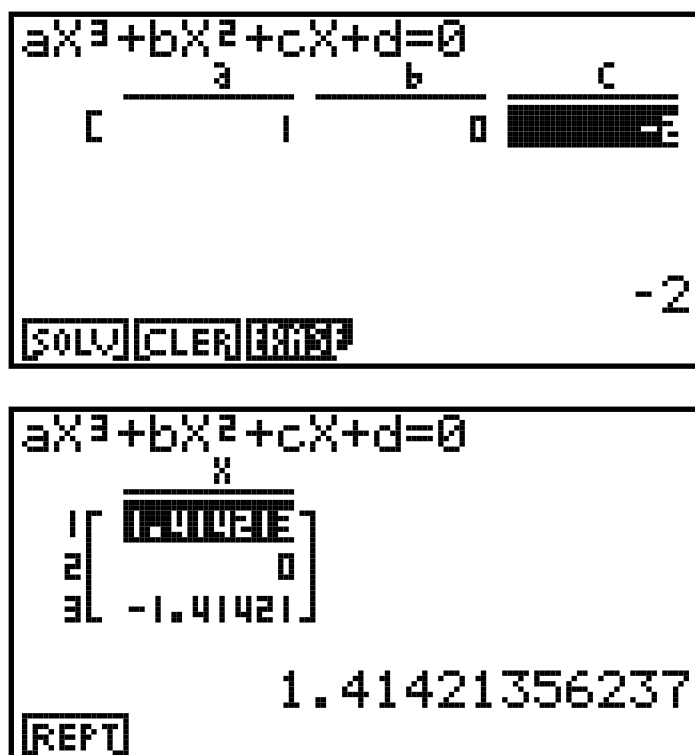
Some students may prefer to solve this equation analytically from the outset, reducing the equation to  $x^3 = 8$ . However, others may use their calculator to get one solution ( $x = 2$ ) and then search for an analytic solution, after noticing that the solution appears to be an integer. (This is not unlike realising that a quadratic expression could have been factored after using the quadratic formula produces rational roots).

*Type 5*

Solve exactly  $x^3 - x + 4 = x + 4$ .

Learning the difference between exact and approximate numbers and thus solutions to equations is rather difficult, and many students do not learn this until a quite advanced stage of mathematics is reached. On many graphics calculators today, one of the exact solutions ( $x = 0$ ) will be readily available through solving or graphical solving routines. Although most calculators do not

in fact give an integer for the result, they *appear* to do so, since there is no decimal point after the zero, which has been rounded to the accuracy of the calculator. Thus, a student using a Casio fx-9700 may produce the screens shown in Figure 13 after rearranging the equation into the form  $x^3 - 2x = 0$ :



**Figure 13:** Approximate solutions to a cubic equation on a Casio fx-9700

However, the other solutions are given as decimals (e.g., 1.41421356237). Although some students would recognise the solutions as  $\pm\sqrt{2}$ , many would not; in addition, many students using a calculator here that required initial guesses to be made for solutions may miss the third solution (or even the second). Only an analytical solution will be a mathematically adequate response to a question of this kind, and it is expected that the student should realise that the command ‘solve exactly’ has this meaning. Although the solution presented by a student in response to this question may not refer to the calculator at all, the students thinking may be aided by the calculator, or the numerical solutions provided by the calculator may be used to reassure them that their analytical solution is correct.

*Type 6*

Solve for  $x$ :  $x^3 - x + 4 = (a - 1)x + 4$ .

Unlike the illustration of Type 5, only an analytic solution is feasible here; students who enter the equation directly into an automatic solve area of the calculator may find that the calculator appears to offer an incorrect solution. In fact, the calculator will regard the variable  $a$  as a constant, use its present numeric value (entered previously) and then proceed to ‘solve’ the equation.

*Type 7*

Use the fact that the solutions to  $x^3 - x = 0$  are  $x = -1, 0$  and  $1$  to explain why  $h(x) = x^2(x^2 - 2)$  has turning points at  $(-1, -1)$ ,  $(0, 0)$  and  $(1, -1)$ .

This question demands that students write an explanation, based on their understanding of the links between the two graphs. Although it may be helpful to graph the two functions to fully understand the question, the response to this question should rely on observing that one function is the derivative of the other.

*Type 8*

A container for liquid is shaped like a rectangular box with a small cylindrical neck in the centre and on top of the box. The neck can hold 4 cc of liquid. The dimensions of the box are such that the length of one of the sides of the base is 1 cm less than its height and the length of the other side is 1 cm more. Express the volume of the container as a function of its height.

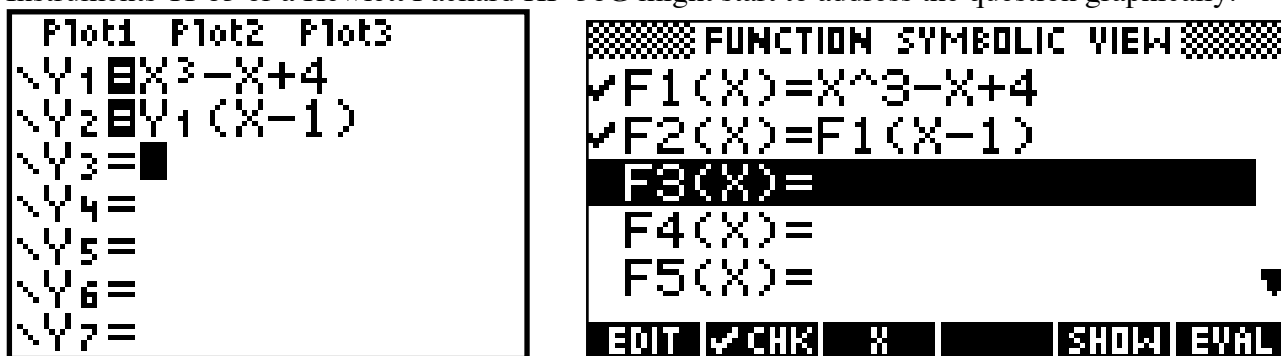
Clearly, questions like the Type 8 example are unaffected by the availability of graphics calculators.

*Type 9*

The solution to  $x^3 - x + 4 = 0$  is -1.796 (to 3 decimal places).

Find the solution(s) to  $(x - 1)^3 - (x - 1) + 4 = 0$ .

This question is intended to determine whether students understand the relationships among graphs and equations and the effect of a horizontal transformations on the graph of a function. Students who do understand these relationships will have little difficulty quickly writing down the solution of  $x = -0.796$ . It would be quite inefficient to use a graphics calculator to deal with this task, even though it is possible to do so. Figure 14 shows how sophisticated users of either a Texas Instruments TI-83 or a Hewlett Packard HP-38G might start to address the question graphically.

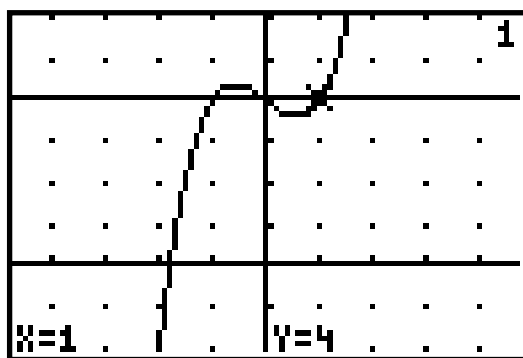


**Figure 14:** Defining functions with horizontal translations on a TI-83 and a HP-38G

For each of these two calculators, the appropriate graphs will be drawn by the commands shown. However, students with enough sophistication to recognise that a horizontal translation is involved would not need to draw the graphs and then manipulate them in order to solve the equation; this would be inefficient for someone with that level of understanding. Similarly, of course, it would be unnecessary and inefficient to enter the equation  $(x - 1)^3 - (x - 1) + 4 = 0$  directly into a graphics calculator in order to solve it.

*Type 10*

The graph below shows a cubic function and the line  $y = 4$  graphed for  $-4.7 \leq x \leq 4.7$  and  $-2 \leq y \leq 6$ . Give a definition for the cubic function.



**Figure 15:** Texas Instruments TI-82 screen to be interpreted

Questions of Type 10 serve a very useful purpose, and can be quite revealing about student misconceptions, although care must be exercised that the answer is unambiguous (or the scoring method is tolerant of a range of correct responses). The traced point (1,4) in Figure 15 is included to allow students to check their answers efficiently. The intention of this question is that students will reason from their knowledge of the shape of the graph, roots and vertical transformations that the function graphed is a vertical translation of 4 units of a function with roots at -1, 0 and 1 (*apparently* so, at least, from the graph). So a likely choice for the function is  $y = x(x - 1)(x + 1) + 4$ . Many students would like to reassure themselves that their reasoning was correct by graphing the function on their graphics calculator.

The above ten illustrations of designing assessment tasks suggest that there is no easy alternative to careful consideration of likely student thinking and response to assessment tasks at the design stage, which of course is also the case for more familiar kinds of assessment than those involving graphics calculators. The likely responses of students to tasks depends on the levels of sophistication of the students, both in terms of mathematical thinking and also in terms of calculator fluency.

Several of the illustrated questions above for which it is expected that a graphics calculator is not expected to be used may well be attempted by students with the aid of their calculators, if they have ready access to them. When assessing student responses to tasks, the only evidence usually available to us is what they write down; as shown above, there are many opportunities for a student's mathematical thinking to be supported, challenged and influenced by disciplined use of a graphics calculator. It is precisely for this reason that graphics calculators are potentially so useful to both teachers and learners of mathematics, in fact. Even though student responses to a question do not necessarily show these effects explicitly, it is necessary to contemplate them at the design stage.

### IMPLICATIONS OF THE USE OF GRAPHICS CALCULATORS IN ASSESSMENT

As suggested earlier, the use of graphics calculators in assessment is an important step towards integrating the technology into the curriculum. Before this step is taken, there are not likely to be substantial effects on the curriculum, since the graphics calculator is likely to be regarded merely as an optional extra, albeit a desirable one. After the step is taken, however, curriculum developers, teachers and their students are likely to see the mathematics curriculum through fresh lenses. Especially for high stakes assessment, the use of graphics calculators for assessment is likely to have the flow-on effect of sanctioning and encouraging calculator ownership and use.

One likely effect is that the availability of graphics calculators may seem to trivialise some mathematical procedures that presently take a long time. A good example is the solution of systems of linear equations. Many students (probably too many, in fact) learn this as a rather complicated

procedure, especially for systems with more than two equations. It takes students a good deal of time to learn such procedures, and a good deal of extra time is spent developing fluency, and maintaining it. Yet few would argue that this time is well spent, in terms of students developing an understanding of the nature of systems of linear equations. Little insight for anything is added by learning how and when to use Gauss-Jordan elimination or Cramer's Rule, and many teachers regard the time spent as a necessary evil, rather than intellectually productive. In fact, it takes so long to develop expertise with solving the systems that many of us have had too little time left to devote attention to the more important task of *constructing* such systems; consequently, we have often taught students well how to solve a system that someone else has constructed, but been very much less successful at teaching them how to construct such systems for themselves. Perhaps we will be able to redress the balance a little if students can use a graphics calculator to handle the routine procedures. The use of the graphics calculator will possibly allow us to recognise better which aspects of mathematics are worth the most attention and which are less worthy of the all too scarce time available for teaching and learning.

A second kind of effect of using graphics calculators in assessment relates to the potential for students to develop mathematical insight. An example concerns drawing graphs of elementary functions. Prior to the use of graphics calculators, students were expected to learn to draw graphs of functions, and often spent a fair deal of time doing so. However, the actual experience of the students was often concerned with the mechanics of drawing the graph, and much more rarely concerned with what a graph actually was, what it was for, and why anyone would want to draw one, except for the obvious reason that the textbook, teacher or examiner had directed that it be done. When drawing graphs by hand, students frequently plot many more points than they need, or many less than they need, because they have developed too little intuition for what a graph should look like. Neither of these is efficient. It is indeed ironic that, before the availability of graphics calculators, so few students seemed to draw a graph of their own volition, as if they did not really believe that they are useful for anything. If many of the mechanical details can be left to a machine, however, students can focus attention more carefully on *using* a graph for some purpose. and there is some prospect that they will come to see that some kinds of questions lend themselves well to graphical support. If time is freed up to work with graphs rather than work at producing them, we can be more optimistic that our students will see the connections between graphs and equations, or will see that the graphs of continuous functions look like straight lines if you look at them closely enough, or that the gradient of a curve changes from negative to positive at a local minimum point, and so on. Graphs have much to tell us, but it seems that students have been too busy drawing them to listen. Using graphics calculators in assessment, and thus integrating them into a course, may raise levels of mathematical insight significantly.

A third kind of implication is that we may take the opportunity provided by the graphics calculator to augment the curriculum. A good example of this occurs with data analysis. Prior to the use of graphics calculators in assessment, it was unreasonable to expect students to engage in data analysis and when it was included in assessment, data sets were always very small (or given in summary form). Frequently, the emphasis was on computation of numerical statistics, with little attention given to relationships among variables and with appropriate inferences from data. Only least squares linear regression was involved, since all else was too complicated. However, if students have a graphics calculator with a number of regression models, as all graphics calculators do at present, it makes sense for the curriculum to be changed slightly to examine data with a view to finding, understanding and using relationships, even though they might not be linear in form. Although the relevant mathematical statistics is too sophisticated for a high school audience, the idea that data may be modelled with various kinds of functions, of which linear is only one, is quite accessible to the great majority.

From the perspective of curriculum development, an implication of the use of graphics calculators in assessment is that learning to choose and use technology might be contemplated as an explicit goal. For example, in recent revisions to senior high school courses in Western Australia to take advantage of the graphics calculator, a new general objective was added: "Students will select

and make use of appropriate technology”. With such an objective made explicit, we might expect that classroom time and energy will be spent trying to help students attain the objective, and that some assessment of the extent to which students are inclined and able to use the graphics calculator sensibly will become normal.

A concern for equity has often been voiced in Australia, regarding the availability of graphics calculators to be used for assessment tasks. Some people have been quite concerned about implications for less affluent communities, less able to afford sophisticated technology. In fact, however, the inclusion of graphics calculators into assessment structures, especially high-stakes assessment structures, provides a powerful argument that the technology is not a frill, but a necessary part of the curriculum. This in turn may make it easier than would otherwise have been the case for school boards to provide funds to reduce the inequities.

A final implication was illustrated in some of the examples in the preceding section. When graphics calculators are available to students for assessment, a multiplicity of methods may be encouraged, rather than the traditional focus on doing mathematics the ‘right’ way. An extensive example of this, in the context of equations, is provided by Kissane (1995). While it could be argued that it is intrinsically better for students to learn several ways of dealing with mathematical problems rather than just one, the main argument for this multiplicity of approaches is that seems likely to strengthen conceptual development to have multiple perspectives accessible to students.

## CONCLUSION

All assessment requires careful thought, if we are to ensure that we are getting useful, reliable information about student achievements and difficulties. The use of graphics calculators in assessment presents some new challenges, which require a good knowledge of the capabilities of the technology and the educational outcomes we seek. Although there are many issues related to the use of graphics calculators in assessment, none of them presents insurmountable obstacles to intelligent calculator use in assessment at all levels. Although some of the issues are related to unfamiliarity with the technology, and the fact that it is not yet universally accessible to students, other issues are more fundamental and less transient. But much is to be gained by integrating graphics calculators into assessment structures. In particular, mathematics education might finally be able to take advantage of the many new opportunities offered by personal technology. The effort to come to terms with the problems can reasonably be expected to reap substantial benefits.

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A copy is available at <http://wwwstaff.murdoch.edu.au/~kissane/papers/1cme8.pdf>