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1. compute $F^{12} = F^{23}$ from corresponding image points and estimate the epipoles (the left and right null vector of the matrix) $\mathbf{e}_l^{12} = \mathbf{e}_l^{23}$ and $\mathbf{e}_r^{12} = \mathbf{e}_r^{23}$.

2. set the 3×4 perspective transformation matrices $P_l^{12} = P_l^{23} = [\, I \mid \mathbf{0} \,]$ and compute $P_r^{12} = P_r^{23} = [\, M \mid \mathbf{e}_r \,]$ where $I$ is the identity matrix, $M = [\, [\mathbf{e}_r]_\times F \mid \mathbf{e}_r \,]$, $F$ is the fundamental matrix and $\mathbf{e}_r$ is its null vector, and $[\mathbf{v}]_\times$ denotes the skew-symmetric matrix of $\mathbf{v}$.

3. compute the projective structure $\mathbf{X}^{12}$ using $P_l^{12}$ and $P_r^{12}$. Then compute the transformation $T$ such that

$$
\begin{aligned}
P_l^{23}\, T \mathbf{X}_i^{12} &= \mathbf{x}_i^2 \\
P_r^{23}\, T \mathbf{X}_i^{12} &= \mathbf{x}_i^3, \quad \forall i = 1...n
\end{aligned}
$$

   where $\mathbf{x}_i^j$ is the $i$-th corresponding point from image $j$.

4. compute $S = T^{-\top}$. Matrix $S$ is known to be similar to the defective matrix $T_E$ having eigenvalues $\lambda(S) = \{e^{i\theta}, e^{-i\theta}, 1, 1\}$ and eigenvectors $e(S) = \{[1, i, 0, 0]^\top, [1, -i, 0, 0]^\top, [0, 0, 1, 0]^\top, [0, 0, 1, 0]^\top\}$.

5. apply the eigen-decomposition to $S$ and retrieve the real eigenvector $\mathbf{w} = [w_1, w_2, w_3, w_4]^\top$ that corresponds to the repeated real eigenvalue.

6. compute $H_\infty = M - \mathbf{e}_r[w_1, w_2, w_3]/w_4$.

7. solve the elements of $K = AA^\top$ in the equality (defined up to a scale) $K = H_\infty K H_\infty$.

8. recover $A$ from the Cholesky decomposition of $K$. The estimated camera matrix $A$ (3×3 upper triangular) is essential for Euclidean reconstruction.

## 2. A sensitivity analysis

Let $B$ be the similarity transformation of $S$ and $T_E$, then the eigenvectors of $S$ are simply $e(S) = \{\mathbf{e}_1 = B\,[1, i, 0, 0]^\top, \ \mathbf{e}_2 = B\,[1, -i, 0, 0]^\top, \ \mathbf{e}_3 = \mathbf{e}_4 = $

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# Euclidean Reconstruction from an Image Triplet: A Sensitivity Analysis

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## Abstract

*This paper studies the sensitivity in Euclidean reconstruction from an image triplet taken by an uncalibrated camera mounted on a robot arm. The idea of such a reconstruction is closely related to that proposed in [4]. In this paper, we focus on an intermediate step of the reconstruction procedure which requires estimating the screw axis that corresponds to the defective eigenvector of a 4×4 matrix. Hundreds of the conducted synthetic tests show that the algorithm is very sensitive to image noise and perturbations on camera motions and that if the matrix is perturbed by Gaussian noise then the reliability of the computed screw axis can be estimated.*

## 1. Introduction

It is often essential for a stereo vision system to reconstruct the 3D map of an environment without any prior knowledge of the characteristics and relative geometry of the cameras. There has been much research conducted in this area [1], and a review to all of past contributions to the literature is not possible in this short paper. In this research work we are interested in reconstructing the 3D map of objects in the workspace of an RTX robot using an uncalibrated camera mounted at the robot's wrist for object recognition. If an image triplet (3 images) that is captured by the camera is such that $F^{12}$ and $F^{23}$ are identical, where $F^{ij}$ denotes the fundamental matrix between the $i$-th and $j$-th images, then a Euclidean reconstruction procedure similar to that proposed by Zisserman et al [4] can be applied. The sensitivity of the reconstruction procedure in the presence of image noise and perturbations of camera motion is the focus of this research. Before going into the details of our conducted sensitivity analysis, the reconstruction procedure is briefly described below:

$B [0, 0, 1, 0]^\top$}. From here on, we denote the true value of an entity $k$ as $\bar{k}$ and its observed value in the presence of noise as $\tilde{k}$. Let $\bar{S}$ be the matrix under the noise-free conditions, $N$ be the noise matrix such that its elements satisfy a Gaussian distribution $\mathcal{N}(0, \sigma^2)$ with the standard deviation $\sigma$ denoting the noise level, and the eigenvalues of $\tilde{S} = \bar{S} + N$ be $e(\tilde{S}) = \{\tilde{\mathbf{e}}_1, \tilde{\mathbf{e}}_2, \tilde{\mathbf{e}}_3, \tilde{\mathbf{e}}_4\}$. Since the eigenvalues associated with $\mathbf{e}_1$ and $\mathbf{e}_2$ are disjoint from that associated with $\mathbf{e}_3$, the subspace spanned by $\mathbf{e}_1$ and $\mathbf{e}_2$ (denoted as $ran([\mathbf{e}_1, \mathbf{e}_2])$) is less sensitive to perturbations and remains relatively invariant. Define the distance between two subspaces $V_1$ and $V_2$ to be (see [2])

$$\text{dist}(V_1, V_2) = \| W_1^\top Z_2 \|_2 = \| Z_1^\top W_2 \|_2 \qquad (1)$$

where $[W_1 \; W_2]$ and $[Z_1 \; Z_2]$ are square and orthogonal matrices; $V_1 = ran(W_1)$; $V_2 = ran(Z_1)$. If the noise level is low then $\text{dist}(ran([\tilde{\mathbf{e}}_1, \tilde{\mathbf{e}}_2]), ran([\bar{\mathbf{e}}_1, \bar{\mathbf{e}}_2]))$ is small. This means that the 2D subspace that contains the screw axis $\bar{\mathbf{w}}$ $(=\bar{\mathbf{e}}_3)$ is also relatively invariant although the 1D subspace $ran([\bar{\mathbf{e}}_3])$ is much less stable.

The perturbed $\tilde{S}$ matrix is usually non-defective and 3 cases can occur when applying the eigen-decomposition to it:

*Case 1:* $\lambda(\tilde{S})$ contains 4 complex eigenvalues (i.e. 2 complex conjugate pairs).

*Case 2:* $\lambda(\tilde{S})$ contains 2 complex eigenvalues in a conjugate pair and 2 different real eigenvalues.

*Case 3:* $\lambda(\tilde{S})$ contains 4 different real eigenvalues.

In all the cases above, a criterion for correctly selecting the eigenvector that is closest to the screw axis is essential. This is, we want to choose an eigenvector $\tilde{\mathbf{e}}$ (and thus $\tilde{\mathbf{w}}$) such that

$$\Delta w = \cos^{-1} | <\tilde{\mathbf{w}}, \bar{\mathbf{w}}> | \rightarrow \min \qquad (2)$$

where $|.|$ denotes absolute value and $<.,.>$ denotes inner product. Since $\bar{\mathbf{w}}$ is the eigenvector of $\bar{S}$ that corresponds to a defective eigenvalue, $\tilde{\mathbf{w}}$ (or $\tilde{\mathbf{e}}$) should be the eigenvector corresponding to the eigenvalue $\tilde{\lambda}$ that is a repeated eigenvalue of a *nearby* defective matrix. Such a $\lambda$ has a large condition number, $C_\lambda$, the definition of which is given as (see [2] for explanations):

$$C_\lambda = 1 / | <\mathbf{x}, \mathbf{y}> |$$

where $\mathbf{x}$ and $\mathbf{y}$ are the left and right eigenvectors associated with $\lambda$. This criterion for determining $\tilde{\mathbf{w}}$ is effective only if the noise level is low. For a significantly high level of noise, $\tilde{S}$ may not be near a defective matrix or, even if it does, the nearby defective matrix may not be $\bar{S}$. We shall assume from here onward that $\|N\|_2 << \|\bar{S}\|_2$.

If the eigenvector $\tilde{\mathbf{e}} = [\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \tilde{e}_4]^\top$ chosen is complex, as is always for case 1, then an approximation of the screw axis must be in $ran([\tilde{\mathbf{e}}, \tilde{\mathbf{e}}^*])$ where $\tilde{\mathbf{e}}^*$ is the complex conjugate of $\tilde{\mathbf{e}}$ [†]. Indeed, it was found that the distance (defined in (1)) between $ran([\tilde{\mathbf{e}}, \tilde{\mathbf{e}}^*])$ and the 2D subspace (computed using the Schur decomposition of $\bar{S}$) containing $ran([\bar{\mathbf{w}}])$ is very small. To estimate the screw axis $\tilde{\mathbf{w}} = [\tilde{w}_1, \tilde{w}_2, \tilde{w}_3, \tilde{w}_4]^\top$ from the complex eigenvector $\tilde{\mathbf{e}}$, we adopted the following procedure:

- If $\| \text{Re}(\tilde{\mathbf{e}}) \|_2 > \| \text{Im}(\tilde{\mathbf{e}}) \|_2$ then $\tilde{w}_i$, $i = 1 \ldots 4$ was estimated as $\text{sign}(\text{Re}(\tilde{e}_i)) \left( \text{Re}(\tilde{e}_i)^2 + \text{Im}(\tilde{e}_i)^2 \right)$.
- Otherwise, $\tilde{w}_i$, $i = 1 \ldots 4$ was estimated as $\text{sign}(\text{Im}(\tilde{e}_i)) \left( \text{Re}(\tilde{e}_i)^2 + \text{Im}(\tilde{e}_i)^2 \right)$.

Here, $\text{Re}(.)$ and $\text{Im}(.)$ denote the real component and imaginary component of the complex number concerned, and

$$\text{sign}(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

To study the sensitivity in estimating the screw axis from $\tilde{S}$ at different noise levels, we set $\sigma$ to values ranging from 0.001 to 0.05 while maintaining the mean value of $\|\bar{S}\|_2$ at $\approx 8$. In each set of the experiments, the transformation matrix $T_E$ with a randomly generated rotation angle $\theta$ (see step 4, Section 1) was first computed. A non-singular matrix $B$ was then randomly generated; the computation of $T = B T_E B^{-1}$ and $\bar{S} = T^{-\top}$ followed. The noise matrix $N$ was then generated for a specified noise level $\sigma$ and $\tilde{S}$ was computed. We define the *condition number ratio* $r$ as follows:

$$r = \left( \max_\lambda C_\lambda \right) \Big/ \left( \min_\lambda C_\lambda \right).$$

For each of the estimated $\tilde{\mathbf{w}}$, we also computed $\Delta w$ as given in (2).

Figure 1 shows the relationship between $r$ and $\Delta w$ for 50 synthetic tests with $\theta = 35°$ and $\sigma = 0.001$. All the $\Delta w$'s for case 1 are very small ($< 0.5°$) even for $r \leq 10$; however, for the same ratio $r$ for case 2, the $\Delta w$'s are much larger (up to $3.5°$). Note that the angle deviations $\Delta w$ for case 2 show an exponential drop with an increase of $r$ and that for a small $\sigma$, case 3 seldom occurs.

Figure 2 shows another 40 synthetic tests with $\theta = 24°$ and $\sigma = 0.05$. Almost all the computed $\Delta w$'s are too large to be acceptable. Case 3 occurs in 3 occasions. Most of the $\Delta w$ angles for case 1 are comparatively better than those for the other two cases. As mentioned above, for a high noise level $\tilde{S}$ may not be near the

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[†]Note that both $\tilde{\mathbf{e}}$ and $\tilde{\mathbf{e}}^*$ have the same condition number.
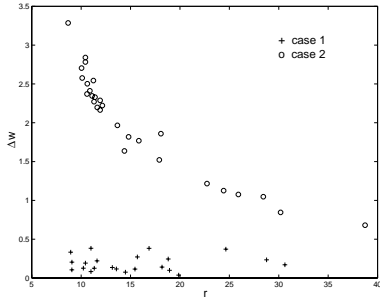
**Figure 1. Relationship between $r$ and $\Delta w$ when $\sigma = 0.001$.**

defective matrix $\bar{S}$. However, the plot shows a general decrease of $\Delta w$ when $r$ increases. Empirically, this demonstrates that if $\tilde{S}$ was near a defective matrix then this defective matrix was likely to be $\bar{S}$. Unfortunately, we have not been able to provide a formal mathematics proof to support this claim.
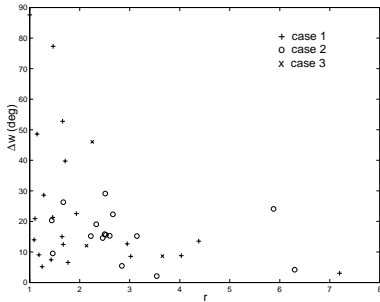


**Figure 2. Relationship between $r$ and $\Delta w$ when $\sigma = 0.05$.**

The above analysis and experiments show that a large value of $r$ is prerequisite to a reliable estimation of $\tilde{\mathbf{w}}$ on which the subsequent Euclidean reconstruction depend. We have simplified our analysis by applying perturbations directly to $S$ from which $\tilde{\mathbf{w}}$ is determined. In real applications, the image noise comes from the errors in the image coordinates of corresponding points, and the behaviour of the noise matrix $N$ is not clear. Although we have not investigated into this further at this stage, our analysis shows that a small $r$ is a good indication that the estimated $\tilde{\mathbf{w}}$ is poor.

The above procedure for computing the screw axis had been applied to many synthetic tests in which the image coordinates or camera motion were perturbed. Due to space limitation, we only outline an experiment that involves adding Gaussian noise of $\mathcal{N}(0, 0.25^2)$ to the coordinates of corresponding points. The procedure for computing the camera matrix for Euclidean reconstruction is given in Section 1, and the fundamental matrices were computed using the method de-

scribed in [3]. In Table 1, $\Delta w < 5°$ yet the errors in focal length and image centre are quite large, and the percentage error of the aspect ratio is around 15%. This result indicates that a good estimate of the fundamental matrix is essential so that optimal estimates of $H_\infty$, and subsequently, $A$ are also guaranteed.

| | |
|---|---|
| $\bar{S}$ | $\begin{bmatrix} 2.228 & 0.542 & 0.246 & 0.253 \\ 1.476 & 2.883 & -0.640 & 1.176 \\ -2.369 & 1.199 & 1.966 & -0.174 \\ -1.843 & -2.244 & 0.704 & -0.000 \end{bmatrix}$ |
| $C_\lambda$ | $1.8363, 1.8363, 7.881 \times 10^6, 7.881 \times 10^6$ |
| $\bar{r}$ | $4.29 \times 10^6$ (case 2) |
| $\bar{\mathbf{w}}$ | $[\, 0.269, 0.405, -0.311, -0.8163 \,]^\top$ |
| $\bar{A}$ | $\begin{bmatrix} -740.655 & 0 & -17.244 \\ 0 & -860.029 & -3.077 \\ 0 & 0 & 1 \end{bmatrix}$ |
| $\tilde{S}$ | $10^4 \times \begin{bmatrix} -0.314 & 0.471 & 0.331 & -0.049 \\ 0.233 & -0.263 & -0.211 & 0.056 \\ 0.478 & -0.714 & -0.494 & 0.072 \\ 0.927 & -1.396 & -0.962 & 0.137 \end{bmatrix}$ |
| $C_{\tilde\lambda}$ | $2.528, 32.073, 32.073, 47.052$ |
| $\tilde{r}$ | $18.608$ (case 2) |
| $\tilde{\mathbf{w}}$ | $[\, -0.259, -0.345, 0.367, 0.823 \,]^\top$ |
| $\Delta w$ | $4.747°$ |
| $\tilde{A}$ | $\begin{bmatrix} -791.091 & -0.000 & -28.782 \\ 0 & -797.402 & -73.835 \\ 0 & 0 & 1 \end{bmatrix}$ |

**Table 1. Estimation of screw axis and camera matrix in the presence of image noise.**

## 3. Conclusion

In this paper, we have focused on the estimation of the screw axis from a 4×4 defective matrix in Euclidean reconstruction. The results of our analysis indicate that if the perturbed matrix is not near a defective matrix then the estimated screw axis is not reliable. We conclude that all the steps involved in the reconstruction procedure must be optimized.

## References

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