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A Hybrid Optimisation Method for Managing Uncertainty in Capacity Expansion Planning

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Abstract. This paper addresses the application of a new fuzzy robust optimisation model in a capacity planning problem with uncertainties in demand and resource constraints. Fuzzy robust optimisation model employs robust optimisation formulation and fuzzy programming to deal with these uncertain variables. A power system capacity expansion plan is used as a case study to test the applicability of the proposed model. The results of this case study show the advantages of the proposed method compared to classical stochastic programming.

Keywords: Robust Optimisation, Capacity Planning, Fuzzy Programming, Uncertainty Analysis.

1 Introduction

Nowadays, optimisation methodologies have applications in many areas such as production planning, scheduling, transportation, financial planning, and chemical and physical systems. Many problems in these areas require that decisions be made under uncertainty; therefore, some methodologies are employed to deal with uncertainty. These include: stochastic programming, fuzzy mathematical programming, and dynamic programming.

Stochastic programming which is the first category of optimisation modelling method for dealing with uncertainty contains four sub-categories: Stochastic Linear Programming (SLP), Stochastic Integer Programming (SIP), Stochastic Non-linear Programming (SNP) and Robust Optimisation (RO). The stochastic programming methods usually partition the uncertain variables into two stages [1]. The first stage solves the problem before the actual realisation of uncertain variables and the second stage, which is called recourse stage, considers the random variables and minimises the total cost of the two stages. Fuzzy Programming (FP) which is the second category in optimisation modelling methods for dealing with uncertainties involves the optimisation of a precise objective function subject to constraints with fuzzy coefficients [2]. Zimmerman [3] and Negoita [4] presented approaches for converting Fuzzy Linear Programming (FLP) to Linear programming (LP). Zimmerman's approach defuzzifies the objective function and adds a constraint to the existing model and Negoita's approach works on defuzzifying the fuzzy constraints by applying different alpha cuts. The third category for optimisation method under uncertainty is Dynamic Programming (DP) presented by Bellman [5] as a mathematical theory for dealing with multi-stage decision making process. A discrete-time system with N-time period containing the state of the system, a control action and a random parameter are applied. It is assumed that the present state of the system is fully determined by its recent history [1].

These three categories show different optimisation modelling approaches in dealing with uncertain variables. Hybrid methods may be required based on the nature of the problems. For example, some problems depend on scenario-based and also ambiguous data; therefore there is a need for a hybrid method to model this kind of problems. This paper focuses on developing a new hybrid optimisation method by using the concept of robust optimisation and fuzzy programming where it is possible to deal with scenario-based and ambiguous data. In section 2, the details of fuzzy robust optimisation modelling method will be described. A case study is introduced in section 3. The application will be discussed in section 4, followed by conclusion in section 5.

2 Methodology

Consider Mulvey's general robust optimization model where it is possible to integrate goal programming formulation with a scenario-based description of problem data [6]:

$$Min \sum_s p_s \xi_s + \lambda(x, y_1, \ldots, y_s) + \omega(z_1, \ldots, z_s) \quad [1a]$$

$$s.t: \quad Ax = b \quad [1b]$$

$$B_s x + C_s y_s + z_s = e_s \quad for\ all\ s \in \delta \quad [1c]$$

$$x \geq 0,\ y \geq 0 \quad and \quad \xi = c_s{}^T x + d_s^T y_s \quad for\ all\ s \in \delta$$

Where $\{y_1, y_2, \ldots, y_s\}$ and $\{z_1, z_2, \ldots, z_s\}$ are control variables and error vectors for each scenario ($S \in \delta$) respectively. The objective function contains the summation of fixed and operational costs with probability p_s ($\sum p_s \xi_s$), the cost variation ($\lambda(x, y_1, \ldots, y_s)$) and feasibility penalty function ($\omega(z_1, \ldots, z_s)$). Eq. (1b) denotes the structural constraints that are fixed and free of noise and Eq. (1c) denotes the control constraints with $\{d_s, B_s, C_s, e_s\}$ coefficient set [6]. Coefficients may be fuzzy or crisp in new fuzzy robust optimisation method therefore two fuzzy programming approaches will be employed as described below.

Zimmermann [3] presented Fuzzy Linear Programming (FLP) with Linear Membership Function (LMF) as follows:

$$\begin{aligned} &Min \quad Z = Cx \\ &s.t. \quad Bx \leq b_0 \qquad [2] \\ &x \geq 0 \end{aligned}$$

Where Z denotes the set of k fuzzy objectives; $Bx <= b_0$ represents the set of constraints and x are the structural variables. Minimising the single worst deviation of Z from Z^* is the final goal. This can be solved by maximising the minimum membership function. A single new variable (ψ) has been added to convert FLP model to LP model by minimising the deviation among the fuzzy objectives.

$$\begin{aligned} & Max \quad \psi \\ & s.t. \quad \psi - f_k(z_k) \le 0 \quad for \ k = 1,..., k \\ & Bx \le b_0, \\ & x \ge 0 \end{aligned} \qquad [3]$$

Zimmerman [3] considered fuzzy sets in the objective function and crisp sets in the constraints. Negoita et al. [4] presented an approach to convert a FLP problem to a LP problem with fuzzy constraints. The fuzzy linear problem is based on the following formulation [7]:

$$\begin{aligned} & Max \quad cx \\ & s.t. \quad k_1x_1 + k_2x_2 + ... + k_nx_n < K \\ & x_j \ge 0 \qquad j = 1,..., n \end{aligned} \qquad [4]$$

Where c is a vector of coefficients, and k_1, k_2 and k_n are fuzzy sets. If $k_1, k_2,..., k_n, k$ has the limited number of membership function values:

$$\mu_k \in \{\alpha_1, \alpha_2,..., \alpha_p\} \ with \ \ 0 \le \alpha_1 \le \alpha_2 \le ... \le \alpha_p \le 1 \qquad [5]$$

then a FLP problem will be altered to a finite set of LP problems as follows:

$$\begin{aligned} & Max \quad cx \\ & s.t. \quad x_1R_{\alpha i}(k_1) + x_2R_{\alpha i}(k_2) + ... + x_jR_{\alpha i}(k_j) + ... + x_nR_{\alpha i}(k_n) < R_{\alpha i}(k) \\ & x_j \ge 0 \qquad j = 1,...,n \qquad i = 1,..., p \end{aligned} \qquad [6]$$

The new Fuzzy robust optimisation model has two main phases. The first phase considers the model (7a) to find the best alpha cut among other alpha cuts for each scenario. This model does not contain the cost variation ($\lambda(x, y_1, ..., y_s)$) and feasibility penalty function ($\omega(z_1,..., z_s)$) which are related to different scenarios in a robust optimisation method but alpha cut sets are associated with only one scenario.

$$Min \quad p_s c^T x + p_s d^T y \qquad [7a]$$

$$s.t. \quad Ax = b \qquad [7b]$$

$$Bx + Cy = e \qquad [7c]$$

$$x \ge 0, \ y \ge 0$$

As stated, two methods (Zimmerman [3] and Negoita [4]) converting Fuzzy Linear Programming (FLP) to Linear programming (LP) have been applied to find the best alpha cut among others. The following model shows the converted fuzzy linear programming:

$$Min \quad \psi \qquad [8a]$$

$$s.t. \quad \psi - p_s c^T x - p_s d^T y = 0 \qquad [8b]$$

$$Ax = b \qquad [8c]$$

$$Bx + Cy = e \qquad [8d]$$

$$x \geq 0,\ y \geq 0$$

Where ψ is the degree of achievement of fuzzy goals. The best alpha cut for each scenario will be selected by running this model for different alpha cuts. Then, the second phase is the robust optimisation phase for the best selected alpha cuts which follows from the robust optimisation model (Eq. [1]).

3 Case study

A power system capacity expansion problem has been adopted to test the proposed method. In this case study, " the capacities of a set of power plants are selected to minimise the capital and operating cost of the system, meet customer demand, and satisfy physical constraints" [6]. Load duration (θ) and level of demand (pl) are the main uncertain variables in defining the future demand. The electric power demand (d_j) is obtained from the load duration curve by rearranging the instantaneous demands in decreasing order [8].

$$d_j = (pl_j - pl_{j-1}) \times \theta_j \qquad [9]$$

It is assumed that θ and pl are fuzzy variables and the membership functions for each variable are given in Figure 1.

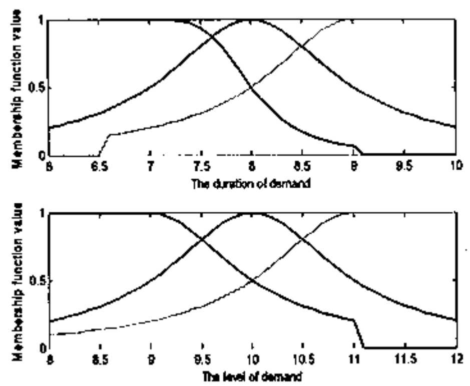

Figure 1–The membership function of fuzzy level and duration of demand

An optimisation model minimising the fixed and operational costs is employed to find the best alpha cut for each scenario. The initial model has fuzzy coefficients in its objective function and constraints. The model was implemented for each scenario by using fmincon function in Matlab optimisation toolbox. Fmincon function attempts to find a constrained minimum of a scalar function of several variables starting at an initial estimate which is referred to as constrained nonlinear optimization or nonlinear programming [9]. 9 (3×3) scenarios were developed based on the 3 categories (low, medium and high) for each variable (θ , pl).

A set of the best alpha cuts for each scenario with their probabilities was imported into the fuzzy robust optimisation method for solving the power capacity expansion problem. Robustness is obtained by varying the penalty parameters (λ, ω) and observing the changes in expected value and expected infeasibility [10].

4 Discussion

Higher amount of ω means more utilisation of all installed capacity and higher λ produces a set of design capacities and allocations whose cost is intensive to whatever scenario evolves [10]. The standard deviation shows the relationship of the cost variation by varying the values of λ and ω. Figure 2 shows the downward trend of standard deviation, average cost and excess capacity for different λ and ω values. The average cost has an upward trend for a given λ and increasing ω while expected capacity reduces to zero (in the limit) [10].

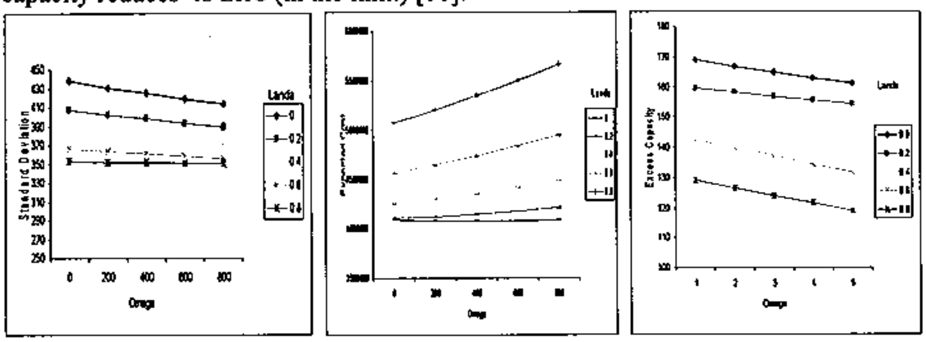

Figure 2 – The standard deviation, average cost and excess capacity for different λ and ω values

Table 1 shows the comparison of intelligent robust optimisation with stochastic programming ($\lambda = 0$ and $\omega = 0$) for the power capacity expansion problem. While the cost in robust optimisation is higher than the cost in stochastic programming (by 4.1% for $\lambda = 0.40$ and $\omega = 200$), there is 9.1% reduction in excess capacity.

Table 1– The Comparison of new fuzzy RO model with stochastic programming

	Expected cost	*Variance of cost*	*Excess capacity*
Intelligent robust optimisation	4253172	385	153
Stochastic programming	4085772	438	169

Decision maker should have a trade off between the increasing amount of cost and reducing amount of excess capacity to find robust solution. The minimum growth in cost and maximum reduction in excess capacity compared with stochastic programming may be an indicator for decision makers to decide about the exact robust solution. For example, there is 0.423% increase in cost with 5.49% reduction in excess capacity for $\lambda = 0.20$ and $\omega = 200$ but decision maker may prefer more increase in cost with more reduction in excess capacity. Therefore, the final decision about the exact point depends on the decision makers' viewpoints and also the additional budget for increasing the cost.

5 Conclusion

This paper presents a fuzzy robust optimisation method applied to a future power capacity planning case study. This model penalises for excess capacity and also identifies trade-offs between shortage and excess capacity based on different scenarios for electricity demand containing fuzzy variables. It is assumed that the model addresses the generation issue based on a given price structure in this framework, however, it has the ability to generate different strategic option if there is a fluctuation in price which will affect the electricity demand.

This research presents a method which gives the opportunity to decision makers to decide the best strategic option. This option may depend on different circumstances such as additional budget, and rules which may be forced by government. The developed framework has the ability to provide a mechanism to recognise the trade off based on decision makers' preferences.

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