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6. Planetary systems

6.1 The Solar System

6.1.1 Structure of the Solar System

[R05 1.2; MD 1]

- → The Solar System is a closed dynamical system. Distance to nearest star (Proxima Centauri, $d = 1.29$ pc $\sim 4 \times 10^{16}$ m) is $\sim 10^4$ times distance of Neptune from the Sun (30 AU $\sim 4 \times 10^{12}$ m).
- → Astronomical unit AU = average Earth-Sun distance $\simeq 1.49 \times 10^{11}$ m.
- → Main constituents of the Solar System: Sun, planets, dwarf planets, satellites, and small Solar-System bodies (asteroids, comets, trojans, centaurs and Trans-Neptunian Objects).

Definitions

- → A *planet* is a celestial body that (a) is in orbit around the Sun, (b) has sufficient mass for its self-gravity to overcome rigid body forces so that it assumes a hydrostatic equilibrium (nearly round) shape, and (c) has cleared the neighbourhood around its orbit (IAU 2006).
- → A *dwarf planet* is a celestial body that (a) is in orbit around the Sun, (b) has sufficient mass for its self-gravity to overcome rigid body forces so that it assumes a hydrostatic equilibrium (nearly round) shape, (c) has not cleared the neighbourhood around its orbit, and (d) is not a satellite (IAU 2006).
- → All other objects, except satellites, orbiting the Sun shall be referred to collectively as *Small Solar-System Bodies* (IAU 2006).
- → A *satellite* of a planet is a small body whose distance from the planet never exceeds the semi-major axis of the planet (Shen & Tremaine 2008).
- → Asteroids: small (size $< 1000\,\mathrm{km}$) rocky bodies orbiting the Sun.
- → Comets: small icy objects orbiting the Sun that develop diffuse gaseous envelopes and tails when sufficiently close to the Sun.

Distribution of bodies in the Solar System

→ Inner planets (Mercury, Venus, Earth, Mars) have semi-major axes $0.38 < a/\,\mathrm{AU} < 1.52$.

→ Asteroid belt with $2 \lesssim a/\,\mathrm{AU} \lesssim 4$ (largest body in the asteroid belt is the dwarf planet Ceres with mass $\sim 10^{-4} M_{\rm Earth} \sim 2\times10^{-3} M_{\rm Mercury} \sim 7\times10^{-2} M_{\rm Pluto}$).

→ Outer planets (Jupiter, Saturn, Uranus, Neptune) have semi-major axis $5.2 < a/\,\mathrm{AU} < 30$.

→ Trojan asteroids: same $a \simeq 5.2\,\mathrm{AU}$ as Jupiter. In L_4 and L_5 of the Sun-Jupiter system.

→ Centaurs: minor bodies (asteroids/comets) with a in the range of the outer planets (between Trojans and Kuiper belt).

→ Trans-Neptunian Objects (TNOs): objects with $a > a_{\rm Neptune} \sim 30\,\mathrm{AU}$. TNOs include: Kuiper belt including Pluto and Plutinos (30-50 AU), Scattered disc objects (50-1000 AU), Oort cloud, a roughly spherical shell of comets (1000-50000 AU, up to 1/4 of distance to Proxima Centauri).

→ Dwarf planets: Ceres ($a \sim 2.8\,\mathrm{AU}$ asteroid belt), Pluto ($a \sim 40$, Kuipert belt), Haumea ($a \sim 43\,\mathrm{AU}$, Kuiper belt), Makemake ($a \sim 45\,\mathrm{AU}$ Kuiper belt) and Eris ($a \sim 68\,\mathrm{AU}$; Scattered disc).

→ Titius-Bode law. It is an empirical law: distance from the Sun in units of Earth distance

$$a_n = 0.4 + 0.3 \cdot 2^n \qquad for\ n = -\infty, 0, 1, 2$$

Mercury ($-\infty$), Venus (0), Earth (1), Mars (2), asteroids (3), Jupiter (4), Saturn (5), Uranus (6). Poor agreement with Neptune.

Planets

→ Solar mass $M_\odot \sim 2\times10^{30}$ kg. Solar radius $r_\odot \sim 7\times10^{8}$ m. Most massive planet is Jupiter with mass $M_{\rm Jupiter} \sim 0.001 M_\odot$ and radius $\sim 0.1 r_\odot$.

→ The rest of the planets add up to less than a Jupiter mass. Second most massive planet is Saturn $M \sim 0.3 M_{\rm Jupiter}$. Mass of Earth $M_{\rm Earth} = 5.97\times10^{24}$ kg$\sim 3\times10^{-6} M_\odot \sim 3\times10^{-3} M_{\rm Jupiter}$

→ Eccentricities of planets are small < 0.1, with the exception of Mercury with $e \sim 0.206$.

→ Inclinations are small $i < 4^o$, with the exception of Mercury with $i \sim 7^o$.

→ See tables with main properties of planets. Masses, semi-major axes, periods, eccentricities, inclinations (Appendices III and IV of R05; FIG CM6.1 and FIG CM6.2)

Satellites

→ Number of satellites. Earth (1: Moon); Mars (2: Phobos, Deimos); Jupiter (67; 16 massive; 4 Galilean: Io, Europa, Ganymede, Callisto); Saturn (62; 1 massive Titan); Uranus (27; 5 massive); Neptune (13; 1 massive Triton)

→ Typically orbits of satellites are almost coplanar (with the planet orbit around the Sun), have small eccentricity, and rotate in the same direction in which the planet revolves around the Sun. There are exceptions (typically more distant satellites). Satellites rotate: typical spin-orbit coupling (rotation frequency and orbit frequency are equal).

→ Moon. Mass $M_{\rm Moon} \sim (1/81) M_{\rm Earth}$. Radius 1700 km $\sim 0.27 r_{\rm Earth}$ ($r_{\rm Earth} \sim 6371$ km)

→ The four outer (and most massive) planets have ring systems, composed of millions of small satellites.

6.1.2 Resonances in the Solar System

[R05 1.2; MD 1]

→ There are several resonances in the Solar System. For instance, mean motion resonances, i.e. simple ratios between mean motions n (and therefore semi-major axes a).

→ Jupiter-Saturn "great inequality":

$$\frac{n_{\rm Jupiter}}{n_{\rm Saturn}} \sim \frac{5}{2}.$$

The ratio is close to 5/2, but significantly different from 5/2. The difference from the exact resonance is of the order of 1%. In the expansion of the disturbing function this gives rise to small denominator and therefore a resonant (long-period) term with period

$$T = \frac{2\pi}{-2n_J + 5n_S} \sim 900\text{yr}.$$

→ Neptune is in $3:2$ mean motion resonance with Pluto (and other TNOs, known as the Plutinos)

$$\frac{n_N}{n_P} \simeq \frac{3}{2}.$$

Neptune is also in $2:1$ resonance with other TNOs known as Twotinos.

→ Laplace resonance among Io, Europa and Ganymede (Jupiter's satellites)

$$n_1 - 3n_2 + 2n_3 = 0,$$

where n is mean motion ($n_1 = 2n_2 = 4n_3$). Phases are such that triple conjunctions are avoided.

→ In the asteroid belt the distribution of the asteroids' semi-major axes matches the positions of the resonances w.r.t. Sun-Jupiter system. For instance, there are gaps in correspondence of 3:1, 2:1 and other resonances (the so called Kirkwood gaps) while there is a peak at the 3:2 resonance. See Fig. 1.7 of MD (FIG CM3.24).

→ Trojan asteroids have same mean motion and a as Jupiter (60 degrees ahead and 60 degrees behind Jupiter, librating around the triangular Lagrangian points L_4 and L_5 of the Sun-Jupiter system).

→ There are also spin-orbit resonance: for instance 3:2 spin-orbit resonance of Mercury. 1:1 spin-orbit resonance of the Moon with respect to the Earth.

6.1.3 Stability and dynamical evolution of the Solar System

[MD 9.10; R05 9]

→ An important question for celestial mechanics is whether the Solar System is stable. Do the current planet orbits remain almost unchanged over long time? Or should we expect, for instance, that some of the planets will be ejected from the Solar System? Or planet collisions? Or collisions of planets with the Sun?

→ Questions. 1) Does the distribution of planetary orbits change substantially over long times? 2) If yes, do the changes happen gradually or suddenly with close encounters? 3) If the Solar System is stable, is this because of the quasi circular orbits, low inclination and near commensurability of mean motions of planets?

→ From radioactive dating of Earth and Moon rocks we know that Solar System is about the age of the Sun 4.5×10^9 yr (i.e. billions of Earth orbits). Studies of fossils suggest that complex life has been present on Earth since 2×10^9 yr ago, implying that Earth orbit must have been almost unaltered since then.

Laplace-Lagrange secular theory

→ A possible approach is *analytic*. Take Lagrange's planetary equations and follow the evolution assuming that only secular terms in the expansion of the disturbing function are important (i.e. neglecting short-term and resonant terms). This is the so called *Laplace-Lagrange secular theory.*

→ We recall the classification of the terms in the expansion of the disturbing function (Chapter 4): *secular terms* (i.e. long-period terms, not depending on the mean longitudes), *resonant terms* (depending on the mean longitudes, but still long period because of resonance), and *short-period terms* (all other terms depending on the mean longitudes).

→ The bottom line of the application of Laplace-Lagrange secular analysis to the Solar System is that the system is, in this sense, stable: orbital parameters of the planets vary only little.

→ Of course the Laplace-Lagrange secular theory is limited because we know that resonant effects and possibly short-period terms might be important. Including these effects require numerical instead of analytic approach.

Numerical studies

→ Two main *numerical methods* have been used to study the stability and long-term evolution of the Solar System.

(1) Direct numerical integration of the orbits. Exact equations. For outer planets, time-step is typically $\sim$ 40 d; including the inner planet a typical time-step is $\sim$ 8d (see Kinoshita & Nagai 1984; Wisdom & Holman 1991; Batygin & Laughlin 2008, ApJ, 683, 1207).

(2) Numerical integration of the averaged planetary equations, after selecting and averaging terms of the disturbing function. Approximated equations. Massive algebraic calculations. Much longer time-step: typically 500 yr. The secular equations are obtained by averaging the equations of motion over the fast angles that are the mean longitudes of the planets (Laskar 1988, 2008).

→ The bottom line of the numerical studies of kinds (1) and (2) is that planets move on chaotic orbits (strong dependence on initial conditions, sudden variations of orbits), but at least the outer planet do not show sign of gross instability over timescales $\sim$ 10 Gyr, of the order of the lifetime of the Sun.

→ When inner planets are considered, some catastrophic behaviors are not excluded by these calculations. Possible that Mercury will collide with either Venus or the Sun and that Mars will be ejected by the Solar System (timescale $\lesssim$ 1 Gyr). Such events appear much more unlikely for the outer planets (Laskar 1994; Batygin & Laughlin 2008).

→ So, the numerical results indicate that, with few possible exceptions (Mercury, Venus, Mars) the planets remain close to their current orbits for up to $\sim$ 10 Gyr. In this sense the Solar System is stable: planets might be an example of bounded chaos (MD 9.10; Lecar et al. 2001, ARA&A).

→ Implication of chaos. Fundamental limits to our ability to predict the positions of the planets for long time intervals.

→ For instance, maximum Lyapunov exponent $10^{-6.7}$ yr^{-1} (Lyapunov time $\sim$ 5Myr for inner planets; Laskar 1988), means that an error of 1 cm in the current position of the Earth propagates so that the position of the Earth 200 million years in the future is basically unpredictable (MD 9.10).

→ Another interesting question is the fate of small bodies orbiting in different zones of the Solar System. For instance, it has been shown that the perturbations of the present-day planets are sufficient to eject nearly all the material (with the exception of the asteroid belt) from between the planets on time scales less than the age of the Solar System (Lecar et al. 2001). It follows, for instance, that the Centaurs are on unstable orbits.

6.1.4 Dynamics of satellites

Moon-Earth-Sun dynamics

[R05 5.12.3; R05 10]

→ *Lunar problem*: orbital motion of a satellite around a planet. It is not just a two-body problem: motion influenced mainly by the Sun, but also significant perturbations from the other planets. Classical lunar problem: planet-satellite-Sun.

→ Focus on Moon-Earth system, but same method for any other planet-satellite system. Mean orbital elements of Moon-Earth orbit:

$$a \simeq 3.844 \times 10^5 \, \text{km}$$

$$e \simeq 0.055$$

$$i \simeq 5^o 09',$$

where the inclination i is measured w.r.t. the plane of the ecliptic.

→ Elements vary periodically due mainly to gravitational interaction with the Sun. Other planets have small, but non-negligible effect on Moon's orbit

→ Dynamics of Earth-Moon-Sun best studied as general three-body problem in Jacobian coordinates. $m_1 = E$, $m_2 = M$, $m_3 = S$. Take $\mathbf{c}$ centre of mass of m_1-m_2 system: $\mathbf{c} = (m_1\mathbf{r}_1 + m_2\mathbf{r}_2)/(m_1 + m_2)$. We define $\boldsymbol{\rho} = \mathbf{r}_3 - \mathbf{c}$ and $\mathbf{r} = \mathbf{r}_{12} = \mathbf{r}_2 - \mathbf{r}_1$ (in general we adopt the notation $\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i$).

→ The equations of motion in Jacobi coordinates can be used to describe M-E-S system exploiting the fact that $r/\rho \approx 1/400 \ll 1$, where r is E-M distance (d_{EM} ∼384400 km) and ρ is distance between S and M-E centre of mass (150×10^6 km). It is possible to expand the equation in terms of the small quantity r/ρ (see Roy 10.8 10.9).

Stability of satellites' orbits

[Shen & Tremaine 2008]

→ Giant (outer) planets in the Solar System have some regular satellites (within $0.05\Delta_{\rm H}$) and irregular satellites (up to $0.6\Delta_{\rm H}$), where $\Delta_{\rm H}$ is the Hill radius.

→ Fraction of irregular/regular and nature of orbit (prograde/retrograde) different in different planets.

→ Numerical integration of orbits suggests that the observed distribution is a consequence of stability/instability of distant satellites of orbits the giant planets (Shen & Tremaine 2008; see FIG CM6.7).

→ Typically, stable orbits are found out to $\sim 0.5\Delta_{\rm H}$ (prograde) and $\sim 0.7\Delta_{\rm H}$ (retrograde), provided the inclination is not large.

→ In all cases instability between $\Delta_{\rm H}$ and $2\Delta_{\rm H}$. At distances larger than $\Delta_{\rm H}$ orbits are unstable for Saturn, but not for the other planets (consequence of Jovian perturbations).

6.2 Extrasolar planetary systems

[CO07 7.4, 23.1; www.exoplanet.eu]

6.2.1 Detection and properties of known extrasolar planets

→ The question of the existence of planets and planetary systems around other stars is very old. Remarkably, Giordano Bruno (1548-1600) envisaged the existence of infinite number of inhabited worlds around other stars.

→ First extrasolar planet around a typical (G type) star discovered in 1995 (Mayor and Queloz 1995): the star is "51 Pegasi" the planet is "51 Peg b". Before (1992) planets were found only around the pulsar PSR 1257+12 (three ∼ Earth-size planets)

→ By May 2013 December 2012 the numbers are: 693 planetary systems / 888 planets / 133 multiple planet systems.

Methods of detection

→ Methods of detection: radial velocity measurements, astrometric wobbles, eclipses (transits), gravitational microlensing (planet orbits around lens star), direct imaging, pulsar timing (anomalies in the timing of the observed radio pulses).

→ Most successful methods: radial velocity measurements (measures of variation of line-of.sight velocity of the star) and transits (measures of variation of the luminosity of the star).

→ Radial velocities as small as 1-3 $\mathrm{m\,s^{-1}}$ can be measured. Corrections due to rotation of Earth, orbit of Earth, rotation of star, pulsation, etc...

→ Analysis of the radial velocity curve allows to put a lower limit to the planet mass (see spectroscopic binaries), because inclination is unknown. What we measure is $m_{planet} \sin i$ (the mass of the star is derived from the star spectral type).

→ The inclination can be measured if the planet is transiting, i.e. if there is an eclipse (from the specific shape of the light curve, which also allows to measure the star size). If transit plus radial velocity measures, then we measure mass of the planet.

→ With transit, beside inclination, it is possible to measure also radius of planet and then density (when combined with radial velocity).

→ With HST planets found also astrometrically (for instance wobbles of Gllese 876, a.k.a. GJ 876)

→ *Multi-planet systems.* Typically detected from radial velocity measurements. The radial velocity curve is perturbed periodically (see fig. 23.2 of CO07; FIG CM6.3).

Example

→ *Measuring mass from radial velocity.* [CO07 23.1] The method is the same as for spectroscopic binaries. Take for example the case of 51 Peg b.

$$T = 2\pi\sqrt{\frac{a^3}{G(M_* + M_p)}}.$$

Using $M_p \ll M_*$ we get

$$a \simeq \left[\frac{GM_*T^2}{4\pi^2}\right]^{1/3}.$$

The measured quantities are the period $T = 4.23$ days and the maximum line-of-sight speed of the star $v_{*,obs} = 56.04\,\mathrm{m\,s^{-1}}$; from the star spectral type (G) we infer $M_* \simeq 1M_\odot$. Writing the above equation in physical units (MKS) we have

$$a \simeq \left[\frac{6.67\times10^{-11}\times1.99\times10^{30}\times(4.23\times24\times3600)^2}{4\pi^2}\right]^{1/3} = 7.65\times10^9\,\mathrm{m} \simeq 0.05\,\mathrm{AU}.$$

We also know (from the shape of the velocity curve) that the orbit is quasi circular, so the intrinsic velocity of the planet is

$$v_p \approx \frac{2\pi a}{T} = \frac{2\pi\times7.65\times10^9}{4.23\times24\times3600} \simeq 131\,\mathrm{km\,s^{-1}}$$

The ratio of star-to-planet intrinsic velocities is

$$\frac{v_*}{v_p} = \frac{v_{*,obs}}{v_p\sin i} = \frac{M_p}{M_*}$$

so

$$M_p\sin i = \frac{v_{*,obs}}{v_p}M_* = \frac{56.04}{1.31\times10^5}M_\odot = 8.48\times10^{26}\ \mathrm{kg} \simeq 0.45M_{\mathrm{Jupiter}}$$

where we used $v_{*,obs} = 56.04\,\mathrm{m\,s^{-1}}$ and $M_{\mathrm{Jupiter}} \simeq 1.89\times10^{27}\,\mathrm{kg}$. The above is a lower limit on the actual mass M_p.

Physical properties

→ *Mass distribution.* Recall that $M_J \sim 0.001M_\odot \sim 300M_{\mathrm{Earth}}$. Mass interval from Earth mass to 30 Jupiter mass. But note that above $\sim 13M_{\mathrm{Jupiter}}$ and below $75M_{\mathrm{Jupiter}}$ the systems can be brown dwarves. Differential distribution

$$\frac{\mathrm{d}n}{\mathrm{d}M} \propto M^{-1}.$$

See plot of mass distribution taken from www.exoplanets.eu (FIG CM6.4).

→ *Semi-major axis distribution.* Typical semi-major axis of known exoplanets is $\lesssim 1\,\mathrm{AU}$. See plot of mass vs. semi-major axis from www.exoplanets.eu (FIG CM6.4). Many of the known extrasolar planets have large masses (Jupiter or more), but orbits much closer to the star (period of days) than Jupiter. With longer monitoring we can measure planets with larger semi-major axis (i.e. longer period).

→ *Eccentricity distribution.* Orbits circular for small semi-major axis (tides) and quite eccentric for some outer planets. Distribution quite different w.r.t. Solar System. 60% of the known extrasolar planets have orbits with eccentricity $e > 0.2$. See plot of eccentricity distribution (fig. 1 of Zakamska et al. 2011; FIG CM6.5).

6.2.2 Extrasolar planetary systems : resonances, stability and habitability

Resonances

→ *Resonances in extrasolar planetary systems.* Mustill & Wyatt (2011): "Mean motion resonances (MMRs) occur when two objects' orbital periods are close to a ratio of two integers [...]. There are also now numerous examples of suspected or confirmed MMRs in extrasolar planetary systems (e.g., GJ 876 b and c in a 2:1 resonance, Laughlin & Chambers 2001)." Origin and interpretation of these findings is currently debated.

→ Mustill & Wyatt (2011): "Although resonant orbits occupy only a small volume of phase space, they are common because of a locking mechanism which can preserve the resonance once attained". This mechanism is known as trapping into resonance.

→ Batygin & Morbidelli (2013): "A considerable fraction of multi-planet systems discovered by the observational surveys of extrasolar planets reside in mild proximity to first-order mean motion resonances. However, the relative remoteness of such systems from nominal resonant period ratios (e.g. 2:1, 3:2, 4:3) has been interpreted as evidence for lack of resonant interactions. [...] It is possible that many planetary systems are actually in resonance even if their orbital periods are apparently not in (strict) commensurability."

Stability

→ *Stability* [see Dvorak et al. 2010] The problem of the stability of planetary systems is a question that concerns only multi-planetary systems that host at least two planets.

→ A planetary system can be stable if planetary orbits are close to stable resonant periodic orbits. We recall that not all resonances are stable. Typically stable systems combine mean motion resonance with spacing of phases such that close encounters are avoided.

→ Given the high fraction of stars that are binaries, a relevant problem is the stability of orbits of planets orbiting a binary star (either around only one member or around both stars; see e.g. Dvorak et al. 2010)

→ *Example of numerical study of stability* (Fabrycky & Murray-Clay 2010). Application to the extrasolar planetary system HR 8799. This system has three planets (detected by direct imaging): the masses are estimated to be 7, 10, $10 M_{\rm Jupiter}$. Numerical orbit integration with Stoer-Bulisrch method shows that if the orbits are not resonant the system is highly unstable (see fig. 1 of Fabrycky & Murray-Clay 2010, FIG CM6.8). Dynamical stability can be used to constrain orbital parameters, in addition to observed quantities.

Habitability

→ A particularly interesting application of the studies of stability in extrasolar planetary systems is the search for Earth-like planets in the habitable zones of these systems.

→ *Habitable zone.* Kano & Gelino (2012, Astrobiology): "The Habitable Zone (HZ) for a given star describes the range of circumstellar distances from the star within which a planet could have liquid water on its surface, which depends upon the stellar properties."

→ For an extrasolar planet to be habitable, it must be on a small-eccentricity, stable orbit within the habitable zone. It must remain for long enough time in the conditions that allow to have liquid water on the surface.

→ Among the planets found up to 2010 none of the planets are Earth-like and have all the necessary properties to develop a biosphere (Dvorak et al. 2010). Recently few candidates have been found: GJ 581g, with mass $\sim 3-4 M_{\rm Earth}$, in the habitable zone of a dwarf (M) star (Vogt et al. 2010) and Kepler 22b with radius $\sim$2.4 the Earth radius, in the habitable zone of a Sun-like (G) star (Borucki et al. 2012).

→ In the search of Earth-like planets, an important field is the study of possible terrestrial (habitable) planets in systems with an observed Jupiter-like giant planet. There are a few possible configurations (see fig. 9 of Dvorak et al. 2010; FIG CM6.6): the dynamical stability of these configurations are studied. An interesting and possibly stable case (depending on specific orbital parameters) is to have Trojan planets, i.e. planets in 1:1 resonance with the giant planet in the L_4 and L_5 of the star-giant planet system.

→ Heng & Vogt (2011): "It is generally accepted (Lammer et al. 2010) that, for stellar masses below $0.6 M_{\odot}$, an Earth-mass exoplanet orbiting anywhere in the habitable zone becomes tidally locked or spin-synchronized within the first Gyr of its origin, such that it keeps one face permanently illuminated with the other in perpetual darkness. Such tidal locking will greatly influence the climate across the exoplanet and figures prominently in any discussion of its potential habitability."

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