UNIVERSITY OF BOLOGNA - FACULTY OF ENGINEERING
INTERNATIONAL MASTER COURSE IN CIVIL ENGINEERING - A.Y. 2012/2013

INTRODUCTION TO NUMERICAL METHODS

LAB 7: ONE-SPACE DIMENSIONAL HEAT EQUATION

a)

Consider the initial-boundary-value differential problem, namely the one-space dimensional heat equation:

$$\begin{cases} u_t = c\, u_{xx} & x_{\min} < x < x_{\max}\ , \quad t > 0 \\ u(x, t = 0) = u_0(x) & x_{\min} \le x \le x_{\max} \\ u(x = x_{\min}, t) = f_1(t) & t > 0 \\ u(x = x_{\max}, t) = f_2(t) & t > 0 \end{cases}$$

where $c \in R^+$, $x_{\min}, x_{max} \in R\ (x_{\min} < x_{max})$ and u_0, f_1, f_2 are the problem data, that is the heat equation parameter (diffusivity), the space domain limits, and the initial and boundary condition functions, respectively. (notice that we are considering Dirichlet boundary conditions!)

In MATLAB, create a script that computes numerically an approximate solution of the above problem by using the finite difference method:

After setting the data of the problem, that is the diffusivity c, the space domain limits x_{min}, x_{max}, the initial condition function $u_0(x)$, the (left and right) boundary condition functions $f_1(x)$, $f_2(x)$ (directly in the script or asking to the user), and the space and time discretization steps h and k (so that the space-time domain mesh for the finite difference method is univocally defined), the script must compute the approximate values of the solution at the mesh nodes. At each time step, the script must show in a Figure the initial condition function, the current boundary condition values and the current approximate solution.

As far as the finite difference method is concerned, implement and test for different problem data and different space-time discretization steps (h and k) the fully explicit, the fully implicit and the Crank-Nicolson schemes (if you prefer, you can implement the general two-levels scheme and obtain the other schemes as particular cases)? What about stability? What about the stationary state solution (solution for $t \to \infty$)? Discuss the results.