

Questions of Mathematical Methods in Engineering

We propose a list of questions in preparation of the oral examination in the course of "Mathematical methods for engineering". It is advisable to answer in a loud voice. Take care of the precision of the answers. Questions with * may be assigned in the written examination.

0 Preliminaries

1. Given f and g measurable, with domain A , what is the definition of the relation $f \sim g$? *
2. What is the equivalence class $[f]$?*
3. What are the main properties of this relation?
4. What is the statement of the theorem of Beppo Levi?*
5. What is the statement of the dominated convergence theorem?*

1 Normed spaces

1. What is a norm ?
2. What is the definition of the norm $\|\cdot\|_\infty$ in $B(A, \mathbf{R})$?*
3. What is the definition of the norm $\|\cdot\|_1$ in $L^1(A)$?*
4. Given $A \subseteq X$, with X normed space, what is the definition of the interior of A and what does it mean that A is open?
5. Given $A \subseteq X$, with X normed, what is the definition of the boundary of A ?*
6. Given $A \subseteq X$, with X normed space, what does it mean that A is closed?
7. Given $A \subseteq X$, with X normed space, what is the definition of the closure of A ?*
8. Given $A \subseteq X$, with X normed space, what does it mean that A is dense in X ?*

9. State the definition of continuous function from $A \subseteq X$ to Y , with X and Y normed spaces.*
10. What is the definition of limit for $f : A \subseteq X \rightarrow Y$, with X and Y normed spaces?*
11. What is the definition of limit of a sequence?*
12. What is a Cauchy sequence?*
13. Give a characterization of the closure in terms of sequences. *
14. What does it mean that a normed space is complete? *
15. Give example of complete normed spaces.
17. What is an inner product?
18. What is the definition of the space $L^2(A)$?*
19. What is the definition of the norm associated with an inner product?
20. What is the Cauchy-Schwarz inequality?
21. What is a Hilbert space?*
22. State a result of existence and uniqueness of a point of minimal distance in A from a given point. *
23. How can the point of minimal distance in A from a given point be characterized, in case A is a closed subspace?*
24. What is an orthonormal system?*
25. How can an orthonormal basis be constructed, starting from an ordinary one?
26. If A is a closed subspace, how can the point of minimal distance in A from a given point be represented, in case A is a finite dimensional linear subspace?*
27. In what case does a sequence of orthogonal projections of x converge to x ?*
28. What is the classical Fourier expansion of an element of $L^2([-\pi, \pi])$?*
29. State a result of uniform convergence for classical Fourier series.*
30. What is a generalized Fourier expansion?*
31. Describe an application of Fourier series to problems with the heat equation.

2 Functions of one complex variable

1. What is the definition of complex derivative?* (Specify the assumption on the function)
2. What is a holomorphic function?*

3. What can one say, concerning sums, products, quotients, compositions of holomorphic functions?
4. What are the Cauchy-Riemann conditions and what do they have to do with holomorphic functions?*
5. What is the definition of the complex exponential functions?*
6. What is a logarithm function?*
7. What is the definition of $\sin(z)$, with $z \in \mathbf{C}$?
8. What is the definition of $\cos(z)$, with $z \in \mathbf{C}$?
9. What is the definition of the complex integral $\int_{\alpha} f(z)dz$?* (Specify the conditions in order that it is defined)
10. How does a complex integral change when passing from a path to another equivalent?*
11. What does it mean that two paths are A -homotopic?* (Specify the required assumptions)
12. What does it mean that A open subset of \mathbf{C} is simply connected?*
13. How does a complex integral of a holomorphic function change when passing from a certain closed path to a A -homotopic one?
14. What is the value of the complex integral of a holomorphic function on a closed path if the domain is simply connected?*
15. State Cauchy's integral formula. *
16. What is a power series?*
17. What is the radius of convergence of a power series?*
18. State the lemma of Abel.*
19. Given its radius of convergence, what can be said, concerning the set of complex numbers such that the series converges?*
20. What can be said for complex numbers z such that $|z - z_0|$ equals the radius of convergence?
21. What is the formal derivative of a power series?*
22. What can be said, concerning its radius of convergence?*
23. State the definition of analytic function.*
24. What is the connection between holomorphic and analytic functions?*
25. What is an isolated singularity of a holomorphic function?*
26. What is the Laurent expansion of a holomorphic function in a neighborhood of an isolated singularity?*
27. What does it mean that an isolated singularity is removable?*
28. What is a polar singularity?*
29. What is an essential singularity?*
30. What is the residue?*
31. What is the definition of the index?*

32. What is the statement of the residue theorem?*
33. What is a harmonic function?*
34. What is the connection between holomorphic and harmonic functions?*
35. State the Dirichlet problem for the Laplace equation.*
36. State the maximum principle.*
37. What is a conformal mapping?*
38. What is a homeomorphism?*
39. What does it mean that two open subsets of \mathbf{C} are conformally equivalent?*
40. What is the statement of the theorem of Riemann?*
41. What does it mean that a point in the boundary of an open subset A of \mathbf{C} is simple?*
42. State a theorem of extension of a conformal mapping to a homeomorphism between the closures.*
43. State a theorem of existence and uniqueness for the Dirichlet problem for the Laplace equation.*

3 Fourier transform

1. What is the definition of the Fourier transform of an element of $L^1(\mathbf{R}^n)$?*
2. State the Riemann-Lebesgue theorem.*
3. State some results of interaction between the Fourier transform and the derivatives.*
4. Illustrate the inversion formula for the Fourier transform.*
5. What is the class $\mathcal{D}(\Omega)$?*
6. What is the connection of this class with the spaces $L^1(\Omega)$ and $L^2(\Omega)$?
7. Describe some examples of elements of $\mathcal{D}(\Omega)$.
8. Describe the formula of Parseval.*
9. State the definition of the Fourier transform of elements of $L^2(\mathbf{R}^n)$.*
10. List the main properties of the Fourier transform in $L^2(\mathbf{R}^n)$.
11. What is a locally summable function (or equivalence class of functions)?*
12. Give examples of classes of functions which are locally summable.
13. Define the α -weak derivative of a locally summable function (if existing).*
14. Explain in what sense it is an extension of the classical α - derivative.
15. State some results of interaction between weak derivatives and Fourier transform. *

16. What does it mean that two locally summable functions are convolvable and what is the definition of convolution?*
17. State sufficient conditions in order that two functions are convolvable.*
18. State a result of connection between Fourier transform and convolution.*
19. Describe some results concerning the Helmholtz equation in \mathbf{R}^n .
20. Describe some results concerning the Cauchy problem for the heat equation.
21. Describe some results concerning the Cauchy problem for the wave equation.

4 Calculus of probability

1. What is a σ -algebra ?*
2. List the main properties of σ -algebras.
3. What is a probability measure?*
4. List the main properties of probability measures.
5. What is a probability space?
6. Describe the construction of a probability space in a finite set.
7. How many are the injective mappings from a set with k elements to a set with n elements ($k \leq n$)?*
8. How many are the subsets with k elements of a set with n elements ($k \leq n$)?*
9. What is the definition of the conditional probability $P(B|A)$?*
10. Describe Bayes' formula.*
11. Give the definition of independent family of events.*
12. Describe a Bernoulli process.*
13. What is the most probable number of successes in a Bernoulli process?*
14. What is a simple random variable?*
15. What is a real random variable (rrv)?*
16. What is an n -dimensional random variable?*
17. Define Borel subsets of \mathbf{R}^n .*
18. Define the distribution function of a rrv.*
19. List its main properties.
20. What is the probability of the union of an increasing sequence of events? (Explain what is the sense of "increasing") *
21. What is the probability of the intersection of a decreasing sequence of events? (Explain what is the sense of "decreasing") *
22. Define the distribution law of a n -dimensional random variable.*

23. List its main properties.
24. What is a discrete random variable?
25. What is the density of a random variable?*
26. What is the expectation of a simple random variable?*
27. What is the expectation of a rrv?*
28. Give conditions, in order that a discrete random variable admits expectation and explain how it can be computed.*
29. Give conditions, in order that a random variable with density admits expectation and explain how it can be computed.*
30. List the main properties of expectation.
31. State some results for the calculation of the expectation of $g \circ X$, with X n -dimensional random variable and $g : \mathbf{R}^n \rightarrow \mathbf{R}$ continuous.*
32. Define the variance of a rrv.*
33. Give the definition of independence of a family of rrv.*
34. What can one say about the density of an n -dimensional random variable with independent components?*
35. What can one say concerning the expectation of the product of independent rrv-s?*
36. What can one say, concerning the variance of the some of independent rrv-s?*
37. State Chebyscev's inequality.*
38. State the weak law of large numbers.*
39. Define convergence in law.*
40. State P. Levy's theorem.*
41. State the central limit theorem.*
42. Define Markov chains.*
43. Describe the problem of the player's ruin.