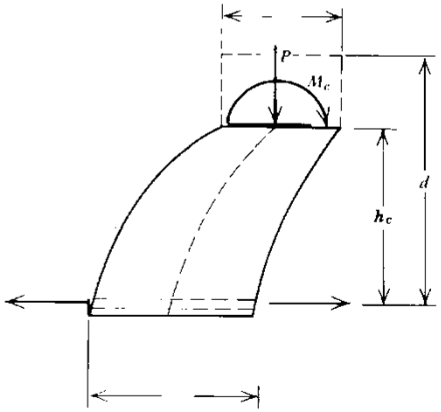
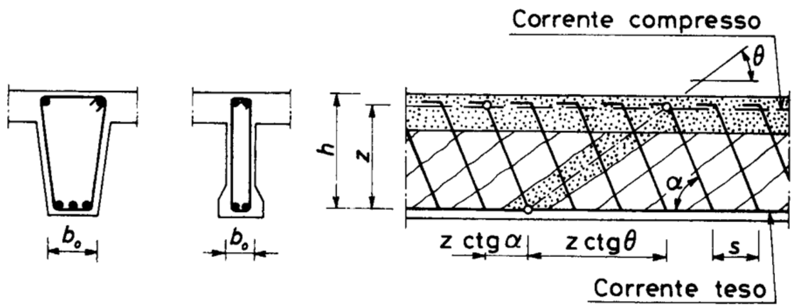
ULTIMATE LIMIT STATE VERIFICATION AGAINST SHEAR



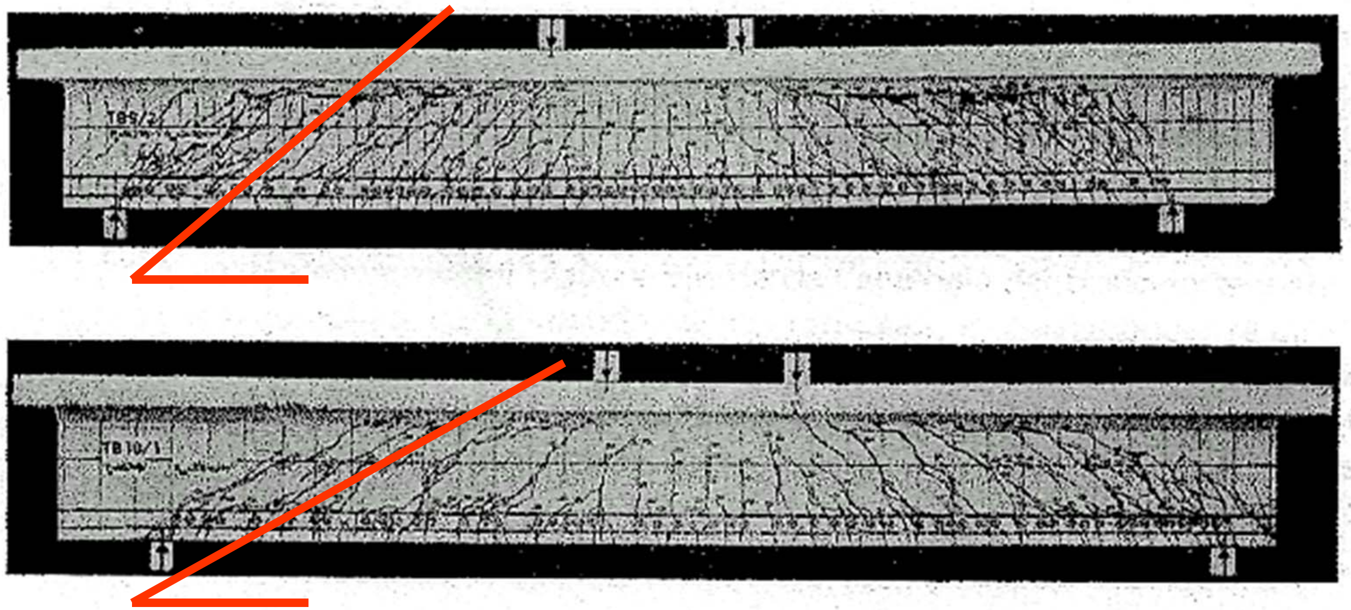


Claudio Mazzotti
DICAM – Faculty of Engineering
University of Bologna

REINFORCED CONCRETE BEAMS – BEHAVIOUR UNDER SHEAR

CRACKING OF BEAMS WITH DIFFERENT STEEL REINFORCEMENT

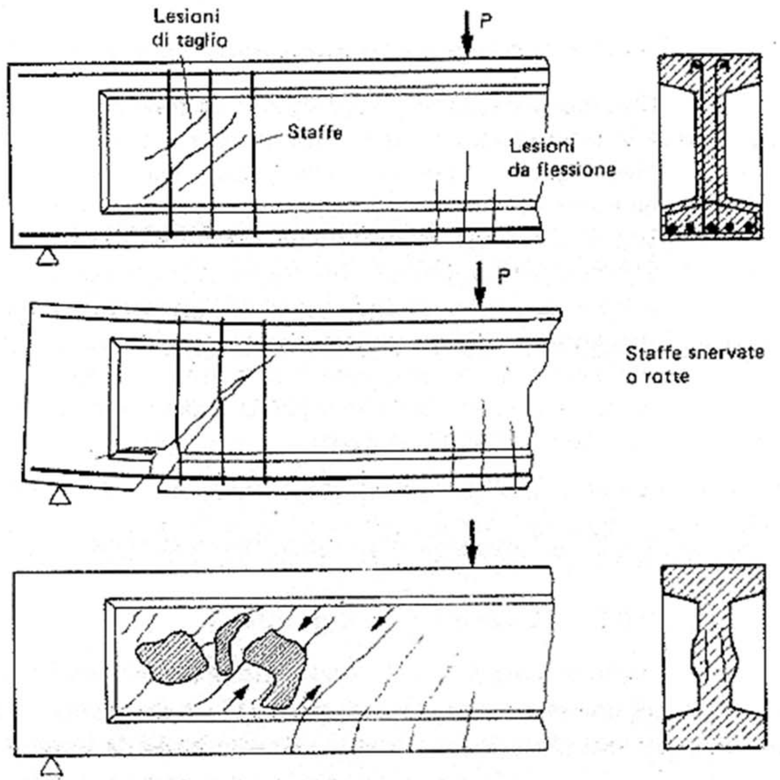
HIGH STEEL REINFORCEMENT



LOW STEEL REINFORCEMENT

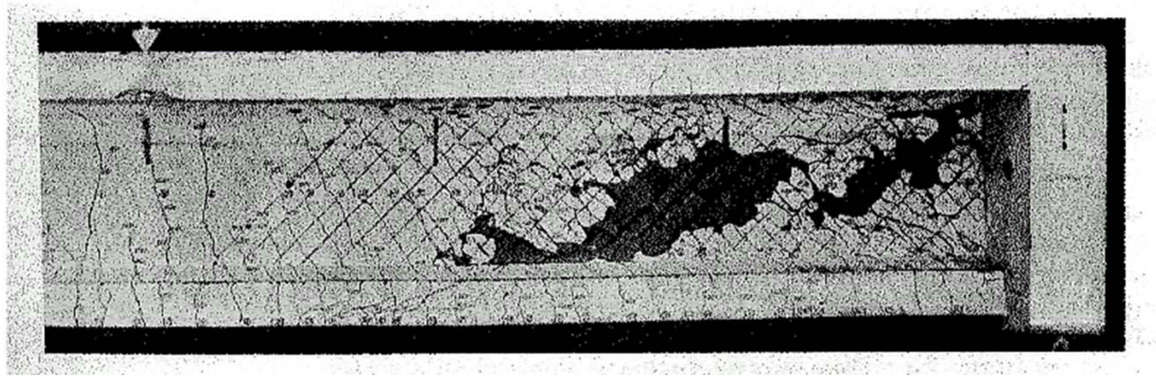
REINFORCED CONCRETE BEAMS – BEHAVIOUR UNDER SHEAR

DIFFERENT FAILURE MECHANISMS FOR BEAMS WITH SMALL WEB THICKNESS



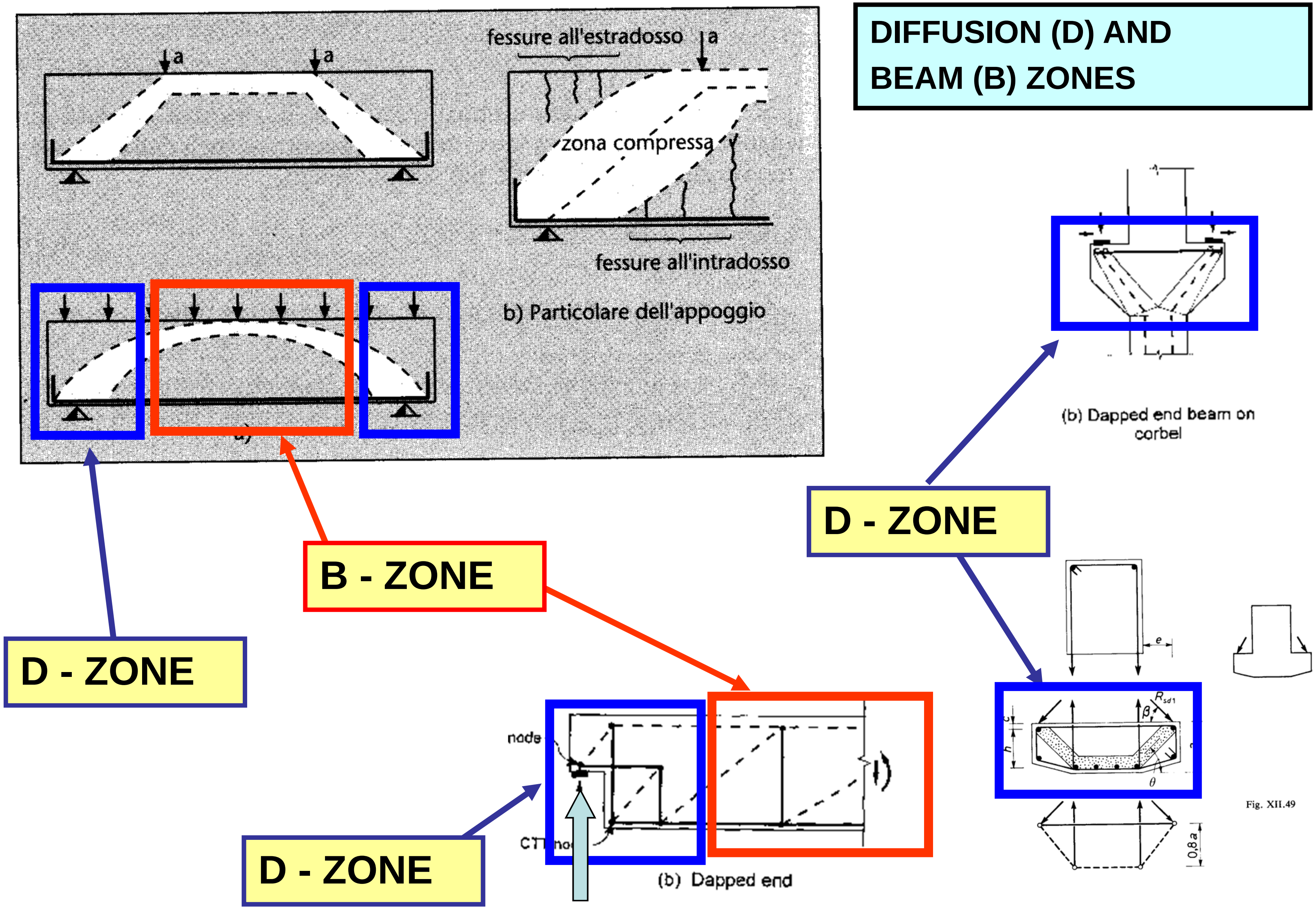
STEEL FAILURE

CONCRETE FAILURE



REINFORCED CONCRETE BEAMS – BEHAVIOUR UNDER SHEAR

DIFFUSION (D) AND BEAM (B) ZONES

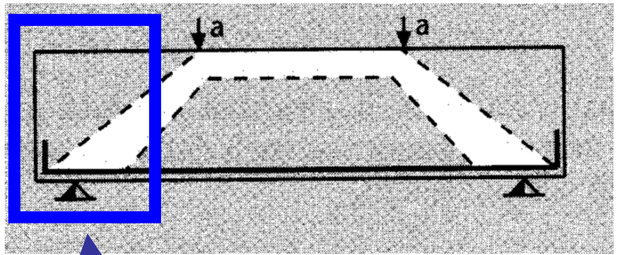


REINFORCED CONCRETE BEAMS – BEHAVIOUR UNDER SHEAR

RESULTS OF AN EXPERIMENTAL CAMPAIGN ON ROOF PRECAST ELEMENTS

REINFORCED CONCRETE BEAMS – BEHAVIOUR UNDER SHEAR

DIFFUSION ZONE – ARCH BEHAVIOUR CLOSE TO THE SUPPORTS



D - ZONE

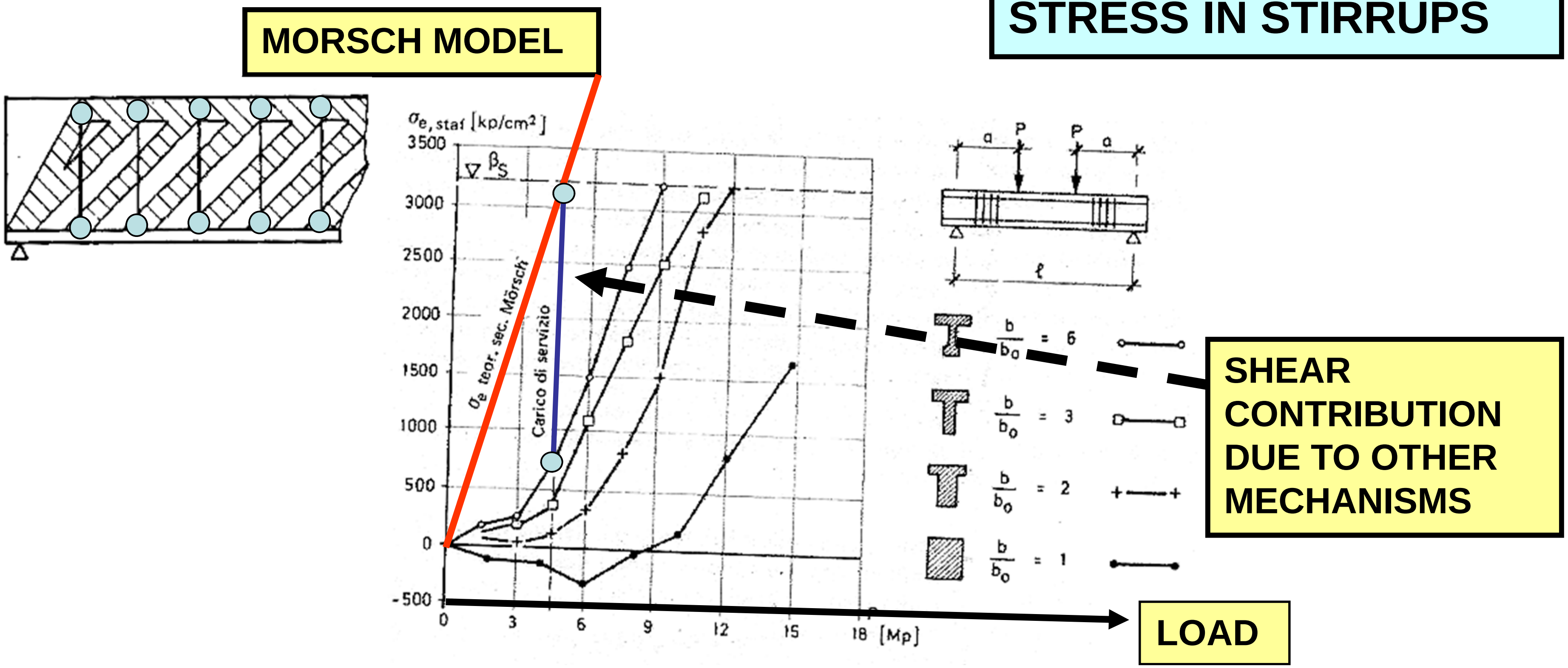
CONCRETE FAILURE IN COMPRESSION

D: Discontinuity/Disturbed

REINFORCED CONCRETE BEAMS – BEHAVIOUR UNDER SHEAR

SECTIONAL FAILURE DUE TO UNCORRECT DETAILING

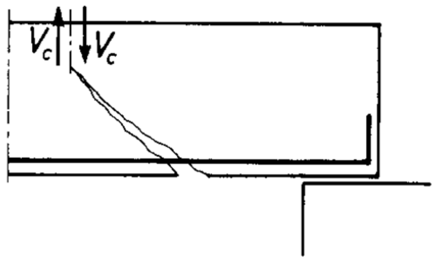
SHEAR STRENGTH FOR A BEAM WITHOUT SHEAR REINFORCEMENT



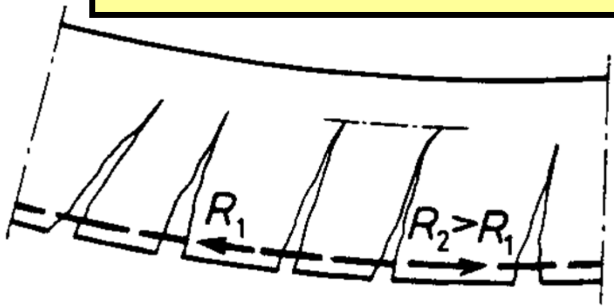
RC BEAMS MAY CARRY LOAD ALSO WITHOUT SHEAR REINFORCEMENT

PRINCIPAL MECHANISMS TO CARRY SHEAR WITHOUT SHEAR REINFORCEMENT

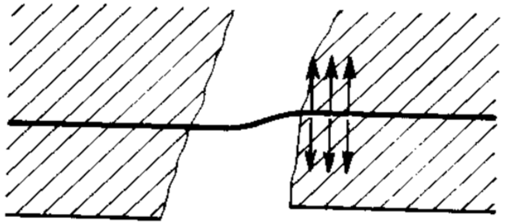
SHEAR STRENGTH OF THE COMPRESSION CHORD



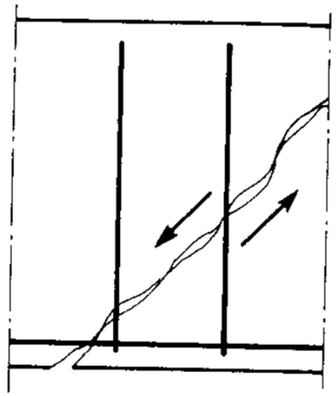
VARIABLE TENSION IN STEEL BARS



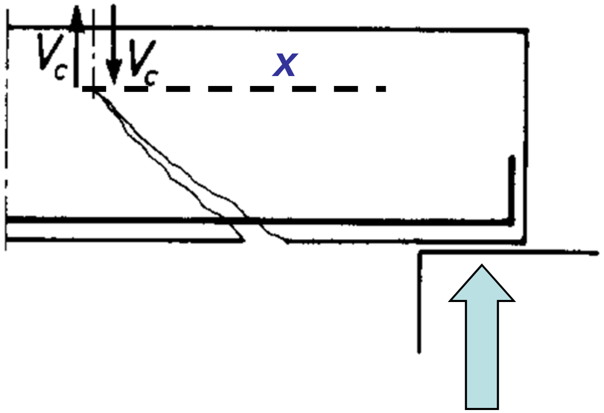
DOWEL ACTION



AGGREGATE INTERLOCK



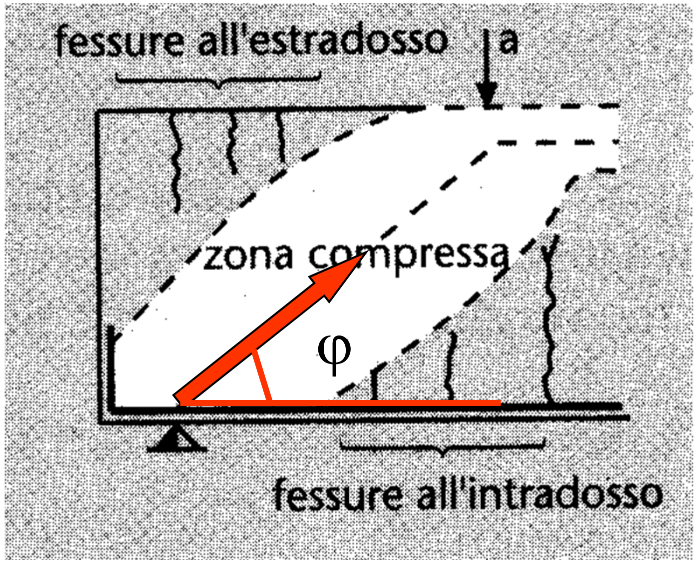
SHEAR STRENGTH OF THE COMPRESSION CHORD



The portion of beams above the neutral axis, which is under compression can carry shear stresses (carrying capacity increased by compressive load)

- **It goes almost to 0 in case of traction!**

$$V_c = b \cdot x \cdot \tau_{\lim}(\sigma_c)$$

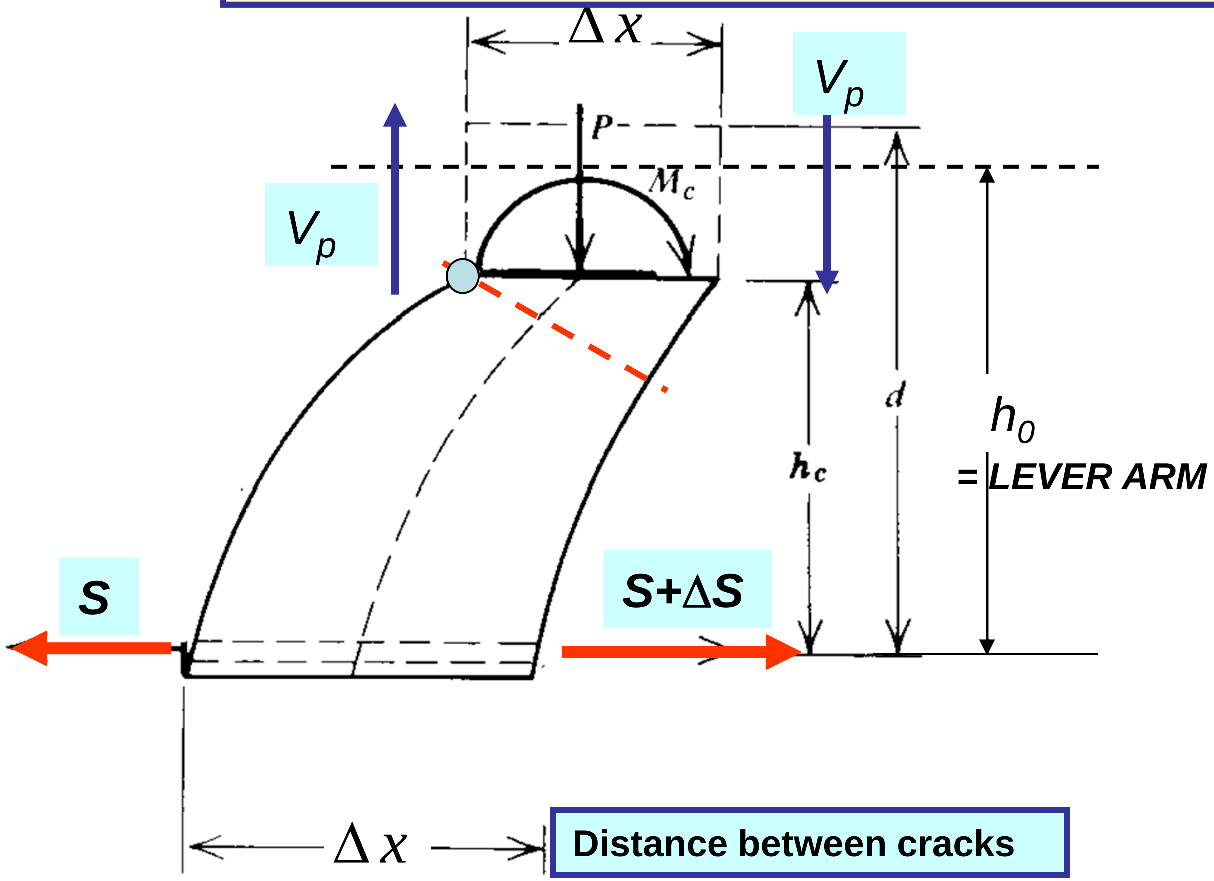


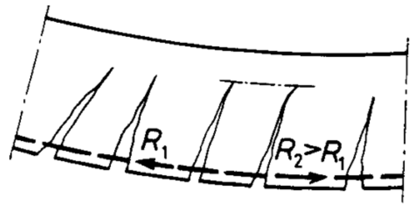
Most important mechanism in "D" zones

$$V_c = b \cdot x \cdot \tau_{\lim}(\sigma_c) \cdot \sin\varphi$$

NOTE: if *x/d<0.16 strength reduced*

2) "VARIABLE STRESS IN REBARS"





The stress ΔS depends on the increments of bending moment.

This mechanism is controlled by the tensile strength of concrete.

$$f_{ctm} = 0.30 \sqrt[3]{f_{ck}^2}$$

$$f_{ctk} = 0.7\ f_{ctm}$$

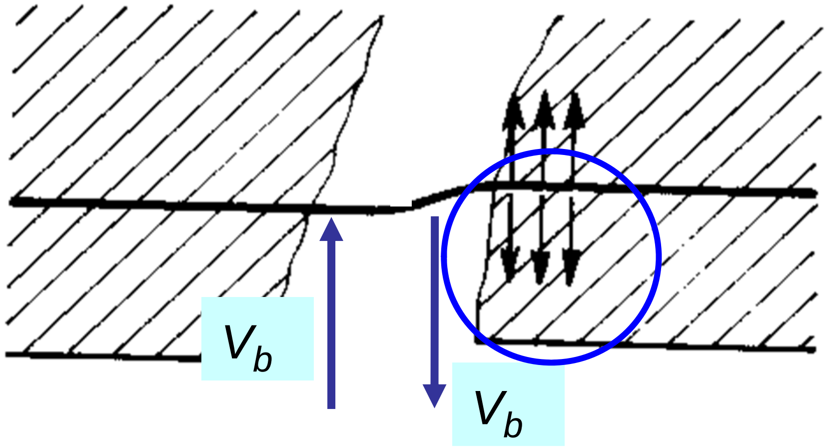
$$f_{ctd} = \frac{f_{ctk}}{\gamma_c}$$

$$f_{ctd} = 0.14 \sqrt[3]{f_{ck}^2}\ \ (in\ N/mm^2)$$

$$V_p \cong 0.28 \cdot b \cdot h_0 \cdot f_{ctd} \cong 0.25 \cdot b \cdot d \cdot f_{ctd}$$

Empirical/theoretical formula

3) DOWEL ACTION



The opening of an inclined crack produces a vertical relative displacement between of the faces of the crack.

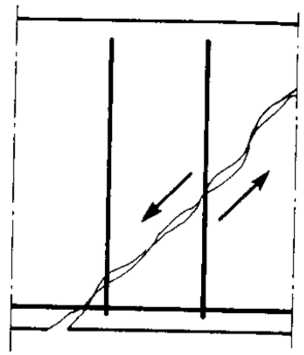
Longitudinal rebars carry a shear controbution contrasting this movement

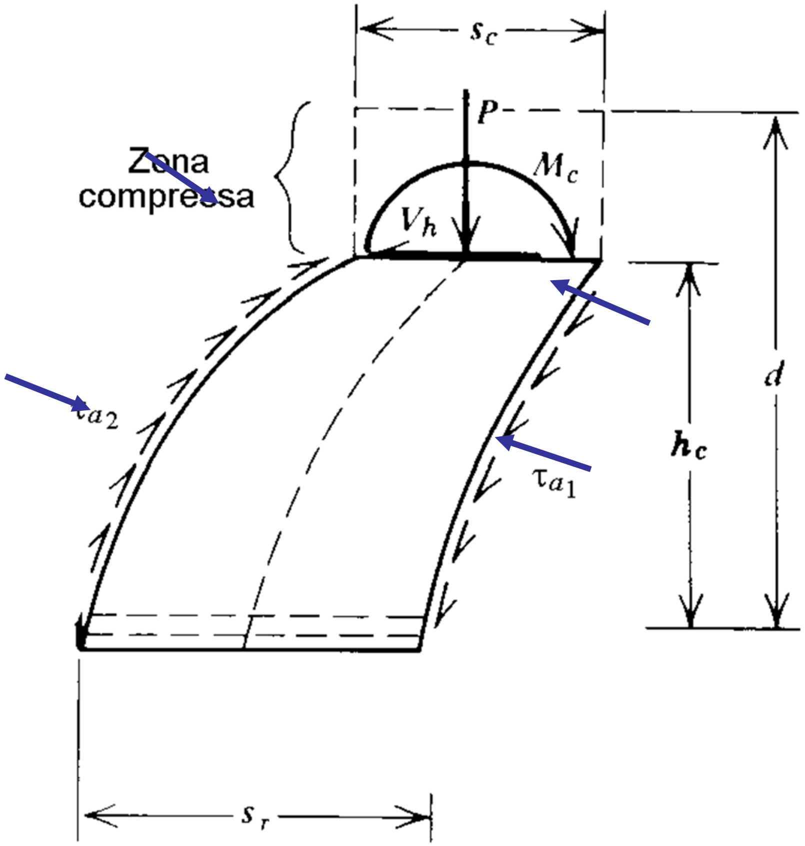
NOTE: if there is no web reinforcement this mechanism depends on the tensile strength of concrete

$$V_b \cong 6.5 \cdot A_s \cdot f_{ctd}$$

Empirical equation

4) AGGREGATE INTERLOCK



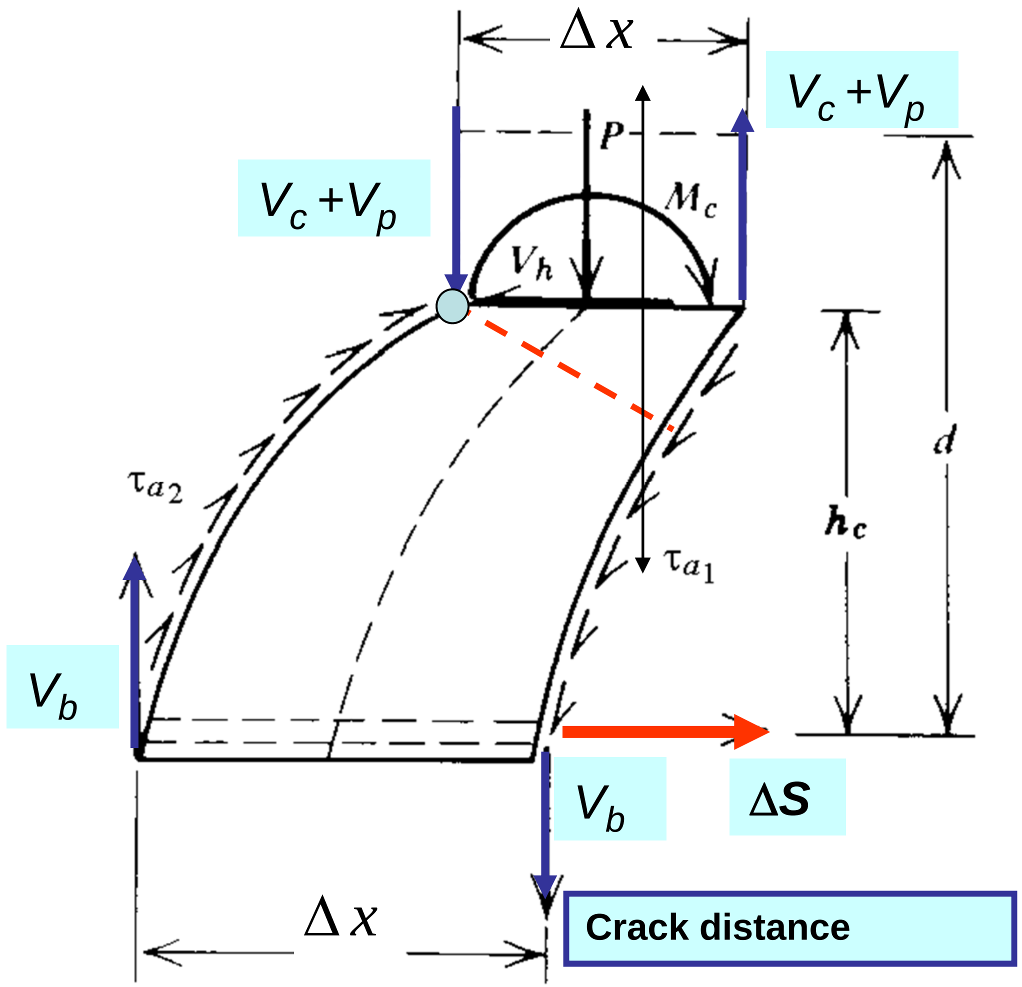


The friction between the uneven faces of cracks can transfer shear stresses.

- **They depend on the presence of compression of the crack surfaces and therefore:**
- **Friction forces become almost zero in case of traction.**
- **Longitudinal and most importantly web reinforcement can increase the magnitude of this effect by bridging the cracks**

4) AGGREGATE INTERLOCK

- **The main effect is that related to the different stress in the rebars at two different cracks**
- **There is interaction among the effects previously described**
- **Web reinforcement has positive effects**
- **Traction reduces the magnitude of all the effects previously described.**

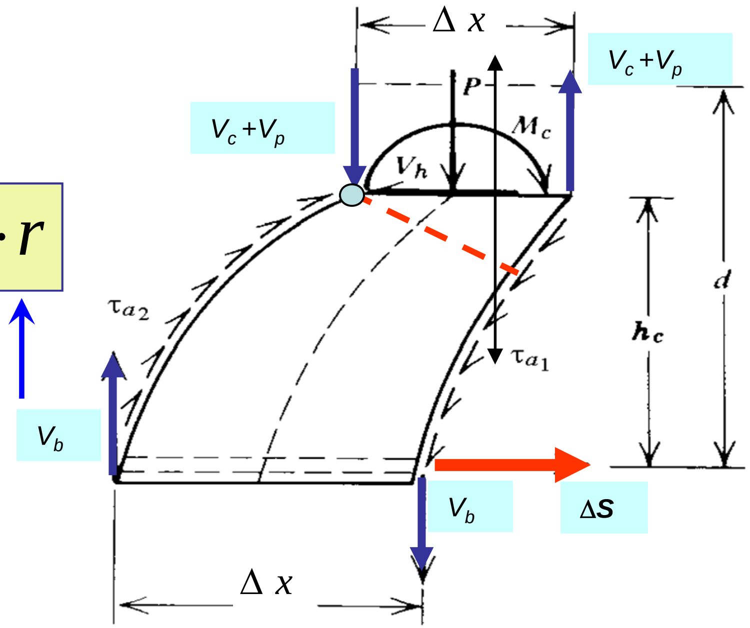


This behaviour has a stabilizing effect of the strength of the mechanism 1

CODES
BEAMS WITHOUT WEB REINFORCEMENT

- **Italian code (D.M. 1996) [OLD]**

$$V_{Rd1} \cong 0.25 \cdot b_w \cdot d \cdot f_{ctd} \cdot \delta \cdot (1 + 50\rho_{sl}) \cdot r$$



V_{Rd1} = shear strength (without web reinforcement);
f_{ctd} = concrete tensile strength;
r = (1,6-d); d in meters ($d \leq 0{,}60$ m);

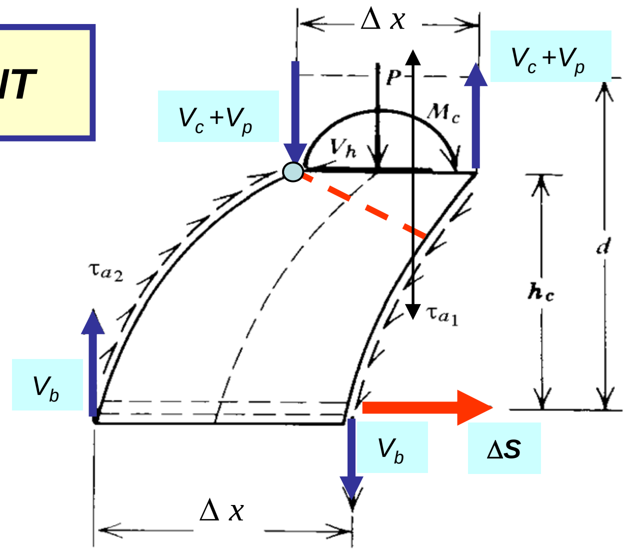
ρ_{sl} = $\dfrac{A_{sl}}{b_w \, d}$ ($\rho_l \leq 0.02$)

b_w = web width;
d = H-c;
δ = 1 no axial forces;
δ = 0 traction;

δ = $1 + \dfrac{M_0}{M_{sdu}}$ compression; M_0 decompression moment M_{sdu};
A_{sl} = area of longitudinal steel bars.

SHEAR STRENGTH WITHOUT WEB REINFORCEMENT

There is interaction among the effects previously described



- **Italian code (D.M. 1996)**

$$V_{Rd1} \cong 0.25 \cdot b_w \cdot d \cdot f_{ctd} \cdot \delta \cdot (1 + 50\rho_{sl}) \cdot r$$

ΔS

DOWEL EFF.

Axial force

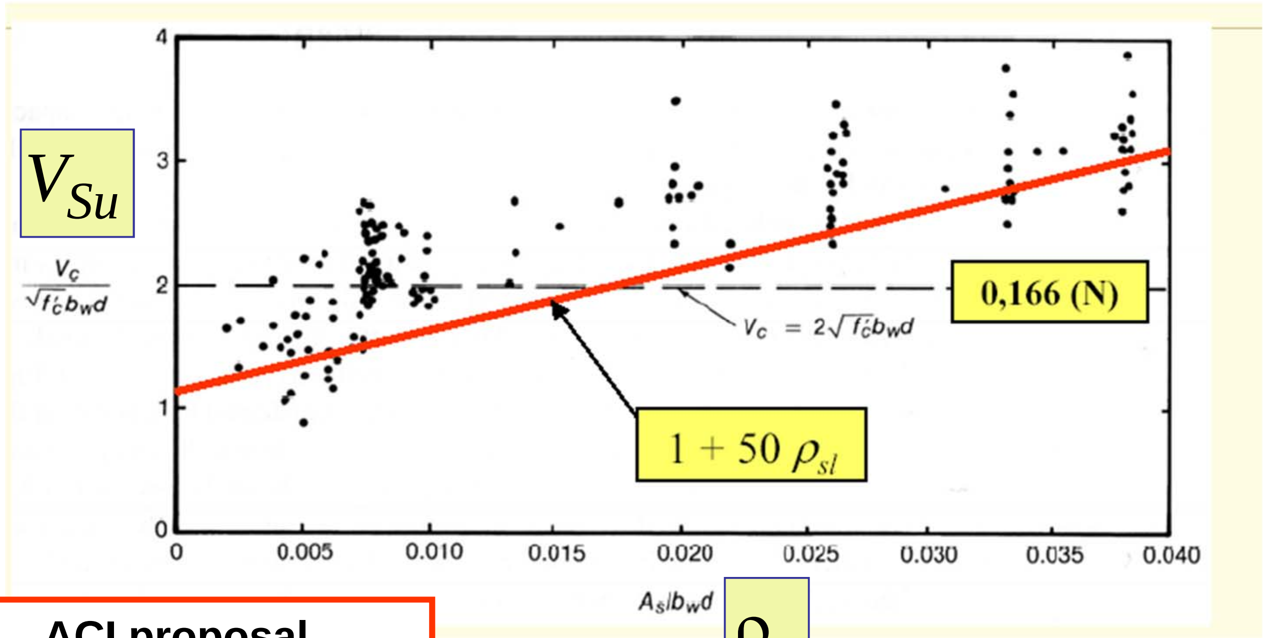
Aggregate interlock

δ= 1 (P =0)
δ= 0 (Traction)
δ > 1 (Pre-stressed)

r = (1,6-*d*) for *d* ≤ 0,60 m
r = *1* per *d* > 0,60 m

NEXT SLIDE

DOWEL EFFECT



ρ_s

ACI proposal

- **Huge dispersion of results**
- **Empirical equations**

For these reasons different codes use different equations

NTC 2008 & EC2 [2005]

FULLY EMPIRICAL EQUATIONS

$$V_{Rd} = \left\{0,18 \cdot k \cdot (100 \cdot \rho_1 \cdot f_{ck})^{1/3} / \gamma_c + 0,15 \cdot \sigma_{cp}\right\} \cdot b_w \cdot d \geq (v_{min} + 0,15 \cdot \sigma_{cp}) \cdot b_w d$$

PRESTRESSING CONTRIBUTION

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0$$

$$v_{min} = 0.035 \cdot \sqrt[3]{k^2} \sqrt{f_{ck}}$$

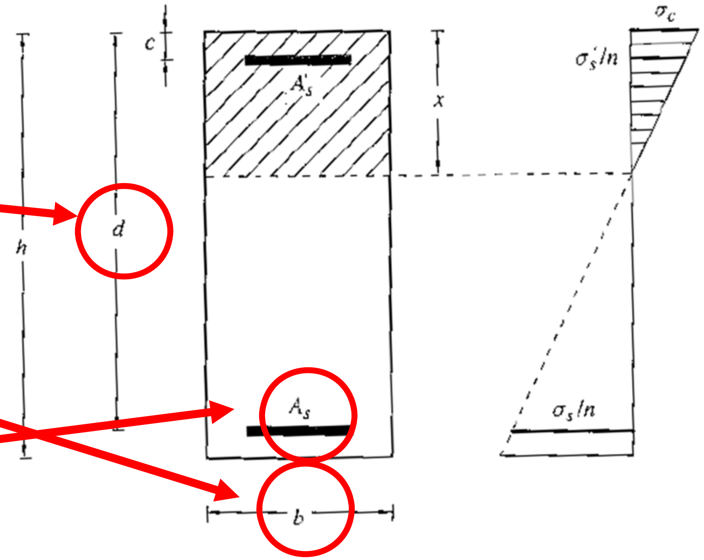
$$\rho_1 = A_{s1} / (b_w \cdot d) \qquad (\leq 0,02);$$

$$\sigma_{cp} = N_{Ed}/A_c \qquad (\leq 0,2\ f_{cd});$$

d: distance from the extreme fibre in compression to the centroid of the longitudinal reinforcement on the tension side (mm).

b_w: width of the web of the beam (mm)

A_{s1}: area of reinforcement near the tension face of the beam (mm*mm)



EXAMPLE: SHEAR STRENGTH WITHOUT WEB REINFORCEMENT

$B = b_w = 30$ cm
$H = 50$ cm $d = 46$ cm
Longitudinal steel bars: 3 ∅ 16 = 603 mm²

$$R_{ck} = 25\,MPa = 25\,N/mm^2 \Rightarrow f_{ctd} = 1\,N/mm^2$$

DM1996

$$V_{Rd1} \cong 0.25 \cdot b_w \cdot d \cdot f_{ctd} \cdot \delta \cdot (1 + 50\rho_{sl}) \cdot r$$

$r = 1.6 - 0.46 = 1.14$ $\rho_{sl} = 603 / (300 \times 460) = 0.0044$ $\delta = 1$

$$V_{Rd1} = 0.25 \times 300 \times 460 \times 1 \times 1 \times (1 + 50 \times 0.0044) \times 1.14 = 48\,kN$$

NTC2008 – EC2

$$V_{Rd} = \left\{0,18 \cdot k \cdot (100 \cdot \rho_1 \cdot f_{ck})^{1/3} / \gamma_c + 0,15 \cdot \sigma_{cp}\right\} \cdot b_w \cdot d$$

r = 1+(200/460)^(1/2)=1.65<2 $\sigma_{cp} = 0$ $V_{min} = v_{min} \cdot b_w \cdot d = 46$ kN

$$V_{Rd} = \left(0.18 \cdot 1.65 \cdot (100 \cdot 0.0044 \cdot 20)^{\frac{1}{3}} \Big/ 1.5\right) \cdot 300 \cdot 460 = 56kN \geq V_{min}$$

SHEAR VERIFICATION

$$V_{Sd} \leq V_{Rd1}$$

YES → ***No web reinforcement required***

Minimum reinforcement

NO → ***Web reinforcement required***

→ ***Calculate beam strength with web reinforcement***

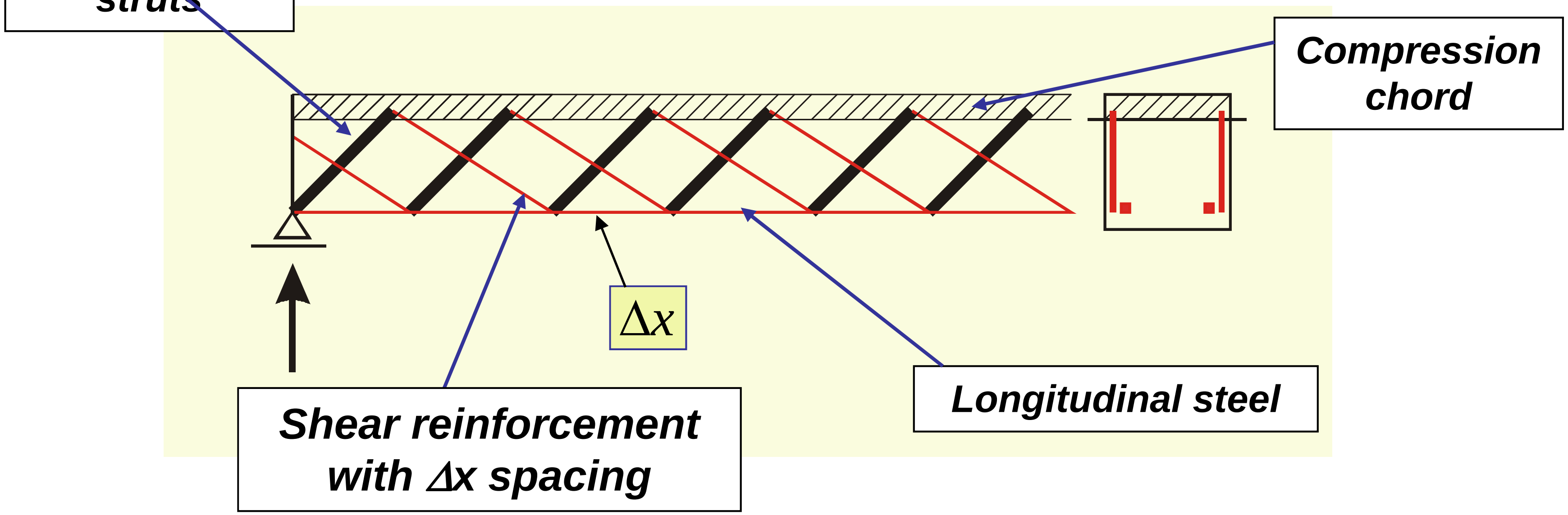
BEAMS WITH WEB REINFORCEMENT

2 POSSIBLE COLLAPSE MECHANISMS

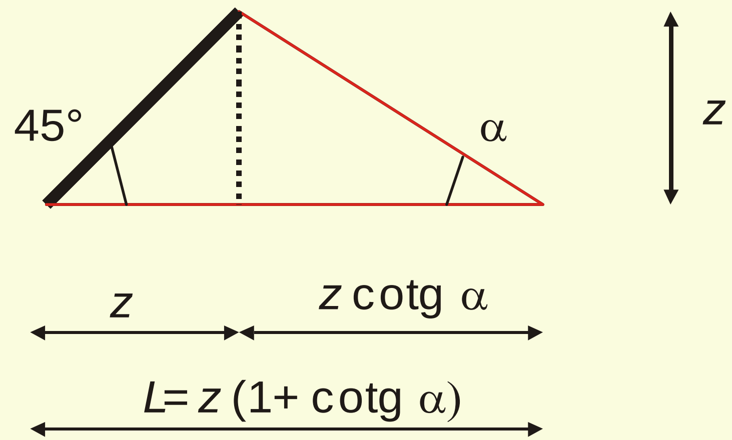
- ***CONCRETE CRUSHING*** → $V_{Rd,max}$
- ***STEEL FAILURE*** → $V_{Rd,s}$

Ritter – Morsch STRUT & TIE model

Classical Hypothesis: Concrete struts with 45° inclination



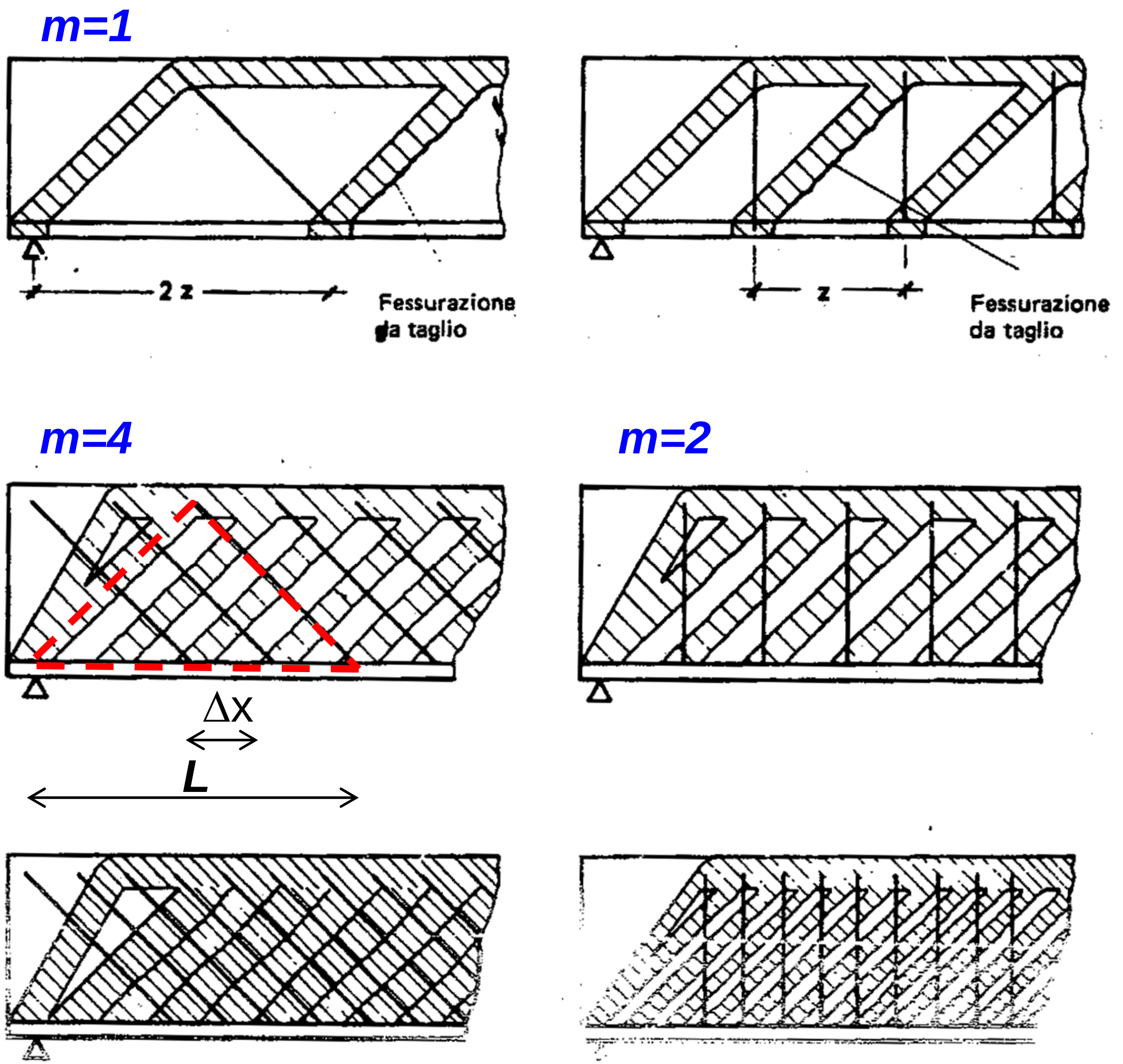
THE GENERAL TRUSS ELEMENT



Multiplicity of the truss

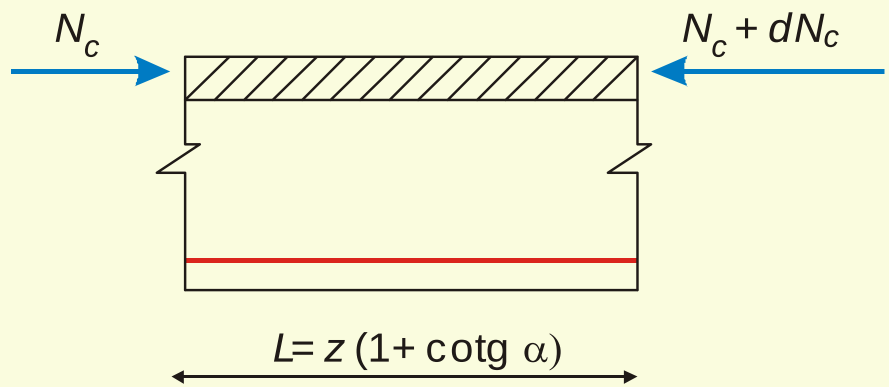
$$m = \frac{L}{\Delta x} = \frac{z(1+\text{cotg}\,\alpha)}{\Delta x}$$

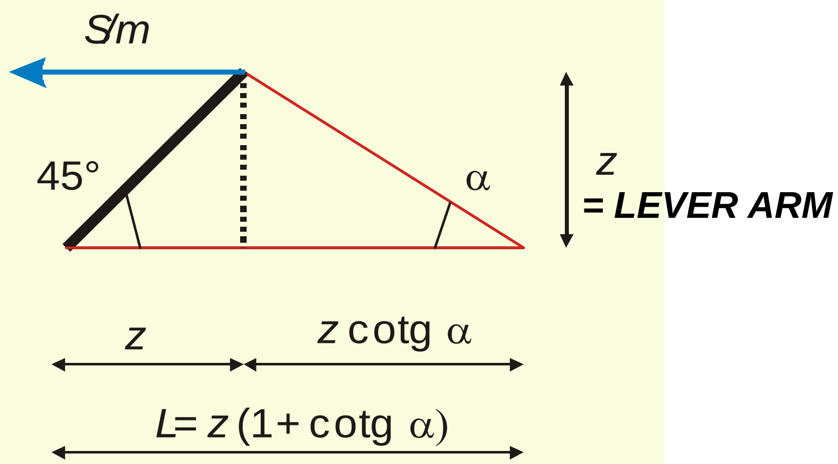
Multiplicity of the isostatic lattice model



$$m = \frac{L}{\Delta x} = \frac{z(1 + \mathrm{cotg}\,\alpha)}{\Delta x}$$

THE SHEAR FORCE TO BE CARRIED OUT BY THE REINFORCEMENT





$$\Delta N_c = S = \tau \cdot b \cdot L = \frac{V\,S}{J\,b} \cdot b \cdot L = \frac{V}{z} \cdot L$$

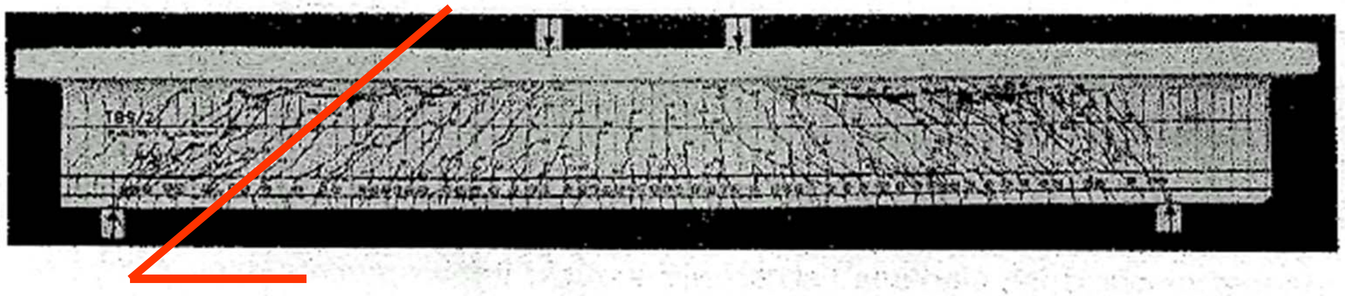
SHEAR FORCE TO BE CARRIED OUT ON THE LENGTH L

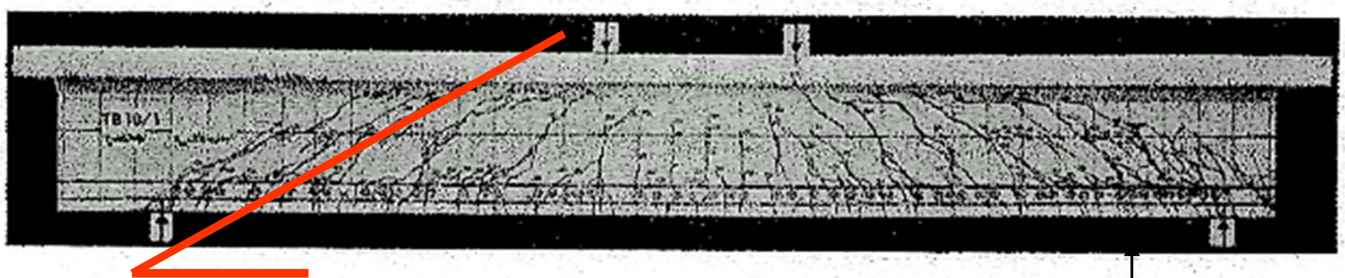
$$S/m = \frac{V\,L}{z}\frac{1}{m}$$

SHEAR FORCE TO BE CARRIED OUT BY THE GENERAL TRUSS ELEMENT

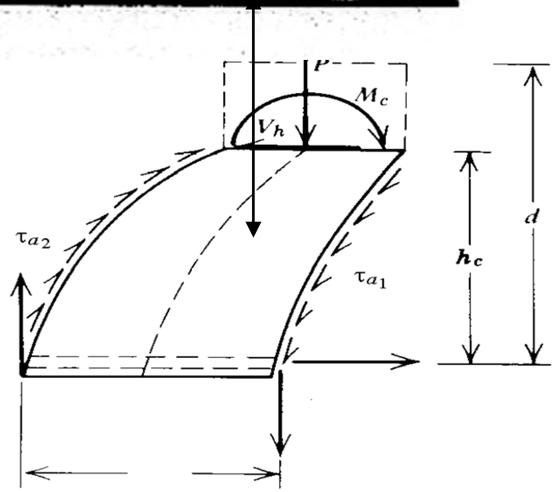
STRUT AND TIE MODEL WITH VARIABLE INCLINATION

HIGH STEEL REINFORCEMENT



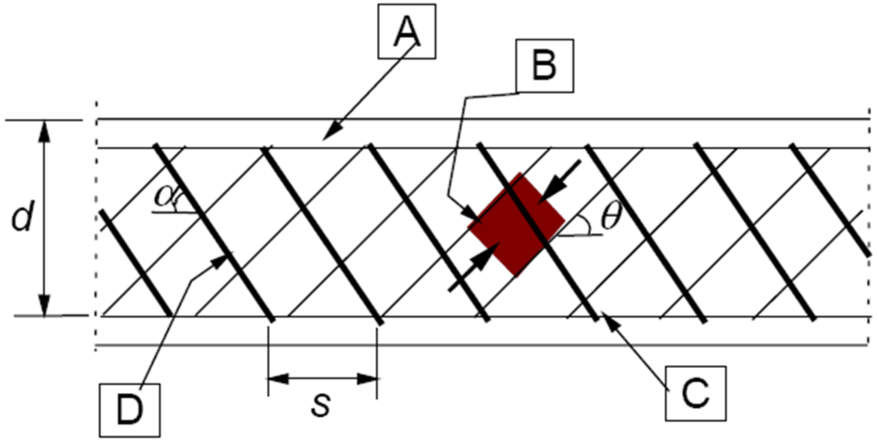


LOW STEEL REINFORCEMENT



INCLINATION OF CONCRETE STRUCTS MAY BE DIFFERENT FROM θ=45°

STRUT AND TIE MODEL WITH VARIABLE INCLINATION



A - compression chord, B - struts

C - tensile chord, D - shear reinforcement

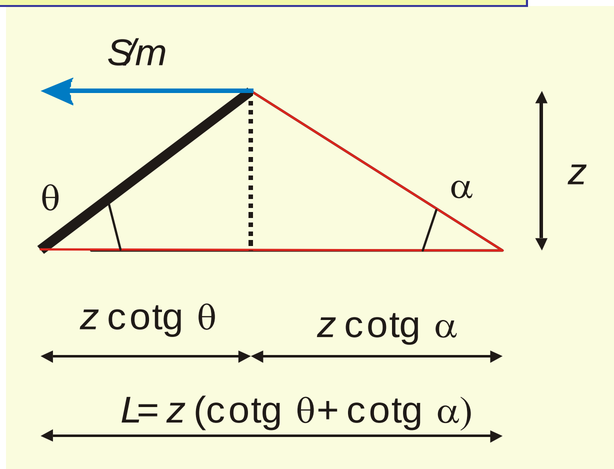
The inclination of the concrete struct is not 45° but θ

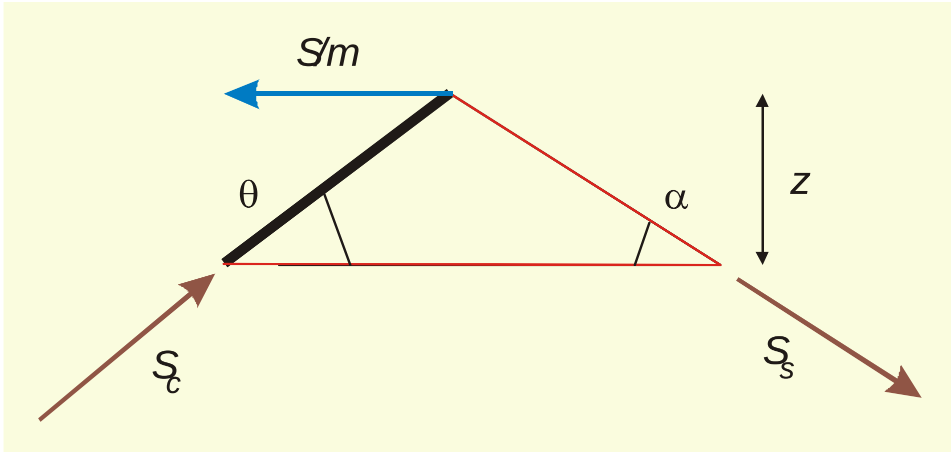
$21{,}8° \leq \theta \leq 45°$

$2.5 \geq \mathrm{cotg}\,\theta \geq 1.0$

Moltiplicity of the truss

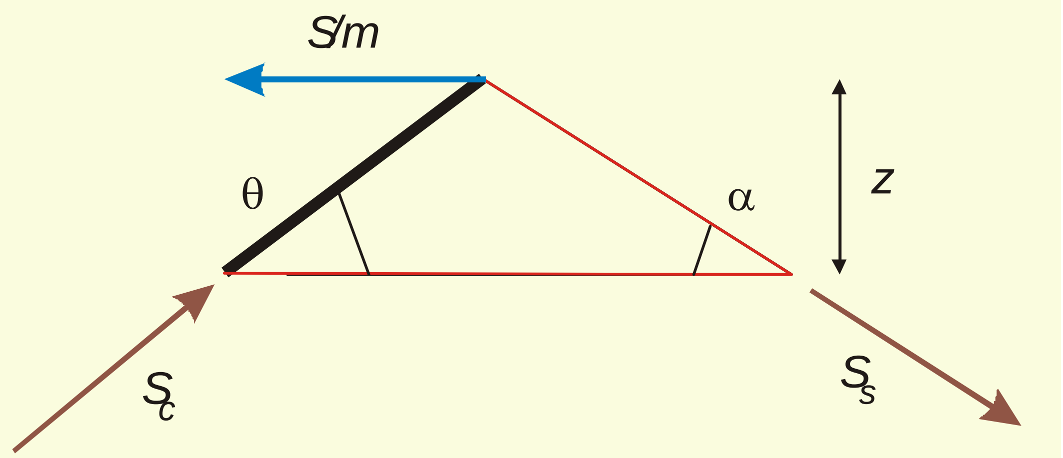
$$m = \frac{L}{\Delta x} = \frac{z(\mathrm{cotg}\,\theta + \mathrm{cotg}\,\alpha)}{\Delta x}$$





APP

SHEAR FORCES ON THE CONCRETE STRUCT AND THE STEEL REINFORCEMENT



$$V_{Rd,\max} = b_w \, z \, \nu \, f_{cd} \frac{\cot g\,\theta + \cot g\,\alpha}{1+\cot g^2\,\theta}$$

RESISTANCE AGAINST CONCRETE FAILURE

$$V_{Rd,s} = A_{sw} \, f_{ywd} \frac{z}{\Delta x} (\cot g\,\theta + \cot g\,\alpha) \sin\alpha$$

RESISTANCE AGAINST STEEL YIELDING

DESIGN SHEAR RESISTANCE

$$V_{Rd} = \min(V_{Rd,\max}, V_{Rd,s})$$

ATTENTION: the concrete contribution is now not considered to calculate V_{wd}

EXAMPLE: STRUT AND TIE MODEL WITH VARIABLE INCLINATION

$$\theta = 45° \Rightarrow \text{cotg}\,\theta = 1$$

$$21{,}8° \leq \theta \leq 45° \quad 2.5 \geq \text{cotg}\,\theta \geq 1.0$$

$$V_{Rcd} = \nu\, f_{cd}\, b_w\, z \frac{1 + \text{cotg}\,\alpha}{2}$$

$$V_{Rcd} = b_w\, z\, \nu\, f_{cd} \frac{\text{cotg}\,\theta + \text{cotg}\,\alpha}{1 + \text{cotg}^2\,\theta}$$

$$V_{wd} = f_{ywd} \cdot A_{sw} \frac{z}{\Delta x} (\sin\alpha + \cos\alpha)$$

$$V_{wd} = A_{sw}\, f_{ywd} \frac{z}{\Delta x} (\text{cotg}\,\theta + \text{cotg}\,\alpha) \sin\alpha$$

IF VALUES OF cotg θ GREATER THAN 1 ARE ASSUMED (INCLINATION OF THE CONCRETE STRUTS SMALLER THAN 45°):

- ***THE SHEAR STRENGTH AGAINST CONCRETE FAILUTE IS SMALLER (but it is usually large for beams with constant web width)***
- ***THE FORCE ON THE STEEL REINFORCEMENT IS REDUCED (and a smaller web reinforcement is required)***

EXAMPLE: STRUT AND TIE MODEL WITH VARIABLE INCLINATION

$1.0 < \cot\theta < 2.0$, $\quad V_{Rd2} = \dfrac{b_w \; z \; \nu \; f_{cd}}{\cot\theta + \tan\theta}$, $\quad \dfrac{A_{sw}}{s} = \dfrac{V_{Sd}}{z \, f_{ywd} \cot\theta}$

$\cot\theta$	V_{Rd2} (kN)	A_{sw}/s (cm^2/m)	Staffe
1.0	480.2	12.4	Φ8/8
1.50	443.3	8.27	Φ8/12
2.0	384.2	6.20	Φ8/16

IF VALUES OF cotg θ GREATER THAN 1 ARE ASSUMED (INCLINATION OF THE CONCRETE STRUTS SMALLER THAN 45°):

- ***THE SHEAR STRENGTH AGAINST CONCRETE FAILUTE IS SMALLER (but it is usually large for beams with constant web width)***

- ***THE FORCE ON THE STEEL REINFORCEMENT IS REDUCED (and a smaller web reinforcement is required)***

STRUT AND TIE MODEL WITH VARIABLE INCLINATION – HOW TO OBTAIN DESIGN FORMULAS

$$t_{Sd} = \frac{V_{Sd}}{b_w\, z\, f_{cd}}$$

NON DIMENSIONAL (EXTERNAL) SHEAR FORCE

$$t_{Rcd} = \frac{V_{Rcd}}{b_w\, z\, f_{cd}} = \nu \frac{(\operatorname{cotg}\theta + \operatorname{cotg}\alpha)}{1+\operatorname{cotg}^2\theta}$$

A NON DIMENSIONAL SHEAR STRENGTH AGAINST CONCRETE FAILURE

$$t_{Rsd} = \frac{V_{wd}}{b\, z\, f_{cd}} = \frac{A_{sw}\, f_{ywd}}{b\, z\, f_{cd}} \frac{z}{\Delta x}(\operatorname{cotg}\theta + \operatorname{cotg}\alpha)\sin\alpha = \omega_{sw}(\operatorname{cotg}\theta + \operatorname{cotg}\alpha)\sin\alpha$$

NON DIMENSIONAL SHEAR STRENGTH AGAINST STEEL YIELDING

WHERE $\omega_{sw} = \frac{A_{sw}}{\Delta x} \frac{f_{ywd}}{b\, f_{cd}}$ ***IS THE NON DIMENSIONAL STEEL RATIO***

STRUT AND TIE MODEL WITH VARIABLE INCLINATION

BALANCE CONDITION $$t_{Rcd} = t_{Rsd}$$

$$\nu \frac{(\text{cotg}\,\theta + \text{cotg}\,\alpha)}{1+\text{cotg}^2\,\theta} = \omega_{sw} \sin\alpha\,(\text{cotg}\,\theta + \text{cotg}\,\alpha) \qquad \nu \frac{1}{1+\text{cotg}^2\,\theta} = \omega_{sw} \sin\alpha$$

Example: vertical stirrups: α = 90°

$$\text{cotg}\,\theta = \sqrt{\frac{\nu}{\omega_{sw}} - 1}$$

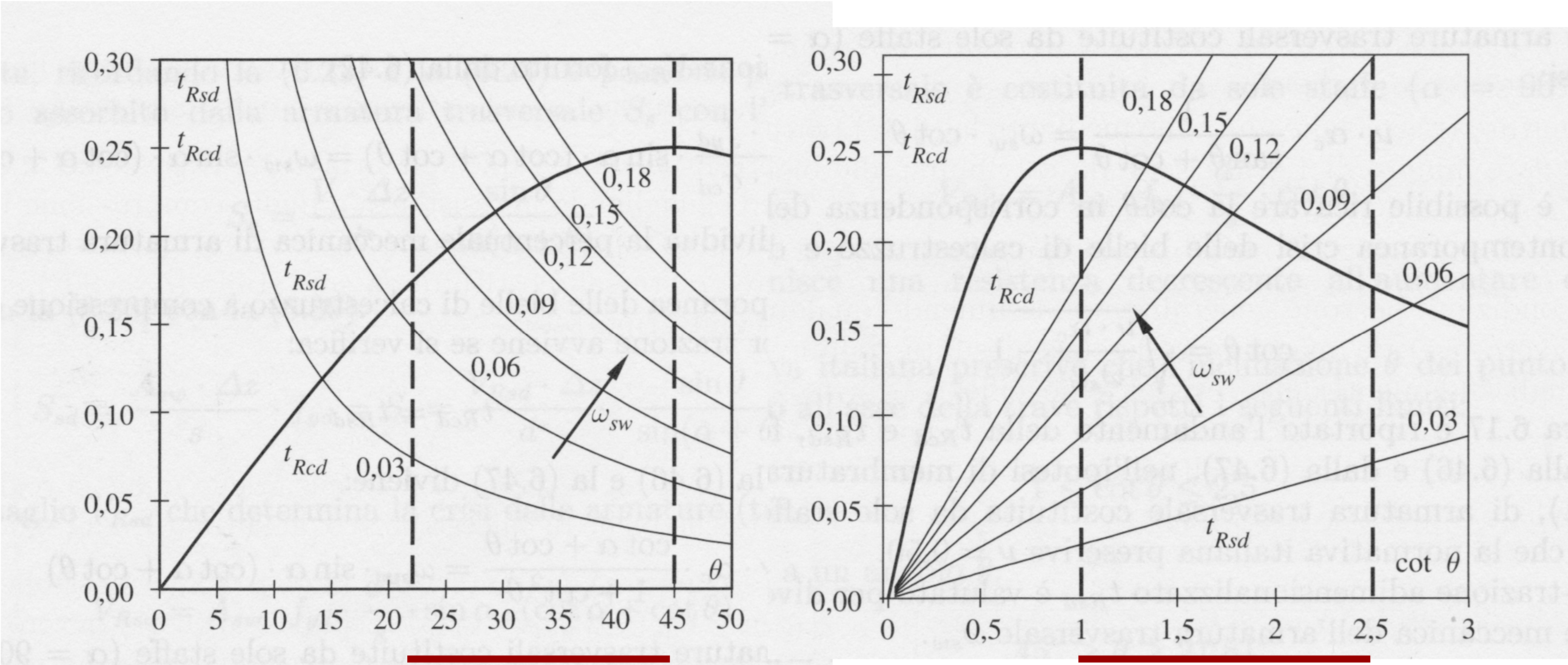
WHERE

$$\omega_{sw} = \frac{A_{sw}}{\Delta x} \frac{f_{ywd}}{b\, f_{cd}}$$

VERIFICATION PROBLEM:	DESIGN PROBLEM:
Input data: Cross-section, Steel reinforcement ω_{sw}	Input data: Cross-section, External shear force t_{Sd}
Calculate: Cotg θ, Design shear strength t_{Rcd}	Calculate: Cotg θ, Steel reinforcement ωsw

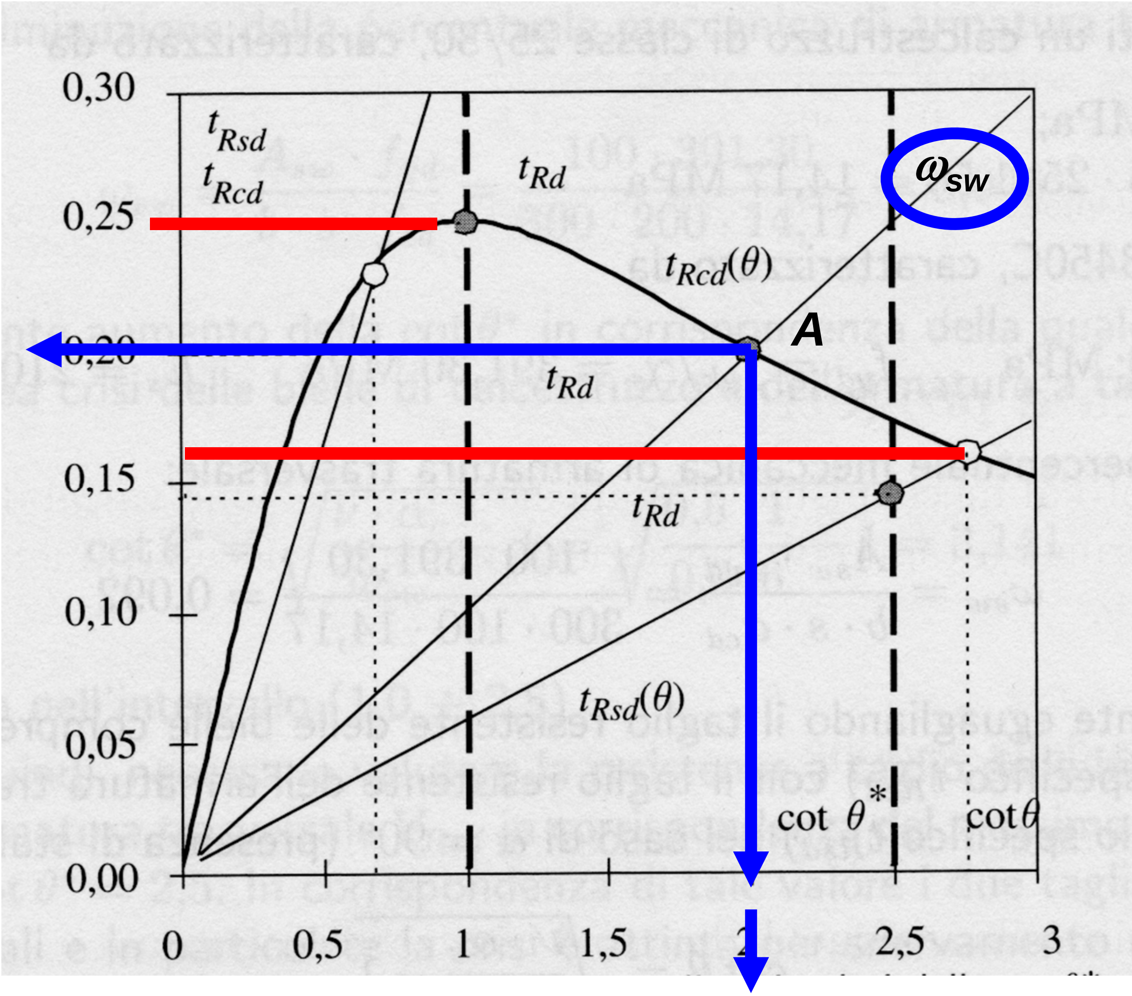
STRUT AND TIE MODEL WITH VARIABLE INCLINATION – DESIGN FORMULAS

Shear action, shear resistance against concrete failure or steel yielding



Limits for θ **Limits for cotg θ**

SHEAR – VERIFICATION PROBLEM
HOW TO CALCULATE THE SHEAR RESISTANCE



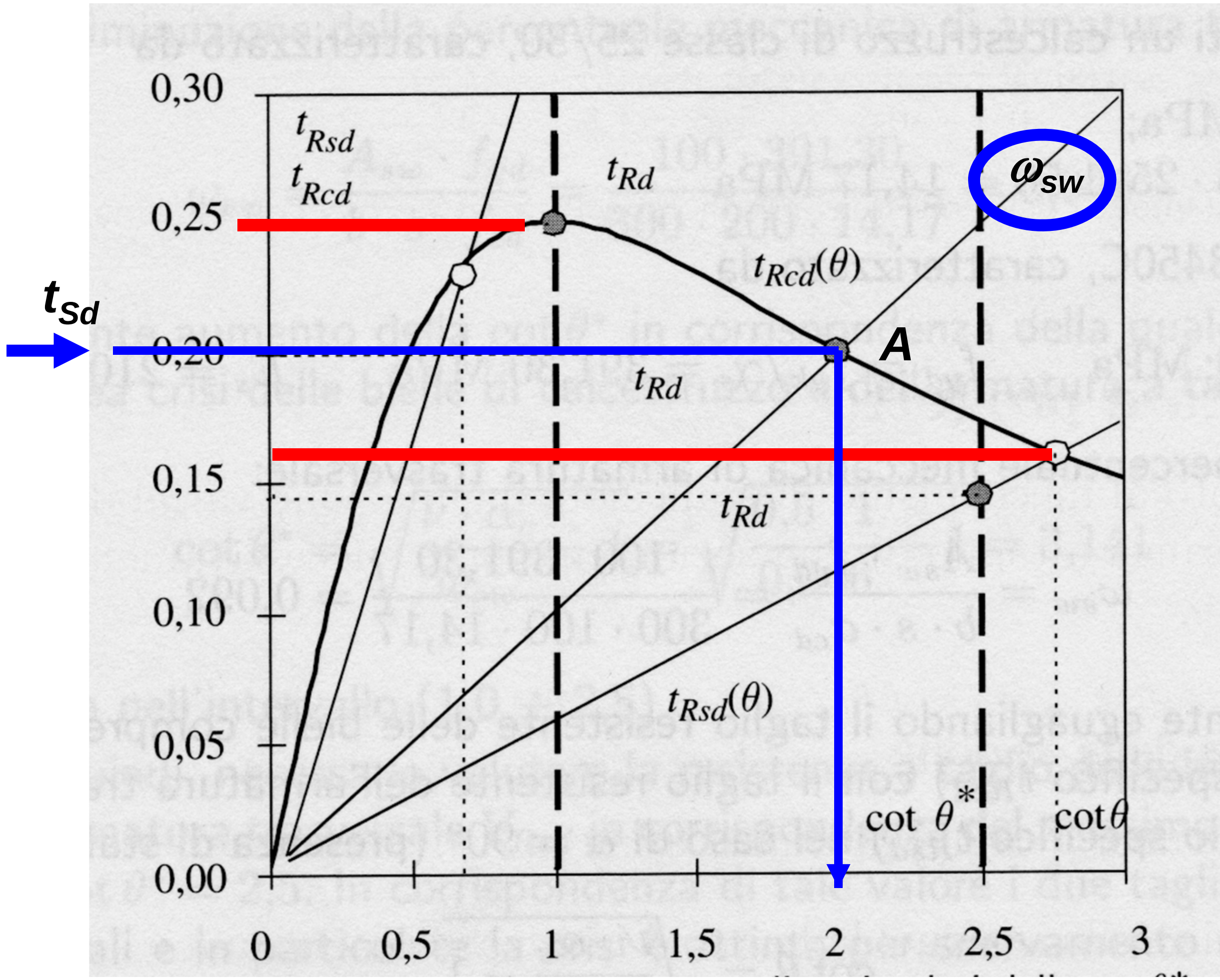
Input data:
Cross-section
Steel reinforcement ω_{sw}

Select the Balance point A

Calculate:
Cotg θ
Design shear strength t_{Rcd}

Shear action, shear resistance against concrete failure or steel yielding

SHEAR - DESIGN PROBLEM
HOW TO OBTAIN THE SHEAR REINFORCEMENT



DESIGN PROBLEM:

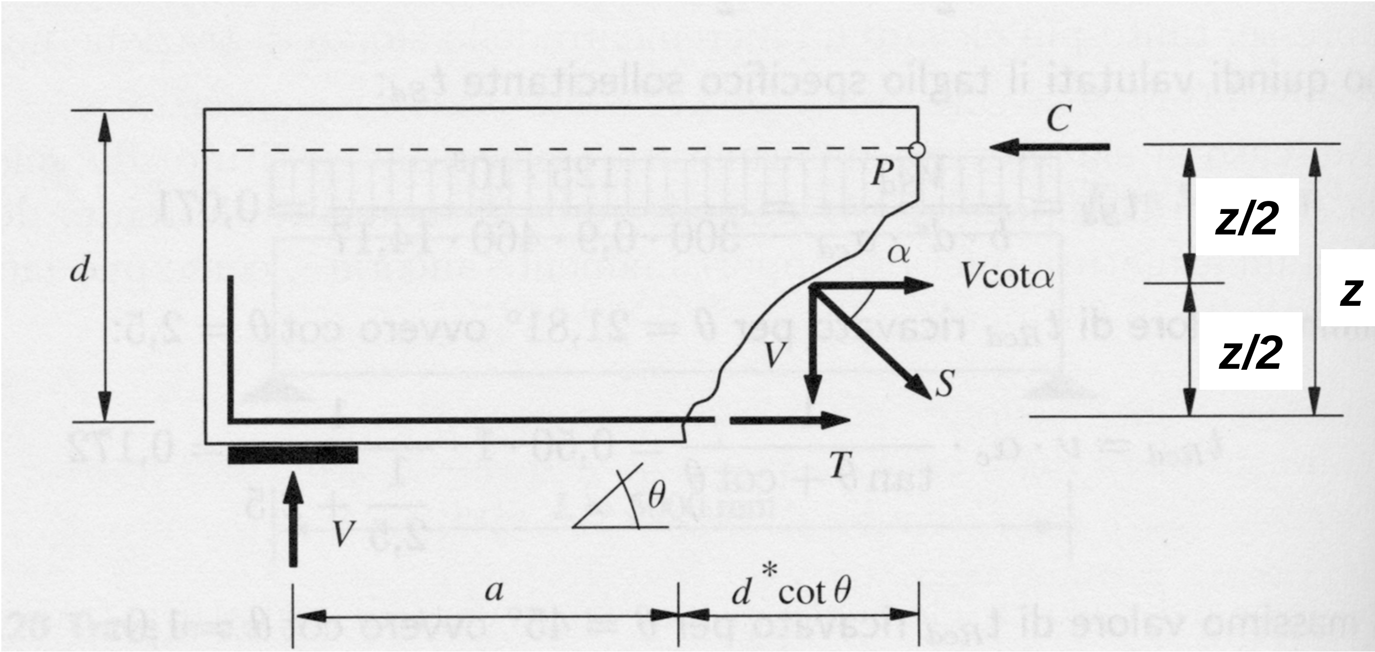
Input data:
Cross-section,
External shear force t_{Sd}

Select the Balance point A

Calculate:
Cotg θ,
Steel reinforcement ω_{sw}

Shear action, shear resistance against concrete failure or steel yielding

THE EFFECT OF CRACK INCLINATION ON THE STRESS ON LONGITUDINAL BARS



α **: Angle of web reinforcement** θ **: Angle of shear crack**

EQUILIBRIUM EQUATION WITH RESPECT TO POINT P

$$T\,z - V\,(a + z\,\mathrm{cotg}\,\theta) + V\,\frac{z}{2}\,\mathrm{cotg}\,\theta + (V\,\mathrm{cotg}\,\alpha)\,\frac{z}{2} = 0$$

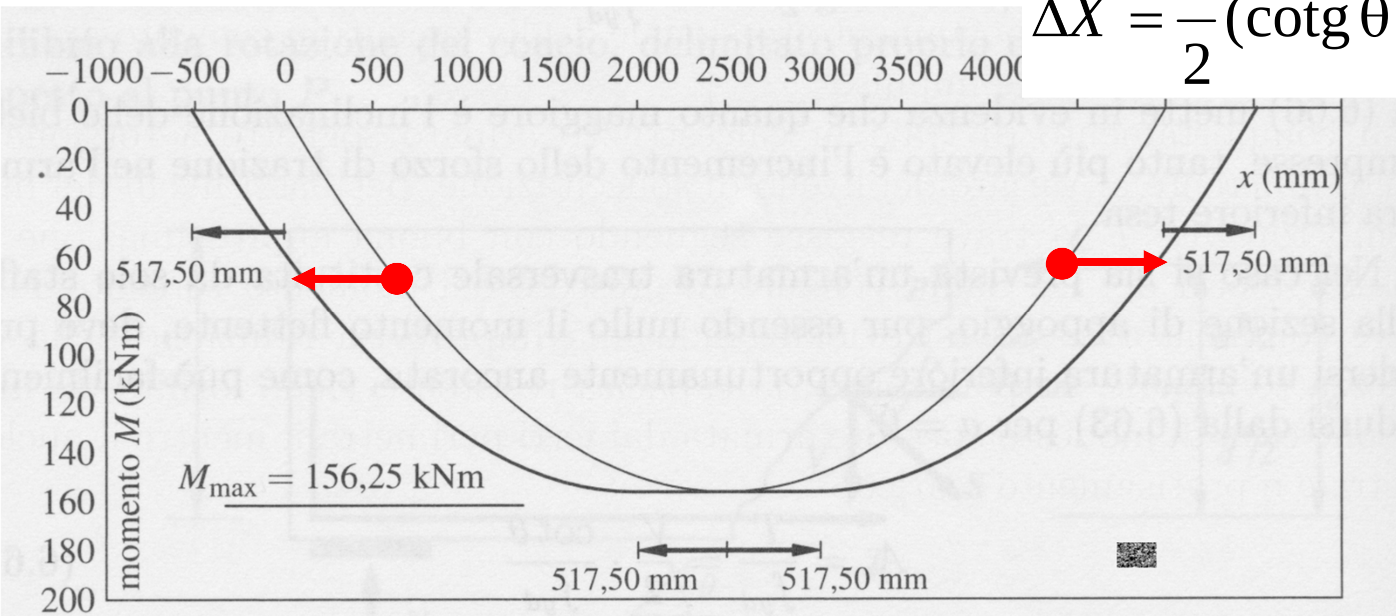
$$T = \frac{V}{z}\left[a + \frac{z}{2}(\mathrm{cotg}\,\theta - \mathrm{cotg}\,\alpha)\right]$$

THE EFFECT OF CRACK INCLINATION ON THE STRESS ON LONGITUDINAL BARS

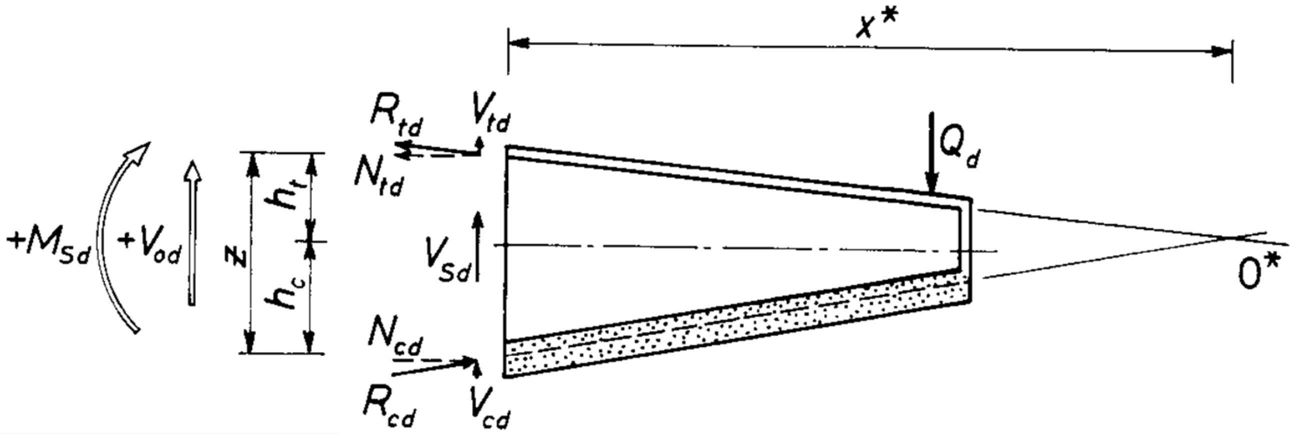
substituting $M = V \cdot a$ $\qquad T = \frac{M}{z} + \frac{V}{2}(\text{cotg}\,\theta - \text{cotg}\,\alpha)$

Hence, due to the inclined crack, the force in the longitudinal bars is greater than that calculated for the section at x=a

$$\Delta X = \frac{z}{2}(\text{cotg}\,\theta - \text{cotg}\,\alpha)$$



SHEAR IN BEAMS WITH VARIABLE SECTION



EXTERNAL SHEAR

$$V_{sd} = V_{0d} - V_{td} - V_{cd}$$

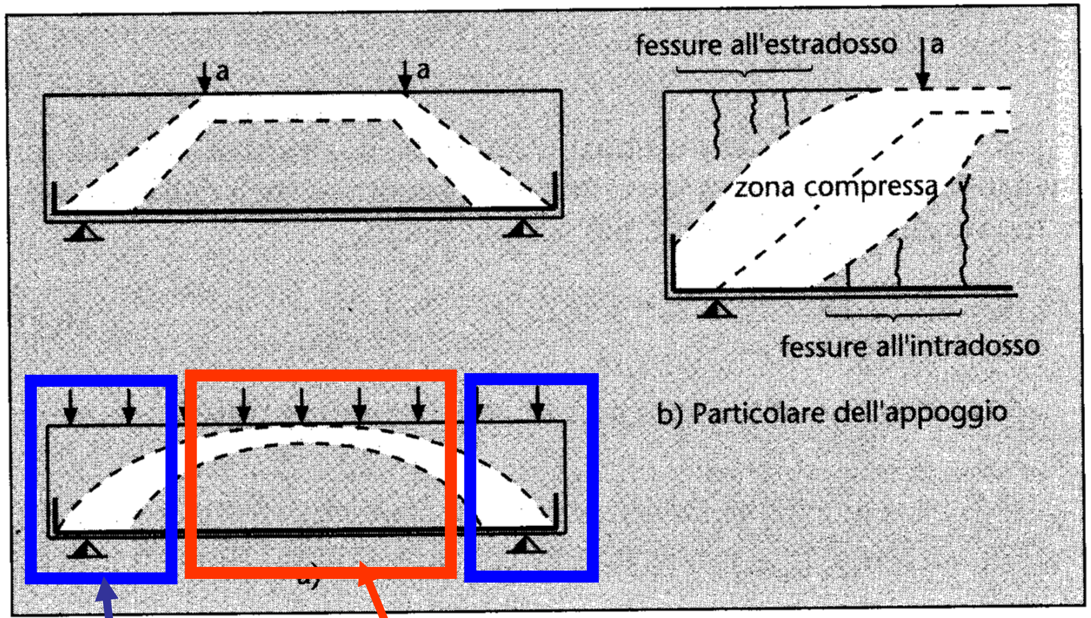
EFFECTIVE SHEAR TO BE USED IN DESIGN

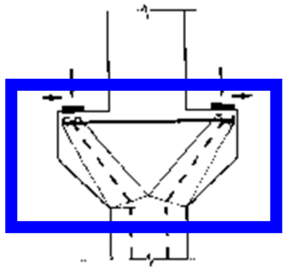
CONTRIBUTIONS OF VERTICAL COMPONENTS OF TENSILE RESULTANT IN STEEL AND COMPRESSION CHORD

EXAMPLES OF STRUT-AND-TIE MODELS FOR DIFFUSION ZONES

D – DIFFUSION ZONE

B – BEAM ZONE





(b) Dapped end beam on corbel

D – DIFFUSION ZONE

B – BEAM ZONE

D – DIFFUSION ZONE

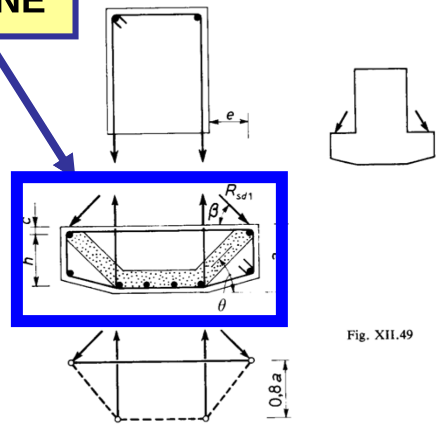
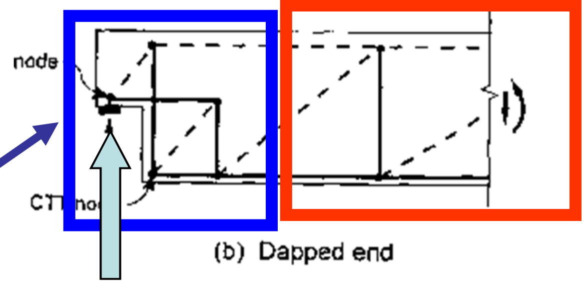


Fig. XII.49



(b) Dapped end

D – DIFFUSION ZONE

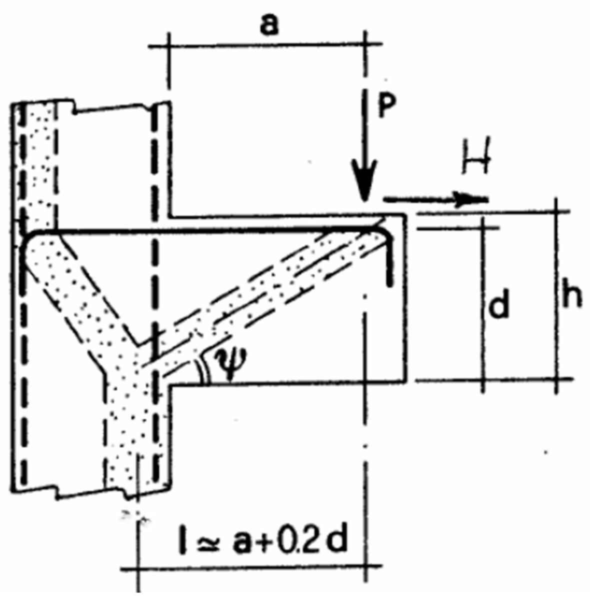
STRUT-AND-TIE MODELS FOR DIFFUSION ZONES

IN THE CASE OF DIFFUSION ZONES, THE STRESS STATE IS VERY COMPLEX, AND CLASSICAL MODELS DO NOT APPLY.

STRUT-AND-TIE MODELS are based on EQUILIBRIUM EQUATIONS, and THE GEOMETRY is defined according to the experience or GuideLines.

The safety verifications are done with respect to:

1. ***Resistance of the tensile elements (steel reinforcement) (***R_s)
2. ***Resistance of the concrete struts (***R_c)
3. ***Anchorage of the steel bars (***R_b)
4. ***Resistance of the nodes (***R_n)



The following inequality is required in order to have a DUCTILE FAILURE MECHANISM

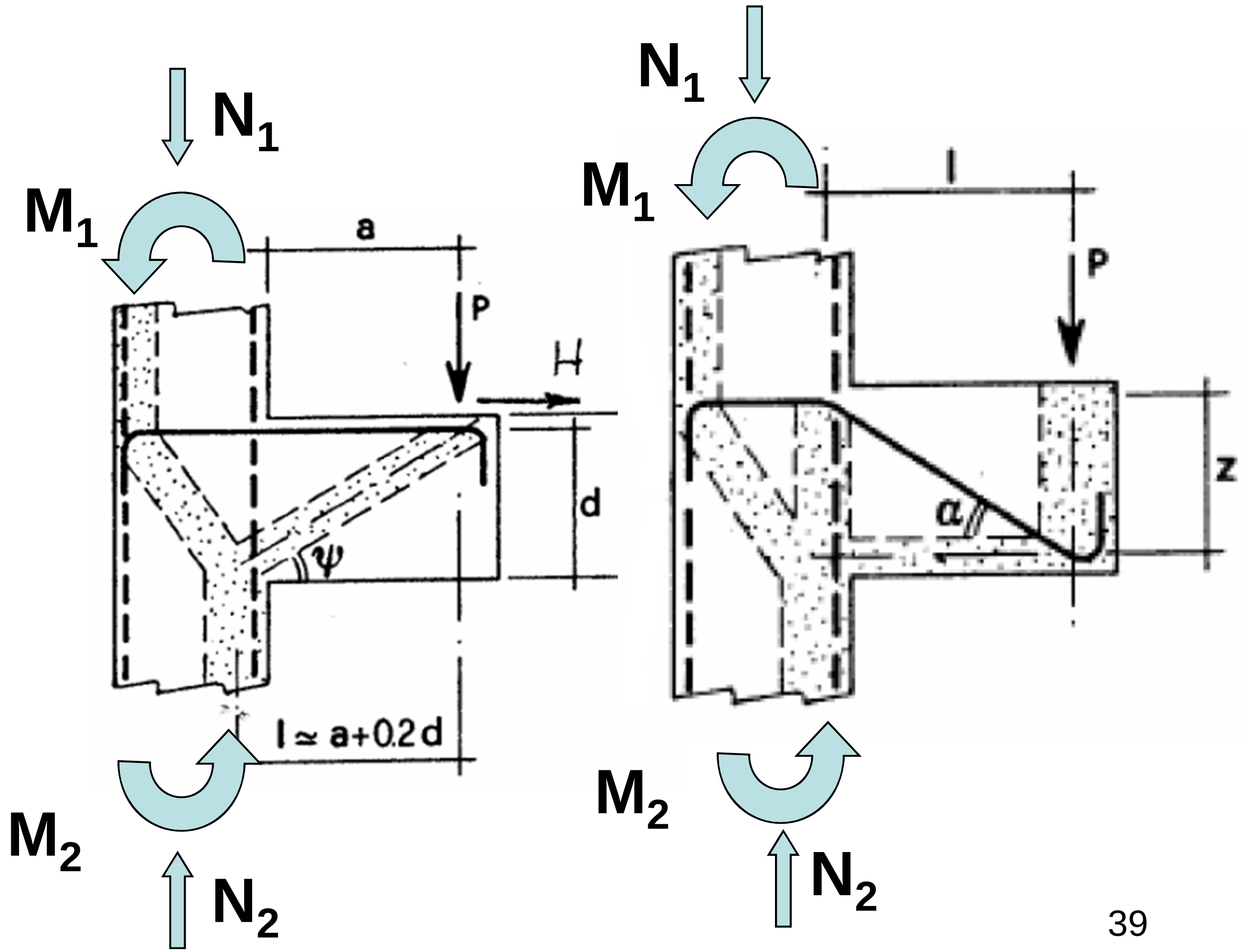
$$R_s < (R_n, R_b, R_c)$$

EXAMPLE: STRUT-AND-TIE MODEL FOR RC CORBELS

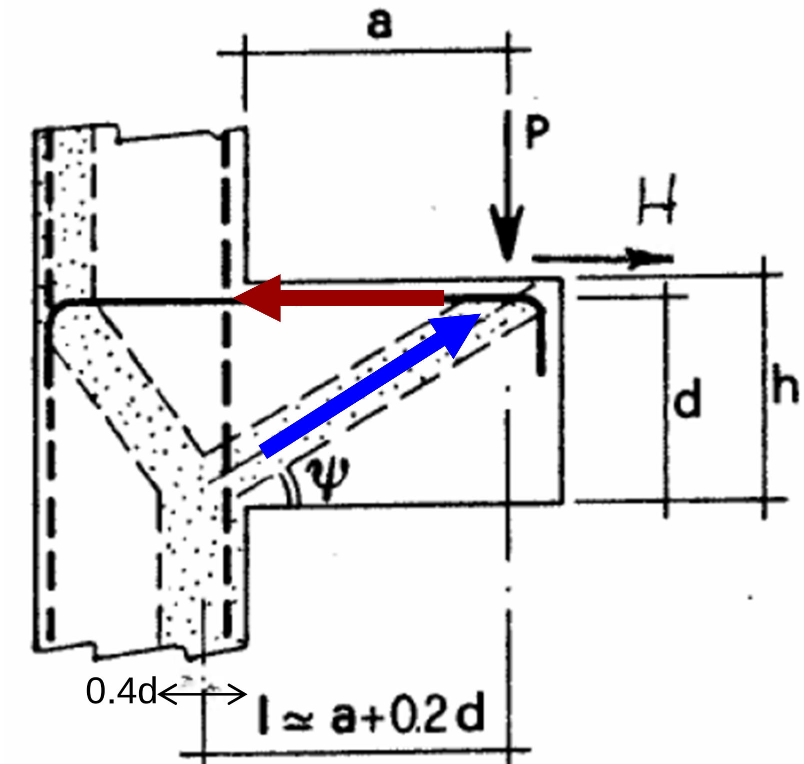
The STRUT-AND-TIE MODEL depends on the positioning of the main steel reinforcement

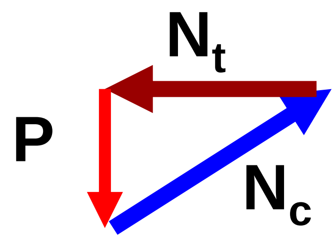
RULES:

1. ***In column, position and width of the concrete struts are obtained from a beam analysis.***
2. ***Ties where steel reinforcement is placed.***
3. ***Equilibrium of the nodes must be satisfied***
4. ***The widths of the concrete struts are defined according to experience rules.***
5. ***Accurate detailing rules are required.***



EXAMPLE: STRUT-AND-TIE MODEL FOR RC CORBELS





EQUILIBRIUM EQUATIONS

$$\begin{cases} N_c \cdot \sin\psi + P = 0 & \rightarrow \quad N_c = -\dfrac{P}{\sin\psi} \\ N_t + N_c \cdot \cos\psi = 0 & \rightarrow \quad N_t = P \cdot \cot\psi \end{cases}$$

ASSUMPTION

Design Value (concrete strength)

$$P_{c,Rd} = N_c \sin\psi = 0.4d\, b \sin\psi \; f_{cd} \sin\psi \geq P$$

Design Value (steel yielding)

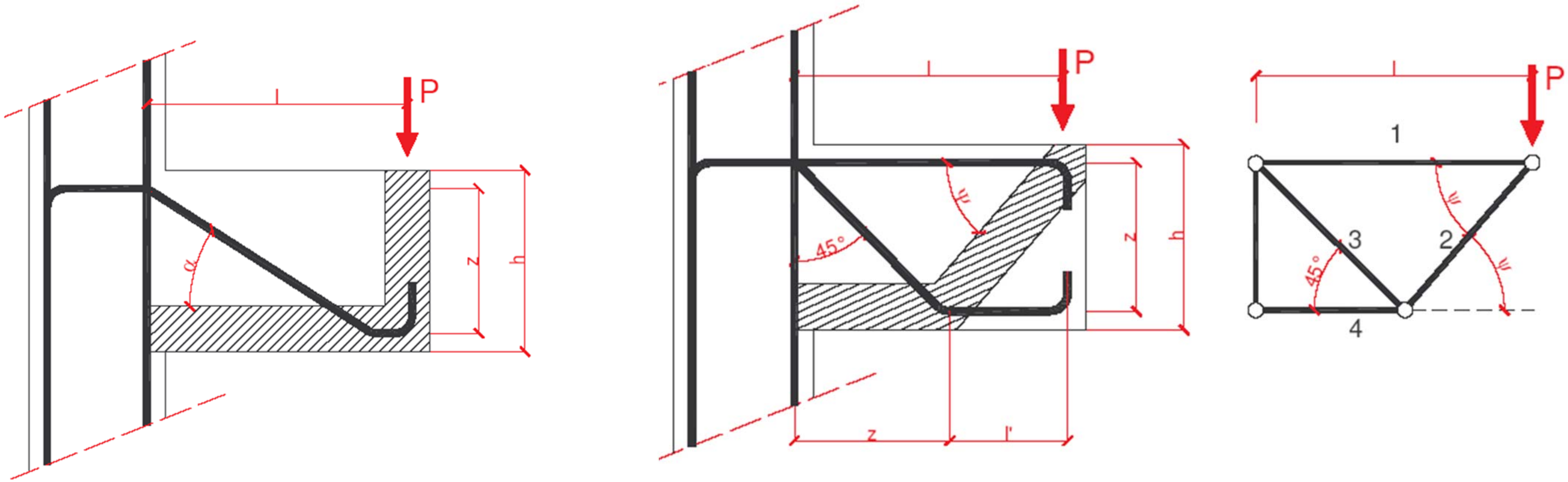
$$P_{t,Rd} = \frac{N_t}{\cot\psi} = A_s \cdot f_{yd} \cdot \frac{1}{\cot\psi} \geq P$$

A CORRECT DESIGN REQUIRES

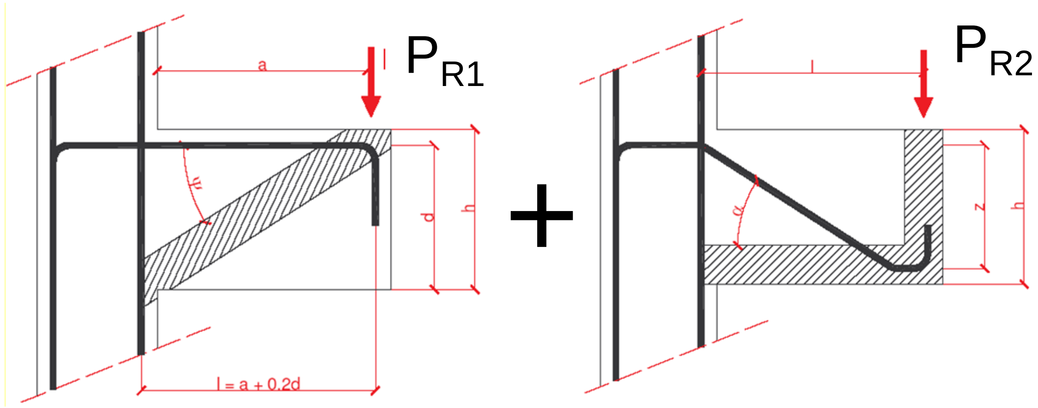
HIERARCHY OF RESISTANCES $\mathbf{P_{t,Rd} < P_{c,Rd}}$

THEN $\mathbf{P_{Rd} = P_{t,Rd}}$

OTHER MODELS FOR RC CORBELS WITH ALTERNATIVE STEEL POSITIONING



SYSTEMS WITH STATICALLY REDUNDANT TRUSS SYSTEMS ARE POSSIBLE (if both steel ties are present, two equilibrated mechanisms are possible)



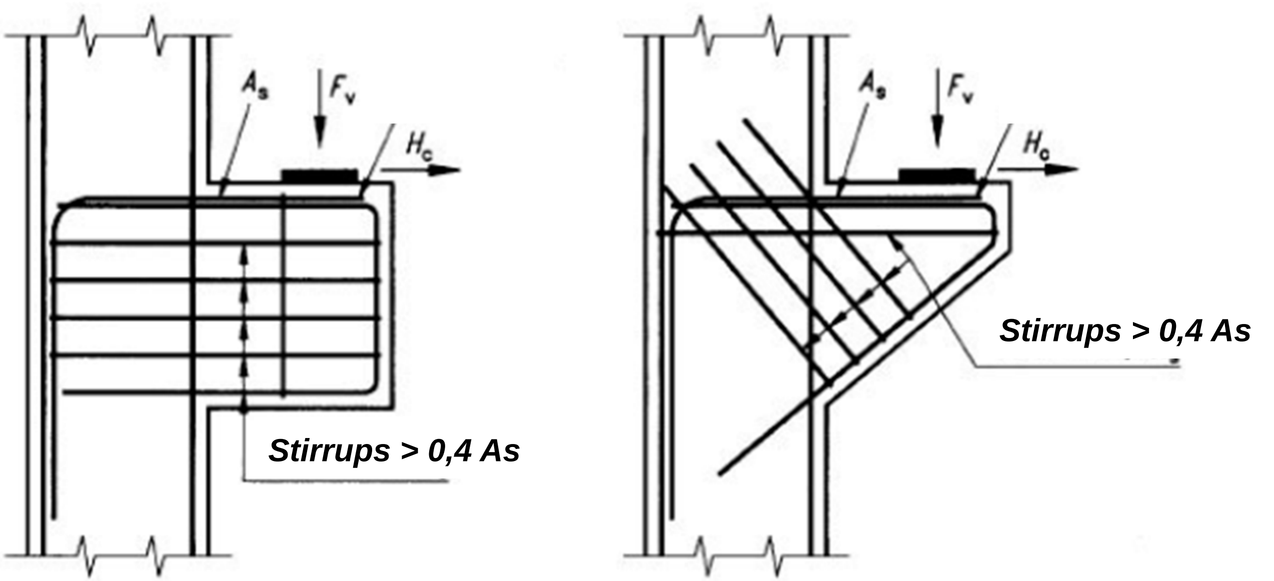
The sum is motivated by the ductile behaviour of the system if the HIERARCHY OF RESISTANCES is satisfied:

$$\mathbf{P_{Rd}} = \mathbf{P_{R1}} + 0{,}8\ \mathbf{P_{R2}}$$

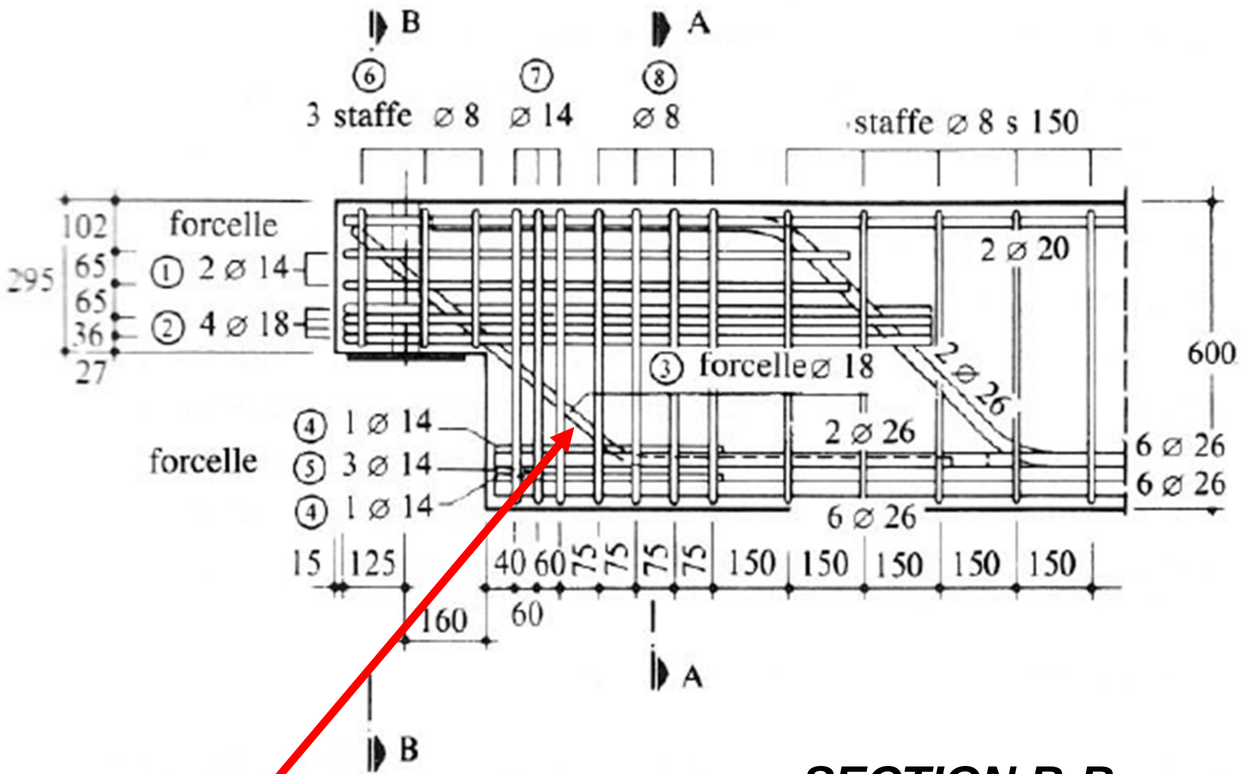
DETAILING FOR RC CORBELS

The steel tie must be adequately anchored

Struct and tie model is an EQUILIBRIUM MODEL: additional distributed steel is required to reduce the cracking of the corbel under service loadings

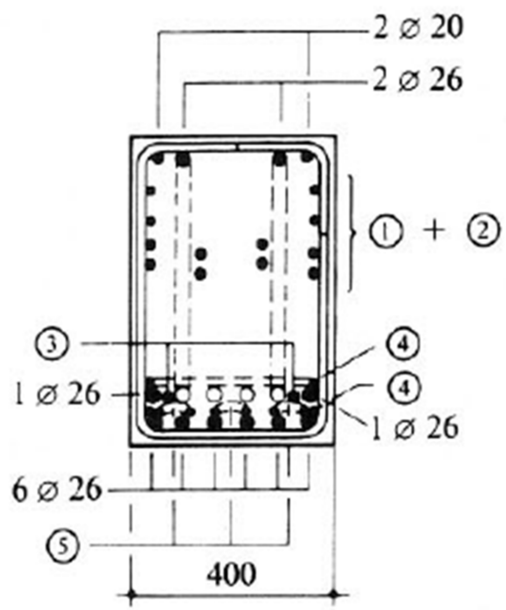


DETAILING FOR GERBER SADDLE

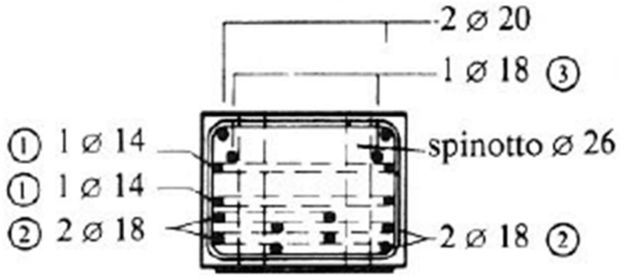


Inclined bars to reduce the width of the concrete crack

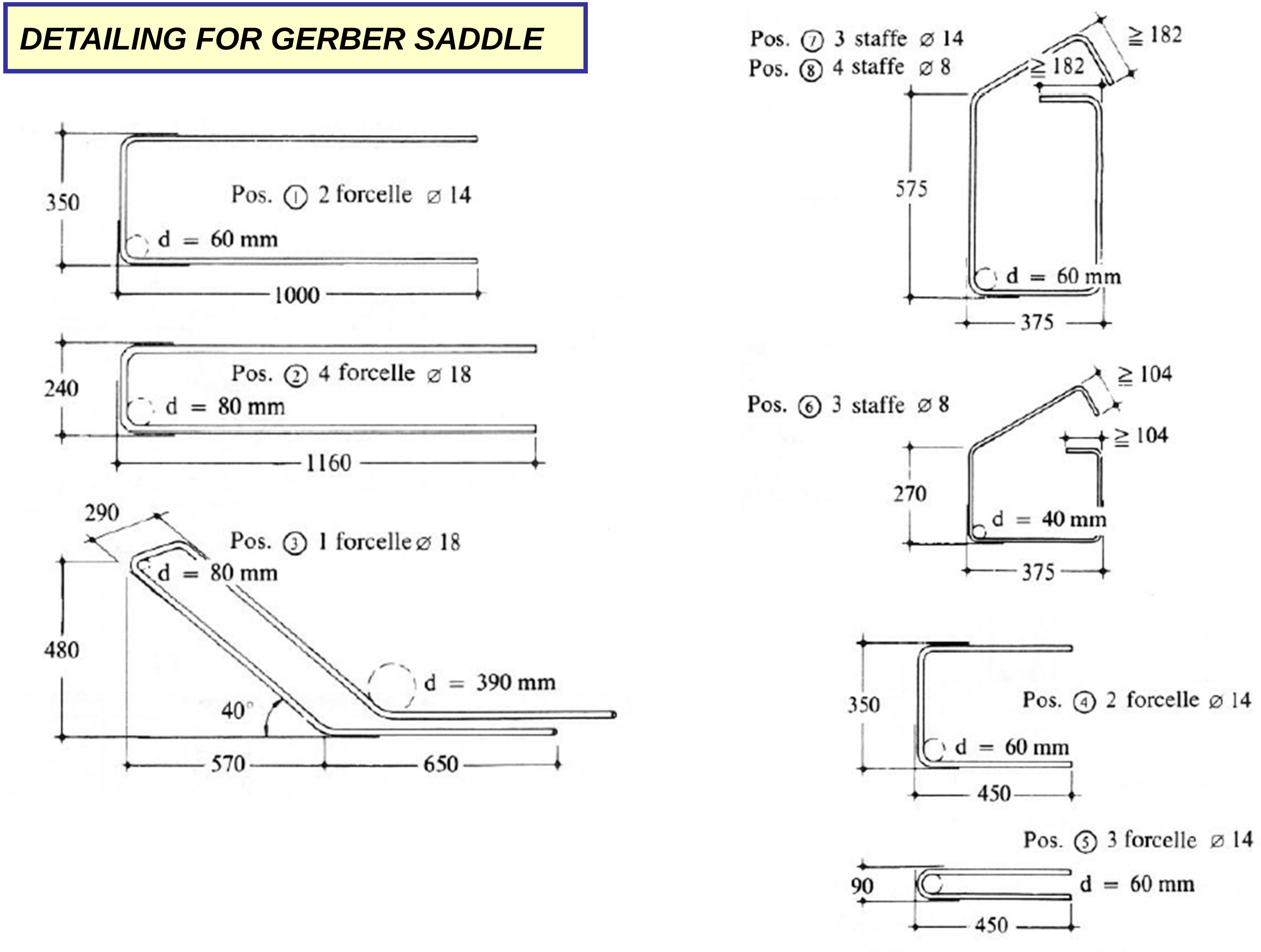
SECTION A-A



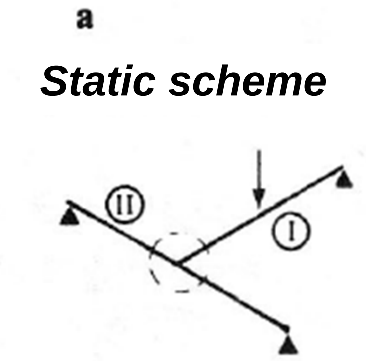
SECTION B-B

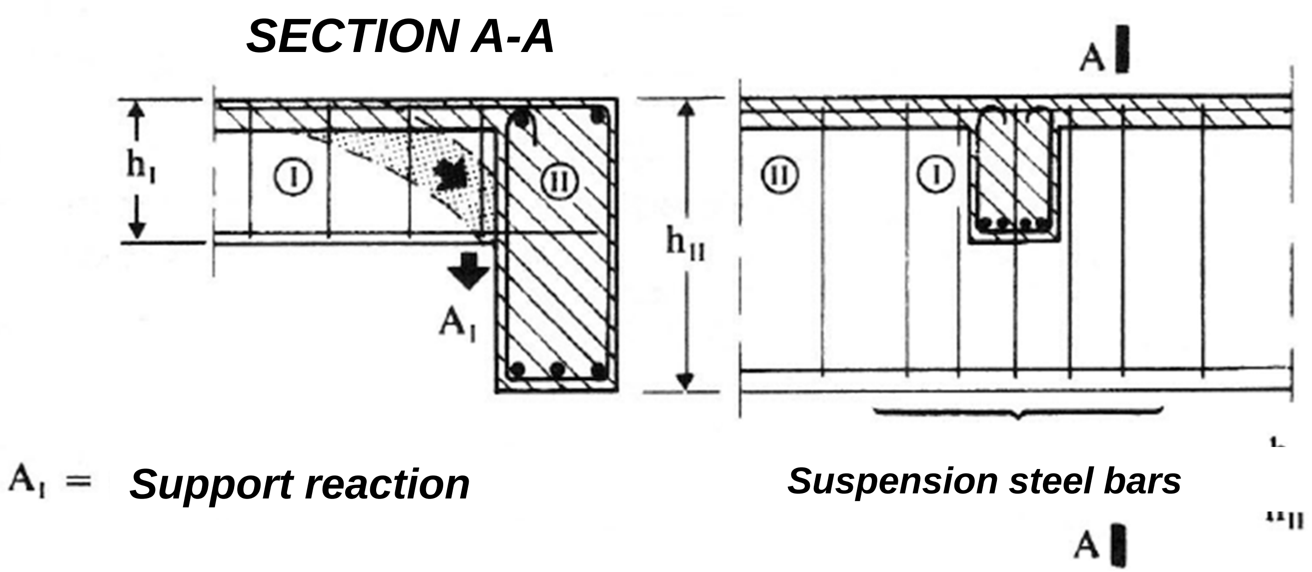


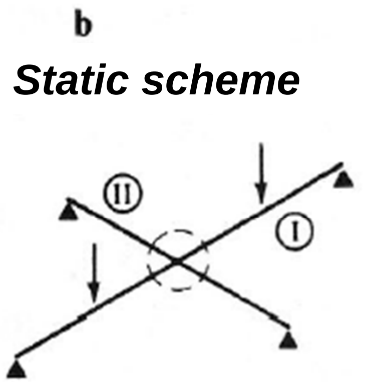
DETAILING FOR GERBER SADDLE

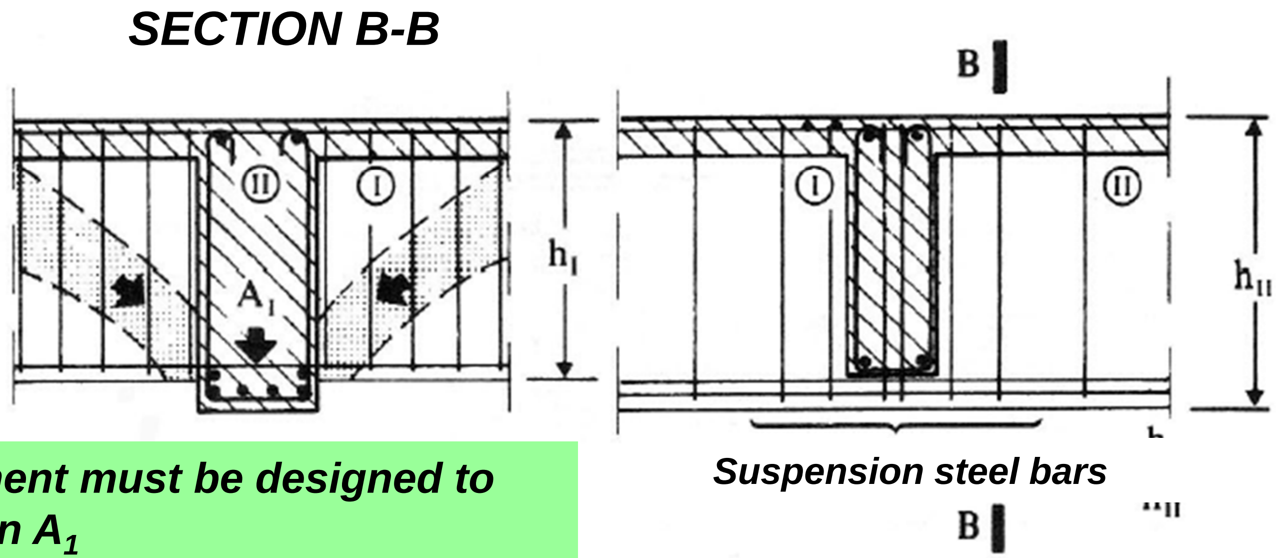


INDIRECT LOADING: SUSPENSION STEEL REINFORCEMENT





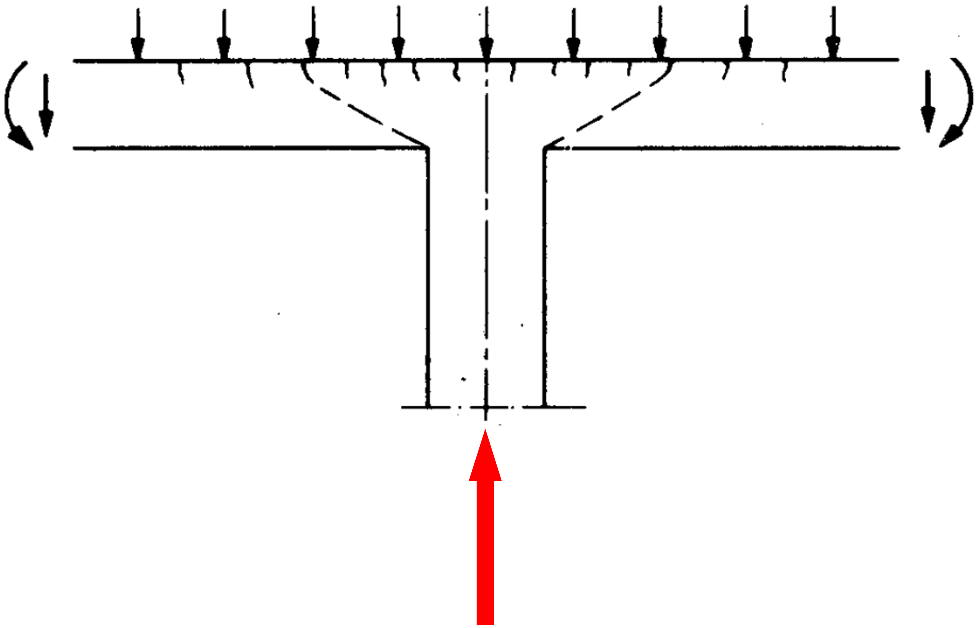




A suspension reinforcement must be designed to carry the support reaction A_1

PUNCHING SHEAR

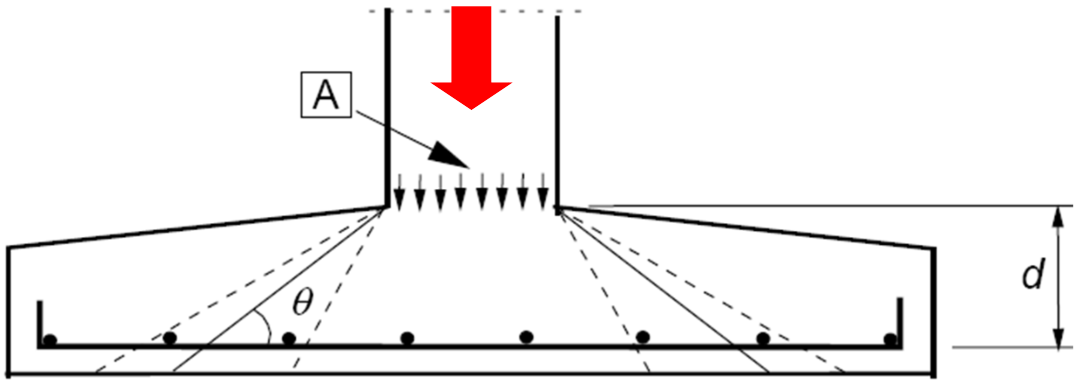
Punching shear results from a CONCENTRATED LOAD or a REACTION acting on a relatively small area (the loaded area A_{load})



Example:
SOLID SLAB OVER COLUMNS

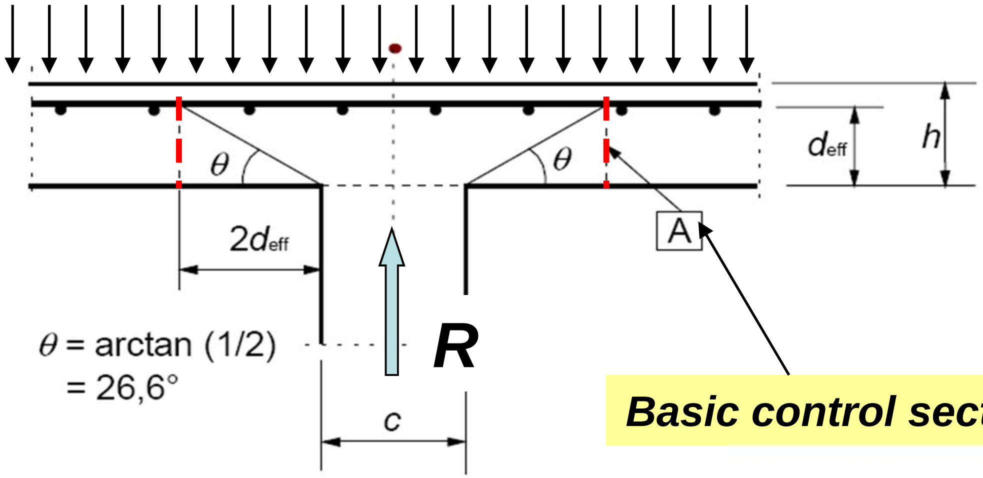
Other examples:
CONCENTRATED LOADS OVER SMALL THICKNESS SOLID SLABS

FOOTINGS



PUNCHING SHEAR

MODEL FOR CHECKING PUNCHING FAILURE AT ULTIMATE LIMIT STATE (EC2) (circular section – axial force only, no bending)



Basic control section

Calculate the load which is transferred over the Basic control Section

The load contribution acting directly on the control area can be subtracted

SHEAR STRESS OVER THE BASIC CONTROL SECTION

$$v_{Ed} = \frac{R - q\,A_{cont}}{A_A} = \frac{q(A_{slab} - A_{cont})}{A_A}$$

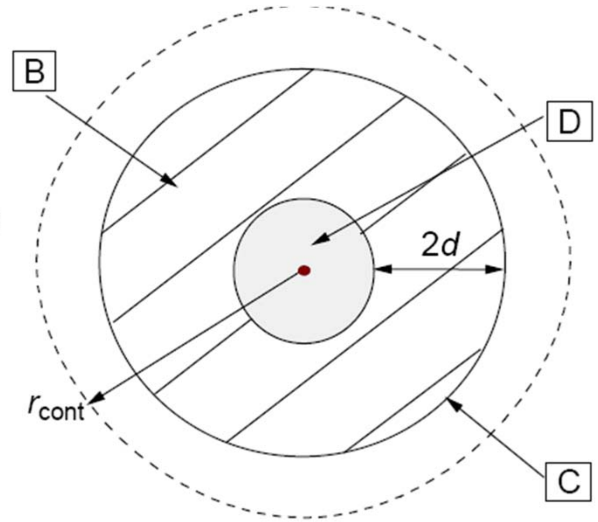
CONTROL SECTION AREA

B - basic control area A_{cont}

C - basic control perimeter, u_1

D - loaded area A_{load}

r_{cont} further control perimeter



$$A_A = 2\pi(2d_{eff} + c/2)\,d_{eff} = u_1\,d_{eff}$$

PUNCHING SHEAR

VERIFICATION OF THE CONCRETE (EC2)

Punching shear around the column

$$v_{Ed} \leq v_{Rd,\max} = 0.5\, \nu\, f_{cd}$$

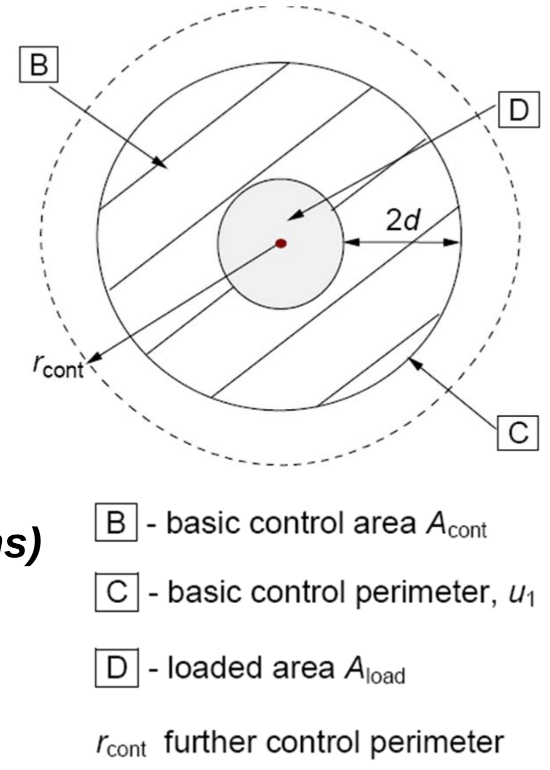
ν = 0.5 cracking of concrete due to shear

VERIFICATION OF THE TENSILE FORCE (EC2)

In control perimeter u_1

Punching shear reinforcement is not necessary if

$$v_{Ed} < v_{Rd,c}$$



Punching shear resistance per unit area (similar to shear in beams)

$$v_{Rd,c} = \frac{0{,}18}{\gamma_c} k (100 \rho_l\, f_{ck})^{1/3} \geq 0.035 k^{3/2} f_{ck}^{1/2}$$

$k = 1 + \sqrt{200/d} \leq 2$ ***Size effect***

$\rho_l = \sqrt{\rho_{lx} \cdot \rho_{lz}} \leq 0{,}02$ ***Tensile steel ratio***

PUNCHING SHEAR

In control perimeter u_1

VERIFICATION WITH REINFORCEMENT (EC2)

PUNCHING SHEAR RESISTANCE (UNIT AREA) WITH SHEAR REINFORCEMENT

$$v_{Rd,cs} = 0{,}75\, v_{Rd,c} + 1{,}5 \frac{d\, A_{sw}}{s_r} f_{ywd} \frac{1}{u_1\, d} \sin\alpha$$

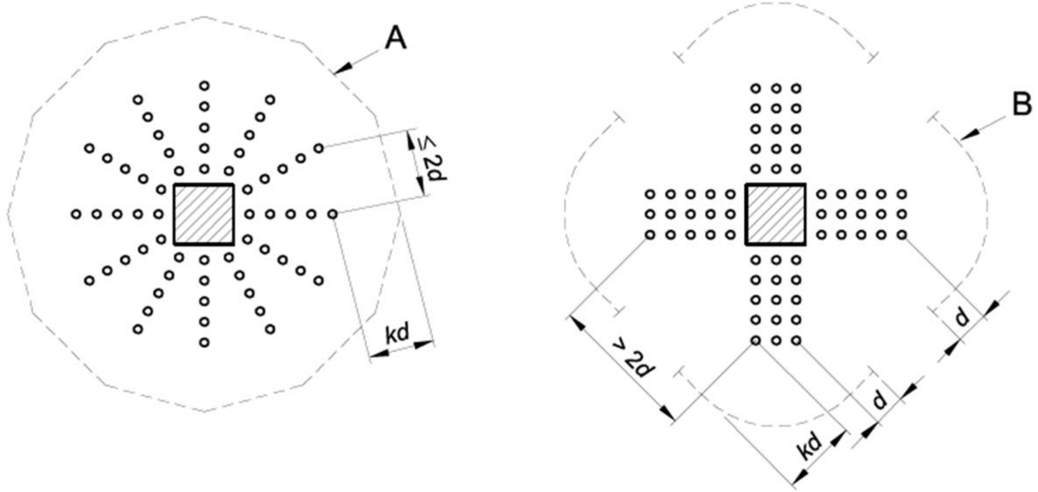
concrete contribution — ***Radial spacing*** — ***Control perimeter***

Perimetri di verifica per pilastri interni

Legenda

A Perimetro u_{out}

B Perimetro $u_{out,ef}$



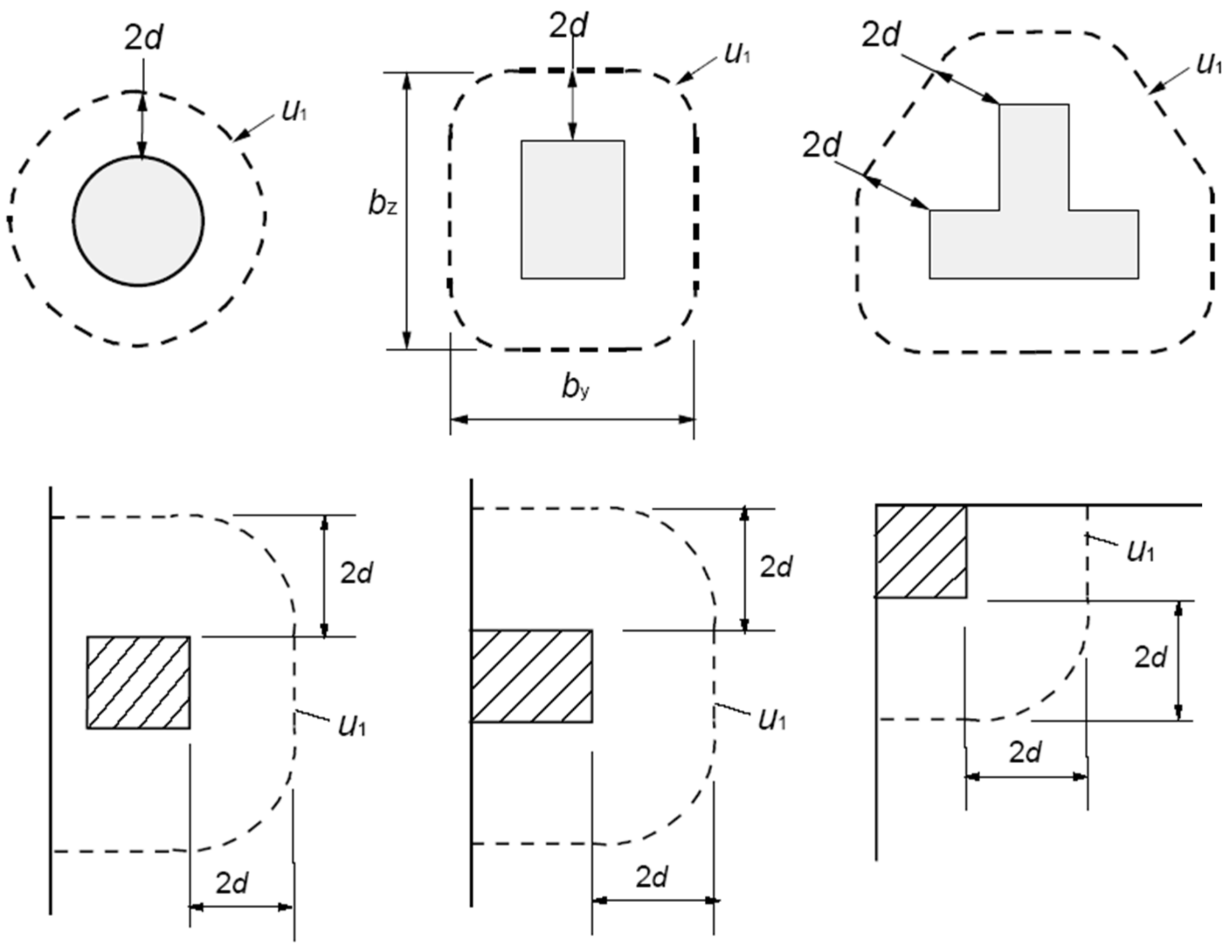
COMPARE WITH BEAMS

$$V_{wd} = A_{sw}\, f_{ywd} \frac{z}{\Delta x} (\operatorname{cotg}\theta + \operatorname{cotg}\alpha) \sin\alpha$$

Definition of the perimenter where the reinforcement is not necessary in order to obtain the extension of the reinforcement

PUNCHING SHEAR

In real cases, the problem can be more complex:



CONTROL PERIMETERS AROUND LOADED AREAS OR COLUMN SECTIONS

PUNCHING SHEAR

FOOTINGS WITH VARIABLE HEIGHT – different control sections must be considered

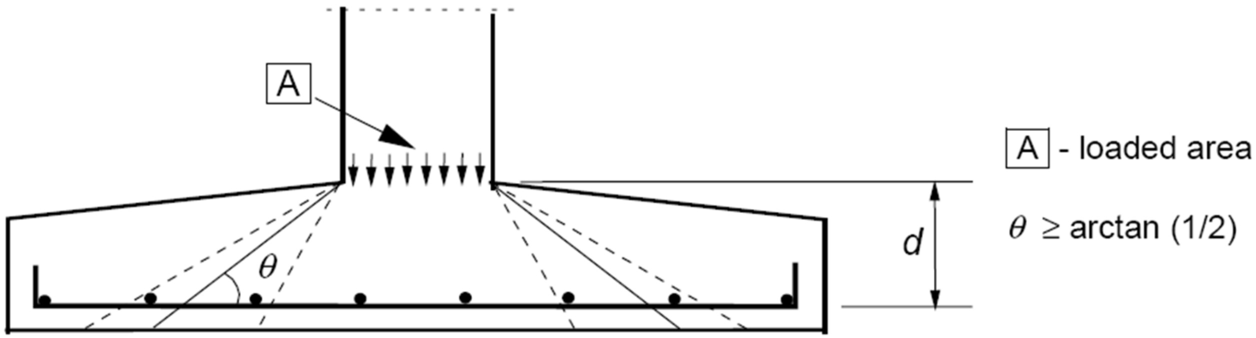
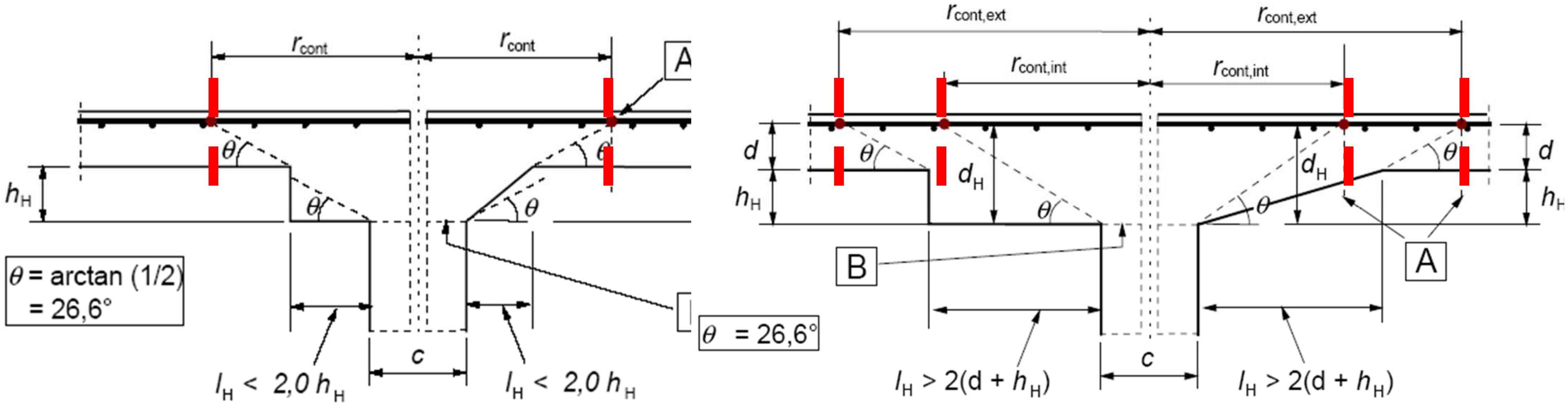


Figure 6.16: Depth of control section in a footing with variable depth

CONCRETE SLABS WITH ENLLARGED COLUMN HEAD

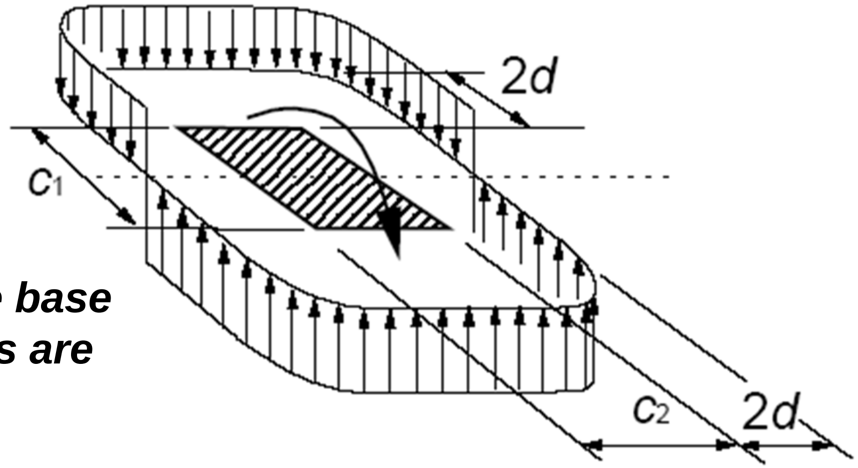


PUNCHING SHEAR

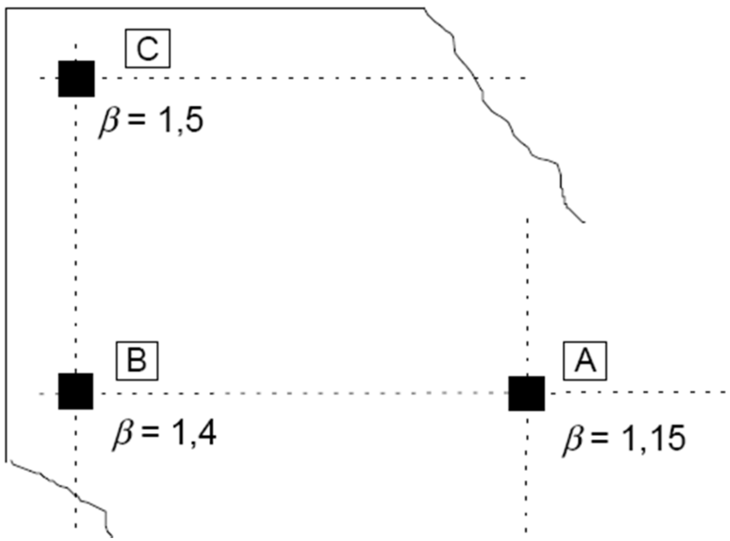
MAXIMUM SHEAR STRESS IN THE CASE OF AN ECCENTRIC REACTION

$$v_{Ed} = \beta \frac{V_{Ed}}{u_1 \, d}$$

$$\beta \geq 1$$



depend on Moment and Shear acting on the base and on geometrical parameters (many cases are explicitly considered in EC2)



A - internal column

B - edge column

C - corner column

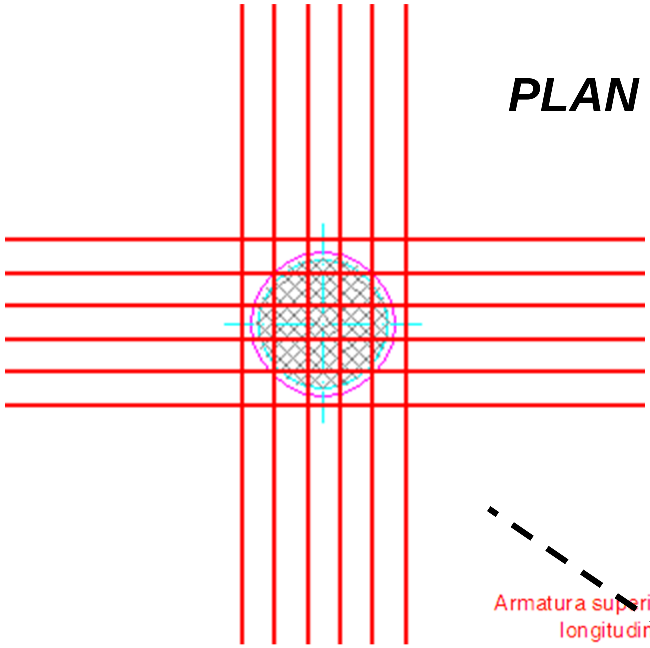
Approximate values for β coefficient

The effect of the moment is greater in the case of edge columns

PUNCHING SHEAR

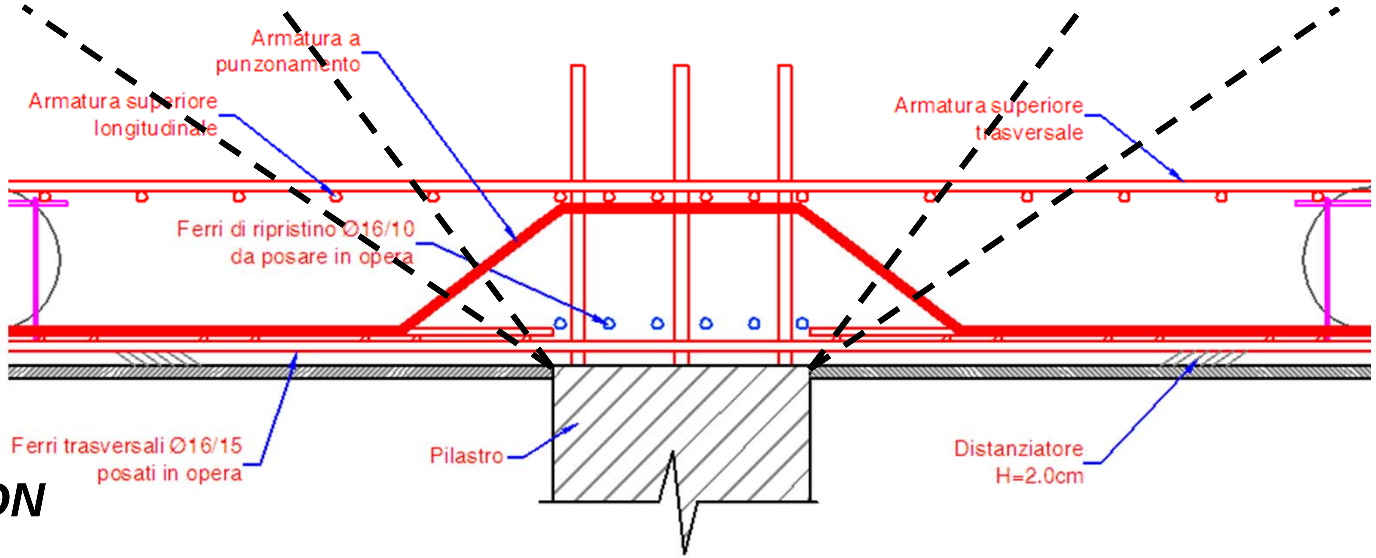
Some examples of steel reinforcement against punching shear

PLAN



- ***The individual contributions of the steel bars crossed by the control section must be computed***
- ***Different control sections must be considered – all of them must be crossed by the shear bars against punching***

$$26{,}6° \leq \theta \leq 45°$$

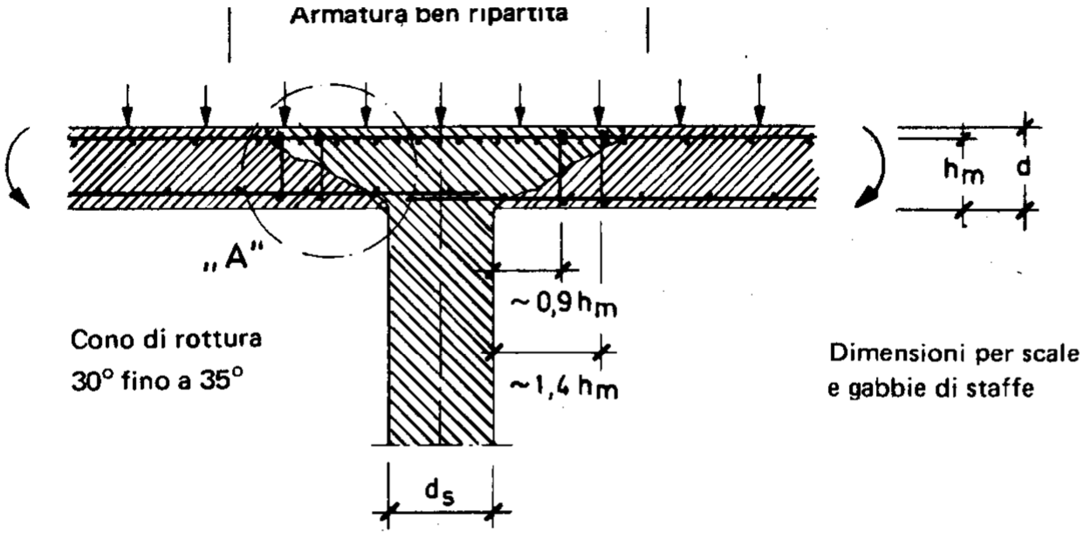


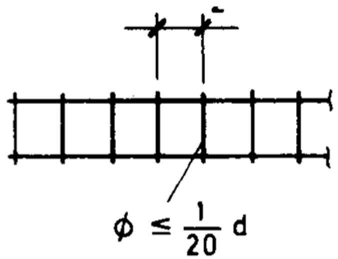
SECTION

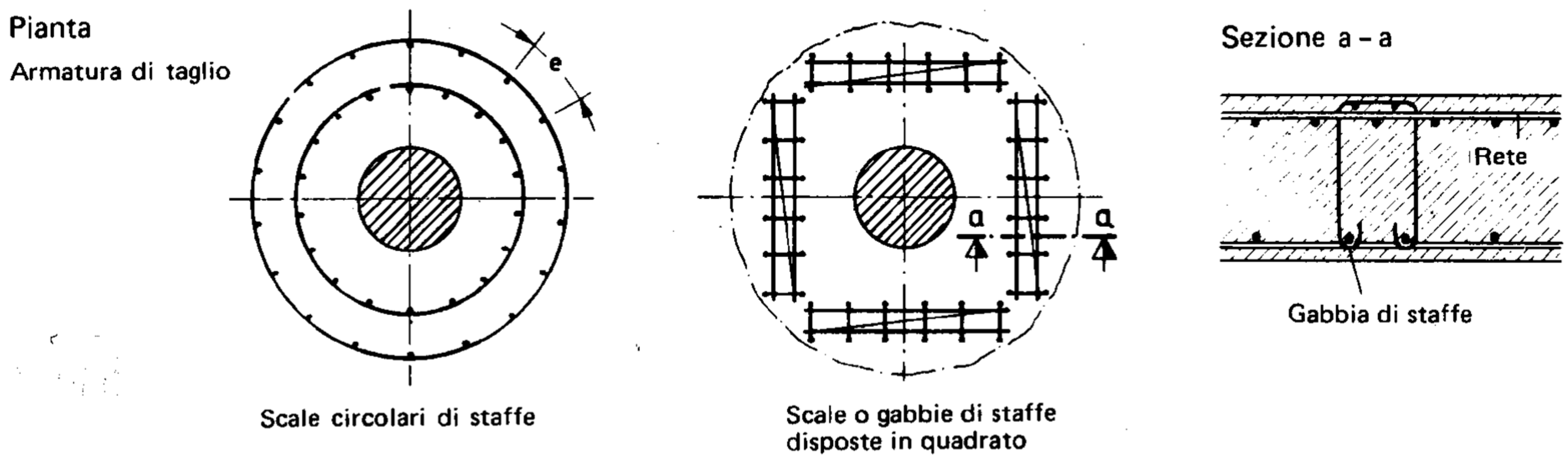
PUNCHING SHEAR

Some examples of steel reinforcement against punching shear

- ***The individual contributions of the steel bars crossed by the control section must be computed***
- ***Different control sections must be considered***

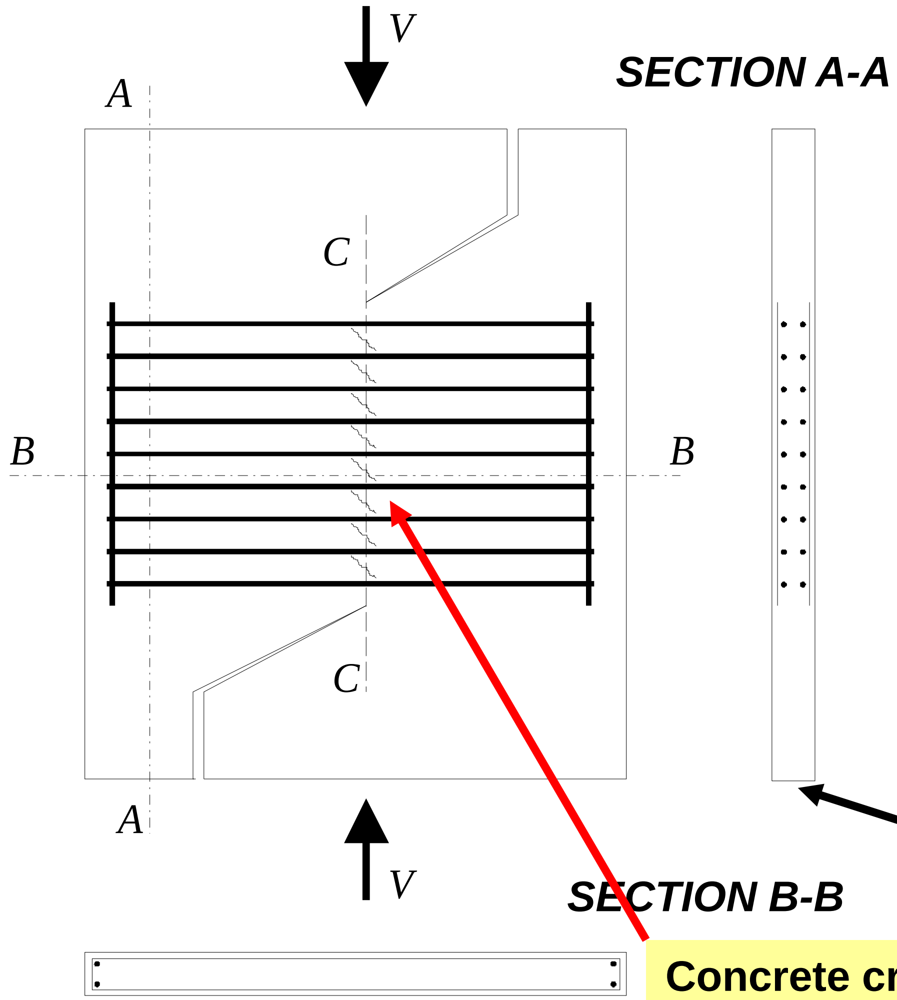




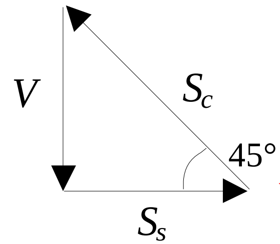


SHEAR AT THE INTERFACE BETWEEN THIN-WALLED RC ELEMENTS

The Walraven tests
Delft Polytechnic of Technology (1971)



Concrete cracking due to shear



Force equilibrium for the left side of the wall
If we assume 45°:

$$S_s = V$$

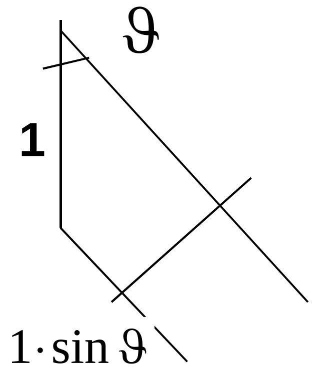
h =wall thickness

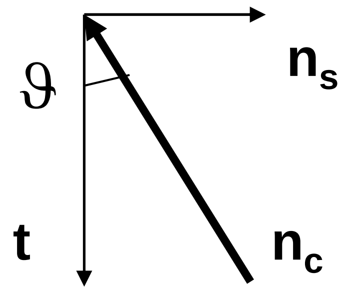
SHEAR AT THE INTERFACE – VARIABLE INCLINATION MODEL (EC2)

Equilibrium of forces per unit length

$$n_c = \frac{t}{\cos\vartheta}$$

$$n_s = t\tan\vartheta$$





$$\sigma_c \cdot h \cdot 1 \cdot \sin\vartheta = \frac{t}{\cos\vartheta}$$

DESIGN OF THE TRANSVERSE REINFORCEMENT (t -> As)

$$\frac{A_s}{s_f} f_{yd} \geq \frac{t}{\mathrm{cotg}\,\vartheta}$$

SHEAR STRENGTH AGAINST CONCRETE FAILURE

$$v_{Rcd} = \nu\, f_{cd} \cdot h \cdot \cos\vartheta \cdot \sin\vartheta$$

$$v_{Rcd} \geq v_{Ed}$$

EFFICIENCY OF CONCRETE STRUTS

$$\nu = 0.6\left[1 - \frac{f_{ck}}{250}\right] \quad \textit{with } f_{ck} \textit{ in MPa}$$

SHEAR AT THE INTERFACE – VARIABLE INCLINATION MODEL (EC2)

MINIMUM TRANSVERSE REINFORCEMENT (BALANCE POINT)

$$v_{Rcd} = \nu\, f_{cd} \cdot h \cdot \cos\vartheta \cdot \sin\vartheta$$

$$v_{Rsd} = \frac{A_s}{s_f} f_{yd} \cot g\,\vartheta$$

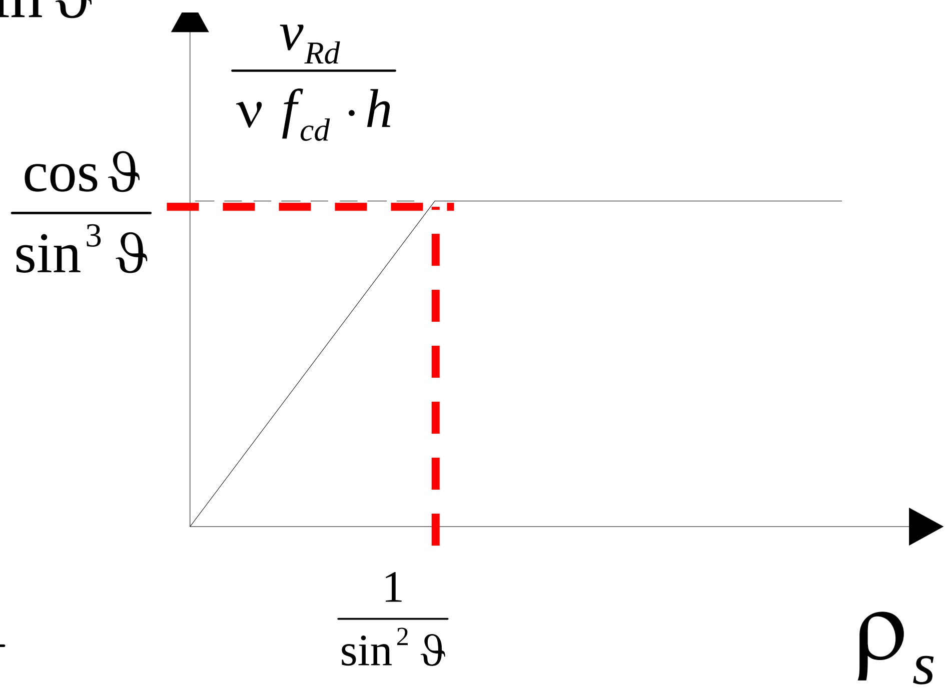
$$\frac{A_{s,\min}}{s_f} f_{yd} \cot g\,\vartheta = \nu\, f_{cd}\, h \cos\vartheta \sin\vartheta$$

Setting (transverse reinforcement ratio)

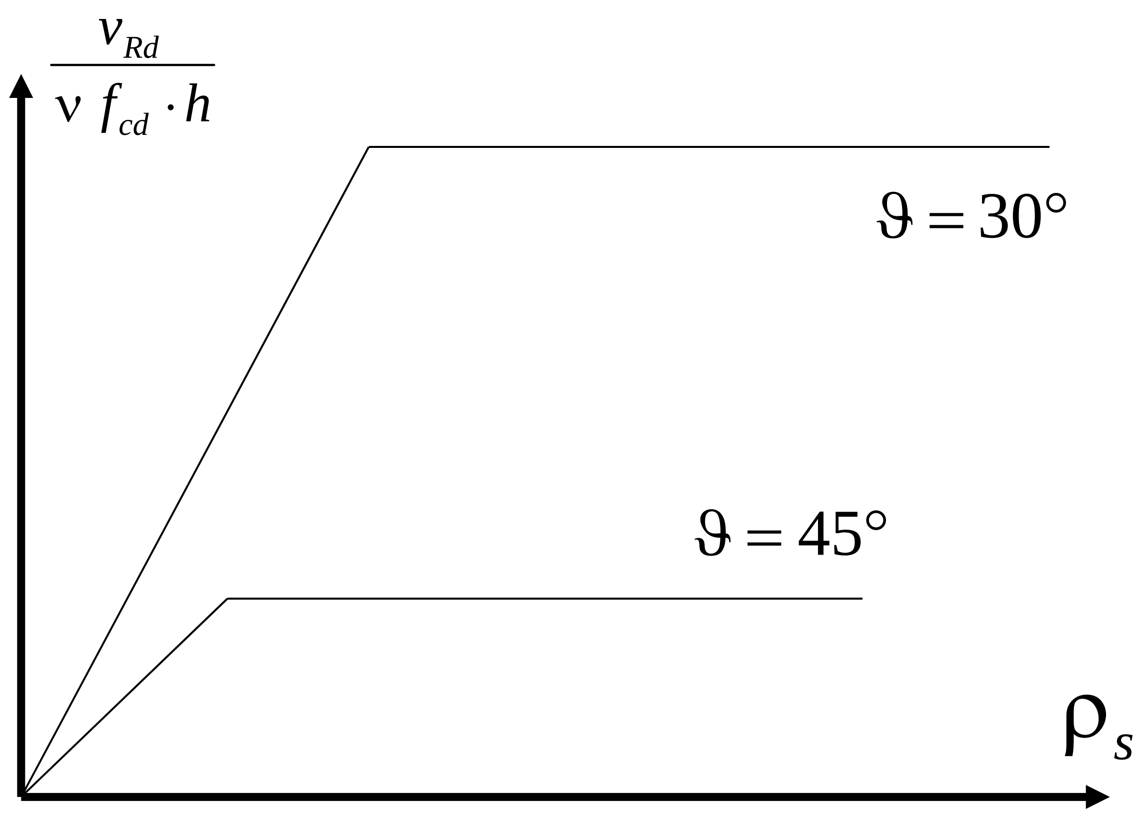
$$\rho_s = \frac{A_s}{s_f} \frac{1}{h} \frac{f_{yd}}{\nu\, f_{cd}}$$

Balance condition

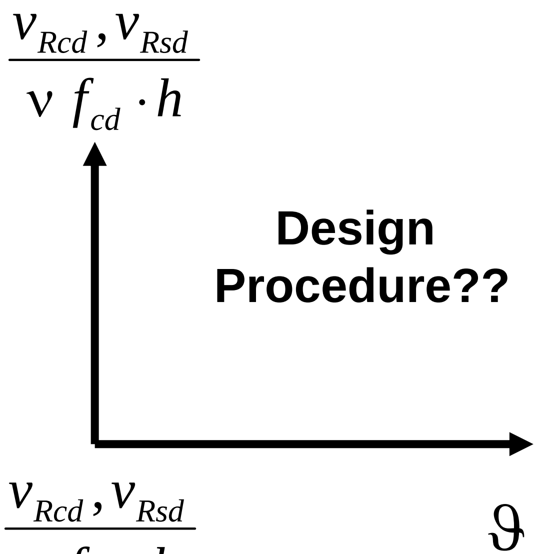
$$\rho_{s,\min} = \frac{1}{\sin^2\vartheta} \qquad \frac{v_{Rd}}{\nu\, f_{cd} \cdot h} = \frac{\cos\vartheta}{\sin^3\vartheta}$$

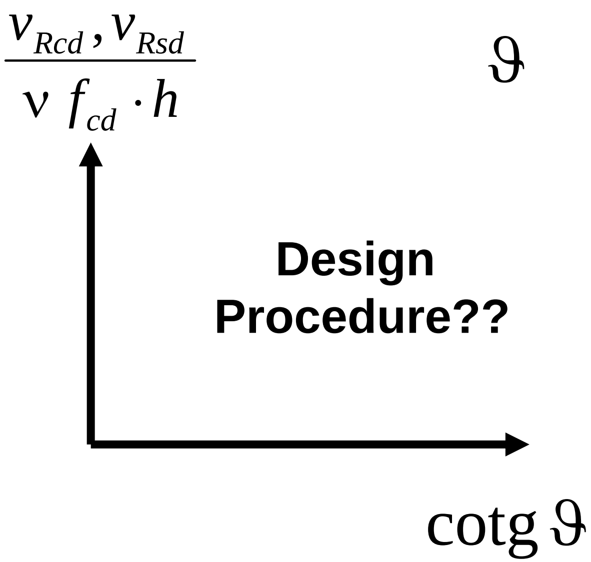


SHEAR AT THE INTERFACE – VARIABLE INCLINATION MODEL (EC2)

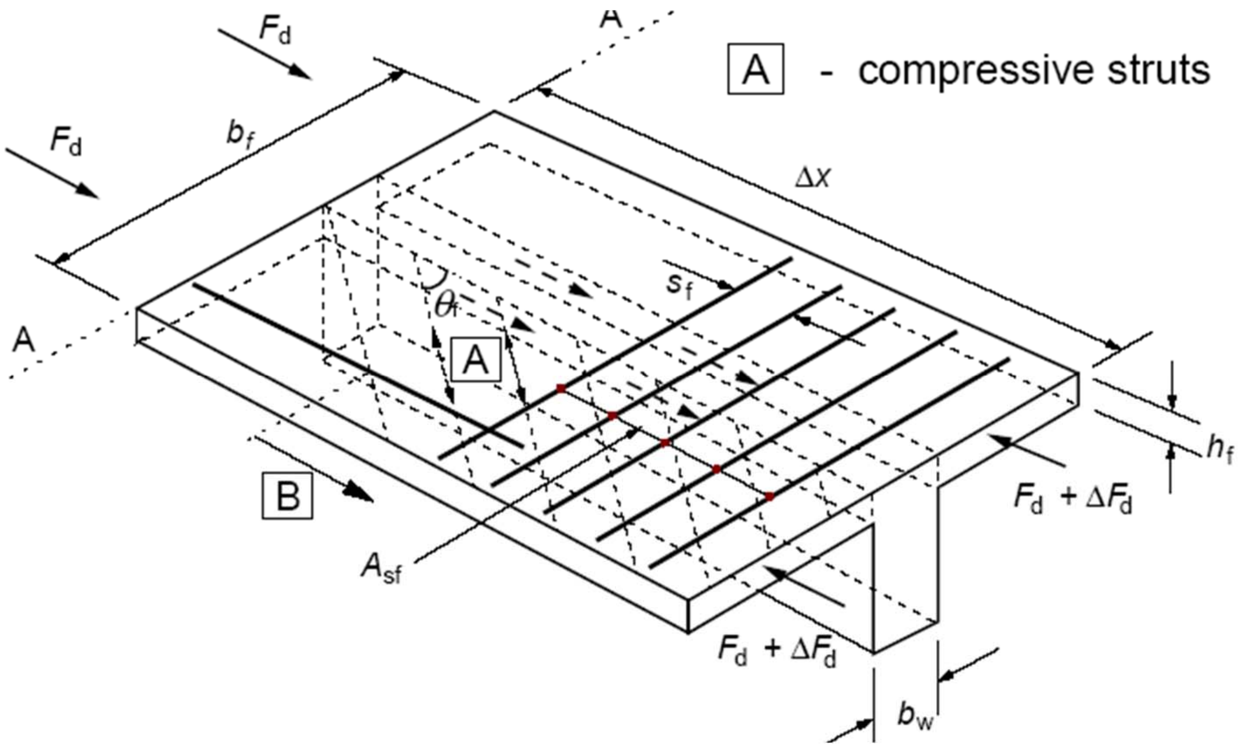


SHEAR AT THE INTERFACE – VARIABLE INCLINATION MODEL (EC2)





APPLICATION OF THE METHOD

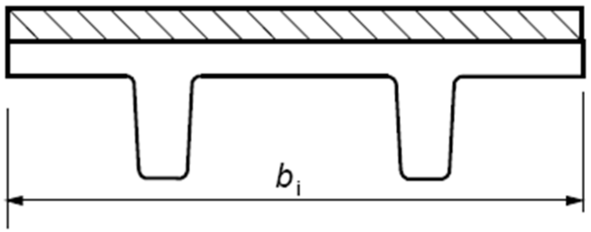


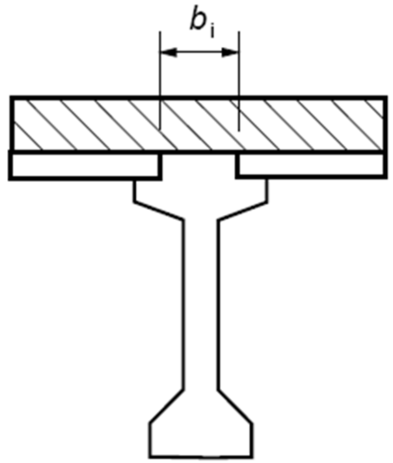
$1{,}0 \leq \cot\theta_f \leq 2{,}0$ for compression flanges ($45° \geq \theta_f \geq 26{,}5°$)

$1{,}0 \leq \cot\theta_f \leq 1{,}25$ for tension flanges ($45° \geq \theta_f \geq 38{,}6°$)

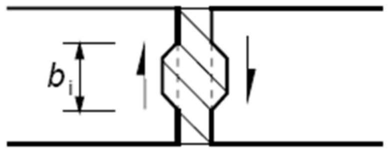
With more details in the following…

OTHER EXAMPLES:
SHEAR AT THE INTERFACE BETWEEN CONCRETES CAST AT DIFFERENT TIMES

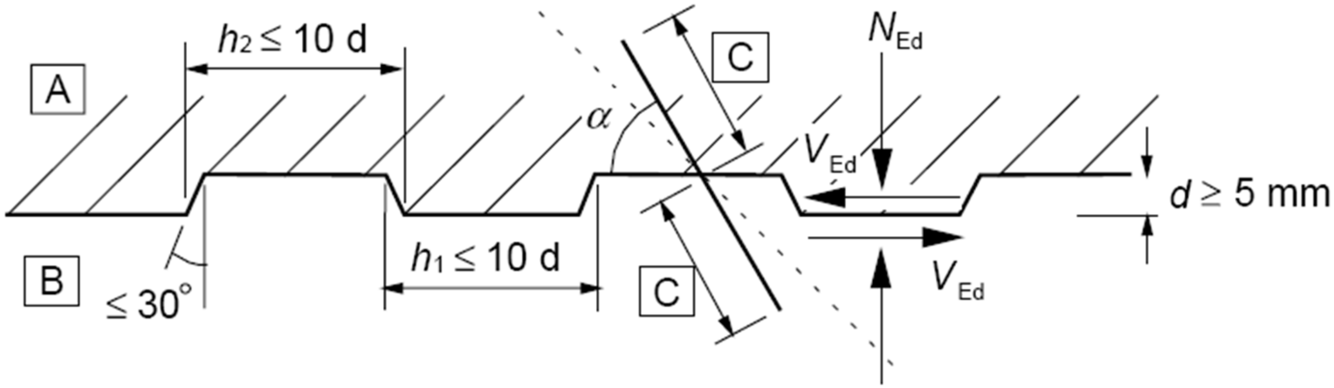




In these cases, the inclination angle must be defined as a function of the friction between the two materials



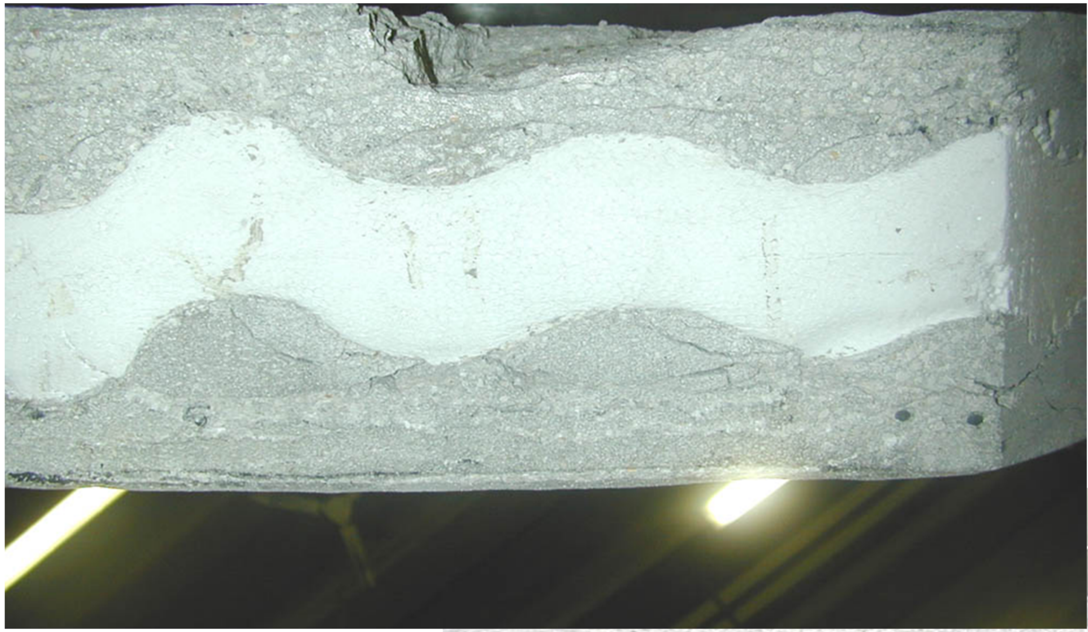
INDENTED JOINT



A - new concrete, B - old concrete, C - anchorage

AN EXAMPLE:
CONCRETE – POLYSTIRENE SANDWICH PANEL

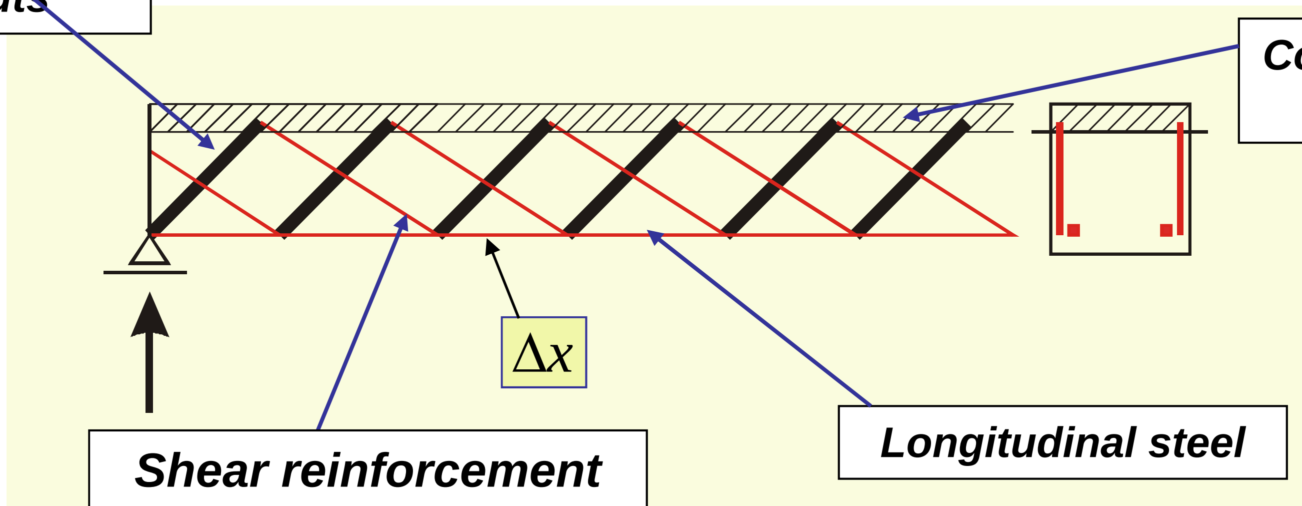
AN EXAMPLE:
CONCRETE – POLYSTIRENE SANDWICH PANEL



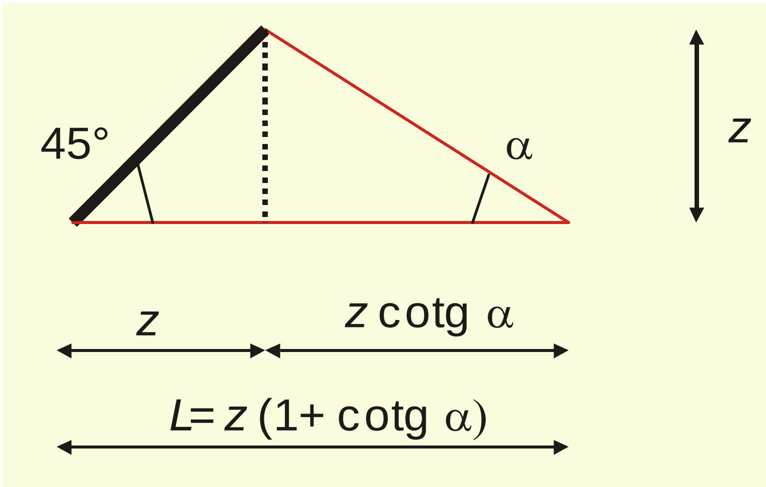
AN EXAMPLE:
CONCRETE – POLYSTIRENE SANDWICH PANEL

Ritter – Morsch STRUT & TIE model

Classical Hypothesis: Concrete struts with 45° inclination



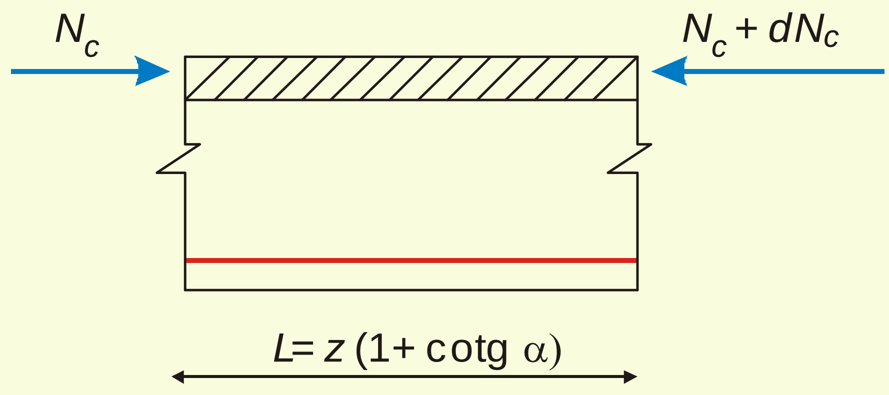
THE GENERAL TRUSS ELEMENT

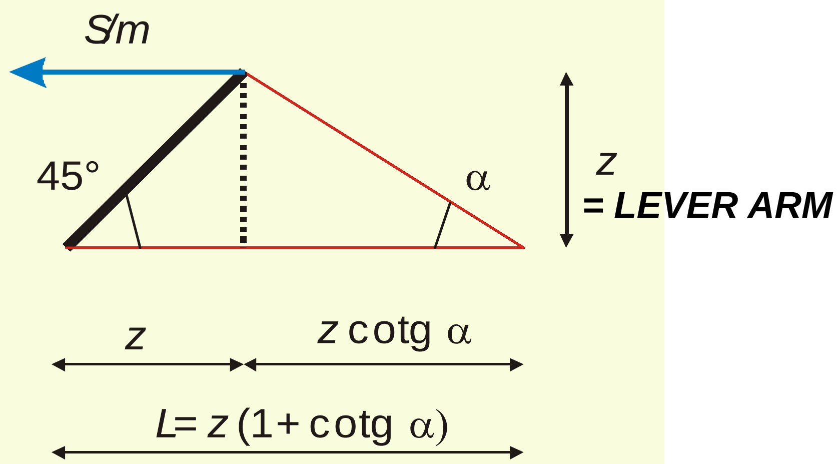


Multiplicity of the truss

$$m = \frac{L}{\Delta x} = \frac{z(1+\operatorname{cotg}\alpha)}{\Delta x}$$

THE SHEAR FORCE TO BE CARRIED OUT BY THE REINFORCEMENT





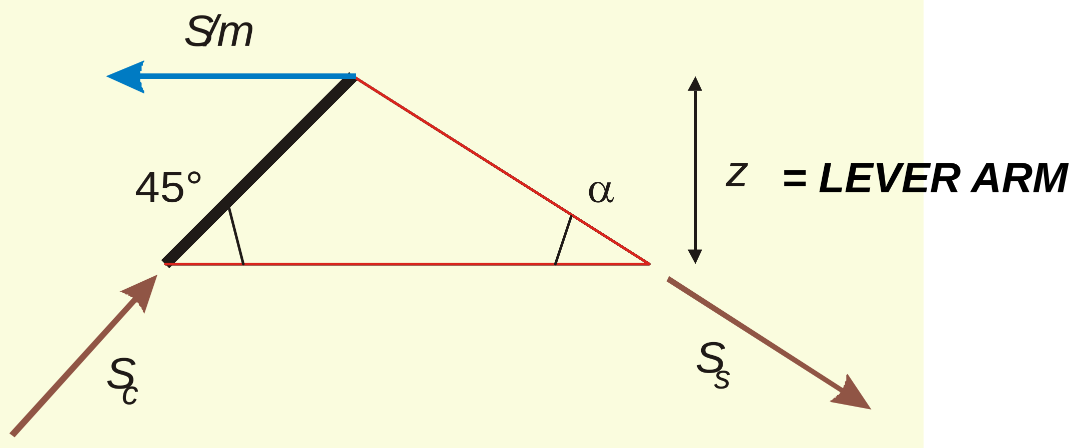
$$\Delta N_c = S = \tau \cdot b \cdot L = \frac{V\,S}{J\,b} \cdot b \cdot L = \frac{V}{z} \cdot L$$

SHEAR FORCE TO BE CARRIED OUT ON THE LENGTH L

$$S/m = \frac{V\,L}{z}\,\frac{1}{m}$$

SHEAR FORCE TO BE CARRIED OUT BY THE GENERAL TRUSS ELEMENT

FORCES ACTING ON THE CONCRETE STRUTS AND THE STEEL REINFORCEMENT



$$S_s = \frac{V\,\Delta x}{z\,(\sin\alpha + \cos\alpha)}$$

Traction force on the steel reinforcement

$$S_c = \sqrt{2}\sin\alpha \cdot S_s = \frac{\sqrt{2}\sin\alpha}{(\sin\alpha + \cos\alpha)}\,\frac{V\,\Delta x}{z} = \frac{\sqrt{2}}{(1 + \cot g\;\alpha)}\,\frac{V\,\Delta x}{z}$$

Compression on concrete struts

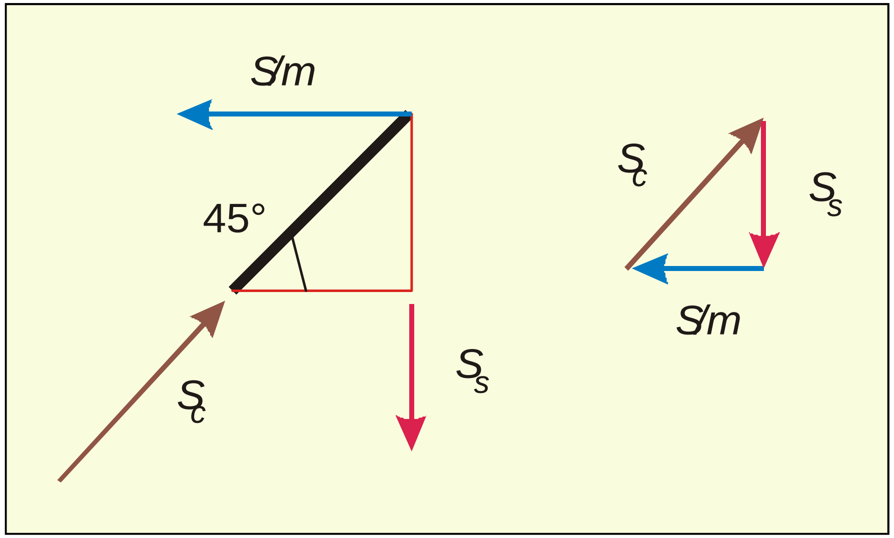
Forces acting on the concrete struts and the steel reinforcement

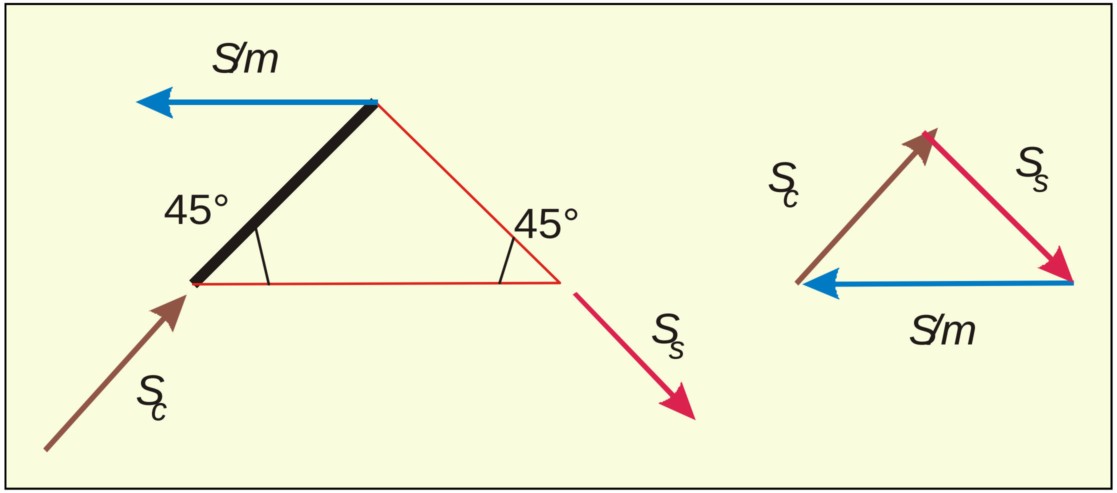
Example: vertical stirrups: $\alpha = 90°$

$$1 + \text{cotg}\,\alpha = 1$$

$$S_s = \frac{V\,\Delta x}{z}$$

$$S_c = \sqrt{2}\,\frac{V\,\Delta x}{z}$$





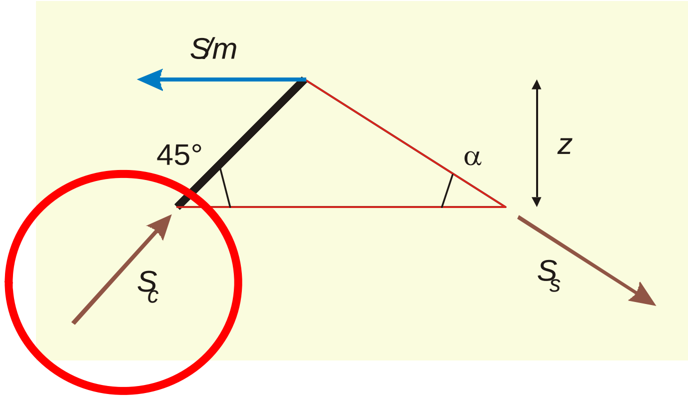
Example: $\alpha = 45°$

$$1 + \text{cotg}\,\alpha = 2$$

$$S_s = \frac{\sqrt{2}}{2}\,\frac{V\,\Delta x}{z}$$

$$S_c = \frac{\sqrt{2}}{2}\,\frac{V\,\Delta x}{z}$$

CHECK OF THE CONCRETE STRUT



Force on the concrete struct

$$S_c = \frac{\sqrt{2}}{1+\cot g\,\alpha}\frac{V\,\Delta x}{z}$$

Ultimate strength of the concrete struct

$$S_{cd} = f'_{cd}\, b_w \frac{\Delta x}{\sqrt{2}}$$

Maximum compression strength

$$f'_{cd} = \nu\, f_{cd}$$

EFFICIENCY COEFFICIENT FOR THE CONCRETE STRUCTS DUE TO FLEXURE

$$\nu = 0.5$$

Italian NTC 2008

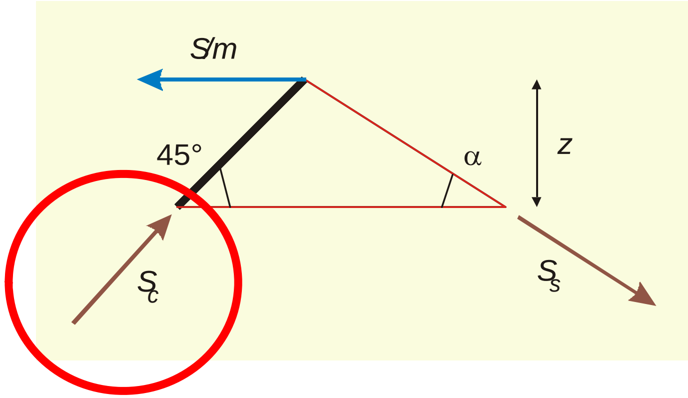
$$\nu = 0.6\left[1-\frac{f_{ck}}{250}\right] \quad with\ f_{ck}\ in\ MPa$$

EC2

WE SET (LIMIT CONDITION):

$$S_c = S_{cd}$$

CHECK OF THE CONCRETE STRUT



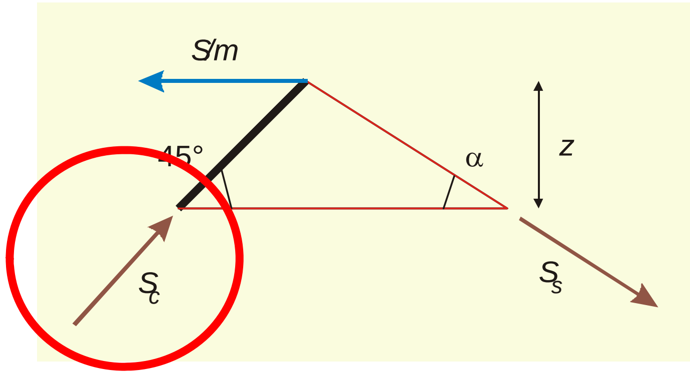
$$S_c = S_{cd} \Rightarrow V_{Rcd}$$

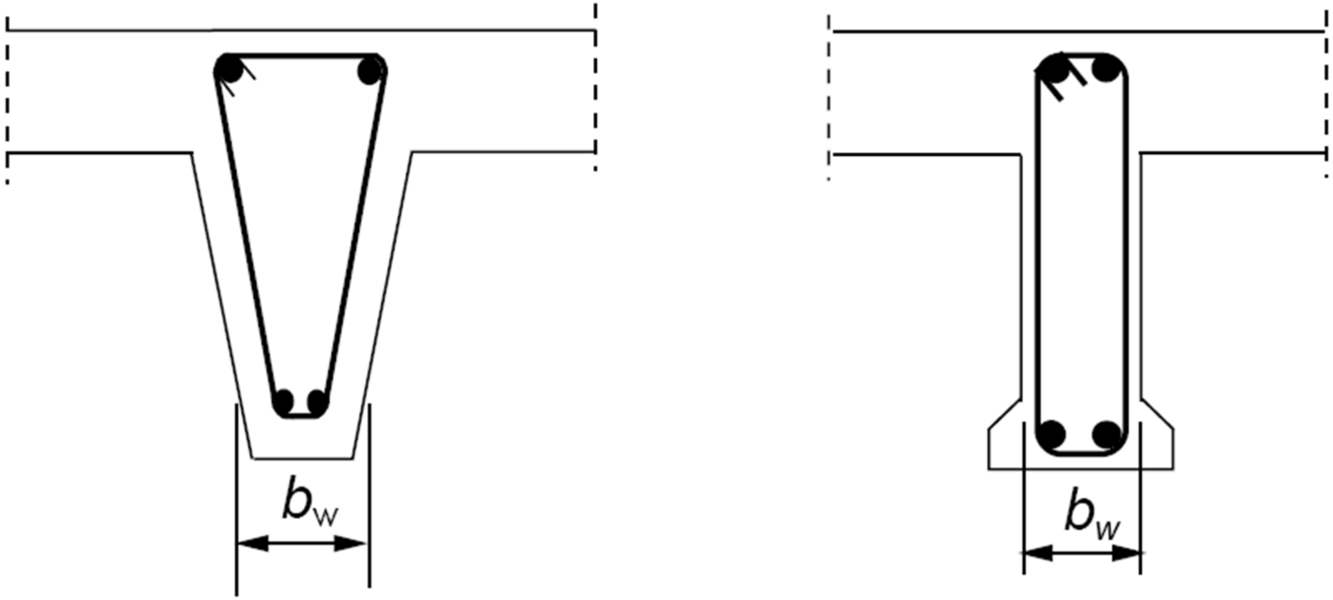
$$\frac{\sqrt{2}}{1+\text{cotg}\,\alpha}\,\frac{V\,\Delta x}{z} = \nu\, f_{cd}\, b_w\, \frac{\Delta x}{\sqrt{2}}$$

SHEAR RESISTANCE FOR A BEAM WITH FAILURE OF CONCRETE STRUTS (with 45° inclination)

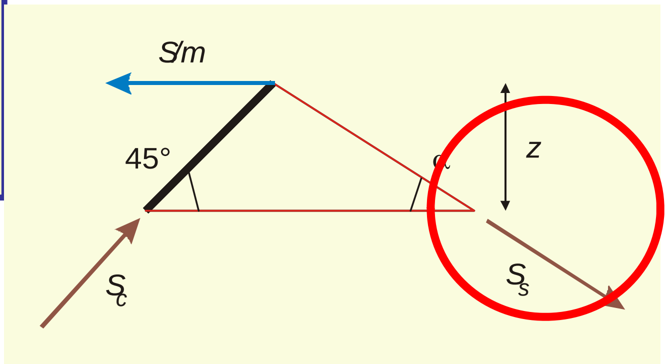
$$V_{Rcd} = \nu\, f_{cd}\, b_w\, z\, \frac{1+\text{cotg}\,\alpha}{2}$$

DEFINITION OF THE WEB THICKNESS





SHEAR RESISTANCE AGAINST STEEL FAILURE – HOW TO CALCULATE THE WEB STEEL REINFORCEMENT (θ=45°)



$$V_{Rd} = V_{cd} + V_{wd}$$

DESIGN SHEAR RESISTANCE

CONCRETE CONTRIBUTION

WEB REINFORCEMENT CONTRIBUTION

$$S_s = \frac{V\,\Delta x}{z\,(\sin\alpha + \cos\alpha)}$$

SETTING $A_{sw}\, f_{ywd} = S_s$

$$V_{cd} = 0.6 \cdot f_{ctd} \cdot b_w\, d \cdot \delta$$

- CONCRETE CONTRIBUTION IS SIMILAR TO THAT OF THE BEAMS WITHOUT SHEAR REINFORCEMENT

- Coefficient 0.6 has been obtained experimentally (aggregate interlock is more favourable)

$$V_{wd} = f_{ywd} \cdot A_{sw} \frac{z}{\Delta x} (\sin\alpha + \cos\alpha)$$

- ITALIAN CODE (D.M. 1996)
- Vcd not allowed by NTC2008 and EC2 (see next section)
- The ACI approach is similar

EXAMPLE: CALCULATE THE WEB REINFORCEMENT

Same example considered before

B = b_w = 30 cm
H = 50 cm d = 46 cm

V_{sd} = 150 KN
STIRRUPS (α=90°)
∅ 8 @ 200 mm

$$R_{ck} = 30\,N/mm^2 \;\Rightarrow\; f_{cd} = 13.2\,N/mm^2 \;\Rightarrow\; f_{ctd} = 1.1\,N/mm^2$$

SHEAR RESISTANCE AGAINST CONCRETE FAILURE

$$V_{Rcd} = 0.5 b_w\, d\; f_{cd}\,(1+\mathrm{cotg}\,\alpha)$$

$$V_{Rcd} = 0.5\times300\times460\times13\times(1+0) = 896\,kN > V_{sd}$$ ✓

$$\boldsymbol{V_{Rd} = V_{cd} + V_{wd}} \;\rightarrow\; \boldsymbol{V_{wd,min} = V_{Sd} - V_{cd}}$$

$$V_{cd} = 0.6\cdot f_{ctd}\cdot b_w\, d\cdot\delta = 0.6\times1.1\times300\times460\times1 = 82\,kN$$

$$V_{wd,\min} = V_{Sd} - V_{cd} = 150 - 82 = 68\,kN$$

$$V_{wd} = f_{ywd}\cdot A_{sw}\,\frac{0.9\,d}{\Delta x}(\sin\alpha+\cos\alpha) = 374\times101\times0.9\times\frac{460}{200}\times1 = 78\,kN$$ ✓

EXAMPLE: WEB REINFORCEMENT CALCULATED WITH THE 1° METHOD

Same section as before

B = b_w = 30 cm
H = 50 cm d = 46 cm

V_{sd} = 192 KN
VERTICAL STIRRUPS (α=90°)
Φ 8 mm

$$R_{ck} = 30\,N/mm^2 \Rightarrow f_{cd} = 13.2\,N/mm^2 \Rightarrow f_{ctd} = 1.1\,N/mm^2$$

CALCULATE THE MAXIMUM STIRRUP SPACING

$$V_{Rd2} = 0.3\,b_w\,d\,f_{cd}\,,\quad V_{wd} = V_{Sd} - V_{cd},\quad \frac{A_{sw}}{s} = \frac{V_{wd}}{0.9\,d\,f_{ywd}}$$

V_{Rd2} (kN)	V_{cd} (kN)	A_{sw}/s (cm^2/m)	STIRRUPS
896	82	9.0	Φ8/11

√

ACCORDING THE THE ALLOWABLE STRESS METHOD

$$\frac{A_{sw}}{s} = \frac{V_{Sd}/1.5}{z\,\overline{\sigma}_s} = 12.1\,cm^2/m$$

√

Stirrups ∅ 8 @ 80 mm