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The competitive dynamics of new DVD releases

Anirban MUKHERJEE
Singapore Management University, anirbanm@smu.edu.sg

Vrinda KADIYALI

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Anirban Mukherjee
Singapore Management University

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Anirban Mukherjee and Vrinda Kadiyali *

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Abstract

In the U.S. motion picture industry, DVDs are increasingly a major source of revenue for movie studios. Two important strategic decisions for DVDs are the release date and price. Industry evidence suggests studios consider various release options as evidenced by their pre-release announcements. When deciding DVD release dates and prices, studios must consider the following. First, weeks of high demand potential, increase sales. Second, intense competition in weeks of peak demand might reduce market share, lower margins and lead to higher costs of release. Third, deferring a DVD release (e.g. to a non-peak demand week) after the movie's theatrical run reduces the potential sales of the title. So how does the competitive equilibrium of release dates and prices evolve?

Such competitive dynamics are typically studied using Markov Perfect Nash Equilibrium methods. Our industry has features that make these methods unsuitable. First, studios make interim release announcements (both action and state vectors in our model), which are not observed by the researcher, and cannot be imputed from the data. Second, the seasonality of demand and competition and other time trends in the data lead to time-varying payoffs and hence time-varying strategy selection rules for firms. That is, states of the world follow a non-stationary (non-homogenous) Markov process. Third, given the variety of possible competitive dynamics at play (strategies might be strategic substitutes or complements, depending on the set of players being analyzed), multiple equilibria might result. There are no existing methods available to analyze dynamic competitive games with multiple equilibriums.

To overcome these modeling challenges, we build a model where agent (studio) beliefs reflect Perfect Bayesian distributions over unobserved state variables. This is analogous to methods in the oblivious equilibrium literature. Due to missing information (interim studio announcements), we cannot identify DVD profits from current period action choices of agent. Instead we compare agent choices over multiple periods using the competitive industry evolution predicted by the iterated Markov kernel. We estimate a model of DVD sales of a title in a period. To calculate future profits, we derive and forecast a sufficient statistic that captures industry competitive structure. Comparing timing and pricing decisions with computed best responses, we identify title and week specific release costs in our model. We show that estimates from our partial information model converge to those from a complete information MPNE model, and find reasonable predictive validity of agent behavior in our dataset.

While the application of our model is to the U.S. DVD market from 2000 to 2005, the model is applicable to competitive industries where researchers are unable to obtain data on intermediate states and decisions of firms and/or where industry and firm profits vary over time.

Key words: dynamic competitive models, censored data, non-stationarity, motion picture industry.

* Comments and suggestions are highly welcome. Anirban Mukherjee is a Ph.D. candidate in Marketing. Vrinda Kadiyali is a professor of Marketing and Economics at the Johnson School of Management, Cornell University. They can be reached at am253@cornell.edu and kadiyali@cornell.edu. Thanks to Ting Zhu for early discussions.

1. Introduction

In the U.S. motion picture industry, DVDs have increasingly become a major source of revenue for movie studios with total DVD sales growing from $1.9 billion in 2000 to $16.3 billion in 2005. The release date and price of a DVD (or title, as it is called in the industry) affect profits because weekly DVD sales vary dramatically with peak weekly sales being seven-fold non-peak sales, and release prices for DVDs varying between $5 and $35 across titles, in the time period of our study, 2000-2005. Therefore, release timing and prices of DVD are important strategic variables. The focus of this study is the competitive dynamics of the setting of these two variables in the U.S. DVD industry in this period. Substantively our objective is to study the optimality of observed decisions. We investigate changes in DVD revenue and release costs due to time varying title and industry forces, and build managerial decision tools to predict future demand and competition levels.

When deciding DVD release strategies, studios must consider the following. First, there is substantial difference in industry demand between peak demand (weeks with highest aggregate industry sales) and non-peak demand weeks. Cēterīs pāribus, any DVD will obtain higher revenues in a peak-demand week than in a non-peak week. Second, despite peak periods having the potential for higher revenues, any one DVD might not realize higher revenue. This is because the higher revenue potential in a peak week is likely to attract more titles that also anticipate higher revenues. Higher competition in peak weeks is likely to reduce sales of the title, and might result in lower prices, resulting in lower profits. As more titles enter peak demand weeks, release costs in these weeks might be higher. For example, promotional allocations and slotting fees to retailers and the costs of advertising media might rise in weeks of peak demand. In addition, titles have their "shelf-life" clocks running from the time that they exit the theatrical channel. The third factor to consider is that deferring a DVD release (e.g. to a non-peak demand week) long after the movie's theatrical run reduces the potential sales of the title in the channel. Studios must therefore make appropriate strategic choices of DVD release timing and pricing. Given these forces affecting release date and price decisions of any one title, how does the competitive equilibrium of release dates and prices evolve? There are three factors that complicate our modeling the equilibrium.

First, key information describing the evolution of the industry is missing in our data. We know that studios consider various release date options as evidenced by their intermediate release announcements.[1] That is, studios can and do announce they will release on a certain date and price but the

[1] Cached web-pages accessed using the internet archive confirm the existence of a steadily changing set of announcements. However, we have been unable to find a data source for past DVD announcements of studios.

final date and price often turns out to be different. Various competitive dynamics are possible in these release date announcements. For example, a title that was successful in the box-office might announce to pre-empt another successful movie of the same genre from releasing the same weekend. At the same time, seeing the release announcement by a high-revenue potential title might persuade a smaller title of the same genre to release in the same week, in the hope of free-riding off promotional monies spent by the larger title, especially if the peak demand favors the particular genre of movies. This might in turn deter another smaller movie of the same genre from announcing the same release date. Therefore, these announcements can serve to preempt, or coordinate release dates with rivals.

We do not observe the interim release announcements in our dataset, and only observe the final release date and time. We build a model that allows for the effect of the missing data on agent actions when finding the profit function, and show estimates from the model converge to the traditional full information MPNE estimator. An alternative choice is to build a model of the release timing and pricing decision taken at the last period prior to release, ignoring interim announcements. Such a dynamic game is fully observed and can be estimated using a Markov Perfect Nash Equilibrium (MPNE) frame work. However, in general, a model ignoring the strategic variables of the game (release timing and pricing announcements) may be misspecified, and lead to biased profit function estimates.

Hence, our paper contributes methodologically by describing an estimator for agent payoffs in a dynamic model with censored (or missing) information. Marketing researchers are often faced with datasets on forward looking firms in which a key strategic variable is unobserved or censored for a part or the entirety of the dataset. For example, firms might scout several locations before choosing a final location, or make a capacity decision and then alter it before reaching the final choice. A typical MPNE estimation needs complete data on current and future state vectors and actions taken by agents to identify the transition matrix and enable use of nested fixed point algorithms. Our model is general enough to be applied to other industries with censored data.

Second, the seasonality of demand and competition leads to time-varying strategy selection rules for firms. Each title is likely to have a release timing and pricing strategy that varies by week. For instance, a title may be more likely to release a DVD title if future periods have decreased demand, than if future periods have increased demand; the seasonality of demand leads to seasonality in the set of entrants. Also, in this market, there was an eight-fold increase in demand between 2000 and 2005. The seasonality of demand itself changed over time (DVDs were gifted not just for Christmas but also increasingly for Mother’s Day, or graduation, or children’s birthdays). Hence, studio profits varied over weeks and over years. In our model, this implies that the states of the world do not follow a first-order stationary (homogenous) Markov process, as is typically assumed in MPNE models.

Third, it is plausible that our setting has multiple equilibria. Release timing (and pricing) strategies in our model may be both the traditional strategic substitutes but also strategic complements. For example, a movie with small theatrical revenues might view a larger movie as a strategic complement if the larger movie can drive traffic to DVD stores. Two larger movies may see their timing strategies as strategic substitutes if the market stealing effect dominates. Allowing for strategic complementarity and substitutability brings about the possibility of multiple equilibria, a situation not tractable in extant dynamic model estimation methodologies.

We describe a novel estimator for dynamic games based on a partial information model (i.e. where the announcement vector is unobservable). Our model is analogous to the Oblivious Equilibrium (OE) model (Weintraub, Benkard and Roy, 2007). OE assumes that an "agent" (studio in our model) in a period is "oblivious" to the current states of others, and instead holds beliefs (distribution) over candidate states possible in the period. Similar to OE, in our partial information model, studios are oblivious to release announcements of other titles in past periods, and instead have beliefs that reflect the Perfect Bayesian distribution over possible release announcements. In appendix 1.2 we show that payoffs estimated in the partial information approach converge to those in a full information model. While our partial information model is unable to identify the drivers of an interim release announcement in the game, the model is able to identify the relevant components of studio profit functions and hence the trade-offs made by the studios between forces.

The estimation methodology proposed in OE is not appropriate for our setting. Extant MPNE models, including OE, identify model primitives by using the revealed preferences of a firm in a period. The methods identify drivers of profit functions by comparing observed choices with computed best responses. The missing announcements prevent us from using extant methods. We show that we can recast the equilibrium condition, as being a fixed point in best responses of agents across multiple periods; theorem (T1) shows that the optimal release announcement strategy of a firm leads to maximum future profits from release, and hence an optimal final release timing and pricing schedule.

We show that a unique industry evolution pathway is consistent with our partial information model. We estimate a market outcome function that describes sales in a period, as a function of the time since theatrical and DVD release, seasonal demand and competition. Employing a logit formulation of market shares, we generate a sufficient statistic to approximate the evolution of the industry equilibrium which accounts for differences amongst titles and studios. We forecast industry evolution using the statistic, and compute payoffs to studios from different release timing and pricing choices. Last we maximize the quasi-likelihood of observed strategy choices, consistent with the other steps of the estimation, to measure the seasonal differences in release costs which rationalize observed actions.

Our estimates of DVD market share are similar to prior findings.[2] We find that net release costs are smaller for movies that were more successful in the box office, indicating that blockbusters both make more money on DVD, and face lower release costs than other titles. Increasing the inter-release time (time between theatrical and DVD release) both reduces the sales potential of the title, and increases release costs. The model predicts competition levels in the industry with reasonable accuracy up to 30 weeks into the future, providing a decision support tool for marketing managers in studios. Within sample, our model shows a good fit with higher prediction accuracy for future release dates and prices of DVDs than alternative model specifications. Finally, our policy simulations investigate how optimal Theater-to-DVD windows depend on seasonal demand, competition and release cost variation. The simulations provide a decision support tool for marketing managers in the entertainment industry, predicting changes in DVD release strategies should there be changes in the industry landscape (market expansion, change in seasonality, change in release costs, etc.).

Our model generalizes the MPNE frame work used to study strategic decisions by multiple forward looking firms, and is applicable to competitive industries with time-varying payoffs or with frequent entry of new products. Examples include technology products, other entertainment products like music, and fashion products. And as described earlier, the framework can be used in industries where researchers are unable to obtain data on firm actions/state space, and need to model using a censored dataset.

2. Conceptual Overview of Equilibrium Forces

As mentioned in the introduction, three forces affect the equilibrium. First, weeks of peak demand increase sales. Second, intense competition in weeks of peak demand reduces market share and lowers margins. Relatedly, peak weeks might have higher costs of release. Third, deferring a DVD release (e.g. to a non-peak demand week in order to avoid competition) to a week long past the movie's theatrical run reduces the potential sales of the title in the channel. In this section we present a conceptual overview of our model, reviewing relevant literature and discussing each of the forces.

2.1: Temporal Variation in Demand

[2] Due to the high computational load of the estimation algorithm, results discussed are preliminary and have been estimated on data from 2000 to 2002. We will shortly estimate the model on the entire 2000-2005 data.

As mentioned in the introduction, demand for DVDs is time varying, affecting the optimal release timing and pricing of titles. We first discuss how existing literature has approached seasonality, and then its impact on our model.

In the economics literature, Einav (2007) and Chiou (2007) study the impact of seasonality on the demand for movies in primary and sequential channels respectively. Einav (2007) presents an empirical analysis of theatrical revenue in the U.S. movie industry, studying both seasonality and competition. Because his model is for theatrical releases, his research question does not include declining potential sales with deferral of entry dates. Chiou (2007) includes the effect of deferral of release dates, controlling for the endogeneity of release date selection, but does not model the process for the evolution of dates and prices. Both papers find strong evidence of seasonal changes in demand, and that firms' account for seasonality in their strategic choices.

The marketing literature has found that revenue in the movie theater, over the same weeks in different years, can be predicted. Krider and Weinberg (1998) discuss competition when faced with seasonal demand variations. Radas and Shugan (1998) outline an approach for including seasonal trends in estimating demand curves by taking a transformation of time. Luan and Sudhir (2007) model the effect on box office revenues of the theater-to-DVD window.

Seasonal demand (and seasonal costs, as we will discuss below) cause seasonal variation in payoffs. Modeling these seasonal payoffs in the framework of dynamic games poses a problem. While dynamic games have been studied for a few years now (e.g. see Rust 1987, Ericson and Pakes 1995), most researchers have typically focused on mature and stable industries. Firms in these industry use time invariant strategy selection criteria, allowing the researcher to assume that the states of the world follow a stationary, first order Markov process. The stationary Markov process leads to the solution concept of a stationary MPNE, using the implicit assumption that profits are only a function of strategic decisions of firms, with no exogenous change in industry profits due to macroeconomic forces or technological change.

The DVD industry in our period of interest showed a rapid increase in sales from $2 billion to $16 billion in 6 years. The growth in sales did not occur symmetrically over various weeks in a calendar year, nor across various calendar years. For instance, the largest growth in DVD sales occurred in the Christmas holidays. Thus, neither over weeks of the same year nor in the same season across multiple years, were revenues and costs, hence profits and firm strategy selection criteria, constant as assumed in the stationary MPNE model.

We utilize the Ericson and Pakes (1995) framework, and add to the MPNE literature by describing a non-stationary (time varying) MPNE model. Our model relaxes the assumption of time homogeneity for MPNE. In appendix 1.2, we discuss the relationship between the non-stationary and stationary MPNE in greater detail. In general, the theory does not guide us on how non-linearities of response functions may translate to decision making rules. We show that the non-stationary MPNE requires a "sufficient statistic" vector describing changes over time, which is essential for identification (see assumption A7 in appendix 1.2 for further details).

Assuming the sufficient statistic for change over time, a non-stationary MPNE can be estimated using extant methods for stationary MPNE. However, the non-homogeneity of model primitives substantially increases the data requirements for estimation and decreases the rate of asymptotic convergence in extant models. In particular, the effect of seasonal change enters non-linearly in the continuation values of the MPNE (see Pakes and McGuire, 2001 for a discussion on calculating continuation values). In our partial (limited) information model, we impute changes in payoffs, through a demand function. The non-linear effects of seasonality are partially accounted for in the demand function. Hence the effect of seasonality enters linearly when calculating continuation values, decreasing computational and data requirements for estimation. We discuss the estimation algorithm used in greater detail in section 4.4.

2.2: Temporal Variation in Competition

Seasonal variations in demand are likely to result in seasonally varying levels of competition (figure 1). Both revenues and costs seasonally due to changes in competition, leading to seasonally varying profitability. We now turn to more detailed discussion of this phenomenon.

Figure 1 – Total DVD Sales and New Releases

Consider first papers on the impact of competition in movies. Swami, Eliashberg, and Weinberg (1999) study multiplex screen allocation decisions and formulate a model to optimize exhibitor scheduling. Ainslie, Dreze and Zufryden (2007) build on the BOXMOD model and study the lifecycle of a movie at the box office, measuring competition within a channel. Einav (2003) models the release timing game in theatrical channels as a sequential game of imperfect information. Foutz-Zhang and Kadiyali (2007) model the release timing game in the theatrical channel and find that pre-announcements of release dates for movies serve a strategic function to deter entry into holiday weeks.

Studios can mitigate competition by making (interim and final) release announcements[3], which allow the studios to compete and cooperate on release schedules. There may be early/late mover advantages to announcing. For instance, as mentioned in the introduction, a title with large theatrical revenue might announce that it is releasing in a high-demand week. This might deter a title of the same genre (and that had a worse theatrical performance) from announcing a release in the same week. Or the similar title might announce the same release date, but set a lower price to undercut the first title. Smaller theatrical-revenue titles might prefer to announce a non-peak demand week where competition is less intense.

Allowing for strategic complementarity and substitutability brings about the possibility of multiple equilibriums Announcements in our model carry the ability to both coordinate and/or pre-empt release timings and pricing. Depending on which pair of competitive interactions are being studied and depending on the revenue potential of the week, strategies in our model may be both strategic substitutes and/or strategic complements. For instance, implicit price collusion due to grim Nash reversion, can be sustained in the model through the presence of a punishment state (low release price), with the Markov kernel capturing the probability of agents entering and exiting the punishment regime. Strategies in this model are strategic complements in the collusive regime, and strategic substitutes in the punishment regime.

While recent advances in game theoretical modeling have lead to estimation techniques for static models with multiple equilibriums, to our knowledge this is the first paper to allow multiple equilibriums to be played in the data. The non-stationary MPNE model allows "equilibrium switches" in a single path of play, as the Markov kernel is not restricted across multiple periods. That is, in each period, agents can choose to play strategies leading to a different equilibrium. Optimal strategies played in the data, are not restricted to being the same across different seasons and different years. For instance, equilibriums played over the summer may be different from equilibriums resulting in the holiday season. Our method describes a consistent estimator for payoffs in a dynamic multi-agent game, accounting for randomization between equilibriums. A caveat is that we do not distinguish between potential equilibriums and cannot identify the probability of choosing a given equilibrium in a period. The Perfect Bayesian evolution of the

[3]The release announcements are not made in consumer outlets and hence do not serve to inform or influence potential consumers. They are reported in industry websites like videoeta.com. Although complete cached data on the history of these announcements is not available, there is strong evidence of titles changing announced release dates and prices.

industry is the result of the equilibrium conditional on equilibrium choice, and the equilibrium choice probabilities.

An important point to consider is whether these announcements reflect actual intentions of titles' release timing and prices, or whether they are strategic lies or simply cheap talk. In formulating a competitive dynamics model for release timing and price, we do not require firms to take decisions influenced by competitor announcements. For instance, if announcements are cheap talk, then in our model the Markov density will reflect the lack of information in the announcements. On the other hand, it is possible that announcements are not cheap talk, and are instead costly commitments to particular strategies. In this case, our model provides consistent estimates of parameters of DVD profits, while allowing for the competitive and cooperative incentives of the release timing game. Thus, the model nests cases where announcements do not shape release and pricing schedules observed, while allowing for the strategic importance of these decisions.

In the literature, entry and post-entry competition have mostly been modeled separately with extant papers on entry and entry timing, typically using two period static entry models (Mazzeo, 2002) in industries where the researcher observes the release of new products (Einav, 2003), or the entry of a firm in multiple locations (Seim, 2006), but not sales post release. A notable exception is Ellickson and Mishra (2007), who use market outcome models to enrich the description of payoffs in a static game.

Our model draws on both product choice and dynamic entry models to recover studio- and season-specific release costs, providing a richer description of the industry. Prior models of release timing in movies have focused exclusively on the seasonality of consumer demand and competition in the week as a source for time-varying profits, and the explanation for observed seasonality in studio actions (Einav, 2003). There are several reasons for expecting unobserved releases costs to vary over the course of the year. 75% of studios' marketing budgets on average are dedicated to broadcast media (Galloway, 2004), where the cost of advertising varies over the course of the year. DVD release costs may vary as a function of the total sales in the week of release, as channel partners may be able to demand a better share of profits in weeks of high demand. The increased competition between DVDs may spill over into within-store promotions (e.g. end cap displays) and other retailer promotional resources. Lastly, the growth in total DVD sales may lead to a change in release costs and retailer margins. We measure how release costs vary over studios, and over time, and incorporate the effect of changing release costs when conducting counterfactuals and simulations, and compare model fits with and without release costs.

2.3: Perishability

We mentioned in the introduction that the lag between a title's theatrical run and its DVD release (which we term inter-release timing) is likely to have implications for the title's profitability. Movies lose appeal as they spend longer times between sequential distribution channels, a demand feature we call inter-release perishability. Additionally, movies lose appeal after release in a channel, a feature we call within-channel perishability. Below, we discuss how researchers have modeled perishability and explain our conceptualization in our competitive equilibrium framework.

Luan and Sudhir (2007) model the impact of cannibalization of box office revenues by sales and rentals of DVDs, accounting for forward looking behavior of the consumer at the theatre. While cannibalization of theatrical sales provides an incentive for a studio to increase inter-release times, the need to release a movie fresh in the minds of a consumer provides an incentive to decrease inter-release times. Thus they study the underlying tradeoffs between earlier and delayed releases in secondary channels on theatrical revenues.

Three papers study optimal firm actions to maximize revenue in the sequential channel, when considering the perishability of a title. These papers suggest that word of mouth, advertising wear-out effects, and network effects can explain perishability. Hennig-Thurau et al. (2007) use conjoint data to study the effect of different configurations of sequential distributional channels on studio profitability, optimizing release timings across channels. Lehmann and Weinberg (2000) develop a model of the optimal time to enter video rentals for a movie, accounting for the cannibalization of sales from theatrical release. Prasad, Bronnenberg, and Mahajan (2004) use an analytical model to study the effect of consumer expectations on the optimality of the timing decision. In their model, the duration between theatrical and DVD releases of earlier movies, shapes the beliefs of a forward looking-customer for a new movie. The studio's decision depends on current beliefs, making it profitable to deviate from the industry standard, and release early. In each model, firm actions in a title are studied in isolation of the presence of other titles, and of seasonality.

Movies in theaters only exhibit within-channel perishability, Ainslie et al (2007) separate revenue patterns in movie theaters into blockbuster patterns and sleeper patterns. Blockbusters peak early in the first weeks post release, and then decline in revenue. Sleepers peak later than blockbusters, and subsequently decline in revenue. Revenue patterns in DVDs are more complex as DVDs exhibit both forms of perishability. A consumer's dynamic decision making process and the network effects in evaluating entertainment products lead to non-linearaties in the relationship between inter-release perishability and within channel perishability. For instance, longer inter-release times may lead to a saturation of the word of mouth, attracting more consumers in earlier weeks, and then showing faster decay post release. Alternatively shorter inter-release times may attract more customers in earlier weeks due to advertising in the box office channel, and then show faster decay post release. In our paper, we do not separate effects leading to wear-out and instead adopt a flexible 3-parameter gamma specification in the demand formulation.

Managerially, our model can be used to understand the effect of shorter/longer average theater to DVD windows on DVD profits, by simulating the competitive equilibrium in the industry for different DVD release strategies. Extant research on the window between channels has focused on the change in revenue in a particular title, without accounting for seasonality and competition (Luan and Sudhir, 2007). Our paper focuses on the competitive and seasonal aspects of release timing, and their effect on DVD profits. The results from simulations in our model may differ from the inference in models that ignore seasonality and competition. For instance, Indiana Jones 4 was released in theaters on May 22, 2008. A model that ignores seasonality may find that shorter theater to DVD windows are optimal and hence suggest a date in August or September 2008 for Indiana Jones 4. However, seasonality in different channels suggests that optimal decisions differ based on time of theatrical release. Indiana Jones 4 may be better served by waiting for the Christmas Holidays, postponing the DVD release by a period longer than the industry average theater to DVD window.

3. Data and Model

3.1: Descriptive Statistics

Our data comprises release dates, quantities and prices of titles per week after release, as well as title-specific descriptors (e.g. box office revenue, etc.) for all DVDs released in the United States between 2000 and 2005. We describe below the sources of these data and issues with them. We also describe data we are unable to obtain, and the restriction this places on our model formulation and estimation.

Nielsen Videoscan collects DVD sales data from retailers at the point of sale. We use the weekly sales and price of all DVDs sold in the United States, aggregated nationally. Other researchers have used this dataset to study DVD sales (Elberse and Oberholzer-Gee, 2007). The dataset does not include Wal-Mart. In our period of interest, Wal-Mart was a major retailer of DVDs that carried a smaller inventory of possible titles than comparable national retailers. Hence, our sample may understate the importance of larger titles and overstate the importance of smaller titles. We supplement this dataset with estimates of print and advertising expenditure on movies at theatrical release from SNL Kagan. We lack data on print and advertising expenditure (P&A) by studios on DVD. Therefore, we use production cost, P&A in the theatrical channel and box office revenue that are likely to be closely correlated with DVD P&A.

We do not observe release costs in our dataset. Costs in the motion picture industry are comprised predominantly of the production costs of a movie and P&A. Production costs are borne upfront prior to release of a movie in the theatrical channel, and do not affect the release timing of the movie. P&A costs vary seasonally, over time and by firm, and thus affect the release timing of the movie. In our model, we assume that release costs may be incurred by a studio both as a fixed fee for in-store promotions, and through retailer margins. For instance, the fixed release costs of releasing titles on DVD include the cost of in-store promotions in the post-release weeks. In-store P&A might cost more in weeks of peak demand, when retailers face maximum demand for in store advertising and shelf space. We assume titles do not face distribution constraints; this assumption is clearly more appropriate for this market than for the theatrical release market. More importantly, for reasons of tractability, we assume that the retailer plays no strategic role. Another piece of missing data is that we do not observe the weekly release announcements of studios in our dataset. Our model approximates an MPNE with announcements, without data on the announcements of studios.

Similar to Luan and Sudhir (2007), we restrict our study to titles released in theatrical channels prior to release on DVD to reduce computational load. We drop older titles released prior on VHS and re-released on DVD, from our sample. Some titles with smaller revenues, either low production cost sequels or children's titles, may be released direct-to-DVD and are dropped from our sample as we expect the dropped titles have a limited competitive effect on the release timing and pricing game.[4] From 2000 to 2002, the subset of data used currently for estimation, we observe the release of 512 titles with 5339 observations of price and quantity post release.

In our model, prices (and release dates) are chosen by firms, given seasonal demand and release costs, and their rivals' announced and actual release dates and prices. There is considerable price variation in DVDs that cannot be predicted from title characteristics; a regression of price against title characteristics has an adjusted r-squared of 0.2237 (Table 1).

Table 1 – Price Regression

The strategic role of price is an important distinction between DVD releases and theatrical releases. In movie theaters, the price of a ticket is fixed regardless of the popularity of the title (Einav and Orbach, 2007). Hence, the two opposing forces when setting theatrical release dates are the lure of a peak demand week and the competition expected in that week. In our paper we study the joint evolution of two strategic variables (controls), release dates and prices, set simultaneously. The trade-offs between two

[4] We can incorporate the effect of dropped titles in the model. However, the computational cost of additional data is overwhelming and the lack of observables on smaller titles makes demand estimates noisy.

strategic choices leads to outcomes that may appear to be anomalies when considering either variable independently. For instance, consider two movies: a blockbuster and a small independent movie (indie). The blockbuster is released in a week of peak demand, and the indie on a week of lower demand. Intuitively, we might expect the blockbuster to be priced higher than the indie. However, we find a negative correlation (-0.15) between total DVD sales in a week and the average release price of a new movie in the week. Thus the indie may be released at a higher price than the blockbuster. The joint modeling of strategic decisions allows for an explanation. In weeks of lower demand, there is lower short run competition. Hence, titles released in these weeks have higher release prices while those released in higher demand weeks, have lower equilibrium release prices.

Two empirical facts simplify our analysis and estimation. First, the (retail) price of a DVD at release is maintained over the first few months after release, with no significant decrease after release. This is significantly different than previous findings for prices of video games and other entertainment media, where the prices of titles after release decrease over time (Nair, 2007)[5]. For instance, regressing log price against time after release and other explanatory covariates, finds that the price of a title decreases by 6% on average over the course of the first 12 weeks (Table 1).

The second industry feature is that prices for DVDs are well approximated by discrete levels, allowing us to treat price as a discrete variable rather than a continuous variable. In appendix 2, we discuss relaxing this assumption and treating price as a continuous variable. Before we begin a formal discussion of the model, we discuss the timeline of firm actions.

3.2: Timeline of firm actions

In our model, each title released in the theater is a potential entrant in the DVD channel. In each week, a potential entrant may choose to either announce a price and week of release of a title, defer the announcement or change its previous announcement, including withdrawing the announcement altogether. Titles update their decisions simultaneously every week. The state of the industry is described by announced release dates and prices and actual release dates and prices. Pre-order forms from the Video Software Dealers Association indicate that final release dates and prices for DVDs are circulated to video stores, 4 weeks prior to the release of the DVD. Hence, we assume that the final release date and price

[5] There are two explanations for the uniform price of a DVD for the first months, post release. Decreasing prices may lead to forward looking behavior from customers, who may wait for a price decrease and not purchase the DVD at the time of release. Store price guarantees, typical of retailers of home entertainment media, might make it unprofitable for a retailer to decrease prices after release.

decision is taken 4 weeks prior to the observed final releases in our implementation but suppress the period in our notation.

A limitation of our study is that we specify a model at the level of a title, and ignore portfolio optimization concerns of a studio: cannibalization, the effect on release costs from multiple releases and the effect of strategic decisions on other formats and channels (such as the theatrical channel and rentals). Studios managing multiple titles may choose to spread DVD release dates to mitigate the effect of cannibalization and substitution, and/or choose to cluster DVD release dates to lower release costs. We measure payoff changes with earlier/later release, independent of revenue on DVD, to capture the net effect of the release decision over all channels. However, we cannot disentangle between the sources of the payoff variation: substitutability with the theatrical channel, changes in future revenue streams, etc. While our framework and estimation methodology allows for these issues, the additional computational burden is overwhelming in our application. Last, we ignore the role of the retailer, and model the studio as the profit-maximizing agent responsible for release strategy choices.

In the model, the value of a choice of an announcement implicitly includes the strategic value (either cheap talk or serious signaling) of making announcements, and accounts for both cooperative and competitive incentives. Titles maximize profits by choosing optimal announcements of release date and prices, in the presence of seasonally varying payoffs, leading to time-varying best responses for any title (as described in the previous section). For instance, a title may be more likely to release a movie if future periods have decreased demand, than if future periods have increased demand; the seasonality of demand leads to seasonality in the set of potential entrants (see Figure 1). In our model, incumbents face no strategic decisions. That is, once a title enters the DVD channel, it becomes part of the absorbing state of the Markov process in the release timing game. In our dataset, a DVD on average collects 75% of revenue in the first 20 weeks post release with post-DVD release. In this period, the price remains remarkably steady, decreasing by less than 10% of the release price (see Table 1). Hence, it is sensible to model only release price setting.

Inter-release perishability implies that a studio only considers a finite number of periods after theatrical release for the DVD release of the movie.[6] Our model does not assume that all titles must be released on DVD and is general enough to identify titles released in theaters which cannot be profitably

[6] Inter-release perishability implies that despite seasonal demand variations for any cost vector it is never profitable to release an unreleased movie after a finite number of periods. See assumption (A7) and lemma (L2) for a more complete treatment.

released on DVD. Thus, while we assume that all titles play the release timing and pricing game, we allow titles to choose to not release on DVD.

There is an important different between timing models and geographic competition models (see Seim (2007) and Vitorino (2007)). Both "classes" of models are interested in separately identifying the effect of (inter temporal and/or geographic) differences in profitability, and the role of competitors. However in a geographic competition game, two agents in a time period, either do or do not have a competitive effect on each other. This effect does not depend on when they entered, and is solely a function of the identity of each agent. In a timing game, competition is asymmetric inter-temporally. Movies released early do not face competition for the first weeks post release, from movies to be released later. Movies released later, face competition from movies released early in the first weeks post release. The level of the competition faced diminishes with the gap between the release dates: older movies have a limited effect on newer movies.

To summarize: in our model, studios evaluate the value of release announcements in terms of resulting release schedules. The model nests a degenerate case of cheap talk where announcements communicate no information between studios. The costs and benefits of announcing are the changes in the industry landscape due to coordinating and competitive responses of other studios. We identify trade-offs between higher demand, competition, release costs and perishability by the choice of a studio to release the title on DVD, concluding the release timing and pricing game.

Our model specification is applicable when agents (firms) adjust dynamic decisions to changing industry landscapes. While unforeseen shocks, are accounted for in dynamic models that consider forward looking behavior (including our model), systematic industry changes of the nature described lead to a non-stationary MPNE. Other examples of such predictable shocks include market expansion, new product diffusion, changes in public policy, and release of complementor products. In the next section we describe the model specification.

3.3: Studio payoffs

We estimate the profit function per title per week. Profit is estimated in the expected two parts- revenue and cost. To identify the costs of releasing a DVD, we need to separate between the positive effect of pricing on profits from the negative effect of price on quantity sold. Reduced form profit functions based solely on the release timing schedule do not allow us to separate the profit into these components. We use a market outcome model to separate the effect of underlying seasonal shocks and competition on demand, from the seasonality of release costs.

In developing the sales (market outcome) model, we have two choices. We can model consumer demand from first principles of utility, accounting for dynamics in consumer demand (as did Luan and Sudhir, 2007). Alternatively, we can use a reduced-form capture of demand. We choose the latter for the following reasons. First, the focus of this paper is dynamics on the supply side. Researchers in this area typically use reduced-form models of revenue to simplify estimation (Bajari, Benkard and Levin, 2007). A model of firm market share in a period, allows for a parsimonious mechanism to account for the effect of competitors on per period profits. Second, our specification captures the relevant dynamics of inter-release and within-channel perishability, which are the two key dynamic elements that studios consider when setting release timing and pricing.[7] Finally, our data are aggregate, not individual-level (unlike Luan and Sudhir, 2007), making it less suited for structural demand estimation.

In appendix 1, we specify our non-stationary MPNE framework, and describe assumptions on model primitives and the resulting equilibrium. Our operationalization of the general frame work is presented below. Let p_{dwt}, x_{dwt} be the price and characteristics vector of DVD d, released in week w in time t.[8] To allow for competitive effects while ensuring computationally tractability,[9] we model the market share of DVD d in week w and year y, ms_{dwt} as:

$$ms_{dwt} = \frac{\exp\left(\delta_{dwt}\right)}{\exp\left(\delta_{dwt}\right) + \sum_{i \in \mathbb{C}_t \backslash d} \exp\left(\delta_{iwt}\right)} \tag{6}$$

$$\text{where} \qquad \delta_{dwt} = \log(p_{dwt})\alpha + \log(x_{dwt})\beta + \xi_{dwt} \tag{7}$$

The market share model allows us to present a richer description of the industry. In estimating dynamic models, the effect of other agents is approximated by a linear function. In practice, the assumption either leads to an exponential increase (due to an increased number of agents) in the number of estimated parameters. Or symmetry restrictions on the profit function: competition being determined by state and not by identity. The use of a market share (market outcome) model alleviates data and computational requirements. In our application, we use descriptors of titles (box office revenue, genre, rating, and studio/distributor identity) when calculating the asymmetric competitive effects of titles.

[7] We are unable to account for cross-channel substitutability; that is outside the scope of this paper.
[8] For convenience we index time as number of weeks since the first week of January 2000.
[9] Market expansion (e.g Einav (2007)) and/or a random coefficients version of the market share model improve predictive capabilities but increase computational burden. In general, a model that fits the light tail conditions described in the paper can be used instead, without affecting the proof of convergence.

Profits from releasing a DVD accrue post release. We write profits to a studio in period t from releasing a dvd d in week w as

$$\pi_{dwt} = p_{dwt} q\left(x_{dwt}, p_{dwt}\right) - f_{dwt}\left(q\left(x_{dwt}, p_{dwt}\right)\right) \tag{8}$$

where $f_{dwt}\left(q\left(x_{dwt}, p_{dwt}\right)\right)$ is the marginal cost for dvd d, released in week w in time t. In our empirical application, we interact the movie and studio characteristics in a linearly additive specification:

$$\pi_{dwt} = \left(\lambda p_d - \gamma_V x_{dwt}\right) ms\left(x_{dwt}, p_{dwt}\right) f_t\left(Q_w\right) \tag{9}$$

where $\left(\lambda p_d - \gamma_V x_{dwt}\right)$ is the net average studio margin for dvd d, and with quantity calculated using the market share model and total weekly sales, $f_t\left(Q_w\right)$.

Perishability of the movie impacts both title payoffs and the competitive impact of the title. We account for diminishing appeal when calculating both the payoffs for the studio and the competitive impact of the movie on other titles. Further, as our titles differ across periods, the model adjusts to the changing sets of titles released.

The majority of a DVD's revenue is garnered in the first months after release. In this period, competition and seasonality are major determinants of sales. Sales into the future are affected by competition from other movies released in the same week, but not from newer releases coming into the market in later periods. Hence, we model residual sales in remaining periods post the first 12 weeks, as a function of the seasonality of the week of release, the competitive set of the week of release and the observables of the movie. Thus, total payoffs to a studio from a title come from the first 12 weeks of profitability and a residual value of the movie:

$$\pi_{dw} = \sum_{j=w}^{w+M} \beta^{j-w} \pi_{dwj} + \beta^M \kappa_d\left(w\right) - \gamma_F x_{dw} f_w\left(Q_w\right) + \upsilon_{dw} \tag{10}$$

where $\kappa_d\left(w\right)$ is the residual sales and $\gamma_F x_{dw} f_w\left(Q_w\right)$ the release costs for dvd d, in week w. The general model allows υ_{dw} to be correlated across movies and weeks υ_{dw}.[10]

Our model payoffs are firm-specific. Ericson and Pakes (1995) specify a payoff function that depends solely on the number of studios in a particular state, and not the identity of the studios in that

[10] The general forms of most extant dynamic models do not admit contemporaneous correlation as contemporaneous correlation biases estimates of the transition function.

state. They model the state space using a set of counting measures to index the number of studios in a particular state. However, titles differ vastly in appeal. In our model the state representation is much richer and allows for observable differences between titles. The identity of a title impacts not only the payoffs of the DVDs but also addresses the impact of the title on other DVDs available concurrently. The affect of the title on the profitability of other titles depends on the composition of the choice set in that week.

4. Model Estimation

4.1: Challenges in solving for the MPNE

As mentioned in the introduction, we face three challenges in solving for the MPNE in our model. First, we lack intermediate release announcements of studios, and only observe the equilibrium final release schedule, leading to an under-identified transition matrix. Identification of the transition matrix and the use of Nested Fixed Point approaches require knowledge of the current and future state vectors, and the actions taken by agents. To ensure identification of the transition matrix, extant dynamic models have considered research questions where the state and action vectors can either be observed or imputed. While in our model we cannot observe all states and actions due to data constraints, in many applications such data remains unobserved due to other institutional details. For instance, privacy laws may prevent a store from identifying prior behavior of customers, censoring information on past decisions and their current state. Thus, we generalize dynamic models to settings where the researcher is faced with the burden of estimating on a censored dataset.

Second, while we prove the existence of a non-stationary MPNE in our model, we cannot solve for the general form of the MPNE as the state transition matrix is under-identified in a non-stationary MPNE. In the Ericson-Pakes (1995) frame work, identification depends on inter-temporal decisions of studios following a stationary Markov process (for a discussion on identification, see Berry and Tamer, 2006). In this frame work, the best response of a studio depends only on the industry state, and the state transition matrix is identified by the responses of studios to different industry states. In appendix 1.2, we describe the assumptions required to identify the model when best responses of firms change over time, and the related change in convergence properties.

Third, multiple equilibriums in a dynamic model require the specification of an equilibrium arbitration process over future equilibriums. The transition kernel and value function is unique to a particular equilibrium. Hence in a dynamic model with multiple equilibriums, agents in a period hold beliefs over which equilibriums will be played in future periods, to form expectations of future payoffs from a strategy. Without a methodology to arbitrate between equilibriums, the expectation over value functions is poorly defined. In general, randomizing between candidate equilibriums is in advisable as it rules out all signaling mechanisms between firms, including those based on observed variables. For instance, firms may know to play a particular equilibrium in periods of peak demand, and a different equilibrium in periods of low demand. It is also computationally intractable to enumerate possible equilibriums in the model and hence solve for the expected value of an action to an agent.

The presence of multiple equilibriums played in the data leads to inconsistent two step estimation (Aguirregabiria and Mira, 2007; Bajari, Benkard and Levin, 2007) due to the lack of a unique reduced form. A prior approach is to assume that while the researcher is unaware of the equilibrium selection process, and despite knowledge of the potential presence of multiple mixed and pure MPNE in the model, a unique equilibrium is played out in the data. This assumption is strong enough to both rule out equilibrium selection and inconsistent estimates of transition kernels (for instance, see assumption (5A) and (5B) in Aguirregabiria and Mira, 2007). In our data such an assumption is overly restrictive as it implies that all potential entrants in the six years of the dataset play the same equilibrium, across different holiday seasons, and in fast growing markets. The non-homogenous Markov kernel and choice function described and estimated in this paper are flexible enough to allow for the presence of multiple MPNEs in the data.

In the next section, we discuss a partial information estimation approach robust to all three issues described.

4.2: Solution Concepts

We draw from the solution concept of Oblivious Equilibrium (OE). In OE, agents are "oblivious" to the state distribution in a period, and optimize using Perfect Bayesian beliefs over candidate states. In the literature OE has been proposed in three separate contexts. First, in a model with a continuum of agents, MPNE and OE have been shown to be equivalent (Chakrabarty, 2003). Second, Krusell and Smith (1998) described a related model where agent behavior is derived from a response to the distribution of aggregate wealth rather than the precise allocation of wealth across agents, defending the solution concept as a behavioral model of agents in large markets. Third, Weintraub, Benkard, and Van Roy (2007) show that OE approximate MPNE models. They present error bounds for a model with a homogenous transition matrix and show that estimates of an OE converge to estimates from a MPNE in the context of large competitive industries where market shares decrease with the number of firms, particularly in a model using a logit market share function. In appendix 1.2 we show that given our choice of a logit market share function, estimates in our model converge to MPNE estimates despite the non-stationarity of the Markov kernel. We validate the model estimation by comparing agent actions forecasted with observed behavior, and compute an upper bound on the difference between OE and MPNE predictions.

Specifically, we replace the current state of the industry with a distribution over candidate state vectors that reflect the probability of observing the candidate vector in the time period. Rewrite the state vector as

$$\delta^t = \left\{ \delta_R^t, \delta_{UR}^t \right\} \tag{11}$$

where δ_R^t is the state vector for all movies released by time t, and δ_{UR}^t is the state vector for all titles unreleased at time t. Thus, we replace δ_{UR}^t, the unobserved state variables, with the distribution over candidate states consistent with Perfect Bayesian equilibriums. Equilibrium beliefs are neither imposed nor recovered from the data due to the under identified transition matrix.

The non-homogenous first order transition matrix requires us to write a period-specific choice function. We integrate over next period choice functions using the current periods' transition matrix $\widetilde{\psi}_t\left(\delta' \mid \delta\right)$ for each candidate vector and over all possible candidate vectors for the current period.

$$V_t^M(x_{it}, s_t, a_t, \upsilon_t; \theta) = \pi_{it}(x_{it}, \delta_t, a_t, \upsilon_t; \theta) + \beta E_{\delta_t} E_{\delta_{t+1}|\delta_t} E_{\upsilon} V_{t+1}^M(x_{i(t+1)}, a_t, \bullet; \theta) \tag{12}$$

(12) specifies a time-varying choice function in the model, due to the time-varying transition matrix. A non-stationary Markov strategy in the model for the studio is a function $\sigma_{it}^M : \Delta \times \nu \rightarrow A$. A non-stationary Markov strategy profile in model, σ_t^M is a set of non-stationary Markov strategies in the model, for each studio, period t. In the model, the necessary and sufficient equilibrium conditions are

$$V_t^M\left(\delta; \sigma_t^M\right) \geq V_t^M\left(\delta; \sigma_{it}'^M, \sigma_{-it}\right), \forall i, \delta, t, \sigma_{it}'^M \in I, \Delta, T, \Sigma^M \tag{13}$$

We draw on a strategy similar to extant static models of entry (eg. Bresnahan and Reiss, 1990) that use necessary conditions common to all equilibriums. The precise difference in strategic behavior of agents in our model and in a specific MPNE cannot be found without solving for the MPNE. Our model uses week-specific distributions over candidate state vectors without separating between MPNE equilibriums being played in the date or specifying the probability of playing a given equilibrium.

In appendix 1.2, we discuss the difference between our model and a stationary Oblivious Equilibrium (OE). Weintraub et al (2007) derive a theorem that shows that payoffs estimated using OE converge to payoffs estimated in a stationary MPNE. We discuss how to extend their results to our setting and show that payoffs found in the partial information estimator, converge to payoffs found in a full information non-stationary MPNE.

4.3: Model Estimation

As stated before, the OE estimation methodology proposed by Weintraub, Benkard and Roy (2007) is not appropriate for our setting due to the censored state space, non-homogenous Markov transition matrix and agent asymmetries in profit function. We first discuss an alternative characterization of the best response function and the resulting equilibrium, and then the use of these conditions in our estimation procedure.

In appendix 1.3, theorem (T1) implies agent playing the best response (making the optimal release announcement) in a period equivalently ensures the choice of an optimal absorbing state (final release date and price), accounting for the competitor's responses over the course of play. Hence (T1) shows that per period best responses can be translated into across period conditions on the choice of absorbing states. Note that the equilibrium description does not assume that other agents do not respond to the out-of-equilibrium actions of an agent. The assumption that actions, conditional on the path of play, are optimal across periods is a result of per period best response strategies of agents.

The re-characterization is intuitive: the underlying purpose of release announcements in the timing game is to ensure a path to the optimal period and price of release. Formally, in maximizing payoffs in a period, an agent engages in play to ensure that the course of play leads to the maximum payoffs for the agent, across multiple periods. The variation in profits from different release dates and prices stems from the seasonality of underlying demand and release costs, the endogenous evolution of competition as a response to the seasonal demand, and the effect of perishability on title profits.

Identification in the model is driven by comparing payoffs from releasing the movie, an absorbing state in the Markov process, with continuing in the game. This identification strategy has parallels to the literature in single agent dynamic programming problems where the agent has to decide the optimal stopping time (Rust, 1987), in an environment where payoffs from stopping vary over time. The optimal strategy in our model either prescribes releasing or deferring the release of the movie in a week, by maintaining a future release date, or choosing to postpone the release. An observed release indicates the studio found it optimal to maintain or choose the week and price as its announced release date and price respectively. Backtracking from the end of the finite planning horizon and recursively defining the value function, we implicitly construct the continuation value of deferring release. Hence, our estimator compares computed best response stopping points in the model and the decisions of studios to estimate trade-offs between equilibrium forces.

Our estimation and identification strategy is different from extant methods. In extant models, agent actions in a period, conditional on the Markov density, are best responses at equilibrium. In our model, the under-identified Markov density cannot identify the precise best response of the agent in the period. Instead, similar to prior Dynamic Stochastic Discrete Choice models (for a summary, see Ackerberg, Benkard, Berry, Pakes, 2007) we use a “two-step” method to calculate profits from conjectured releases. We forecast the industry evolution of the market, by modeling the evolution of the sufficient statistic. Titles that have been released have no further strategic decisions associated to them and are absorbing states in the Markov chain. The presence of an absorbing state and the forecasts of future industry environments, allow us calculate optimal release dates and prices. Next, we use estimates from the first period to find optimal release dates and prices of studios and maximize the quasi-likelihood.

However, in our model, unlike extant optimal stopping time models, the stopping decision of a firm depends on actions of other agents. In our approach, an agent makes an optimal decision while accounting for the behavior of other agents under the oblivious assumption. Thus, agent behavior in our model may differ from agent behavior in a MPNE. The use of a distribution instead of the information of actual agent state and hence future behavior implies an increased uncertainty which is manifest in the model as the difference in equilibrium outcomes in the MPNE and our model. The long term variance of the forecast is the sum of the long term or average variance of the true forecast generated in the MPNE, the residual variance of the forecasting equation and the average variance of the difference between the partial information approach and a complete information MPNE specification. Thus, the average difference between our model and the MPNE is bound by the mean sample variance of forecast errors.

While we can bound the degree of imprecision introduced by censored information (when compared to full information predictions), we cannot characterize the loss of efficiency in our model over a full information model. Our first stage estimates may be inefficient as they do not use the structural elements of the model. And we estimate the model without conditions on equilibrium actions, specifying the appropriate release announcement strategy, in each period. (T1) does not imply that our approach is econometrically efficient.

As the best response in our model is a unique strictly dominant strategy, the found equilibrium is a pure strategy Nash equilibrium in dominant strategies. Hence, econometrically an important difference between extant solutions of the MPNE model and our approach is that our estimators are econometrically complete in the presence of multiple equilibriums (Tamer, 2003). In extant dynamic MPNE models, multiple equilibriums make the MPNE model, even when the complete state and action space is observed, incomplete econometrically. In contrast, our estimators are consistent for all equilibriums and can be used without identifying equilibriums.

The estimation method described in the next section, uses (T1) and is general enough for any game with accrual of payoffs in periods after the choice of the absorbing state. This is a natural assumption in a game of release timing, where payoffs accrue post entry, but may not be a valid assumption in other games.

4.4: Estimation Algorithm

Our estimation algorithm has 4 steps:

Step 1: Market share Estimation

Estimate the market share model to scale. Let $\tilde{s}_{dwt}$ be the market share of dvd d in time t, in quantities. Define the geometric mean of in group market shares as $\ln(s_t^g) = \frac{1}{N_t}\sum_{i\in C_t}\ln(\tilde{s}_{iwt})$. Then from (7):

$$\ln(\tilde{s}_{dwt}) - \ln(s_t^g) = \ln(\delta_{dwt}) - \frac{1}{N_t}\sum_{i\in C_t}\ln(\delta_{iwt}) + \xi_{dwy} - \frac{1}{N_t}\sum_{i\in C_t}\xi_{iwt} \tag{14}$$

Coefficients of the market share and residual sales models in our application are estimated using Ordinary Least Squares[11]. Identification and regularity conditions of the market outcome function have been well established in the literature. Parameter estimates from the first step are consistent in release timing games, but may not be consistent in entry/exit games. For instance, Pakes, Ostrovsky and Berry (2004), first estimate outcome values from exit decisions and then impute them in the second step of the estimation routine. The selective exit of firms in an entry/exit model may lead to a selection bias in the market outcome equation (first step), if estimated separately.[12] In our release timing game, almost all titles released in theaters are released on DVD and hence are present in the first stage of estimation, ensuring consistency.

Step 2: Forecast the sufficient statistic

We forecast a sufficient statistic to describe the effect of other agents on an agent's profits from release. An infeasible estimator can use the iterated Markov kernel to compute the described profit values

[11] Clustering errors by week and using White's correction for heteroskedasticity does not improve fits and/or predictions.

[12] Entry selection bias can be corrected by using a control function of consistent estimates of the timing decisions of firms.

of an agent across periods, under the assumption of optimal play. The kernel is under-identified in our application. Hence, instead we form a reduced form forecast of the evolution of the industry and assess the optimality of actions of studios when faced with the evolution of the industry.

A consistent forecast of a sufficient statistic can be formed in our research problem by looking at the seasonality of demand in a finite set of future periods. Inter-release perishability allows us to assume the existence of a finite end of the game, beyond which release is no longer profitable (see Lemma 2 in appendix 1.1). In our empirical application, we set nine months as the end of the release game and assume that titles which were not released nine months after theatrical release, exit the release game. By implication, agent decisions involve seasonality over the planning horizon, and the current level of the sufficient statistic.

In our application, we use $s_{fs} \triangleq s_t = \sum_{i \in \mathbb{C}_t} \exp\left(\delta_{it}\right)$ and forecast $E\left[Es_t\left(\mathbb{C}_t\right) | \delta_{UR}^t\right]$. The summary statistic is a measure of the number and strength of competitors, but is independent of the identity of competitors. Figure 2 shows empirical validation of the chosen summary statistic. In periods of peak DVD sales, the summary statistic is higher for released movies, indicating that the best movies were released. In contrast in periods of non peak sales, the summary statistic is higher for non released movies, indicating that the best movies were retained by studios for later release, in coming weeks of higher demand. The evolution of the industry is regressed on the current industry state, seasonality and future entrant vector:

$$\log(s_{dt}) = \theta_{fs} \log\left(s_{d(t-1)}\right) + z_t \theta_z + \tau \tag{15}$$

Figure 2: Total DVD Sales and Industry Evolution

Market share parameter (step 1) estimates are root-n consistent. Hence, forecasts of the sufficient statistic in our model are root-n consistent, and converge in probability to the true sufficient statistic.

<u>Step 3: Compute sales from release dates and prices</u>

We construct the empirical analog of the conjectured profits when releasing in a period. Forecasts from step 2, allow us to define expected payoffs from future actions. Using the market share model and a forecast of sales, we can compute expected total quantities of products sold for every given choice of release date and price. Hence for each agent, in every time period that the agent was in the timing game, we compute sales for feasible release date, price combination for the agent within the planning horizon.

<u>Step 4: Maximize the Quasi-Likelihood</u>

Our first stage estimates of the summary statistic are consistent, but in a finite sample are (with probability 1) not true parameter values. Using the sales estimated from step 3, we specify a quasi-likelihood estimation approach using a parametric specification of the payoff shock. Regularity conditions and other assumptions for the estimation are discussed in appendix 1.4. We use Richardson simplification to find $A(\theta_{ss})$ and the Eicker-Huber-White estimator for $B(\theta_{ss})$. If imputations of the summary statistic s_{fs} are heteroskedastic or autocorrelated, then standard errors of the sandwich estimator can be corrected by appropriately weighting the estimation function (Zeilies, 2006).We choose to use a quasi-likelihood-based method to maximize efficiency and ensure consistency of the standard error estimates. In an under identified model, similar to Bajari, Benkard and Levin (2007) one can instead follow Chernuzhov, Hong and Tamer (2007). Their estimator uses set identification to find parameters that describe difference equilibriums supported by the data minimizing a criterion function that penalizes violations of the best response function. The likelihood based approach is more efficient in the point-identified model, and produces precise standard errors of estimated parameters. In general, finding equilibriums in dynamic game models is computationally demanding. Most MPNE solutions increase exponentially in computational complexity and cost, with the number of agents in the model. In contrast, we are able to estimate on sets of potential entrants (on the order of 40 potential entrants in a period) larger than prior work on release timings as our model increases linearly in computational load, with the number of agents. The derived quasi-likelihood in our application is globally concave with closed form derivatives, further reducing computational load.

4.5: Identification

While the general framework of the model admits under-identified models, our model specification is point-identified. The identification of release costs comes from the effect on release decisions, of inter-release time, seasonal industry demand, and revenue from the title post release. Comparing across titles that could achieve the same revenue, we can identify differences specific to the attributes of the title. Specifically, studio margins are a function of the movie’s characteristics (including inter-release time and time since release). The quantity sold in our model from a release date and price combination, is the product of the market share and the seasonal size of the market $ms(x_{smwt}, p_{smwt})\, f_t(Q_w)$. A change in margins, affects profits depending on the revenue from the title. Hence the coefficients of studio margin are identified through the change in revenue with different choices of release strategies. γ_F is the vector of coefficients of the release cost function, identified through the change in industry sales of DVDs in the weeks post release. Studio differences in margins and release costs are identified in the model through differences in release behavior for similar titles in similar weeks, across studios.

The underlying variation identifying release costs are the different market shares and seasonal market sizes across different weeks, for different release dates and prices. The variation is induced by the underlying seasonality of demand, endogenous evolution of competition (choices of other studios) and perishability. There are two limitations to this approach: we cannot identify any release costs that are constant across the different weeks as they do not figure into the release timing optimization, and we are only identified to scale.[13]

$E_{\nu}(\nu_{it} x_{it}) = 0$ Our specification is similar to specifications used in complete information models. The described model frame work is general enough to allow variables unobserved by the econometrician but observed by agents (common un-observables). Common unobservables lead to decisions of agents being contemporaneous correlated. Similar to complete information models (Gallant, Hong and Khwaja, 2008), our estimators remain consistent under the assumption that common unobservables are orthogonal to observables but potentially correlated with private information shocks. However unlike extant complete information models we maintain restrictions on unobservables and shocks being independent over time. A complete information model assumes away elements of pre-emption and learning. The presence of private information potentially correlated with the common un-observables, implies that allowing serial correlation may lead to "learning" in the game described. The resulting model is beyond the scope of our research. Further, we restrict out attention to identification and estimation of our model in this paper for the parametric form. In future research, we plan to show that our model is semi-parametrically identified, and can be estimated using an extension of the approach of Hong and Shum (2007).

4.6: Results

We find that market share is well predicted by the print and ad spending of a movie, the user ratings and critics' ratings of a movie, all of which are positively correlated with larger box office revenues. Larger number of weeks since theatrical release significantly decreases the attractiveness of the

[13] One can identify scale through the assumption on a discount factor that is less than 1. Identification to scale is adequate to build the counterfactuals and simulations that form the major substantive contribution of the paper.

movie.[14] As in Lehmann and Weinberg (2000), we find that larger box office revenue and greater screen-weeks exposure predicts higher consumer utility. Similar to Luan and Sudhir (2007), we find that longer inter-release times between channels decreases consumer utility. (See Table 2)

Table 2: Coefficients of DVD Market Share Model

We regress the residual sales (sum of revenue in week 13 to week 24 after DVD release) on industry and movie characteristics (see Table 3). While the DVD market share in the first weeks after release is negatively affected by better movies, residual DVD market share is positively affected by the release of better movies at the same time as the DVD. The measured complementarity of titles may arise due to better releases increasings store visits to DVD retailers, and hence the number of older titles sold.

Table 3: Coefficients of Residual Sales

Estimates of DVD release costs (structural supply parameters estimated in the second stage) are in Table 4.[15] Our results indicate that release costs are seasonal, and are higher in weeks of peak demand. We find that movies that performed better at the box office, controlling for the increased sales on DVD, face lower net DVD release costs. We also find that movies that spend a longer time between the channels, have a longer inter release period, sell fewer copies due to the decreased market potential and incur higher release costs. Hence, a model ignoring release costs would overcluster optimal release predictions in weeks of peak demand, as it would ignore changes in release costs. Finally, both marginal release costs and fixed release costs differ across studios, genres and ratings. Our simulation results corroborate our prior explanation of the pricing anomaly. Regressing optimal release prices suggested by the model on seasonal demand variation shows that the model predicts lower prices for movies in weeks of peak demand. For every standard deviation increase in industry demand for the week of release, the simulation suggests a decrease of 16 cents in DVD release price.

Table 4: Coefficients of Release Costs

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[14] We try different time specifications and do not see a difference in fit across different specifications, including higher order polynomials of time spent in channel.

[15] Due to the computational burden results have been estimated on data for 200-2002 for now. We will shortly estimate the model on the entire 2000-2005 data.

4.7: Model Fit and Validation

The estimators of industry evolution are fairly accurate, indicating a reasonable model fit. In both Figure 3 and Figure 4, residual variance and forecast error is limited. The R squared of the 10 week future forecast equation is 0.9267 and R squared of the 30 week future forecast equation is 0.9114.

Figure 3: Forecasting 10 Weeks Into the Future

Figure 4: Forecasting 30 Weeks Into the Future

We compare our in-sample model fits, with two alternative specifications:

i. M0: Reduced form model of prices and Theater-to-DVD window as a function of title characteristics
ii. M1: Dynamic model with release costs set to zero.

The mean absolute error in predicting release dates for M0 is 4.73 weeks. Our model has a MAE of 4.05 weeks while M1 has a MAE of 4.27 weeks over the entire sample. Figure 5 is a histogram of absolute errors in release date prediction for our model, across the entire sample. For short term predictions (observed release in the coming 10 weeks), the model has a MAE of 2.86 weeks, while M1 has a MAE of 2.91 weeks. As expected the model performs better on nearer term than longer term predictions. We predict the release price with an accuracy of 52%[16].

Figure 5: Histogram of Release Date Forecast Errors

5. Conclusion

DVD sales are a major source of studio profitability. As weekly sales vary dramatically over the year and the majority of sales for a title are made in the first weeks post release, the timing and pricing of a DVD release is a major strategic decision for studios. In this paper, we model the dynamic game of pre-emption and coordination played by studios when deciding the joint decisions of release date and price on DVD. In particular, we study the impact of seasonally varying demand, competition and release costs on the evolution of competition and release timing and pricing decisions, in the industry. An issue in

[16] We use 7 levels of price: \$0-\$5, \$5-\$10, …\$30 and above.

estimation is that when setting release dates, studios use weekly announcements to mitigate competition in setting release and pricing schedules. Not accounting for these announcements might lead to biased estimates of the release timing and pricing game. We show how to account for these unobserved announcements to obtain robust estimates of this competitive timing and pricing games among DVD titles.

Substantively, we are able to measure unobserved release costs, allowing for both firm and title heterogeneity in the release cost function. We contribute to the literature methodologically by developing estimation routines for models in which extant estimation methodology cannot solve for MPNE. We do not observe release announcements, and hence estimate the model on a censored state space. The policy functions of studios for determining release strategies change due to the variation in payoffs and the growth of the industry, leading to a non-stationary Markov process. Agent asymmetries prevent the use of counting measures for states to account for the impact of competition on studio profitability. Our estimators are econometrically complete, computationally tractable, and show reasonable predictive accuracy despite these constraints.

A limitation of our paper is that we assume that firms seek to maximize profits on DVD, ignoring positive network externalities on future channels and optimization over multiple titles. While theoretically the model scales to both multiple channels and portfolio optimization, a lack of data on other channels and the accompanying dramatic increase in computational cost, limit the empirical application.

A technical limitation of the model is that in using a simultaneous game of incomplete information, we are subject to the regret critique. Studios in our model make decisions on the basis of their own private information and beliefs on the actions of other agents, and cannot revisit their decisions. In contrast, in a sequential game, actions of rivals reveal private information, and hence may potentially lead to different best responses. These questions deserve further exploration in future research.

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Appendix 1: Non-Stationary MPNE Framework

In this section, we begin by describing the model primitives and the resulting equilibrium.We show the conditions for identifying the $f(\bullet)\{f_i(\bullet)\}_{i=1}^{n}$ non-stationary MPNE and discuss the flexibility of the model when describing multiples equilibriums being played in the data.

1.1: Model Description

Following prior empirical work (Doraszelski and Pakes, 2006), we restrict our attention to symmetric and anonymous equilibriums. A set of functions, is symmetric if $f_i(x_i, x_{-i}, \delta_i, \delta_{-i}) = f_j(x_i, x_{-i}\delta_i, \delta_{-i}), \forall i, j$. Hence we abstract from the identity of the agent in the payoff function, and write. $f(\bullet)$ The function, is anonymous if $f(x_i, x_{-i}, \delta_i, \delta_{-i}) = f\left(x_i, x_{perm(-i)}, \delta_i, \delta_{perm(-i)}\right)$, where $perm(-i)$ is any permutation of the indices of other studios.

Note that symmetry and anonymity restrictions do not assume that studios are identical, but instead that studio and product differences are observed. Reduced-form game payoffs for all agents at equilibrium are a function of its characteristics, state vector and competitive set.

We assume model primitives are common knowledge to potential entrants and incumbents:

$$\left\{\pi_{it}(x_{it}, p_{it}, \delta_t, a_t, :), E_t, \Psi_t(\delta' \mid \delta, a_t, :), v_{it}, \beta\right\}_{(i, x_i, p_i, t, \delta, a) \in I \times \aleph \times \mathrm{P} \times T \times \Delta \times A} \tag{1}$$

We describe these objects and then present the assumptions required for the model.

The state space is $I \times \aleph \times \mathrm{P} \times T \times \Delta$, where $I \in \mathbb{Z}_+$ is the set of all titles ever released in movie theaters, $\aleph$ is the Cartesian product of observed movie characteristics, $\mathrm{P} \subset \mathbb{Z}_+^{\infty}$ is the set of prices, $T \in \mathbb{Z}$ is the set of time periods over which the game is played, $\Delta \subset T^{\infty} \times \mathrm{P}$ is the state vector describing announced release dates and prices and $A \subset T^{\infty} \times \mathrm{P}$ is the announcement vector[17]. $\{\Delta^C, \Delta^A\}$ is a partition of Δ such that Δ^C is the set of continuation states and Δ^C is the set of absorbing states. $\pi_{it}(x_{it}, p_{it}, \delta_t, a_t, :)$ is the profitability of title i, in time period t, when it has characteristics $x_{it} \in \aleph$, price $p_{it} \in \mathrm{P}$, δ_t is the state vector at time t and a_t is chosen by agents. E_t is set of all potential entrants, and $\Psi_t(\delta' \mid \delta, a, :)$ is the transition function that determines state transitions. As mentioned previously, we assume this function is time-varying. We adopt the convention of using primes to denote subsequent period variables, e.g. δ' to denote δ_{t+1}, δ'' to denote δ_{t+2}. In an abuse of notation, we also use agent subscripts to denote the partition of the state and action vector describing an agent, e.g. δ_i to denote the state of agent i.

Incomplete information models simplify the analysis of the equilibrium (Seim, 2007), and are more likely to accurately represent the industry (e.g. given the non-standard contracts for sharing revenue and for deciding the promotional expenditure). Hence, similar to entry models, we assume that prior to making a release decision studios receive a private payoff shock ν_{it}, drawn independently over time, from a distribution $G_{\nu}(\bullet \mid \delta_t, x_{it})$ with support on $\mathbb{R}^{|E_t|}$, and ν_t the vector of private shocks for all titles in period t. Private information shocks (in a timing game) describe payoffs variations from different announcements and capture seasonal and studio specific differences in release costs across periods, including changes in the costs of advertising, promotional expenditure and the manufacturing cost for the DVD. We formalize this intuition in (T1) in section 4.3. Our formulation of private information allows shocks to be correlated across the industry, and heteroskedastic over time. Finally we specify a discount factor β.

We impose further restrictions on the model primitives.

(A1) The state space is finite ($I < \infty; x < \infty, \forall x \in \aleph; p < \overline{P} < \infty, \forall p \in \mathrm{P}; T < \infty; \delta < \infty, \forall \delta \in \Delta$).

(A2) Profits are bounded ($\underline{\pi} < \sum \pi_{it}(\bullet) < \overline{\pi}$). Profits accrue post release (entry) of the title.

(A3) Studios discount future payoffs $\beta \in (0,1)$.

[17] The lack of an announcement, or its withdrawal, is encoded as the origin.

(A4) Private information appears additively in profit function $\pi_{it}(x_{it}, p_{it}, \delta_t, a_t, \nu_t, :) = \pi_{it}(x_{it}, p_{it}, \delta_t, a_t, :) + \nu_{it}$. F_ν is distributed absolutely continuous to the Lebesgue measure.

(A5) State transition, follows a non-stationary first order Markov process with non-homogenous transition function $\Psi_t(\delta' \mid \delta, a, :)$. In general, future states are a time varying stochastic function of past states and actions $\mu_{st}\left(\delta_{t+1}\right) = f_t\left(\delta_t, a_t\right), s.t. \forall \left\{\delta' \in \Delta, \delta \in \Delta^C\right\} \exists a$ with $\mu_{st}\left(\delta'\right) = f\left(\delta, a\right) > 0$.

Assumption (A1) stipulates the finiteness of the state space. First, as we restrict our attention to titles released in movie theaters and ignore direct-to-DVD sales, our set of agents is always finite. Second, in practice, characteristics of a movie have finite range. Third, perishability implies that in any time period, a firm only considers a finite number of future periods for release. Correspondingly, we restrict the decision vector and action vector of potential entrants to be finite. Assumptions (A2), (A3) and (A4) are features of commonly-used profit functions in empirical studies. The second part of (A2) is appropriate in a game of release timing where the agent can only make profits post release of the title.

Assumption (A4) describes the stochastic monotonicity and continuity requirements on payoffs, fundamental to the existence of MPNE. Relaxing distributional assumptions on private information increases the number of mixed MPNE supported in the model. Assumption (A5) requires the state of the world, dates and prices chosen by studios in every week, to evolve in a first order Markov process. The evolution of the next period's states, conditional on the actions and states of agents in the current period, is stochastic. In keeping with extant papers, we require states in future periods to be accessible from continuation states. In our application, the state space is a history of past actions, and evolves deterministically conditional on past states and actions. (A4) and (A5) are implicitly equivalent to requiring additive separability of controls and errors, and conditional independence. Finally our state space is finite (unlike Ericsson and Pakes (1995), who study a problem with an infinite state space), and hence, we do not require agent payoffs to be bounded at the extremums of the state space.

In appendix 2, we describe an extension to the model that allows us to model price as a continuous variable. Thus, we extend the model to both continuous and discrete controls, from discrete controls. As the extension is computationally more expensive and requires additional assumptions (A11) and (A12), we maintain the assumption of discrete controls for the remainder of this paper. Assumptions (A1 – A5) lead to the following lemma:

Lemma (L1): Generically, the best response function is a unique mapping from $I \times \Delta \times T \rightarrow A$.

Proof: (L1) follows naturally under assumptions (A1), (A2), (A3), (A4), (A5) and the solution concept of a Perfect Bayesian Equilibrium (PBE). In a PBE, a studio considers expected profits from each strategic choice. The expectation on the profit function includes probabilities on the decisions of incumbents and potential entrants in future periods, described by the transition kernel specified in (A5). When taking expectations, indifference between two actions, making the best response function a correspondence occurs on a set of measure zero due to the continuity restrictions imposed in (A4).

Corollary (CL1):

If a firm is in a continuation state, $\delta_i \in \Delta_i^C$, then all future states of the firm have positive probability.

$$\int_{a_t} \mu_s\left(\delta_{t+1} \mid a_t, \delta_t\right) \mu_a\left(a_t\right) > 0$$

Proof: (A4) implies that from each state, all actions are played with positive probability. (A5) implies that generically, all future states are achievable from continuation states.

For a particular equilibrium, (L1) implies a unique mapping from a combination of the state vector and observables to a future state. In our model, the state vector is the history of past agent actions, making the evolution of the state space conditional on agent actions, purely deterministic. In general, if agent actions lead to stochastic state changes, then agent beliefs in the PBE are rational, as (L1) implies (A5).

Our non-stationary formulation extends extant frameworks of dynamic games, to allow for multiplicity of equilibriums played in the data. If a particular equilibrium is played in the data, then the specification of the stationary MPNE (with a time homogenous transition kernel) is complete without specifying additional beliefs on equilibrium arbitration. If multiple MPNEs are possible in a model, then different equilibriums may lead to different transition kernels. The presence of multiple equilibriums in a dataset means that the transition kernel is time-varying: equilibrium choice in a particular period determines state transitions in the period. The non-homogenous transition kernel defined in (A5) is agnostic on the source of the non-stationarity and hence is broad enough to allow for the effect of multiple equilibriums on state transitions. A stationary Markov model assumes that best responses depend only on the current state of the agents. Doraszelski and Satterthwaite (2007), prove the existence of a MPNE, and under certain conditions, the existence of a Pure Strategies MPNE. Assuming a stationary MPNE, they write the choice value function as

$$V(x_{it}, a_t, \delta_{it}, \upsilon_t; \theta) = \pi(x_{it}, a_t, \upsilon_t; \theta) + \beta E_{\delta_{t+1} \mid \{\delta_t, a_t\}} E_{\upsilon} V(a_t, \bullet; \theta) \tag{2}$$

where expectations on future value functions are taken over possible next period states, using the transition matrix.

A stationary Markov strategy for a studio is a function $\sigma_i : \Delta \times \nu \rightarrow A$. A stationary Markov strategy profile σ is a set of stationary Markov strategies for each studio in a period. The necessary and sufficient equilibrium conditions in a stationary MPNE are

$$V\left(\delta;\sigma\right) \geq V\left(\delta;\hat{\sigma}_i,\sigma_{-i}\right), \forall i,\delta,\hat{\sigma}_i \in I,\Delta,\Sigma \tag{3}$$

A non-homogenous first order transition matrix requires us to rewrite the choice value function. We write (3) as a period-specific choice value function, taking expectations over the next period choice value functions using the current periods' transition matrix

$$V_t(x_{it},a_t,\delta_{it},\upsilon_t;\theta) = \pi_t(x_{it},a_t,t,\upsilon_t;\theta) + \beta E_{\delta_{t+1}|\{\delta_t,a_t\}} E_\upsilon V_{t+1}(a_t,\bullet;\theta) \tag{4}$$

(3) specifies a time invariant choice value function, while (4) specifies a time-varying choice value function[18]. Time-varying choice value functions, particularly when lacking estimates of the transition matrix, cannot be analyzed using extant methods without arbitrary restrictions on V_t. As current decisions are affected by future seasonality, for instance to control for the effect of seasonality one would need to make V_t a function of future periods.

A non-stationary Markov strategy for a studio is a function $\sigma_{it} : \Delta \times \nu \rightarrow A$. A non-stationary Markov strategy profile σ_t is a set of non-stationary Markov strategies for each studio in period t. In a non-stationary MPNE, the necessary and sufficient equilibrium conditions are

$$V_t\left(\delta;\sigma_t\right) \geq V_t\left(\delta;\hat{\sigma}_{it},\sigma_{-it}\right), \forall i,\delta,t,\hat{\sigma}_{it} \in I,\Delta,T,\Sigma \tag{5}$$

1.2: Proof of Existence of a Non-Stationary MPNE

[18] Blackwell's theorem does not apply to the general class of non-stationary Markov Perfect Nash Equilibriums. For instance, consider an infinite period game in which the market grows faster than the discount rate. In our problem, we assume an upper bound on the profit function and requirement the existence of an absorbing state. These two conditions, used with backwards induction arguments, guarantee the existence of such a function.

To prove the existence of a non-stationary MPNE, we assume:

(A6) Let t_e be the time an agent enters the game. Agent payoffs $\pi(t, t_e, \bullet)$ are a strictly decreasing function of t $\pi(t, t_e, \bullet)$ with $\exists \infty > \bar{t} > t_e, s.t. \forall t > \bar{t}, \pi(t, \bullet) < 0$.

Assumption (A6) [19] is stronger than the standard waiting costs assumptions in extant frameworks due to underlying market growth; firms in markets growing fast enough may prefer to defer release indefinitely despite convex waiting costs. Estimates of inter release perishability from the market share model in our application, support (A6) with longer inter release periods leading to a sharp decline in movie appeal.

Lemma (L2): Agents have a finite planning horizon $F_M < \infty$, where M is the number of periods in which profits accrue, post release.

Proof: Define $f_t^M = \min_n, s.t. \beta^n M \bar{\pi} < \sum_{i=0}^{M-1} \pi_{t+i}$ where $\bar{\pi} = \sup_{m > t+n} \pi_{im}$. The left hand side of the inequality decreases geometrically indicating $f_t^M < \infty$. By construction, $t' > t + f_t^M$, $\beta^{t'-t} \sum_{i=0}^{M-1} \pi_{t'+i} < M\bar{\pi} < \sum_{i=0}^{M-1} \pi_{t+i}$ as (A6) implies a decreasing profit function. If firms never receive more profits in periods beyond f_t^M than in the current period, then they have a finite planning horizon (f_t^M) for the current period. $F_M = \min_{t \in T} f_t^M < \infty$ is the finite planning horizon for the agent across all periods.

Following Dutta and Sundaram (1994), define an extended state space, $\lambda = \{t, \delta\}$ and an extended transition matrix, $\iota(\lambda' | \lambda)$. Firms within a period only consider a finite number of future periods for release. A game in each period can be replaced with an equivalent finite game if we drop unconsidered states (strictly dominated states) from the extended state space creating a finite extended state space. Assumptions A1 – A5, translate in the extended state space to a stationary Markov chain. The conditions in the extended model match those in Doraszelski and Satterthwaite (2007), proving the existence of a

[19] Without (A6), the existence of the equilibrium can be proven by defining the extended model using countable states. Such a model with countable states is harder to identify than the extended model presented.

Pure Strategy Nash Equilibrium. As the extended model notation is a one to one re-parameterization of the original model with each week and state in the original model corresponding to a state in the extended model, an equivalent equilibrium exists in the original model.

While the extended state-space notation is useful for proving the existence of non-stationary MPNE, it is not helpful to the econometrician because by construction, the cardinality of non-zero elements of the state transition matrix is always larger than the number of observations in the data. Thus, the transition matrix remains under-identified in the re-parameterization as in the original model.

We cannot compare the unidentified general non-stationary MPNE with our partial information model. Hence we assume:

(A7) The integrated value function $E_{\delta'|\delta}V_t(x_{it},\delta_t,\upsilon_t;\theta)=E_{\delta'|\delta}V(x_t,\delta_t,\upsilon_t,\wp_V(t);\theta')$, and the Markov kernel, $\Psi_t(\delta'|\delta,a_t,:)=\Psi\left(\delta'|\wp_\psi(t),\delta,a_t,:\right)$, are both stochastic functions of a $\wp_\psi(t)$ and $\wp_V(t)$, finite cardinality function vectors of the effect of time t. Define the augmented characteristics vector as $x_t^A=\left\{x_t,\wp_\psi(t),\wp_V(t)\right\}$.

Assumption (A7) limits the effect of time varying payoffs to a sufficient statistic of any finite cardinality, and integrates these vectors into the vector of descriptive characteristics. While the MPNE specified in section 3 is not identified due to the non stationarity of the transition function, the augmented model (defined above) is identified. Identification requirements from the data scale with the length of the sufficient statistic used. A longer sufficient statistic remains identified in population, but increases the data requirements of the empirical implementation.

(A7) is reasonable in our model, where each agent has a finite planning horizon with a finite number of payoff periods post release. The integrated value function is well approximated in our model using the projected seasonal demand in the near future. The Markov kernel, using Perfect Bayesian restrictions, in turn is well approximated by exogenous shocks in a finite number of future periods, fulfilling the second half of the requirement. In general, non-stationary MPNE may not be well approximated by these assumptions.

Additionally, we make assumptions presented in section 5.1 (assumptions on sequences of profit functions) in Weintraub et al (2007). The demand models discussed prior (logit share, nested logit share and random coefficient logit share) are consistent with these assumptions. We get

$$\lim_{n\to\infty} E_{\delta^{(n)}}\left[V_{(n)}^A\left(x_t^A,:\right)-\tilde{V}_{(n)}^A\left(x_t^A,:\right)\right]=0 \tag{16}$$

where $\tilde{V}^{A}_{(n)}\left(x^{A}_{t},:\right)$ is the model approximation to the identified augmented model using state distribution assumptions outlined earlier, for a market with size n. Assuming light tail conditions, as specified in section 5.4 of Weintraub et al (2007), leads to the main observation in their paper that the discounted sum of differences between actual and oblivious single period profits, converges to zero.

Non-stationary MPNE that follow (A10) may not possess a recurrent class of states, a property required for many well known (and popular) extant MPNE algorithms. For instance, approximating MPNE using the method of Pakes and McGuire (2001), requires the presence of a recurrence class for the adaptive updates to converge to true values. As the long term distribution of a non-homogenous Markov process is not well defined, adaptive learning processes may never converge to the true value, when modeling a non-stationary MPNE. However, capturing the effect of time in the finite vector may allow the use of popular two-step estimation processes, such as Bajari, Benkard and Levin (2007).

Last, (A10) generalizes the convergence result for Oblivious Equilibriums to games with multiple equilibriums played in the data, and an equilibrium arbitration process on future play. A multiplicity of equilibriums played in the data can be described by a finite family of homogenous Markov kernels, driving a non-homogenous Markov process. Our non-stationary MPNE representation is a sufficient descriptor of such a game. The game admits (A10) as an appropriate assumption, if the equilibrium arbitration mechanism (refer to Aguirregabiria and Mira, 2007 for a discussion) is driven by observed strategic and descriptor variables.

$$\pi_{it}\left(x_{it},p_{it},\delta_{t},a_{t};\theta\,|\,q\left(x_{it},p_{it}\right)\right)=\Psi_{it}\left(x_{it},p_{it},\delta_{t},a_{t}\,|\,q\left(x_{it},p_{it}\right)\right)\bullet\theta\ \Psi_{it}\left(\bullet\right)$$

1.3: Alternative Representation of Equilibrium

For ease of exposition, we focus on the partial information model. Formally, first define the iterated (non-homogenous) unconditional Markov density as $\psi_{t}^{1}\left(\delta'\,|\,\delta\right)=\int_{a}\psi_{t}\left(\delta'\,|\,\delta,a\right)\mu_{a}\left(a\right)$ and $\psi_{t}^{n}\left(\delta''\,|\,\delta\right)=\sum_{\delta'\in\Delta}\psi_{t}^{n-1}\left(\delta''\,|\,\delta'\right)\int_{a}\left(\psi_{t+n-1}\left(\delta'\,|\,\delta,a\right)\right)\mu_{a}\left(a\right)$ for $n\geq 2$. Within the equilibrium for a specific state vector δ, the iterated kernel $\psi_{t}^{n}\left(\delta'\,|\,\delta\right)$ reflects beliefs on the state distribution n periods into the future. The iterated Markov density in the partial information model $\widetilde{\psi}_{t}^{n}\left(\delta'\,|\,\delta\right)$ reflects agent beliefs of the evolution of the industry, n periods into the future. Within the partial information model, integrating the iterated kernel over the beliefs on the current period's state distribution $\tilde{\varsigma}\left(\delta_{UR}^{t}\right)$, gives us future state distributions. This distribution $\int_{\left\{\delta_{UR}^t\right\}} \tilde{\psi}_t^n\left(\delta' \mid \delta_{UR}^t, \delta_R^t\right)\tilde{\varsigma}\left(\delta_{UR}^t\right)$ is a function of past states and past actions due to the Perfect Bayesian restrictions on $\tilde{\varsigma}\left(\delta_{UR}^t\right)$.

Lemma (L3): $$\underset{a_{it}\in A_i}{Max}\left\{EV_t\left(a_{it},\bullet\right)\right\} = Max\left\{\begin{array}{l}\underset{a\in A_{it}^a}{Max}\left\{E\pi_t\left(a,\bullet\right)\right\}, \underset{a\in A_{i(t+1)}^a}{Max}\left\{\beta E\pi_{t+1}\left(a,\bullet\right)\right\}, \\ \ldots, \underset{\delta\in A_{i(t+F_M)}^a}{Max}\left\{\beta^{F_M} E\pi_{t+F_M}\left(a,\bullet\right)\right\}\end{array}\right\}$$

where A_{it}^a are actions leading to absorbing states for agent i, and $A_{it}^c = A - A_{it}^a$, and expectations are taken over the equilibrium state distribution in the period, $\tilde{\varsigma}\left(\delta_t\right)$ and private information shock in each period.

Proof: By definition the two partitions of the action space $\left\{A_t^a, A_t^c\right\}$ are mutually exclusive and collectively exhaustive. Hence we get

$$\underset{a_{it}\in A_i}{Max}\left\{EV_t\left(a_{it},\bullet\right)\right\} = Max\left\{\underset{a_{it}\in A_t^a}{Max}\left\{EV_t\left(a_{it},\bullet\right)\right\}, \underset{a_{it}\in A_t^c}{Max}\left\{EV_t\left(a_{it},\bullet\right)\right\}\right\} \tag{6}$$

By continuing in the game, a firm obtains no profits in the current period, but gains the ability to either release in the next period, or choose a different announcement strategy. In a current period, the expected choice value of releasing is the present value of profits in the next periods. Hence, we get

$$\begin{aligned}&\underset{a_{it}\in A_{it}^c}{Max}\left\{EV_t\left(a_{it},\bullet\right)\right\} = \underset{a_{i(t+1)}\in A_{i(t+1)}}{Max}\left\{\beta EV_{t+1}\left(a_{it},\bullet\right)\right\} = \\ &Max\left\{\underset{a_{i(t+1)}\in A_{i(t+1)}^a}{Max}\left\{\beta EV_{t+1}\left(a_{it},\bullet\right)\right\}, \underset{a_{i(t+1)}\in A_{i(t+1)}^c}{Max}\left\{\beta EV_{t+1}\left(a_{it},\bullet\right)\right\}\right\}\end{aligned} \tag{7}$$

Implicitly the iterated Markov density allows us to take expectations over candidate states in a period. At the end of the planning horizon, a firm chooses between release in that period and continuing in that game. Substituting iteratively (7) into (6) recursively until the planning horizon we get the expression of the lemma with an additional term in the choice set of the continuation value past the horizon. For each absorbing state, the continuation value is the profits from releasing the movie. Substitute the profit function for the continuation value. From (L2) we know that profits past the horizon are lower than current period release profits, and hence can never be the argmax. Hence, (6) leads to (L3).

(L3) formalizes the earlier discussion. In expectation, the search for the maximizing release announcement strategy in a period is equivalent to a search for optimal stopping points across periods.

Agents seeking to maximize expected revenues post release, maximize expected payoffs from strategic choices in a period.

In our game, this equivalence as stated is not useful as a private information shock is defined for each potential action. The search for the maximizing strategy in a period is the search for the choice value of each action including the private information shock.

Theorem (T1):

Let $\widehat{\upsilon}$ be the vector of payoff shocks, of cardinality $\sum_{i=0}^{F_M-1}\left|A_{it}^{a}\right|$, to the expected payoffs from choosing an absorbing state. These shocks are distributed with a density μ_{π}, absolutely continuous with respect to the Lebesgue measure. Then:

(i) Private information shocks υ can be considered to be a linear combination of $\widehat{\upsilon}$.

(ii) A search for the optimal absorbing state across periods is equivalent to a search for the optimal strategy in a period.

Proof:

The choice of an action influences the transition of states in the period. The state density n periods in the future conditional on the action vector is $\tilde{\varsigma}\left(\delta_{t+n} \mid a\right)=\sum_{\delta'}\tilde{\psi}_{t+1}^{n-1}\left(\delta'' \mid \delta'\right)\int_{\delta_t}\tilde{\psi}_t\left(\delta' \mid \delta_t, a\right)\tilde{\varsigma}\left(\delta_t\right)$. Note that payoffs in the model accrue to the firm post release. Hence, the choice value of an action $V_t(a)=\sum_{j=0}^{F_M-1}\left(\int_{\delta\in\Delta_{i(t+j)}^{abs}}\pi_t\left(\delta_{t+j},\bullet\right)\tilde{\varsigma}\left(\delta_{t+j} \mid a\right)\right)$ is the expectation over resulting future absorbing states, distributed $\tilde{\varsigma}\left(\delta_{t+n} \mid a\right)$. The iterated Markov kernel has full rank and can be inverted. We can write $V_t(a)=EV_t(a)+\varphi_{\upsilon t}\widehat{\nu}$, where the mean choice value of the absorbing state is perturbed by payoff shocks $\widehat{\upsilon}$. $\varphi_{\upsilon t}$ is the matrix defined through the inverse iterated Markov kernel integrated over the state density in future periods. The mapping is unique allowing us to compare with (A4) and get $\nu_t=\varphi_{\upsilon t}\widehat{\nu}$.

In (L3), consider the case where $\widehat{\upsilon}$ is added to the expected choice value of each absorbing state. Using (T1i), the resulting change in expected choice value can be captured by the private shocks to the choice value of actions in the current period. Hence, we get

$$\underset{a_{it}\in A_i}{Max}\left\{EV_t\left(a_{it},\bullet\right)+\upsilon\left(a_{it}\right)\right\}=Max\left\{\begin{array}{l}\underset{a\in A_{it}^a}{Max}\left\{E\pi_t\left(a,\bullet\right)+\hat{\nu}\left(a\right)\right\},\underset{a\in A_{i(t+1)}^a}{Max}\left\{\beta E\pi_{t+1}\left(a,\bullet\right)+\hat{\nu}\left(a\right)\right\},\\ \ldots,\underset{a\in A_{i(t+F_M)}^a}{Max}\left\{\beta^{F_M}E\pi_{t+F_M}\left(a,\bullet\right)++\hat{\nu}\left(a\right)\right\}\end{array}\right\} \quad (8)$$

Define $\sigma_{it}^n:\Delta^n\times\nu^n\rightarrow A^n$ as the n-period non-stationary MPNE strategy of an agent, for any finite n. A n-period non-stationary Markov strategy profile σ_t^n is a set of non-stationary n-period Markov strategies in period t. The non stationary MPNE equilibrium conditions (5) can be rewritten as

$$V_t\left(\delta;\sigma_t^n\right)\geq V_t\left(\delta;\hat{\sigma}_{it}^n,\sigma_{-it}^n\right),\forall i,\delta,t,\hat{\sigma}_{it}^n\in I,\Delta,T,\Sigma^n \quad (9)$$

(9) states that the equilibrium condition of agents choosing a maximizing strategy in each period subject to the strategies of others, is equivalent to agents seeking the maximizing n period strategy subject to equilibrium n-period strategies of others. From (L2), we know a finite planning horizon exists for the firm. Set F_M as n and substitute in(9). The equilibrium F_M -period strategies can be considered equivalently to be those leading to the payoff maximizing absorbing state in the planning horizon. Hence, the per period equilibrium conditions of the MPNE are equivalent to agents choosing the F_M period strategy which leads to the payoff maximizing absorbing state in the finite planning horizon. Intuitively, agents searching for a strategy to maximize the choice value are searching for the F_M period strategy that leads to the maximizing absorbing state.

1.4: Estimation

Similar to Bajari, Benkard and Levin (2007), to reduce computational load we assume:

(A8) The profit function, conditional on the demand function defined, is linear in unknown parameters. $\pi_{it}\left(x_{it},p_{it},\delta_t,a_t;\theta\,|\,q\left(x_{it},p_{it}\right)\right)=\Psi_{it}\left(x_{it},p_{it},\delta_t,a_t\,|\,q\left(x_{it},p_{it}\right)\right)\bullet\theta$, where $\Psi_{it}\left(\bullet\right)$ is a finite dimension vector of "basis functions" (including polynomial and interaction terms).

(A8) allows us to approximate the payoff function locally. A violation of (A7) does not prevent estimation or affect identification of the model and the described estimation methodology is robust to the use of a non-linear specification. As observed in Bajari, Benkard and Levin (2007), having a payoff function that is linear in unknown parameters implies that the constructed value functions are also linear in unknown parameters, simplifying estimation.

To calculate the payoffs post release, we utilize a sufficient statistic for the impact of the evolution of the industry and require:

(A9) Competition in the industry is described by an industry summary statistic set (s_{fs}). There exists a consistent estimator $\hat{\mu}\left(s_{fs}\right):\hat{\mu}\left(s_{fs}\right)\xrightarrow{p}\mu\left(s_{fs}\right)$, where $\mu\left(s_{fs}\right)$ is the true distribution of the summary statistic in a future period.

Assumption (A9) is similar to assumptions made in Bajari, Benkard and Levin (2007).[20] Instead of assuming a finite parameter vector in the first stage of estimation, we assume the forecasted variables resulting from the first-stage converge to the rational beliefs of agents.[21] (A8) can be used in other two-step dynamic models to allow the first stage regression to be non-parametric, as the summary statistic set is not limited in scope (and may be uncountable). For instance, the summary statistic vector may include the transition kernel and policy functions defined in Bajari, Benkard and Levin (2007). (A8) also naturally follows when a consistent parametric first stage estimator is used to estimate both, the transition kernel and the policy functions as in most dynamic game estimation methodologies.

We assume the following regularity conditions:

(R1) $\theta_{ss}\in\Theta_{SS}$ is a compact subset of $\Re^{|\theta_{ss}|}$ and true value $\theta_{ss}^{0}\in\operatorname{int}\Theta_{SS}$.

(R2) The quasi-likelihood function $\Upsilon\left(\bullet\,|\,\mu\left(s_{fs}\right)\right)$ is uniquely maximized at θ_{ss}^{0}, and $\Upsilon\left(\bullet\,|\,\mu\left(s_{fs}\right)\right)$ is twice continuously differentiable in $\theta_{ss}\in\Theta_{SS}$ with probability 1.

(R1) and (R2) are common regularity conditions for quasi-likelihood estimation, met by the iid Gumbel specification of absorbing state payoff shocks in our application. Under (A8), the argmax of the quasi-likelihood function is a consistent estimator of the second stage structural parameters. From (A4), the second stage quasi likelihood function $\Upsilon\left(\bullet\,|\,\mu\left(s_{fs}\right)\right)$ is continuous, leading to $\Upsilon\left(\bullet\,|\,\hat{\mu}\left(s_{fs}\right)\right)\xrightarrow{p}\Upsilon\left(\bullet\,|\,\mu\left(s_{fs}\right)\right)$. Maximizing the quasi-likelihood yields second stage structural

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[20] In non-stationary MPNE, (A8) implicitly requires (A7). The sufficient statistic in (A8) can only be predicted if the non-homogenous Markov process can be modeled as a homogenous Markov kernel and exogenous time varying variables.

[21] Additional rate of convergence and local smoothness assumptions are required if using a criterion function for estimation as in Bajari, Benkard and Levin (2007).

parameters (θ_{ss}) whose variance is the sandwich estimator $A(\theta_{ss})^{-1} B(\theta_{ss}) A(\theta_{ss})^{-1}$, where $A(\theta_{ss})$ is the Hessian of the log quasi-likelihood and $B(\theta_{ss})$ is the variance of the quasi-score.

For completeness we discuss the endogeneity of observables and a method for correcting for the endogeneity bias. In our research question, endogeneity is not a concern as observables in our payoff function are lagged variables, not affected by current private information shocks. Formally, we assume:

(A10) $E_{\nu}(\nu_{it} x_{it}) = 0$

A violation of (A10) would bias the coefficients estimated due to endogeneity. We can correct for endogeneity bias in our model using two-step estimation. In the first stage, bias correction follows methods for instruments in discrete choice models. In the second stage, violation of (A9) implies that both individual agent errors and forecast errors are correlated with agent observables. Grouping the error terms leaves a single error term correlated with observables. To estimate the model, define a set of moment conditions by matching the best response with calculated best responses and interact the conditions with instrumental variables. While this method corrects for any potential endogeneity bias it is econometrically less efficient in the second stage than maximizing the best response quasi-likelihood.

Appendix 2: Modeling Title Price Choices as a Continuous Variable

In our model we make the simplifying assumption that prices are discrete. In this section, we discuss how to model a continuous strategic variable in conjunction with release date timing. Profitability in a week is given by (10) and (11). To model prices as being continuous we make two assumptions:

(A11) $q(x_{smwt}, p_{smw})$ and $f_{smwt}(q(x_{smwt}, p_{smw}))$ are continuous and differentiable function of p_{smw}.

(A12) $q(x_{smwt}, p_{smw})$ and $f_{smwt}(q(x_{smwt}, p_{smw}))$ are quasi-concave in prices.

(A11) and (A12) are common assumptions on the profit function of a firm, which allow the researcher to formulate first order optimality conditions, and guarantee the existence of a unique maximum. Hence, they are more restrictive than (A1-A5), which lead to (L1). The assumed parametric demand function in our model satisfies both (A11) and (A12).

Under (A11) and (A12), the objective function of the firm is continuous in mixed strategies. As the strategy space is bounded, mixed strategies are a compact subset of a Euclidean space. Hence by Glicksberg's Theorem (Glicksberg, 1952), a Nash Equilibrium exists in mixed strategies.

Estimators for the model can be formulated by either maximizing the probability of joint decisions or by minimizing a criterion function. The probability of seeing a joint decision can be found by using a closed form analytical solution, or through numerical simulation. To specify the criterion function, first take derivatives of (9):

$$E\frac{\partial \pi_{smwt}}{\partial p_{smw}} = \lambda q(x_{smwt}, p_{smw}) + [\lambda p_{smw} - \gamma x_{smwt}]\frac{\partial q(x_{smwt}, p_{smw})}{\partial p_{smw}} \tag{17}$$

To obtain first order conditions, take derivatives of (10) and substitute results of (17):

$$E\frac{\partial}{\partial p_{smw}}\pi_{smw} = \sum_{j=w}^{w+M}\beta^{j-w}E\frac{\partial}{\partial p_{smw}}\pi_{smwj} + \beta^{M}E\frac{\partial}{\partial p_{smw}}\kappa_{sm}(w) \tag{18}$$

A studio maximizes profits by setting release timing dates and prices. If we assume the existence of an interior solution, conditional on a set of dates, the first order conditions allow us to specify a Lagrangian using (18) to find the maximizing price. The difference in criterion function estimators when using only discrete controls versus joint controls, is that while the best response function for the discrete levels problem requires the researcher to enumerate value functions of all strategies, for a continuous control, the researcher first enumerates possible choices of the timing variable, and then conditional on each choice of the timing variable, solve the first order conditions of the problem to find the maximizing price. Neither approach implies a decision hierarchy; the continuous controls algorithm is identically to jointly considering all joint strategic decisions. To allow for boundary solutions, one has to consider solving the relevant Kuhn-Tucker conditions instead of specifying a Lagrangian.

Unfortunately, it is computationally expensive to simulate the probability of a release strategy and to solve using (18) in our model. The derivative of the demand function is non linear in prices and hence requires numerical minimization. As the maximizing price is found in the inner loop for every conjecture of parameters and for every choice of release dates, the computation costs outweigh benefits of implementation in our model.

Figure 1: Total sales and new releases

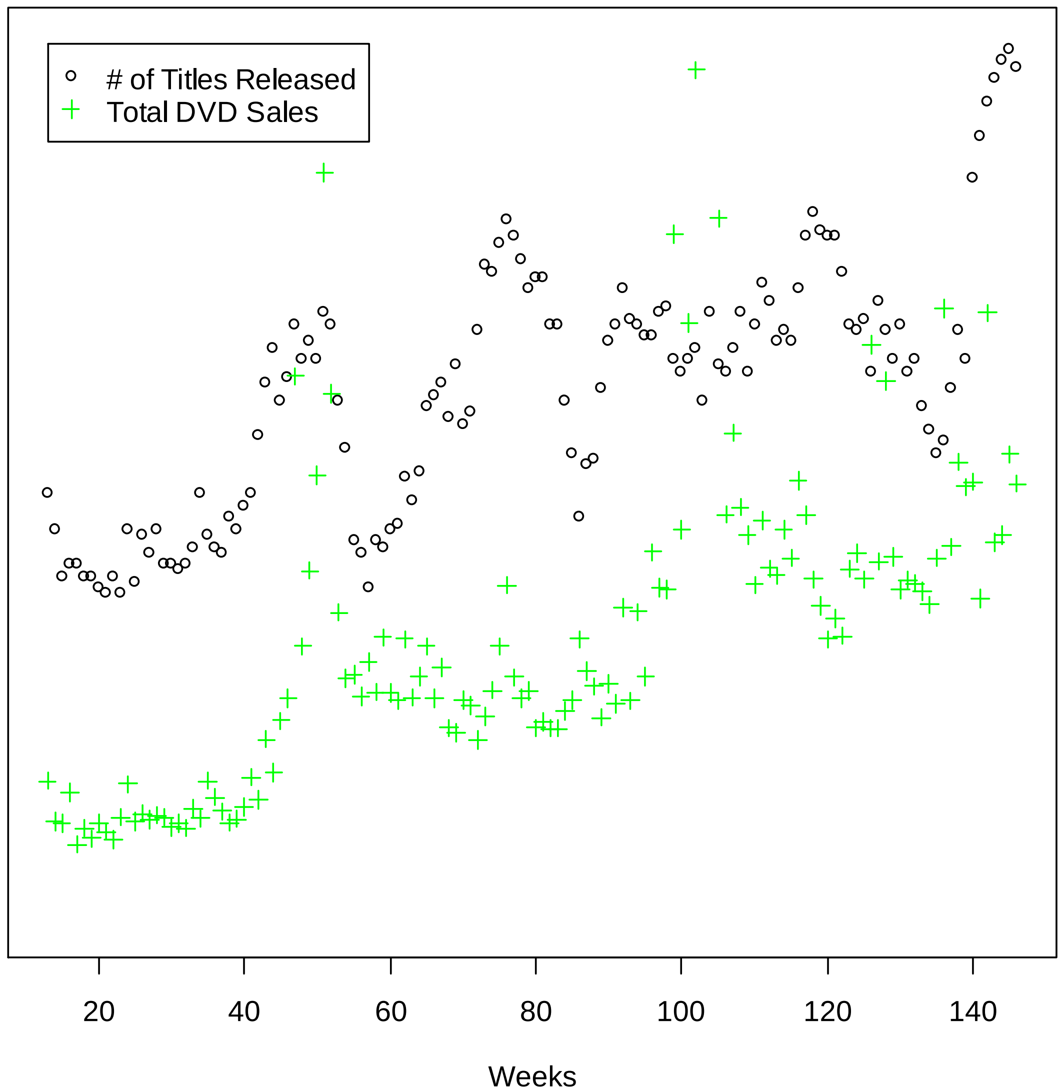

Figure 2: Summary statistic of competition

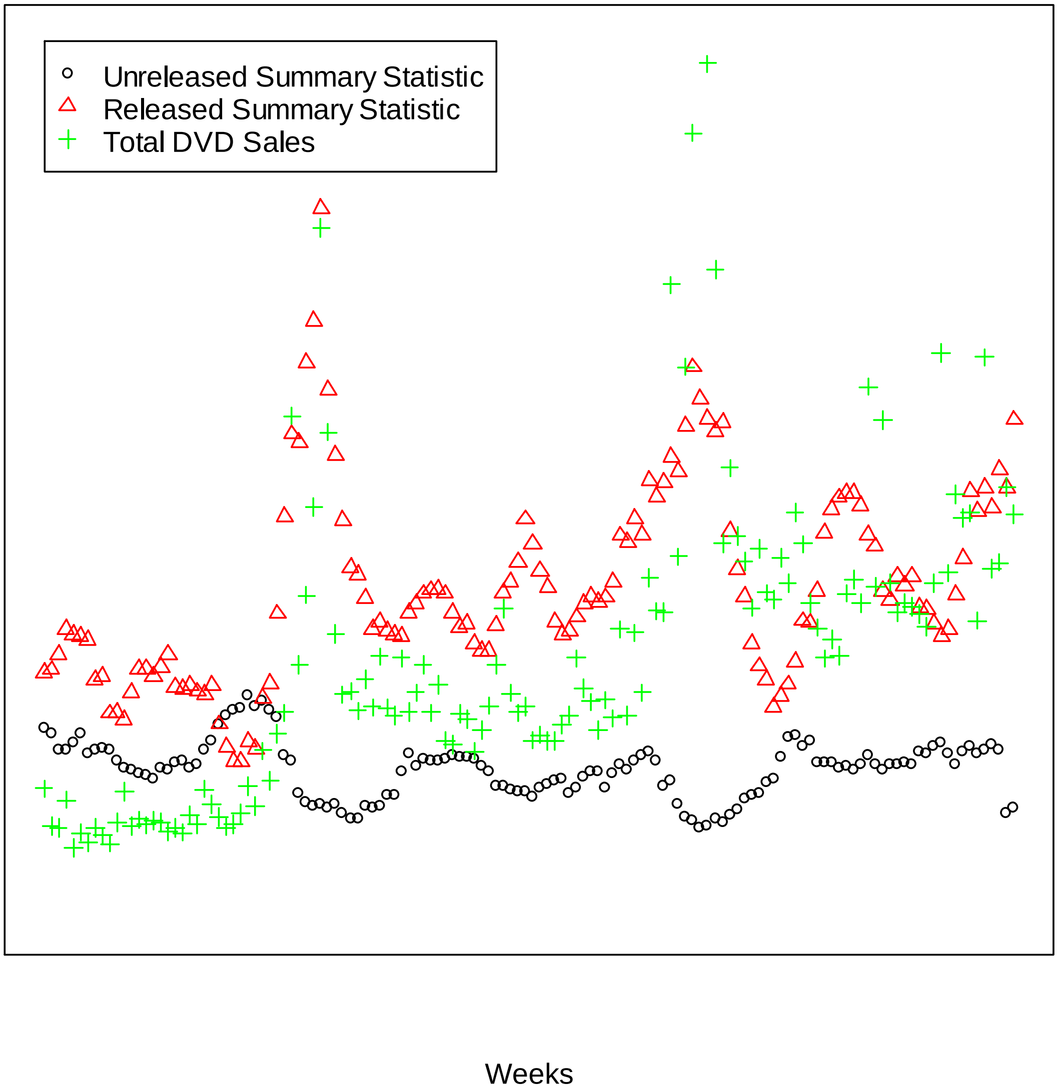

Figure 3: Competition forecast, 10 weeks into the future

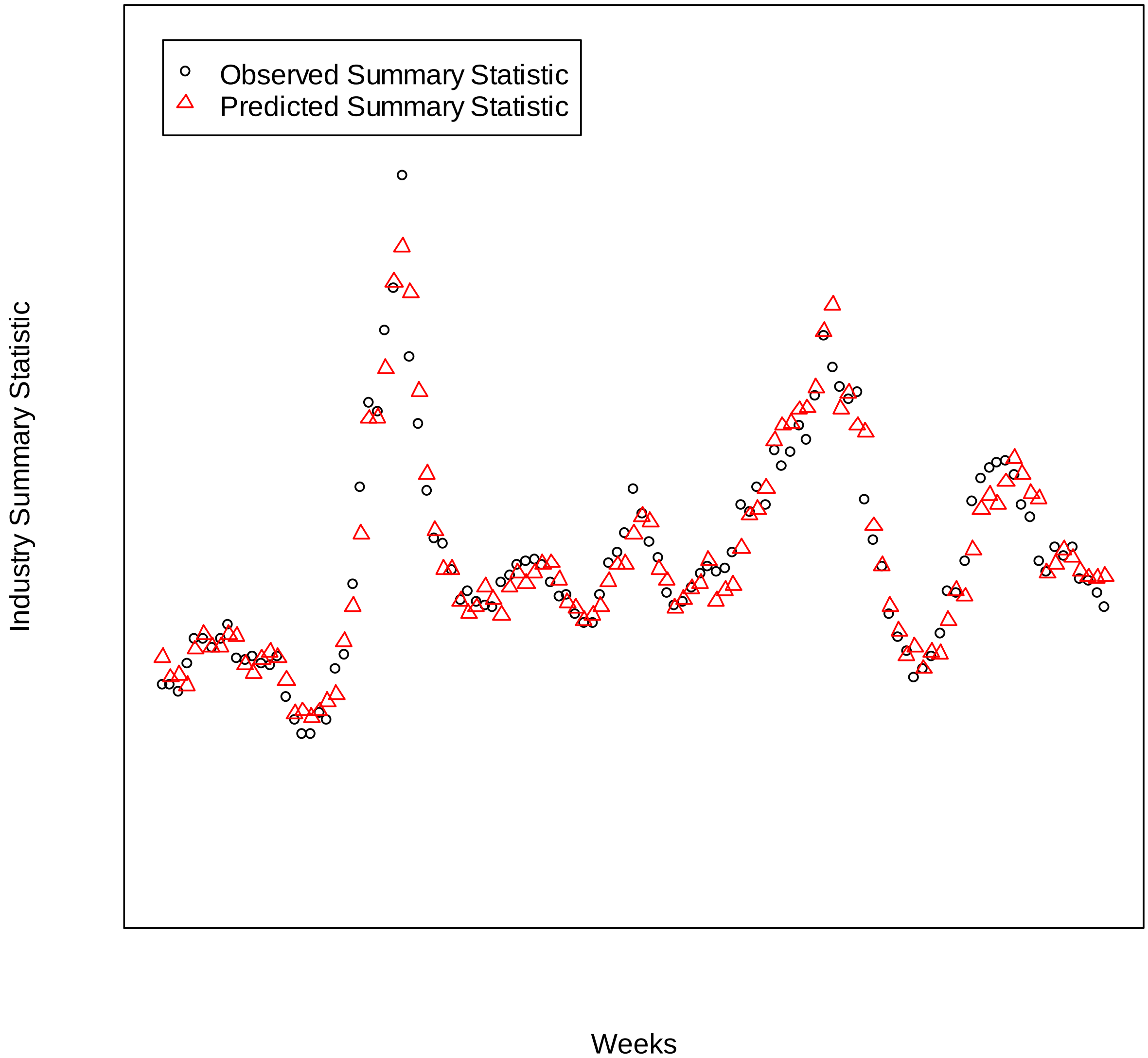

Figure 4: Competition forecast, 30 weeks into the future

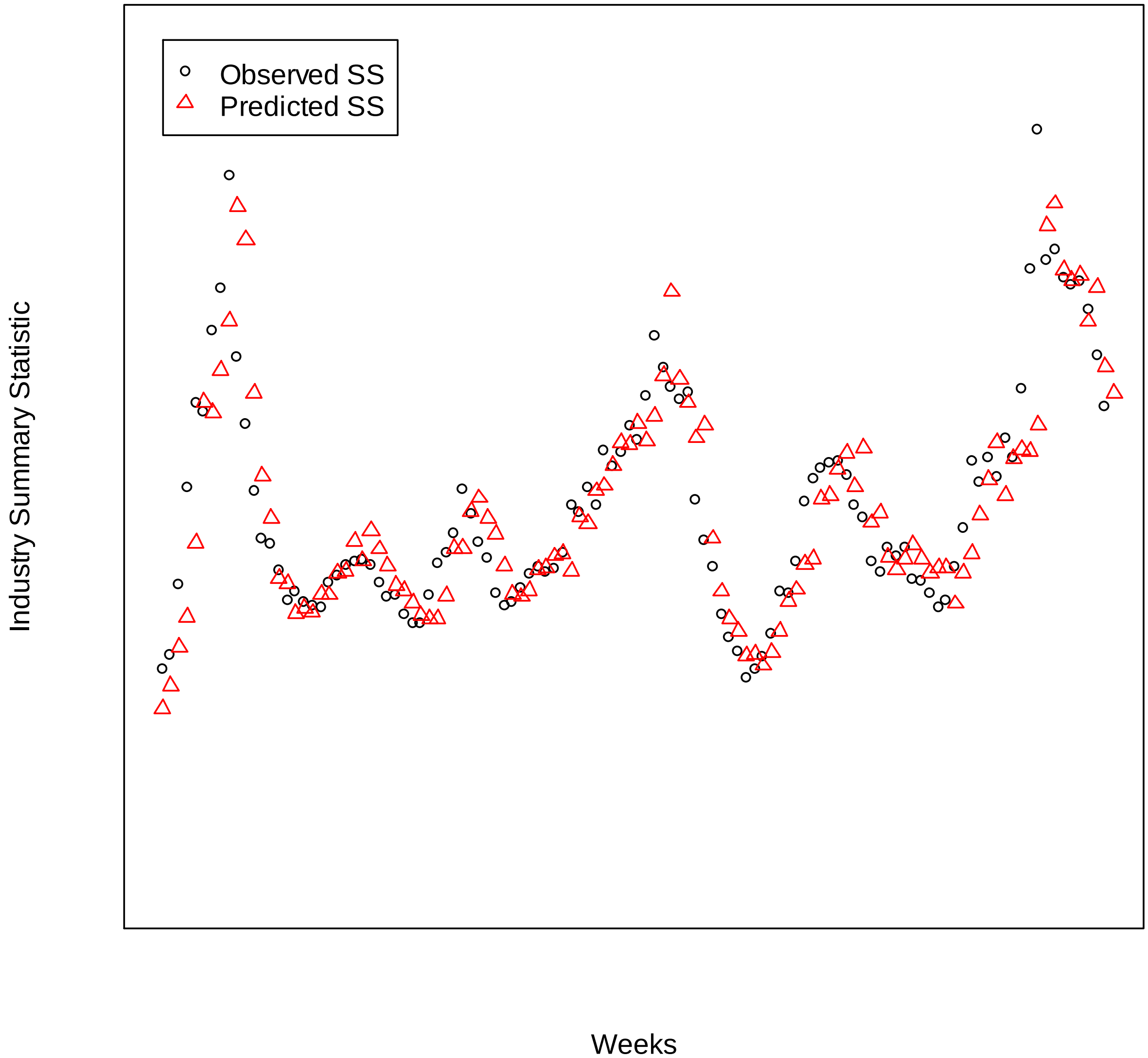

Figure 5: Histogram of Release Date Forecast Errors

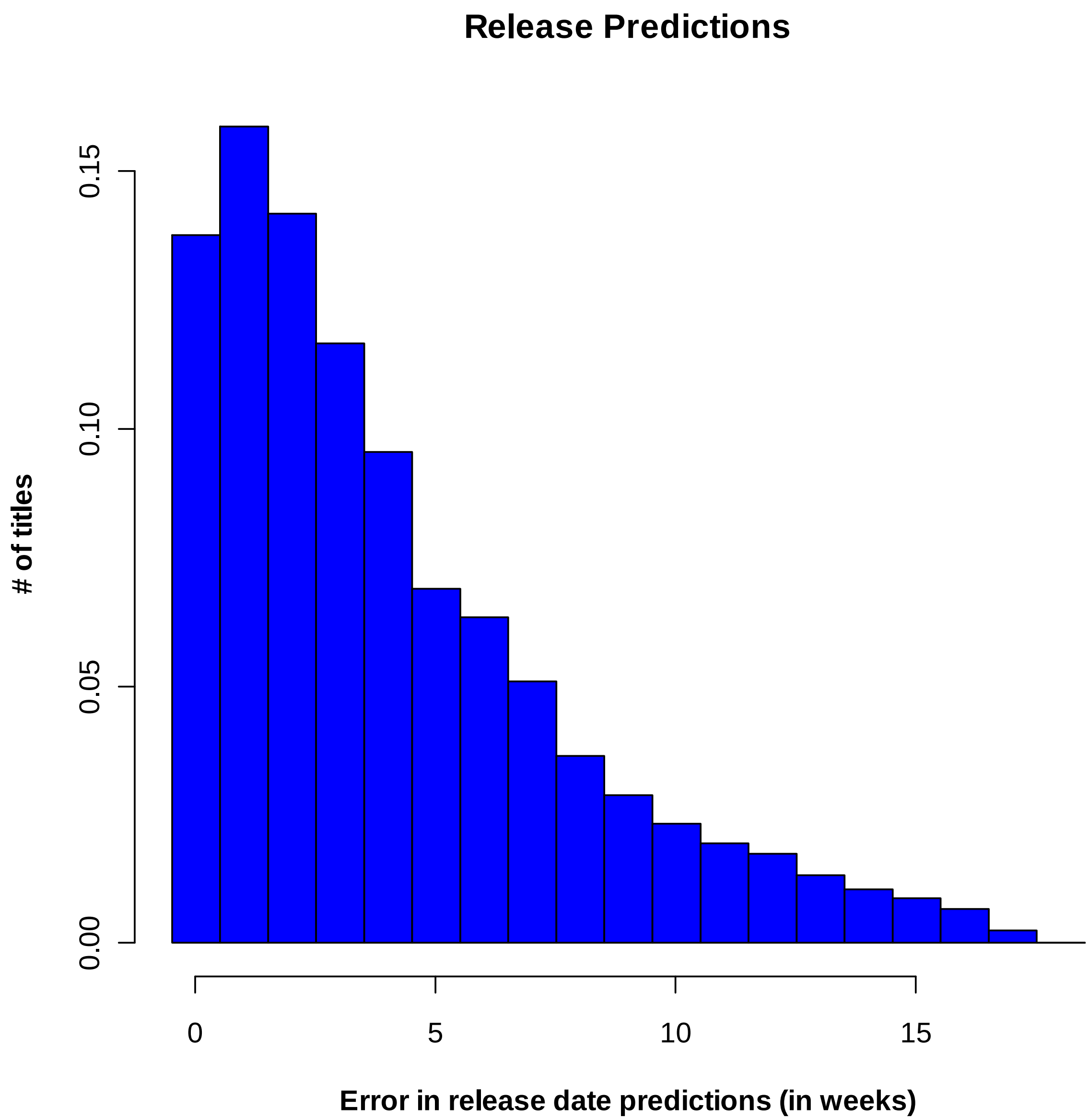

Table 1: Price Regression

	Estimate	Std. Error
Intercept	3.29E+01 ***	5.82E-01
Weeks Since Release (WR)	-3.29E-02 ***	9.09E-03
WR^2	5.61E-03 ***	1.64E-03
WR^3	-2.73E-04 **	8.45E-05
log(Box Office)	3.72E-01 ***	2.74E-02

Signif. codes: 0 '***' 0.001 '**' 0.01 '*'0.05 '.' 0.1 ' ' 1
Multiple R-Squared: 0.2263, Adjusted R-squared: 0.2237
We suppress coefficients for movie characteristics, weekly fixed effects, distributor fixed effects

Table 2: Coefficients of DVD Market Share

	Estimate	Std. Error
(Intercept)	-4.060 **	1.347
Price	-0.199 ***	0.021
log(Price)	1.674 ***	0.342
Weeks Since Release (WR)	-0.754 ***	0.026
Inter-release (IR)	-0.160 ***	0.019
log(WR)	3.702 ***	0.077
log(IR)	3.724 ***	0.496
log(Box Office)	0.909 ***	0.018
WR*IR	-0.0008	0.0009

Signif. codes: 0 '***' 0.001 '**' 0.01 '*'0.05 '.' 0.1 ' ' 1
Multiple R-Squared: 0.6019, Adjusted R-squared: 0.5871
We suppress coefficients for movie characteristics, weekly fixed effects, distributor fixed effects.

Table 3: Coefficients of Residual DVD Sales

	Estimate	Std. Error
(Intercept)	5.31E+00 **	1.79E+00
Industry Sales (week released)	9.39E-08 *	4.23E-08
Industry Sales (11 weeks after release)	-4.74E-08	4.79E-08
Industry Sales (Average over weeks 13-23)	1.30E-07 **	4.10E-08
Industry Competition (week released)	1.63E-07	3.07E-07
Industry Competition (11 weeks after release)	1.53E-06 **	4.62E-07
Unreleased Industry Competition (week released)	1.59E-05 **	6.05E-06
Unreleased Industry Competition (11 weeks after release)	1.57E-05	1.22E-05
Price	-5.78E-02 *	2.64E-02
log(Price)	8.21E-01 *	3.66E-01
Inter-release (IR)	-1.64E-02	2.58E-02
log(IR)	-3.60E-01	7.07E-01
log(Box Office)	9.63E-01 ***	3.07E-02

Signif. codes: 0 '***' 0.001 '**' 0.01 '*'0.05 '.' 0.1 ' ' 1
Multiple R-Squared: 0.826, Adjusted R-squared: 0.809
We suppress coefficients for movie characteristics, weekly fixed effects, distributor fixed effects.

Table 4: Coefficients of Release Costs

		Estimate	Std. Error
Margins	Seasonal variation	14.58 ***	9.02E-01
	lg(Box Office)	-2.65 ***	9.15E-02
	Inter-Release (IR)	23.37 ***	5.13E+00
	IR^2	-6.69 ***	1.04E+00
Residual Sums	Seasonal variation	0.11 ***	2.81E-02
	lg(Box Office)	-0.008 .	4.74E-03
	IR	1.90 ***	1.22E-01
	IR^2	0.09 ***	2.49E-02
Fixed Release Costs	Seasonal variation	66.71 **	2.32E+01
	lg(Box Office)	-14.93 ***	2.52E+00
	IR	-222 .	1.32E+02
	IR^2	42.92 *	2.11E+01
IR	Seasonal variation	26.68 ***	3.16E+00
	lg(Box Office)	-2.72 *	1.12E+00
IR^2	Seasonal variation	-4.29 ***	4.94E-01
	lg(Box Office)	0.39 *	1.80E-01

Signif. codes: 0 '***' 0.001 '**' 0.01 '*'0.05 '.' 0.1 ' ' 1

We suppress coefficients for movie characteristics, weekly fixed effects, distributor fixed effects.