1-2016

What difference do the new factor models make in portfolio allocation?

Frank J. Fabozzi

Dashan HUANG
Singapore Management University, DASHANHUANG@smu.edu.sg

Fuwei Jiang

Jiexun WANG
DOI: https://doi.org/10.2139/ssrn.2752822

What Difference Do New Factor Models Make in Portfolio Allocation?

Frank J. Fabozzi[*] **Dashan Huang**[†] **Fuwei Jiang**[‡] **Jiexun Wang**[§]

First draft: June 2015
Current version: January 2019

[*]EDHEC Business School, 393, Promenade des Anglais BP3116, 06202 Nice Cedex 3–France. E-mail: fabozzi321@aol.com.

[†]Lee Kong Chian School of Business, Singapore Management University, Singapore, 178899; E-mail: dashanhuang@smu.edu.sg.

[‡]School of Finance, Central University of Finance and Economics, Beijing, China, 100018. E-mail: jfuwei@gmail.com.

[§]Khoo Teck Puat Hospital (KTPH), 90 Yishun Central, Singapore, 768828. E-mail: wangjiexun@gmail.com.

What Difference Do New Factor Models Make in Portfolio Allocation?

Abstract

This paper examines the economic implications of new factor models and shows that the Hou-Xue-Zhang four-factor model outperforms the Fama-French five-factor model for investing in anomalies in- and out-of-sample. The difference in certainty-equivalent returns between the two models can be more than 6% per year under modest model uncertainty and margin requirements. The outperformance of the Hou-Xue-Zhang four-factor model appears to come from its better ability to describe the mean rather than the covariance matrix of asset returns.

JEL Classification: G11; G12; C11

Keywords: Portfolio allocation; Mean-variance analysis; Factor model; ETF; Smart beta

1 Introduction

Much of asset pricing research involves searching for factors that improve the understanding of the cross-section of expected stock returns. Based on the neoclassical Tobin q-theory of investment, Hou, Xue, and Zhang (HXZ, 2015) propose a four-factor model that can explain 29 out of 36 significant anomalies. Concurrently, motivated by the dividend discount model, Fama and French (FF5, 2015a, 2016) propose a competing five-factor model that explains anomalies such as the low market beta, share repurchases, and low stock return volatility. While many investors agree that stock returns can be described by a multifactor model, there is little agreement regarding the exact identification of the factors; "it takes a model to beat a model" is a frequently used adage. For example, Hou, Mo, Xue, and Zhang (2019) show that the HXZ model explains more anomalies and produces smaller alphas than the FF5 model, concluding that "the FF[5] five-factor model is in essence a noisy version of the q-factor model."

Barillas and Shanken (2017, 2018) address the issue of how to compare models under the classic Sharpe ratio improvement metric, in the spirit of Gibbons, Ross, and Shanken (GRS, 1989), and show that model comparison is driven by the extent to which each model is able to price the factors in the other models. A surprising result is that the test assets drop out of the analysis and are irrelevant for model comparison. However, in a non-nested model comparison setting, the approach in Barillas and Shanken (2017, 2018) cannot tell which model is better when both model are rejected (i.e., neither model is able to explain factors in the other model).

"Essentially, all models are wrong, but some are useful." Statistician George Box.

The anomalies literature is the scientific foundation ETFGI, an independent research consultancy firm, reports that total assets under management of ETFs and other exchange traded products (ETPs) reach over four trillion dollars worldwide and over 1.5 trillion dollars in the U.S. as of May 2017. As factor investing becomes increasingly important, the financial press has rightfully called into question the reliability of the underlying academic research. A Bloomberg article by Coy (2017) writes: "Most investors have a vague sense they're being ripped off. Here's how it happens.... Researchers have more knobs to twist in search of prized anomaly–a subtle pattern in the data that looks like it could be a moneymaker. They can vary the period, the set of securities under consideration, or even the statistical method. Negative findings go in a file drawer; positive ones get submitted to a journal (tenure!) or made into an ETF whose performance we rely on for retirement."

This paper asks whether the HXZ is a better model for *investing* than the FF5. There are four reasons for this alternative question. First, as shown by Pastor and Stambaugh (2000), a model that is better for pricing is not necessarily better for investing, because investors are usually subject to model uncertainty and margin requirements that prevent them from implementing certain extreme investment strategies suggested by asset pricing models. Second, investing involves both the mean and covariance of asset returns. A model that is worse for pricing is not necessarily worse for investing. There could be factors that account for substantial return comovements, but they are not priced or have very low risk premiums (Constantinides, 1980). Although these factors do not improve the description of average asset returns, they are important for an investor to control portfolio risk.

Third, factor investing is a widely explored strategy of the current investing canon. For example, Ang (2014) shows that 70% of active returns of the Norwegian Government Pension Fund–Global can be explained by exposures to risk factors. Within the exchange traded fund (ETF) marketplace, almost one quarter (24%) of institutional decision makers currently use factor-based products,[1] which allows investors to effectively manage the risk and return trade-off without the need to buy or sell individual securities. Finally and most importantly, as any model is an approximation of the true return-generating process, "it is extremely difficult to evaluate factor pricing models based solely on their pricing performance" (Kogan and Tian, 2017). Instead, investing provides an economic criterion for model comparison that accommodates pricing errors and allows one to compare asset pricing models out-of-sample, which is advocated by MacKinlay (1995) and Ang (2014), among others. They suggest out-of-sample performance as a criterion for identifying a variable as a risk factor.

This paper focuses on the portfolio allocation problem in the standard mean-variance framework. Because the distribution of asset returns is unknown, a Bayesian investor imposes a factor model, such as the HXZ or the FF5 model, to reduce the dimension of the estimation problem and to allocate her wealth among the factors. Although this restriction often improves the portfolio performance (MacKinlay and Pastor, 2000), the investor faces uncertainty regarding the model's pricing ability. Following Pastor and Stambaugh (2000) and Wang (2005), we assume that the investor has a prior belief, specified with varying degrees of confidence in the factor model, and computes the optimal portfolio with her posterior belief, which is

[1] "The Evolution of Smart Beta ETFs", Cogent Research, 2014. Other studies indicate an even greater use or future commitment to factor-based investing by institutional investors. See, for example, "Beyond Active and Passive: Advanced Beta Comes of Age." Research report, State Street Global Advisors (SSGA), 2014. Several state pension funds have made the commitment (Missouri State Employees' Retirement System, Jefferson City; New Mexico Public Employees Retirement Association, Santa Fe, and; Arizona State Retirement System, Phoenix, AZ).

updated by the data. Moreover, to make the optimal portfolio implementable, the investor is subject to a margin requirement that ranges from 0% to 50% (Pastor and Stambaugh, 2000). This assumption is justified by Fama and French (2015b) who show that, without a short selling constraint, it is easy for an investor who is investing in two anomalies to have a leverage ratio of more than 300, which is apparently unrealistic in practice.

Our objective is to examine the extent to which investors' prior beliefs about alternative pricing models impact the utility derived from the implied portfolio choices, and "is not to choose one pricing model over another" (Pastor and Stambaugh, 2000, p. 336).

We choose the long-short spread portfolios of 15 well-known anomalies in Novy-Marx and Velikov (2016) as the non-benchmark risky assets in which to invest. We classify the anomalies into two groups. The first group consists of five anomalies that can be explained by the HXZ model but not the FF5 model, i.e., the alpha of each anomaly is insignificant with the HXZ model but is significant with the FF5 model. The average alphas for the two models are 0.13% ($t = 0.63$) and 0.67% ($t = 3.45$). The second group consists of 10 anomalies that cannot be explained by the HXZ or the FF5 model. In this case, the average alphas for the two models are 0.72% ($t = 4.11$) and 0.75% ($t = 4.44$). These two groups of anomalies are intentionally chosen to explore whether the HXZ model is better for investing when it performs better than or the same as the FF5 model for pricing, respectively.[2]

We first compare the in-sample investing performances between the two models. When an asset pricing model cannot explain the average returns of risky assets with significant alphas, imposing the model on the return-generating process can lead to biased estimates for the predictive mean and covariance matrix of asset returns, and therefore, results in certainty-equivalent return (CER) losses relative to the case without imposing any model. As such, an asset pricing model is better for investing if it generates smaller CER losses. Based on the two groups of anomalies, we find that the HXZ uniformly outperforms the FF5. For example, the CER loss for an investor with a perfect confidence in the HXZ model is over 6% per year less than the CER loss for an investor with a perfect confidence in the FF5 model when the two Bayesian investors are both subject to a 10% margin requirement. The outperformance of the HXZ model is more pronounced when the margin requirement is relaxed. In addition, if an investor with perfect confidence in the

[2] In Robert Novy-Marx's data library, there are 32 anomalies (more comprehensive than the anomalies in Ken French's data library), including size, value, asset growth, and profitability that are used as factors in HXZ and FF5. Excluding anomalies that are factor-related and are insignificant with both HXZ and FF5, 15 are left (since some of them are highly correlated, Novy-Marx and Velikov (2016) use 23 out of 32 anomalies, including size, value, asset growth, and profitability). The results are quantitatively the same when we use the anomaly net returns that consider transaction costs.

HXZ model is forced to accept the portfolio chosen by another investor with an equally strong belief in the FF5 model, the first investor perceives a CER loss of more than 7% per year when the margin requirement is 10%.

If trading is costless, leverage can scale returns without limits. Using the words of Shapre (2011):

"If an investor can borrow or lend as desired, any portfolio can be leveraged up or down. A combination with a proportion k invested in a risky portfolio and $1-k$ in the riskless asset will have an expected excess return of k [times the excess return of the risky portfolio] and a standard deviation equal to k times the standard deviation of the risky portfolio. Importantly, the Sharpe Ratio of the combination will be the same as that of the risky portfolio."

The impact costs is small on long-only portfolios, but rises quickly with leverage, reducing returns quickly. As risk aversion declines to zero, both the expected return and volatility diverge, but so does the impact of trading costs. [Plot a frontier without transaction costs, and with different transaction costs, varying leverage]

In frictionless markets, two perfectly correlated assets with equal Shapre ratio generate the same efficient frontier, and in fact the same payoff space. This equivalent fails in the presence of trading costs: as the more volatile asset has a proportionally higher return, it can be traded to generate higher returns with lower leverage ratios, resulting in an efficient frontier that dominates (for high returns) the one generated by the less volatile asset.

We then compare the out-of-sample investing performances with two exercises. The first exercise assumes that asset returns are independent and identically distributed (i.i.d) over time, and uses a bootstrap simulation to compare the out-of-sample CERs between the two models. Surprisingly, when an investor has a confidence of at least 90% and the margin requirement is 10%, the HXZ model generates 6% more CER per year than the FF5 model. The second exercise relaxes the i.i.d assumption and compares the out-of-sample CERs with real time data. We use an expanding window approach. At the end of each month, we estimate the predictive mean and covariance matrix with the most up-to-date data, and apply the resulting optimal portfolio to the next month's returns. Then, we calculate the out-of-sample CERs with the realized portfolio returns. The results show that, regardless of the margin requirement, the HXZ model performs much better than the FF5 when the investor has a high confidence level, say 90%.

Finally, we explore the source of difference between the HXZ and FF5 models. The better investing performance of the HXZ model is a result of its better ability to describe the mean, the covariance matrix, or both. We compare the performances of the global-minimum-variance portfolios between the two models, which solely use the predictive covariance information for portfolio allocation. The result shows that the two models perform equally well, no matter which group of anomalies is used as the non-benchmark assets. Therefore, the better performance of the HXZ model for investing appears to stem from its better ability to capture the mean of stock returns.

The studies that are most closely related to this paper are Pastor and Stambaugh (2000) and Wang (2005), which incorporate model uncertainty, measured with investors' varying beliefs about asset pricing models, into the framework of portfolio allocation. Pastor and Stambaugh (2000) focus on the Fama and French (FF3, 1993) three-factor model and the Daniel and Titman (1997) characteristic model, and find that the two models generate indistinguishable performances under model uncertainty and margin requirements. This paper concentrates on the two most recent competing factor models and finds that the HXZ model uniformly outperforms the FF5 model, even in the case when they have the same degree of pricing ability on the non-benchmark assets. Wang (2005) focuses on model uncertainty and does not consider the effect of margin requirements. Moreover, despite its importance for investing, these two papers do not consider out-of-sample performance.

This paper is also related to the literature in portfolio allocation with factor-based asset pricing models. To evaluate the performance of different factor models for the covariance structure of individual stock returns, Chan, Karceski, and Lakonishok (1998, 1999) show that the FF3 model does a fair job constructing the global-minimum-variance portfolio. Also focusing on the estimation of the covariance structure, Briner and Connor (2008) explore the trade-off between estimation errors and model specification errors.

Olivares-Nadal and DeMiguel (2018) show that incorporating transaction costs in the mean-variance portfolio problem may help to reduce the impact of estimation error.

Brandt, Santa-Clara, and Valkanov (2009)

Barroso and Santa-Clara (2015), Chen and Velikov (2019), Olivares-Nadal and DeMiguel (2018), Frazzini, Israel, and Moskowitz (2018)

The remainder of the paper is organized as follows. Section 2 reviews the HXZ and FF5 factor models and discusses the importance of comparing them from the perspective of investing. Section 3 presents a framework for making Bayesian portfolio allocation under model uncertainty and margin requirements, and shows that the HXZ model outperforms the FF5 model for investing in anomalies, in- and out-of-sample. Section 4 concludes the paper.

2 New Factor Models

This section reviews the HXZ and FF5 factor models and discusses the importance of comparing them from the perspective of investing.

The HXZ model is motivated by the neoclassical q-theory of investment and consists of four factors: a market factor (MKT), a size factor (ME), an investment factor (I/A), and a profitability factor (ROE). The first factor is the market excess return and the last three factors are constructed from a triple $(2 \times 3 \times 3)$ sort on size, investment-to-assets, and return-on-equity. More specifically, size is the market equity, which is stock price per share times shares outstanding from the Center for Research in Security Prices (CRSP), I/A is the annual change in total assets (Compustat annual item AT) divided by one-year-lagged total assets, and ROE is income before extraordinary items (Compustat quarterly item IBQ) divided by one-quarter-lagged book equity.

The FF5 model is based on the dividend discount valuation theory and adds an investment (CMA, conservative-minus-aggressive) factor and a profitability (RMW, robust-minus-weak) factor to the FF3 model, which consists of market, size (SMB, small-minus-big), and value (HML, high-minus-low) factors. More specifically, CMA is defined as the difference between the returns on diversified portfolios of low and high investment stocks and RMW is defined as the difference between the returns on diversified portfolios of stocks with robust and weak profitability.

Table 1 presents summary statistics for the HXZ and FF5 factors in the sample period of 1972:01–2013:12. Panel A reports the average return (mean), t-statistic from the test that the average return of the factor is zero, standard deviation (Std), skewness (Skew), kurtosis (Kurt), first-order autocorrelation (AC(1)), and annualized Sharpe ratio (SR). Among the seven descriptive statistics, mean and Std are reported in percent per month. The average monthly returns on the factors are all more than two standard errors above zero, except for the FF5 size factor, SMB, which has an average monthly return of 0.23% ($t = 1.71$). The HXZ size factor, ME, has a higher average monthly return of 0.31%, with a t-statistic of 2.20.

Although both HXZ and FF5 use annual asset growth as the proxy for investment, the investment factor

(I/A) in the HXZ model has a higher average monthly return (0.44% versus 0.37%) and a lower standard deviation (1.87% versus 2.00%) than the investment factor (CMA) in the FF5 model. As a result, I/A has a higher annualized Sharpe ratio than CMA (0.82 versus 0.66). Moreover, I/A is less persistent than CMA and their first-order autocorrelations are 0.06 and 0.12, respectively.

The most striking difference between HXZ and FF5 is the profitability factor. First, the HXZ profitability factor (ROE) uses monthly earnings data, whereas the FF5 profitability factor (RMW) uses annual operating profit data. HXZ (2015) argues that the ROE factor is designed to capture anomalies, such as price momentum, earnings surprise, and financial distress, that are all studied at a monthly frequency. Next, since the ROE factor in the HXZ model contains the most up-to-date information about future ROE, its standard deviation is slightly higher than the CMW factor (2.62% versus 2.25%), and its average monthly return almost doubles (0.57% versus 0.29%). This is reflected directly in the annualized Sharpe ratio, which is 0.75 for ROE and 0.44 for RMW. Finally, ROE has a more negative skewness value (-0.75 versus -0.44) and a smaller kurtosis value (8.01 versus 14.4) than RMW.

Panel B of Table 1 reports the contemporaneous correlations of all of the factors. The market factors in the HXZ and the FF5 models have a perfect correlation of 1, and the size factors have a correlation of 0.98. Together with the descriptive statistics in Panel A, we assume that the market and size factors in the two models are indistinguishable and use the returns of MKT and SMB in the FF5 model for portfolio allocation throughout the paper.[3] The negative correlations of MKT with the investment and profitability factors suggest the necessity of new factors that can hedge the market risk. The two investment factors (I/A in the HXZ and CMA in the FF5) have a correlation of 0.90, and the two profitability factors (ROE and RMW) have a correlation of 0.67. An interesting observation is that the value factor (HML) in the FF5 model has a high correlation with the investment factor (I/A or CMA) and a low correlation with the profitability factor (ROE or RMW), suggesting that the redundancy of HML for pricing, shown in FF5 (2015a), is mainly due to the investment factor (I/A or CMA). Since ME and SMB, RMW and ROE, and CMA and IA are different versions of the same underlying construct, to avoid overfitting, we only consider models that contain at most one of the factors in constructing portfolios.

Table 1 raises a question about the main difference in the two models in explaining the cross-section of stock returns. Using factor regressions as FF3 (1993), Hou, Xue, and Zhou (2016) compare the two models

[3]In portfolio allocation, when assets i and j are highly correlated, the estimation of the covariance is highly volatile with extreme entries on $(i,i),(i,j),(j,i)$ and (j,j), resulting in extreme portfolio positions in assets i and j that swing dramatically over time.

on conceptual and empirical grounds, finding that the HXZ model explains more anomalies (29 vs. 17 out of 36) and has less average monthly alphas (0.20% vs. 0.36%). As such, Hou, Mo, Xue, and Zhang (2019) conclude that "the FF[5] five-factor model is in essence a noisy version of the *q*-factor model".

However, alpha is not an appropriate metric for model comparison and can generate counterintuitive results. For example, over the sample period of 1972:01–2013:12, the monthly alpha of the momentum factor is 0.78% ($t = 3.94$) for the CAPM model but is 0.95% ($t = 4.82$) for the FF3 model, which is in stark contrast to most studies, if not all, that the FF3 is a better pricing model. Another example is from Fama and French (2016) c who find that the FF5 model exaggerates, instead of shrinking, the accrual anomaly. The FF5 alpha is 0.31% ($t = 2.27$) and the FF3 alpha is 0.27% ($t = 1.96$) over the sample period of this paper. In general, Barillas and Shanken (2015) show that zero alpha for a non-benchmark asset is neither a sufficient nor a necessary condition for model comparison.

In terms of investing, Figure 1 plots the mean-variance frontiers for investing in the factors of the HXZ, the FF5, or both, where the market and size factors in the two models are assumed to be the same and refer to the MKT and SMB factors in the FF5. Three observations follow the figure immediately. First, the frontier of the FF5 does not lie inside or overlaps that of the HXZ. Second, the global minimum-variance of investing in the HXZ is different from that of the FF5. Specifically, the standard deviation and mean of the global minimum-variance for investing in the HXZ are 1.03% and 0.44%, which are in contrast to 0.97% and 0.34% for investing in the FF5. This difference suggests that the HXZ factors cannot mimic the global minimum-variance portfolio of FF5. Third and lastly, if one invests in both the HXZ and FF5 factors (FF5's five factors plus HXZ's I/A and ROE factors), the standard deviation and mean of the global minimum-variance are 0.94% and 0.38%, respectively. Therefore, this strategy can reduce the minimum-variance and improve its expected return, relative to investing in the HXZ or the FF5 alone.

According to Barillas and Shanken (2017), a factor model is better for pricing if it can price the factors in the competing model with zero alphas. Hou, Mo, Xue, and Zhang (2019) show that the HXZ outperforms the FF5 for investing by using the mean-variance efficiency test of Gibbons, Ross, and Shanken (GRS, 1989). Similarly, a factor model is better for investing if it can mimic the performance of the competing model, i.e., it outperforms *any* portfolio spanned by the competing model in terms of the Sharpe ratio. As such, we turn to Huberman and Kandel (1987) and run a mean-variance *spanning* test on the hypothesis that whether the competing factors' returns can be spanned or replicated in the mean-variance space of the factor model.

Following Kan and Zhou (2012), we carry out six spanning tests: Wald test under conditional homoscedasticity, Wald test under independent and identically distributed (i.i.d.) elliptical distribution, Wald test under conditional heteroscedasticity, Bekerart-Urias spanning test with errors-in-variables (EIV) adjustment, Bekerart-Urias spanning test without the EIV adjustment and DeSantis spanning test.

The six spanning test results reported in Table 2 strongly reject the hypothesis that the FF5 factors are inside the mean-variance frontier of the HXZ factors. Delving deeper, we also test whether the FF5's investment and profitability factors (CMA and RMW) can be replicated by the HXZ factors, and find that the answer is negative. Hence, it not clear whether the HXZ model is better for investing, which is the focus of this paper. For comparison, in the last column of Table 2, we include the GRS statistics and confirm the finding of HXZ (2016) that their model can price the FF5 factors well.

3 Comparing Factor Models in Portfolio Allocation

This section presents the mean-variance portfolio allocation problem under model uncertainty and margin requirements. The objective is to compare asset pricing models from the perspective of investing. For a given investment universe, we calculate the portfolio that is selected by a Bayesian investor who bases her prior belief in the HXZ or the FF5 model, and compare the performances of the two models in- and out-of-sample.

3.1 Portfolio allocation under model uncertainty and margin requirements

Consider the portfolio allocation problem in a universe with a risk-free asset and n risky assets. Without loss of generality, we assume that the risk-free rate r_f is constant over time throughout the paper. Let $r_t = [r'_{1t}, r'_{2t}]'$ be the long-short spread returns, as in Pastor and Stambaugh (2000), where the long side is a risky asset and the short side is either a risky asset or a risk-free asset, r_{1t} is the first m non-benchmark assets, and r_{2t} is the last k $(= n - m)$ benchmark assets. For example, when the HXZ is the benchmark model, r_{2t} is the HXZ four-factor returns and the HML, CMA, and RMW factor returns are included in r_{1t}, which then has $n-4$ elements. Similarly, when the FF5 is the benchmark model, r_{2t} is the FF5 five-factor returns and the I/A and ROE factor returns are simply included in r_{1t}, which then has $n-5$ elements. DeMiguel, Nogales, and Uppal (2014)

Suppose r_t follows a multivariate normal distribution and is i.i.d. over time. The true mean and covariance matrix are denoted as follows corresponding to the m assets and k factors:

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \qquad V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}, \tag{1}$$

which can be summarized in a regression model:

$$r_{1t} = \alpha + B r_{2t} + u_t, \tag{2}$$

where u follows a multivariate normal distribution with mean zero and covariance matrix equal to Σ. With this factor structure, one can write the mean and covariance matrix of the risky assets as

$$\mu = \begin{bmatrix} \alpha + B\mu_2 \\ \mu_2 \end{bmatrix}, \qquad V = \begin{bmatrix} BV_{22}B' + \Sigma & BV_{22} \\ V_{22}B' & V_{22} \end{bmatrix}. \tag{3}$$

The asset pricing model is true if and only if $\alpha = 0_{m\times 1}$, where $0_{m\times 1}$ is an $m \times 1$ vector of zeros.

In the portfolio allocation framework using asset pricing models, the mean-variance investor chooses to believe or not to believe the asset pricing model. If she does not believe the asset pricing model at all, she estimates μ and V without restricting α to zero. The maximum likelihood estimates of α, B, and Σ are denoted by $\hat{\alpha}, \hat{B}$, and $\hat{\Sigma}$, respectively. The investor estimates μ and V in (3) as:

$$\hat{\mu} = \begin{bmatrix} \hat{\mu}_1 \\ \hat{\mu}_2 \end{bmatrix} = \begin{bmatrix} \hat{\alpha} + \hat{B}\hat{\mu}_2 \\ \hat{\mu}_2 \end{bmatrix}, \qquad \hat{V} = \begin{bmatrix} \hat{B}\hat{V}_{22}\hat{B}' + \hat{\Sigma} & \hat{B}\hat{V}_{22} \\ \hat{V}_{22}\hat{B}' & \hat{V}_{22} \end{bmatrix}, \tag{4}$$

where $\hat{\mu}_2$ and $\hat{V}_{22}$ are the sample mean and covariance matrix of r_{2t}.

When the investor has a dogmatic belief about the asset pricing model, she estimates μ and V by imposing $\alpha = 0_{m\times 1}$. Let $\bar{B}$ and $\bar{\Sigma}$ be the maximum likelihood estimates of B and Σ with the restriction. The estimates of μ and V are

$$\bar{\mu} = \begin{bmatrix} \bar{B}\hat{\mu}_2 \\ \hat{\mu}_2 \end{bmatrix}, \qquad \bar{V} = \begin{bmatrix} \bar{B}\hat{V}_{22}\bar{B}' + \bar{\Sigma} & \bar{B}\hat{V}_{22} \\ \hat{V}_{22}\bar{B}' & \hat{V}_{22} \end{bmatrix}. \tag{5}$$

In this paper, we assume that the investor places a confidence level of ω in the asset pricing model, in the spirit of Wang (2005). Let $R = \{r_t, t = 1, \cdots, T\}$ and $\hat{S}^2 = \hat{\mu}_2' \hat{V}_{22}^{-1} \hat{\mu}_2$ be the squared Sharpe ratio of the ex ante tangency portfolio with the same mean and covariance matrix of the k factors. With a Bayesian approach, Wang (2005) shows that the investor estimates the predictive mean and covariance matrix as:

$$\tilde{\mu} = \mathrm{E}(r_{T+1}|R,\omega) = \begin{bmatrix} \hat{\mu}_1 + \omega(\bar{B}\hat{\mu}_2 - \hat{\mu}_1) \\ \hat{\mu}_2 \end{bmatrix}, \tag{6}$$

$$\tilde{V} = \mathrm{Var}(r_{T+1}|R,\omega) = \begin{bmatrix} \psi_0 + \omega\psi_1 + \omega^2\psi_2 & b[\omega\bar{B} + (1-\omega)\hat{B}]\hat{V}_{22} \\ b\hat{V}_{22}[\omega\bar{B} + (1-\omega)\hat{B}]' & b\hat{V}_{22} \end{bmatrix}, \tag{7}$$

where

$$\psi_0 = b\hat{B}\hat{V}_{22}p\hat{B}' + h\hat{\delta}\hat{\Sigma}, \tag{8}$$

$$\psi_1 = b(\bar{B} - \hat{B})\hat{V}_{22}\hat{B}' + b\hat{B}\hat{V}_{22}(\bar{B} - \hat{B})' + h(\bar{\delta} - \hat{\delta})\hat{\Sigma} + h\hat{\delta}(\bar{\Sigma} - \hat{\Sigma}), \tag{9}$$

$$\psi_2 = b(\bar{B} - \hat{B})\hat{V}_{22}(\bar{B} - \hat{B})' + h(\bar{\delta} - \hat{\delta})(\bar{\Sigma} - \hat{\Sigma}), \tag{10}$$

and where $\bar{\delta}, \hat{\delta}, b$, and h are scalars and are defined as follows:

$$\bar{\delta} = \frac{T(T-2)+k}{T(T-k-2)} - \frac{k+3}{T(T-k-2)} \cdot \frac{\hat{S}^2}{1+\hat{S}^2}, \tag{11}$$

$$\hat{\delta} = \frac{(T-2)(T+1)}{T(T-k-2)}, \tag{12}$$

$$b = \frac{T+1}{T-k-2}, \tag{13}$$

$$h = \frac{T}{T-m-k-1}. \tag{14}$$

From (6) and (7), the HXZ and FF5 models imply different restrictions on α and yield different predictive means and covariance matrices. As a result, their optimal portfolios are different. When $\omega = 0$, the predictive mean and covariance are the sample mean and covariance matrix, which are unbiasedly estimated without the restriction on α. When $\omega = 1$, the predictive mean and covariance matrix are fully determined by the estimates that restrict α to zero.

Let x denote the n-vector with the ith element x_i. With $\tilde{\mu}$ and $\tilde{\Sigma}$, the Bayesian investor is assumed to choose x to maximize the mean-variance objective function:

$$\max_{x} \quad x'\tilde{\mu} - \frac{\gamma}{2} x'\tilde{V}x, \tag{15}$$

where γ is the coefficient of relative risk aversion. For simplicity, we assume that γ is equal to three throughout the paper. Without any constraint, the optimal portfolio weight $\tilde{x}$ is

$$\tilde{x} = \frac{1}{\gamma}\tilde{V}^{-1}\tilde{\mu} = \frac{1}{\gamma}\begin{bmatrix} \tilde{\Sigma}^{-1}\tilde{\alpha} \\ \tilde{V}_{22}^{-1}\tilde{\mu}_2 - \tilde{B}'\tilde{\Sigma}^{-1}\tilde{\alpha} \end{bmatrix}. \tag{16}$$

One important characteristic in (16) is that if some of the non-benchmark assets in r_1 have non-zero alphas, the investor with benchmark assets of r_2 should improve her portfolio Sharpe ratio by changing her portfolio weights on the non-benchmark assets in proportion to their alphas. The alpha of a non-benchmark asset, calculated with respect to a given asset pricing model, measures the change in the portfolio's Sharpe ratio that is driven by a marginal increase in the asset weight of the portfolio. Thus, the sign of alpha is the direction of the marginal adjustment in portfolio weight space that yields the maximal increase in the portfolio's Sharpe ratio. Therefore, alphas explain the optimal way to marginally adjust the portfolio relative to the benchmark: increase the weights of non-benchmark assets with positive alphas, and decrease the weights with negative alphas.

In the framework of asset pricing, by the mathematical definition, the adjustment to the portfolio weight can be *infinitesimal*. However, in the framework of investing, the adjustment is actually *finite* as the investor is usually subject to portfolio constraints Almazan, Brown, Carlson, and Chapman (2004). Certain risky assets are not tradable because the investor cannot sell short with full use of the proceeds. Fama and French (2015b) show that, without a short selling constraint, it is easy for an investor who is investing in two anomalies to have a leverage ratio of more than 300, which is apparently unrealistic in practice. DeMiguel, Garlappi, Nogales, and Uppal (2009)

Following Pastor and Stambaugh (2000), we assume that the mean-variance investor in the optimization problem (15) is subject to the following margin requirements:

$$\sum_{j\in\Lambda} 2|x_j| + \sum_{j\notin\Lambda} |x_j| \leq c, \tag{17}$$

where Λ denotes the set of positions in which the short position for the spread return j is risky, and c is the maximum permitted total value of risky long and short positions per dollar of the investor's wealth. For example, $c = 2$ corresponds to a margin requirement of 50% and $c = 10$ corresponds to a margin requirement of 10%. When $c = \infty$, there is no margin requirement and the investor will invest in the capital market line with the maximum Sharpe ratio that she can obtain.

Another reason for using constraint (17) is that it has a good statistical property when controlling for estimation risk. For simplicity, suppose the short position of any spread return j in constraint (17) is a risk-free asset. Then, (17) reduces to

$$\sum_{j=1}^{n} |x_j| = \|x\|_1 \leq c. \tag{18}$$

Given the true mean μ and and covariance matrix V, the utility loss of the optimal portfolio $\tilde{x}$ from using the predictive $\tilde{\mu}$ and covariance $\tilde{V}$ has an upper bound as:

$$\begin{aligned} \left|(\tilde{x}'\tilde{\mu} - \frac{\gamma}{2}\tilde{x}'\tilde{V}\tilde{x}) - (\tilde{x}'\mu - \frac{\gamma}{2}\tilde{x}'V\tilde{x})\right| &\leq \left|\tilde{x}'\tilde{\mu} - \tilde{x}'\mu\right| + \frac{\gamma}{2}\left|\tilde{x}'\tilde{V}\tilde{x} - \tilde{x}'V\tilde{x}\right| \\ &\leq \|\tilde{\mu} - \mu\|_\infty \|\tilde{x}\|_1 + \frac{\gamma}{2}\|\tilde{V} - V\|_\infty \|\tilde{x}\|_1^2, \end{aligned} \tag{19}$$

where $\|\tilde{x}\|_1$ is the L_1 norm of vector $\tilde{x}$, and $\|\hat{\mu} - \mu\|_\infty$ and $\|\hat{V} - V\|_\infty$ are the maximum component-wise estimation errors (Fan, Zhang, and Yu, 2012).[4] Therefore, if $\|\tilde{x}\|_1$ is bounded above (economically, it is a margin requirement), the utility loss resulting from estimation errors is controlled by the largest component-wise errors of $\|\tilde{\mu} - \mu\|_\infty$ and $\|\tilde{V} - V\|_\infty$. As long as each element is estimated well, the overall utility is approximated well without the accumulation of estimation errors.

3.2 Anomalies that are considered for investing

We choose 15 anomaly long-short spread portfolio returns from Novy-Marx and Velikov (2016) as the non-benchmark risky assets and report their descriptive statistics in Table 3, which include the annualized Sharpe ratio, alpha, t-statistic, and R^2 of regressing each anomaly return on the factors in the HXZ and the FF5, respectively.

We intentionally classify the anomalies into two groups to explore whether the HXZ model is better for

[4] If x is an N-dimensional vector, $\|x\|_1 = \sum_{j=1}^{N} |x_j|$ and $\|x\|_\infty = \max_j |x_j|$. If X is an $M \times N$-dimensional matrix, $\|X\|_\infty = \max_i \sum_{j=1}^{N} |x_{i,j}|$, where $x_{i,j}$ is the element in row i and column j of X.

investing when it performs better than or the same as the FF5 model for pricing. Panel A of Table 3 consists of five anomalies: return-on-book equity (RetBE), ValMom, idiosyncratic volatility (IVOL), momentum, and return-on-market equity (RetME). These anomaly long-short portfolio returns can be explained by the HXZ model with insignificant alphas, but cannot be explained by the FF5 model with significant alphas. The average alpha is 0.13% ($t = 0.63$) for the HXZ model and is 0.67% ($t = 3.45$) for the FF5 model. A striking result is that the average R^2 statistics of these two models are 50% and 53%, respectively, implying that although the HXZ outperforms the FF5 in terms of alpha, there are half variations in the average anomaly returns that are left unexplained by both of them.

Panel B of Table 3 consists of 10 anomalies: accruals, net issuance (rebal.:A), investment, gross margins, ValMomProf, industry momentum (IndMom), industry relative reversals (IndRelRev), high-frequency combo (HighFreqCom), seasonality, and industry low volatility (IndLowVol). None of these anomalies cannot be explained by the HXZ model or the FF5 model, and the average alphas are 0.72% ($t = 4.11$) and 0.75% ($t = 4.44$), respectively. The average regression R^2s of the two models are 18% and 22%, suggesting that there are more than three quarters of variations in the average anomaly returns left unexplained by both models. Compared with Table 1, the high average annualized Sharpe ratio, 0.71, is slightly smaller than that of I/A and ROE in the HXZ model (0.82 and 0.75), but it is larger than any other factors.

3.3 Predictive means and standard deviations

Before analyzing the portfolio decisions, we examine the predictive means and standard deviations of risky assets in Table 4. The results for the covariances of individual assets are similar to the standard deviations and are omitted to save space. If there are no substantial differences in these parameter estimates, it is unlikely that there are dramatic differences in portfolio allocations.

According to (6) and (7), imposing an asset pricing model on the return-generating process with a confidence of ω has a first-order effect on the predictive mean and a second-order effect on the predictive variance. Panels A and B of Table 4 report the predictive means. It is apparent that by varying the confidence ω and asset pricing model, the predictive mean of each asset is dramatically changed. For example, the predictive mean of the return-on-book equity (RetBE) anomaly return is 0.71% per month if the investor is agnostic about the HXZ and the FF5 by setting $\omega = 0$ (the sample mean in this case). In contrast, if the investor believes dogmatically in one of them ($\omega = 1$), the predictive mean is 0.70% for the HXZ model and is 0.25% for the FF5 model. The dramatic estimation bias for using the FF5 model is due to the fact that the FF5 model cannot explain the RetBE anomaly. When one imposes a constraint by setting the alpha equal to zero when estimating the predictive mean, the estimate is dramatically biased. Instead, since the HXZ model explains the anomaly with an insignificant alpha, the model is likely to capture the true mean of the return-generating process and the potential estimation bias is negligible, as the imposed constraint is nearly slack. This argument is supported by Panel B of Table 4. Since each anomaly in this panel cannot be explained by the two models, the estimates with them for the predictive mean dramatically deviate from the unbiased estimate, the sample mean by setting $\omega = 0$. For instance, the unbiased predictive mean of the accrual anomaly is 0.26% per month, whereas it is -0.07% for the HXZ model and -0.02% for the FF5 model when $\omega = 1$.

Panels C and D of Table 4 report the predictive standard deviations. In contrast with the results in Panels A and B, the difference in estimates between the two models is relatively small. This result is consistent with Chan, Karceski, and Lakonishok (1999) who show that various models for forecasting covariances generally perform quite similarly. In Panel C, although each anomaly can be explained by the HXZ model but not the FF5, the estimates of the predictive standard deviations with the two models are virtually the same. When $\omega = 1$, the biggest difference in the estimated predictive standard deviations between the two models is 0.09% in the momentum anomaly, and amounts to only 1% of the estimate when $\omega = 0$ (the case with an unbiased predictive variance estimate). In Panel D, when $\omega = 1$, the biggest difference is 0.08% in the ROE factor, which amounts to approximately 3% of the estimate of $\omega = 0$.

A striking characteristic in Panel D is that the differences between $\omega = 1$ and $\omega = 0$ for both models are generally larger than that in Panel C. For example, regarding the HighFreqCom anomaly, the predictive standard deviations with the two models are 4.04% and 4.05% when $\omega = 1$, and are both 3.77% when $\omega = 0$. The biases, 0.27% and 0.28%, amount to approximately 7% of 3.77%. We attribute this larger bias between $\omega = 1$ and $\omega = 0$ to the larger proportion of variations in the average anomaly returns that are left unexplained by the two factor models, as shown by the lower regression R^2s in Table 3. Therefore, when there is a significant mispricing error, imposing an asset pricing model leads to a large bias in the estimation of the predictive standard deviation. To some extent, our finding is consistent with MacKinlay and Pastor (2000) that when a risk factor is missing from an asset pricing model, the resulting mispricing is embedded within the residual covariance matrix.

3.4 In-sample comparison

Table 5 reports optimal allocations per \$100 of wealth when prior beliefs are centered on either of the two asset pricing models, with varying degrees of confidence ω. The risky assets include the five anomalies that can be explained by the HXZ but not the FF5 model (see details in Panel A of Table 3), five factors in the FF5, and the investment and profitability factors in the HXZ model. Hence, the investment universe consists of 12 risky assets and one risk-free asset. When the investor employs the HXZ model, the non-benchmark assets are the five anomalies plus the HML, CMA, and RMW factors in the FF5. Similarly, when the FF5 is the asset pricing model, the non-benchmark assets are the five anomalies plus the I/A and ROE factors in the HXZ model.

As confidence decreases, the optimal portfolio converges to the portfolio based on the sample mean and covariance matrix of anomaly returns (the case of $\omega = 0$), regardless of the asset pricing model. The aim here is to explore the extent to which this behavior occurs at interesting confidence levels of ω. The results in Table 5 are reported for $\omega = 0.75$ and 0.5 as well as the limiting cases $\omega = 1$ (exact pricing) and $\omega = 0$ (no use of a pricing model).

We consider three levels of margin requirements, $c = 2, 10$, and ∞. Panel A reports the results of $c = 2$, which is a constraint with a 50% margin requirement. The first two columns, with $\omega = 1$, display the allocations corresponding to the dogmatic beliefs in each of the two asset pricing models. In this case, each asset pricing model estimates the parameters of the non-benchmark assets by restricting alphas equal to zero. According to Table 3 and HXZ (2016), ex post, the HXZ model explains the average returns of the five anomalies and the HML, CMA, and RMW factors. Hence, the zero alpha constraint for the HXZ model is nearly slack and innocuous. With this tight margin requirement constraint, the optimal portfolio under the HXZ model includes four assets: RetBE (19.8), ValMom (37.3), momentum (12.4), and MKT (61.0), where only MKT is a benchmark asset.

The allocation under the FF5 factor is different, investing in two anomaly assets with small positions, ValMom (12.6) and IVOL (6.5), and in the CMA and MKT factors with large positions (44.7 and 72.5). This result in this panel is in stark contrast to Pastor and Stambaugh (2000), who show that, with a 50% margin requirement, it makes no difference whether the mean-variance investor uses the FF3 model or the Daniel-Titman's characteristic model, even though the non-benchmark assets are constructed to exploit differences between them. One potential explanation is that the two models in Pastor and Stambaugh (2000) have the same power in explaining the non-benchmark assets, whereas the HXZ and the FF5 in this paper have different powers in explaining the non-benchmark assets.

For a given c, let $\tilde{x}$ be the optimal portfolio under the predictive mean $\tilde{\mu}$ and covariance $\tilde{V}$. We compute the in-sample expected utility or certainty-equivalent return (CER) as:

$$\text{CER}(\tilde{x}, \tilde{\mu}, \tilde{V}) = \tilde{x}'\tilde{\mu} - \frac{\gamma}{2}\tilde{x}'\tilde{V}\tilde{x}. \tag{20}$$

Moreover, we calculate the Sharpe ratio as:

$$\text{SR}(\tilde{x}, \tilde{\mu}, \tilde{V}) = \frac{\tilde{x}'\tilde{\mu}}{\sqrt{\tilde{x}'\tilde{V}\tilde{x}}}. \tag{21}$$

The investing problem (15) treats the risky assets as on an individual basis, ignoring the fact that they are constructed as portfolios of individual stocks, and a given stock can appear in a non-benchmark portfolio and in each of the benchmark factors. For this reason, one can argue that the returns on the risky assets are correlated; large differences in position-by-position allocations need not necessarily produce economically significant differences in the overall portfolio characteristics. As a result, the CER and SR are more sensible measures for model comparison.

The last two rows of each panel in Table 5 report the in-sample CER and SR. To facilitate understanding, we multiply the monthly CER by 1,200 to express it as percent per year and multiply the monthly SR by $\sqrt{12}$ for an annual value. The last two rows of Panel A demonstrate that, even with a 50% margin requirement, the CER and SR with different asset pricing models are apparently different. With a dogmatic belief, $\omega = 1$, the investor with the HXZ model can realize a CER of 9% per year. In contrast, she can only obtain a CER of 6.2% with the FF5 model. The difference in CERs suggests that the investor can obtain 23 more basis points per month using the HXZ model. The annualized SR is 0.93 for the HXZ model and 0.75 for the FF5 model.

When the confidence level ω decreases, the optimal portfolios for the two asset pricing models changes accordingly. However, the changes for the FF5 are more dramatic. From (6) and (7), as ω decreases, the predictive mean and covariance matrix of the risky assets approach the mean and covariance matrix estimated without the constraint of alphas equal to zero. As the HXZ can explain all of the non-benchmark assets with insignificant alphas, its estimates of the mean and covariance matrix are close to the estimates without the constraint. Hence, a change in ω has a smaller effect on the optimal portfolio chosen by the

HXZ model. For example, when ω decreases from 1 to 0.75, the investment in the ValMom anomaly slightly decreases from 37.3 to 34.3 per \$100. In stark contrast, the investment with the FF5 model increases dramatically from 12.6 to 29.6.

The case of $\omega = 0.5$ deserves special discussion. In a Bayesian framework, given the residual covariance matrix Σ of r_{1t} in (3), if the prior distribution of α is chosen to satisfy $\text{Var}(\alpha|\Sigma) = \text{Var}(\hat{\alpha}|\Sigma)$, Wang (2005) shows that the required confidence ω should be set to 0.5. Panel A shows that, in this case, the optimal investment in ValMom is 31.3 with the HXZ and is slightly different with the case of $\omega = 0.75$. The optimal investment with the FF5 model is 40.2 per \$100 of wealth, which is significantly different from the case of $\omega = 0.75$.

When the investor is agnostic about any asset pricing model and sets ω equal to 0, the predictive means and covariance matrices with the two models are the same, which are the estimates without restriction on the alphas of the non-benchmark assets. The optimal portfolio is reported in the last column, and the first four anomalies and the MKT factor receive non-zero weights. Since the estimates in this case are unbiased, and when the sample size increases, the portfolio should unbiasedly converge to the one with true parameters.

Panel B of Table 5 reports the results with $c = 10$. That is, for each dollar of investment, the maximum value of risky positions is at most \$10 or the margin requirement is 10%. Three observations follow this panel. First, as the margin requirement is relaxed, the investments in the factors of either asset pricing model dramatically increase. For example, with a dogmatic belief of $\omega = 1$, the investor with the HXZ model allocates 189.3, 134.2, 38.2, and 121.8 to the I/A, ROE, SMB, and MKT factors for each \$100 of wealth; these values are in sharp contrast to the case of $c = 2$, in which the only one non-zero allocation on the factors is MKT with a position of 61. Similarly, the investor with the FF5 model allocates 259.4, 126.5, 38.8, and 126.2 to the CMA, RMW, SMB, and MKT factors, which are significantly larger than the case of $c = 2$. Second, among the 12 risky assets, only the MKT short side is a risk-free asset and the investment in this asset is equal to 100 minus the investment in MKT. From Panel B, for any value of ω, all of the investments in MKT are larger than 100. As such, the investments in the risk-free asset in these cases are negative, implying that the investor is borrowing money. Third and lastly, the investing performances, CER and SR, significantly improve relative to Panel A. The differences in CERs between the HXZ and the FF5 also become larger, and they are 7.2%, 6.8%, and 5.4% per year when $\omega = 1, 0.75$, and 0.5, respectively.

Panel C of Table 5 is the case in which there is no margin requirement, i.e., $c = \infty$. When $\omega = 1$, each asset pricing model estimates the parameters of the non-benchmark assets with zero alphas, the portfolio weights for these assets are zero. As a result, the investor allocates her wealth among the factors of the model that she employs. However, when the investor does not have a dogmatic belief but places a confidence of $\omega = 0.75$, she will allocate across all of the risky assets, regardless of the asset pricing model. An interesting result with this panel is that, while the portfolio is more diversified when the investor's confidence level decreases, the CER and SR do not improve substantially. Instead, the allocations on the non-benchmark assets are generally much smaller than the investments in the factors. In terms of the alternative interpretation of the margin requirements in (19), the small portfolio weights on the non-benchmark assets are partially due to estimation errors.

Table 6 reports the optimal portfolio choices, CERs, and SRs when the investment universe of risky assets are the 10 anomalies, five factors in the FF5, and I/A and ROE in the HXZ model. The key difference between Tables 5 and 6 is that each of the anomaly returns in Tables 6 cannot be explained by the HXZ or the FF5 model. From Table 3, both models have similar degrees of mispricing errors for the 10 anomalies. In this sense, they could have similar investing performance.

Panel A of Table 6 shows the results of $c = 2$. The differences in CERs between the HXZ and the FF5 are 1.5%, 1.6%, and 1.2% per year when $\omega = 1$, 0.75, and 0.5. An interesting finding is that the investor does not invest in the benchmark assets at all when $\omega = 0$. Panel B shows that when the margin requirement is relaxed by setting $c = 10$, the optimal portfolio covers more assets and increases allocations on the factors significantly. In this case, the differences in CERs between the HXZ and the FF5 are 6.9%, 6.6%, and 4.6% per year when $\omega = 1$, 0.75, and 0.5. Panel C strengthens this result and allocates to all of the risky assets with non-zero weights when the investor is uncertain about the asset pricing model. In addition to the difference in CERs between the two models being increasing, the level of CER for each model increases dramatically. The reason is intuitive as all of the risky assets have non-zero alphas, and the investor invests more in the anomalies by enjoying the "arbitrage" opportunities.

As a mean-variance investor, if the distribution of asset returns is known, imposing an asset pricing model by setting alphas equal to zero can lead to biased estimates of the portfolio parameters, which gives rise to a CER loss. A better asset pricing model is the one that yields smaller biases in the predictive mean and covariance matrix. As a result, it should have a smaller CER loss. Suppose the true mean and covariance matrix are μ and V, and x_o is the resulting optimal portfolio for a given c. We calculate the CER as $\text{CER}_o = x_o'\mu - \frac{\gamma}{2}x_o'Vx_o$. Then we calculate the CER of a suboptimal allocation x_s as $\text{CER}_s = x_s'\mu - \frac{\gamma}{2}x_s'Vx_s$, where x_s is an allocation that is optimal for the same c and ω under the predictive distribution from imposing an asset pricing model. For example, if an asset pricing model is imposed, x_s is the optimal allocation from using the predictive mean and covariance matrix, $\tilde{\mu}$ and $\tilde{V}$, that are estimated according to (6) and (7). The difference $\text{CER}_o - \text{CER}_s$ provides an economic measure of CER loss from imposing a pricing constraint on the distribution of asset returns.

Figure 2 displays the annualized CER losses for an investor who believes in the sample mean and covariance matrix that are estimated without imposing any asset pricing model, but is forced to hold a portfolio chosen by another investor with a confidence ω in the HXZ or the FF5 model. The risky assets are the same as Table 5, including five anomalies that can be explained by the HXZ but not the FF5 model, five factors in the FF5, and the investment and profitability factors in the HXZ model. For each of four values of c, the figure plots the CER loss versus confidence ω. Losses are calculated for portfolios from the HXZ model and from the FF5 model. The goal is to explore whether the HXZ is a better model for investing when it is a better model for pricing, as shown in Panel A of Table 3.

Figure 2 makes three statements. First, the CER loss increases monotonically with respect to the confidence level ω. When $\omega = 0$, the investor does not believe in the asset pricing model and the predictive mean and covariance matrix are the same as the sample mean and covariance matrix. In this case, there is no CER loss. When ω increases, the investor places a larger weight on the mean and covariance matrix that are estimated by setting the alphas of non-benchmark assets equal to zero. The resulting predictive mean and covariance matrix are more likely to be biased, and therefore, the CER loss is more likely to increase. Second, the CER loss increases with respect to c. As c increases, the constraint of margin requirements becomes less likely to be binding and the optimal portfolio is closer to the one suggested by the predictive mean and covariance matrix. Hence, the CER loss increases. Third and finally, the CER loss for imposing the HXZ model is negligible and essentially plots as a flat line at zero, regardless of the values of c and ω. When $\omega = 1$, the CER loss increases from 0.4% per year at $c = 2$, to 1.3% per year at $c = 10$, and to 2.5% per year at $c = \infty$. In this case, the CER loss for imposing the FF5 model is dramatic, and it increases from 2.7% per year at $c = 2$, to 8.2% per year at $c = 10$, and to 16.0% per year at $c = \infty$. Hence, when $\omega = 1$, the HXZ outperforms the FF5 by 2.3% at $c = 2$, by 6.9% at $c = 10$, and by 13.5% at $c = \infty$. As such, one can conclude that the HXZ is a better model for investing in these non-benchmark assets.

Figure 3 displays the annualized CER losses for the case in which 10 anomaly spread returns cannot be explained by the HXZ or the FF5 model. As in Figure 2, the investor is assumed to believe in the sample mean and covariance matrix but is forced to hold the portfolio chosen by another investor with confidence ω

in the HXZ or the FF5 model. The risky assets are those in Table 6, including 10 anomalies that cannot be explained by the HXZ or the FF5 model, five factors in the FF5, and the investment and profitability factors in the HXZ model. The goal here is to explore whether the HXZ is a better model for investing even when it performs similarly to the FF5 model for pricing the anomalies in Panel B of Table 3.

Figure 3 shows a similar pattern to Figure 2. As c increases or ω increases, the CER loss increases monotonically. The difference between these two figures is that the magnitude of CER losses in Figure 3 is much larger than that in Figure 2 for both models. Given that $\omega = 1$, when $c = 2, 10$, and ∞, the CER losses for the HXZ model are 5.3%, 26.4%, and 67.6%, and they are 8.2%, 34.8%, and 81.1% for the FF5 model, respectively. Thus, when $c = 2$,10, or ∞, the outperformance of the HXZ in terms of CER is 2.9%, 8.4%, or 13.5%, which is surprisingly similar to that in Figure 2. Summarizing these two figures, one can conclude that the HXZ outperforms the FF5 model for investing in anomalies, regardless of their pricing abilities.

Figure 4 displays precisely the same analysis except that the CER losses are computed for an investor who believes in the HXZ model with confidence ω but is forced to hold the portfolio chosen by another investor with the same degree of confidence in the FF5 model (left two panels), and vice versa (right two panels). The upper two panels correspond to the risky assets in Figure 2 and the lower two panels correspond to the risky assets in Figure 3. When $\omega = 1$ and $c = 2$, the CER loss for the investor who believes in one model but is forced to use another model is always less than 2% per year. When $c = 10$, the CER losses for the investor who believes the HXZ but is forced to use the FF5 are more than 7% per year, which is an economically large magnitude, regardless of the risky assets. Similarly, the CER losses for the investor who believes in the FF5 but is forced to use the HXZ are about 6% per year. When $c = \infty$, all of the CER losses are more than doubled, in comparison to $c = 10$.

3.5 Out-of-sample comparison

A model with better in-sample performance for investing does not necessarily mean it has better out-of-sample performance because of estimation errors. For example, DeMiguel, Garlappi, and Uppal (2009) report that the Sharpe ratio is 0.219 for the mean-variance model with assets MKT, SMB, and HML, whereas the Sharpe ratio is 0.096 with assets MKT, SMB, HML, MOM, 10 book-to-market portfolios, and 10 size portfolios. This example suggests that comparing models out-of-sample is important in that adding more assets could reduce portfolio performance if the estimation errors are not controlled. Kan and Wang (2017) explicitly consider the out-of-sample utility loss using the sample mean and covariance matrix.[5]

3.5.1 Pseudo out-of-sample analysis

We follow Kozak, Nagel, and Santosh (2018) and perform a bootstrap simulation for out-of-sample comparison, which maintains the i.i.d property over time, an assumption made in the main framework. We randomly sample (with replacement) $T+300$ returns on the risky assets and use the first T to calculate the portfolio weights, which are used for the remaining 300 observations to calculate the out-of-sample CER ($\mathrm{CER_{OS}}$),

$$\mathrm{CER_{OS}} = \hat{\mu}_{\tilde{x}} - \frac{\gamma}{2}\hat{\sigma}^2_{\tilde{x}}, \tag{22}$$

where $\hat{\mu}_x$ and $\hat{\sigma}^2_{\tilde{x}}$ are the sample mean and variance of the 300 out-of-sample excess returns of portfolio $\tilde{x}$ that is based on the first T observations. Similarly, we calculate the out-of-sample Sharpe ratio $(\mathrm{SR_{OS}})$ as

$$\mathrm{SR_{OS}} = \frac{\hat{\mu}_{\tilde{x}}}{\hat{\sigma}_{\tilde{x}}}. \tag{23}$$

We repeat the procedure 1,000 times and report the average $\mathrm{CER_{OS}}$. As the sample size is important for out-of-sample performance, we consider five values of T: 60, 120, 240, 360, and 600.

Table 7 reports the annualized $\mathrm{CER_{OS}}$ for investing in the anomalies that can be explained by the HXZ but not the FF5 model. In contrast to the in-sample comparison in Table 5, this table considers two more values of ω, 0.95 and 0.90, in addition to 1, 0.75, 0.50, and 0. Table 7 makes four statements. First, for given values of c and ω, the $\mathrm{CER_{OS}}$ increases as the sample size T increases. For example, when $c=10$ and $\omega=1$, the annualized $\mathrm{CER_{OS}}$ of the HXZ model is 18.1% when $T=60$, and it increases monotonically to 26.4% when $T=600$. Similarly, the annualized $\mathrm{CER_{OS}}$ of the HXZ model increases from 12.2% to 19.8% in this case. Second, when $c=2$, since the HXZ model is a "correct" pricing model and explains all of the anomalies, its $\mathrm{CER_{OS}}$ is flat and does not change as ω decreases. This pattern holds true regardless of the sample size T. In contrast, since the FF5 model cannot explain the anomalies and its estimates for the predictive mean and covariance matrix are biased, the $\mathrm{CER_{OS}}$ increases in general when ω decreases. For

[5] In the literature, there is a large number of papers that explain why the out-of-sample performance could be poor and how to improve it, such as MacKinlay and Pastor (2000), Jagannathan and Ma (2003), Siegel and Woodgate (2007), Garlappi, Uppal, and Wang (2007), DeMiguel, Garlappi, and Uppal (2009), Tu and Zhou (2011), and DeMiguel, Plyakha, Uppal, and Vilkov (2013), among others.

example, when $T = 60$, the annualized CER_{OS} increases from 4.6% at $\omega = 1$ to 6.2% at $\omega = 0$; and when $T = 600$, it increases from 7.3% at $\omega = 1$ to 9.8% at $\omega = 0$. Third, fixing ω, the CER_{OS} does not necessarily increase when the margin requirement is relaxed (increasing c). When $\omega = 1$ and $T = 60$, the annualized CER_{OS} values at $c = 2, 10$, and ∞ are 6.3%, 18.1%, and 14.8% for the HXZ model, and they are 4.6%, 12.2%, and -4.0% for the FF5 model. Finally and more importantly, the HXZ outperforms the FF5 model with large CER_{OS}s when $\omega \geq 0.5$. Even with a modest confidence of $\omega = 0.75$, when $T = 600$, the HXZ outperforms the FF5 by 5.2% at $c = 10$ and by 8.0% at $c = \infty$.

Table 8 reports the annualized SR_{OS} with the same investment setting as Table 7. Consistent with Table 7, the HXZ model outperforms the FF5 model in terms of SR_{OS}, and the outperformance achieves the maximum when $\omega = 1$ and is slightly more pronounced when T is small. An interesting result is that the SR_{OS} increases when c increases for given T and ω.

Tables 9 and 10 report the annualized CER_{OS} and SR_{OS} for investing in anomalies that cannot be explained by the HXZ or the FF5 model, whose in-sample results are reported in Table 6. From Panel B of Table 3, the HXZ and FF5 models have equal pricing abilities in terms of the average alpha and regression R^2. However, with respect to investing, the HXZ outperforms the FF5 significantly out-of-sample. For example, when $\omega = 1$ and $T = 600$, the annualized CER_{OS} values with the HXZ model are 2.6%, 8.4%, and 13.8% larger than the corresponding values of the FF5 at $c = 2, 10$, and ∞. When $\omega = 0.90$, the corresponding counterparts are still 2.1%, 6.8%, and 12.4%, respectively. Different from Table 7, the CER_{OS} values with non-zero ω values for both models are less than the values of $\omega = 0$, because the estimates for the predictive mean and covariance matrix by imposing either of the two models are biased.

3.5.2 Real-time out-of-sample analysis

The previous analysis assumes that the risky returns are i.i.d over time. In practice, however, this assumption does not hold and expected returns are varying over time,[6] which means that the expected returns are moving targets and can never be estimated accurately (Garleanu and Pedersen, 2013). As such, the outperformance of the HXZ model in the previous subsection may not exist in real time.

We use an expanding window approach to compare their out-of-sample performances. With an initial window of 120 months, in each month t, we use data from month 1 to month t to compute the various

[6]We keep the normality assumption as it works well in evaluating portfolio performance in a mean-variance framework (Tu and Zhou, 2004).

portfolio rules, and apply them to determine the investments in the next month. For instance, let $\tilde{x}_t$ be the estimated optimal portfolio in month t and r_{t+1} be the excess return on the risky assets realized in month $t+1$. The realized excess return on the portfolio is $r_{\tilde{x},t+1} = \tilde{x}_t' r_{t+1}$. We then compute the average value of the realized returns, $\hat{\mu}_{\tilde{x}}$, and the variance, $\hat{\sigma}^2_{\tilde{x}}$. The out-of-sample CER can be calculated accordingly.

Figure 5 plots the CER_{OS}s for investing in anomalies that can be explained by the HXZ but not the FF5 model. As shown in Table 7, when ω is high, the difference in CER_{OS}s between the HXZ and the FF5 is economically significant. For example, when $\omega = 1$, the CER_{OS} values for the two models are 22.2% and 16.3% at $c = 10$, and 28.3% and 20.6% at $c = 20$, respectively. The difference in CER_{OS}s suggests that the HXZ outperforms the FF5 by 5.8% or 6.7% per year when the investor is subject to a 10% or 5% margin requirement. Figure 6 plots the CER_{OS} values for investing in the anomalies that cannot be explained by the HXZ or the FF5 model, as in Table 9. When $\omega = 1$ and $c = 20$, the CER_{OS} values for the two models are 34.8% and 27.5% with a difference of 7.3% per year. Even in the case of $c = 10$, the difference is 3.2% per year. Therefore, summarizing these two figures, one can conclude that, under modest model uncertainty and margin requirements, the difference in CER_{OS}s between the two models can be more than 6% per year.

3.6 Source of difference between the HXZ and the FF5

In the mean-variance framework, the only two parameters are the predictive mean and covariance matrix of risky assets. The better investment performance of the HXZ model must come from its better ability to capture the mean, the covariance matrix, or both.

This subsection considers the global-minimum-variance portfolio allocation. That is, the investor chooses portfolio x to minimize $x'\tilde{V}x$ with the margin requirement (17), where $\tilde{V}$ is the predictive covariance matrix and is given in (7). Mathematically, this is an extreme case of (15) with $\gamma = \infty$. The goal here is to show that whether the estimates of $\tilde{V}$ with the HXZ and the FF5 are different enough to yield different portfolios.

Figures 7 and 8 plot the in-sample annualized Sharpe ratio losses from the perspective of a global-minimum-variance investor, who knows the true mean and covariance matrix but is forced to hold the global-minimum-variance portfolio chosen by another investor who places a confidence of ω in the HXZ or the FF5 model, where the sample mean and covariance matrix are assumed to be the true mean and covariance matrix. In addition to the five factors in FF5 and investment and profitability factors in HXZ, the risky assets also include five anomalies that can be explained by the HXZ but not the FF5 model (Figure 7), or 10 anomalies that cannot be explained by the HXZ or the FF5 (Figure 8).

A striking pattern in Figure 7 is that the Sharpe ratio loss is negligible and is always less than 0.1 for both models, regardless of c and ω. In fact, the Sharpe ratio losses are virtually the same when c exceeds 10. This suggests that imposing one of the two asset pricing models does not lead to a significant bias for estimating the risky assets' covariance matrix. In Figure 8, the Shapre ratio loss is large; it is at least 0.2 for both models. However, the losses for the HXZ and FF5 are similar for given c and ω, suggesting that the predictive means and covariance matrix with the two pricing models are biased and similar. The evidence in these two figures is similar to Chan, Karceski, and Lakonishok (1999), who focus on individual stocks and find that the FF3 model performs the same as a nine-factor model under the global-minimum-variance criterion.

With this simple exercise, one can conclude that the better performance of the HXZ model for investing appears to come from its better ability to describe the average returns of risky assets.

4 Conclusion

This paper shows that the HXZ model outperforms the FF5 model for investing in anomalies in- and out-of-sample. Based on the well established framework of Pástor and Stambaugh (2000) and Wang (2005), the out-of-sample certainty-equivalent return of the HXZ model can be 6% per year more than the FF5 under modest model uncertainty and margin requirements. The finding in this paper is a complement, in an economic way, to concurrent studies by Hou, Mo, Xue, and Zhang (2019) and Barillas and Shanken (2017, 2018) who focus on pricing rather than investing.

This paper assumes that asset returns are i.i.d over time. It is of interest to relax this assumption and explore how conditional information affects the investing performance of the two models. While the investment framework does incorporate the investor's varying beliefs in the asset pricing models, it does not address the linkage between the priors and the economic objectives, which can significantly improve the investing performance (Tu and Zhou, 2010). From the perspective of investing, it is undoubtedly interesting to include more competing models, such as Stambaugh and Yuan (2017).

References

Almazan, A., Brown, K. C., Carlson, M., Chapman, D. A., 2004. Why constrain your mutual fund manager? Journal of Financial Economics 73, 289–321.

Ang, A., 2014. Asset Management: A Systematic Approach to Factor Investing. Oxford University Press.

Barillas, F., Shanken, J., 2017. Which alpha? Review of Financial Studies 30, 1316–1338.

Barillas, F., Shanken, J., 2018. Comparing asset pricing models. Journal of Finance 52, 715–754.

Barroso, P., Santa-Clara, P., 2015. Beyond the carry trade: Optimal currency portfolios. Journal of Financial and Quantitative Analysis 50, 1037–1056.

Brandt, M. W., Santa-Clara, P., Valkanov, R., 2009. Parametric portfolio policies: Exploiting characteristics in the cross-section of equity returns. Review of Financial Studies 22, 3411–3447.

Briner, B. G., Connor, G., 2008. How much structure is best? A comparison of market model, factor model and unstructured equity covariance matrices. Journal of Risk 10, 3–30.

Chan, L. K. C., Karceski, J., Lakonishok, J., 1998. The risk and return from factors. Journal of Financial and Quantitative Analysis 33, 159188.

Chan, L. K. C., Karceski, J., Lakonishok, J., 1999. On portfolio optimization: Forecasting covariances and choosing the risk model. Review of Financial Studies 12, 937–974.

Chen, A. Y., Velikov, M., 2019. Accounting for the anomaly zoo: A trading cost perspective. Working paper.

Constantinides, G. M., 1980. Admissible uncertainty in the intertemporal asset pricing model. Journal of Financial Economics 8, 71–86.

Daniel, K., Titman, S., 1997. Evidence on the characteristics of cross sectional variation in stock returns. Journal of Finance 52, 1–33.

DeMiguel, V., Garlappi, L., Nogales, F. J., Uppal, R., 2009. A generalized approach to portfolio optimization: Improving performance by constraining portfolio norms. Management Science 55, 798–812.

DeMiguel, V., Garlappi, L., Uppal, R., 2009. Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? Review of Financial Studies 22, 1915–1953.

DeMiguel, V., Martin-Utrera, A., Nogales, F. J., Uppal, R., 2018. A portfolio perspective on the multitude of firm characteristics. Working paper.

DeMiguel, V., Nogales, F. J., Uppal, R., 2014. Stock return serial dependence and out-of-sample portfolio performance. Review of Financial Studies 27, 1031–1073.

DeMiguel, V., Plyakha, Y., Uppal, R., Vilkov, G., 2013. Improving portfolio selection using option-implied volatility and skewness. Journal of Financial and Quantitative Analysis 48, 1813–1845.

Fama, E. F., French, K. R., 2015a. A five-factor asset pricing model. Journal of Financial Economics 116, 1–22.

Fama, E. F., French, K. R., 2015b. Incremental variables and the investment opportunity set. Journal of Financial Economics 117, 470–488.

Fama, E. F., French, K. R., 2016. Dissecting anomalies with a five-factor model. Review of Financial Studies 29, 69–103.

Fan, J., Zhang, J., Yu, K., 2012. Vast portfolio selection with gross-exposure constraints. Journal of the American Statistical Association 107, 592–606.

Frazzini, A., Israel, R., Moskowitz, T. J., 2018. Trading costs. Working paper.

Garlappi, L., Uppal, R., Wang, T., 2007. Portfolio selection with parameter and model uncertainty: A multi-prior approach. Review of Financial Studies 20, 41–81.

Garleanu, N. B., Pedersen, L. H., 2013. Dynamic trading with ppredictable returns and transaction costs. Journal of Finance 68, 2309–2340.

Gibbons, M. R., Ross, S. A., Shanken, J., 1989. A test of the efficiency of a given portfolio. Econometrica pp. 1121–1152.

Green, J., Hand, J. R. M., Zhang, X. F., 2017. The characteristics that provide independent information about average u.s. monthly stock returns. Review of Financial Studies 30, 4389–4436.

Hou, K., Mo, H., Xue, C., Zhang, L., 2019. Motivating factors. Reivew of Finance 23, 1–35.

Hou, K., Xue, C., Zhang, L., 2015. Digesting anomalies: An investment approach. Review of Financial Studies 28, 650–705.

Hou, K., Xue, C., Zhang, L., 2019. Replicating anomalies. Review of Financial Studies, forthcoming.

Huberman, G., Kandel, S., 1987. Mean-variance spanning. Journal of Finance 42, 873–888.

Jagannathan, R., Ma, T., 2003. Risk reduction in large portfolios: Why imposing the wrong constraints helps. Journal of Finance 58, 1651–1683.

Kan, R., Wang, X., 2017. On the economic value of alphas. Working paper.

Kan, R., Zhou, G., 2012. Tests of mean-variance spanning. Annals of Economics & Finance 13.

Kogan, L., Tian, M. H., 2017. Firm characteristics and empirical factor models: A model-mining experiment. Working paper.

Kozak, S., Nagel, S., Santosh, S., 2018. Interpreting factor models,. Journal of Finance 73, 1183–1223.

MacKinlay, A., 1995. Multifactor models do not explain deviations from the capm. Journal of Financial Economics 38, 3–28.

MacKinlay, A. C., Pastor, L., 2000. Asset pricing models: Implications for expected returns and portfolio selection. Review of Financial Studies 13, 883–916.

Novy-Marx, R., Velikov, M., 2016. A taxonomy of anomalies and their trading costs. Review of Financial Studies 29, 104–147.

Olivares-Nadal, A. V., DeMiguel, V., 2018. Technical note–A robust perspective on transaction costs in portfolio optimization. Operations Research 66, 733–739.

Pastor, L., Stambaugh, R. F., 2000. Comparing asset pricing models: an investment perspective. Journal of Financial Economics 56, 335–381.

Siegel, A. F., Woodgate, A., 2007. Performance of portfolios optimized with estimation error. Management Science 53, 1005–1015.

Stambaugh, R. F., Yuan, Y., 2017. Mispricing factors. Review of Financial Studies 30, 1270–1315.

Tu, J., Zhou, G., 2004. Data-generating process uncertainty: What difference does it make in portfolio decisions? Journal of Financial Economics 72, 385–421.

Tu, J., Zhou, G., 2010. Incorporating economic objectives into bayesian priors: Portfolio choice under parameter uncertainty. Journal of Financial and Quantitative Analysis 45, 959–986.

Tu, J., Zhou, G., 2011. Markowitz meets talmud: A combination of sophisticated and naive diversification strategies. Journal of Financial Economics 99, 204–215.

Wang, Z., 2005. A shrinkage approach to model uncertainty and asset allocation. Review of Financial Studies 18, 673–705.

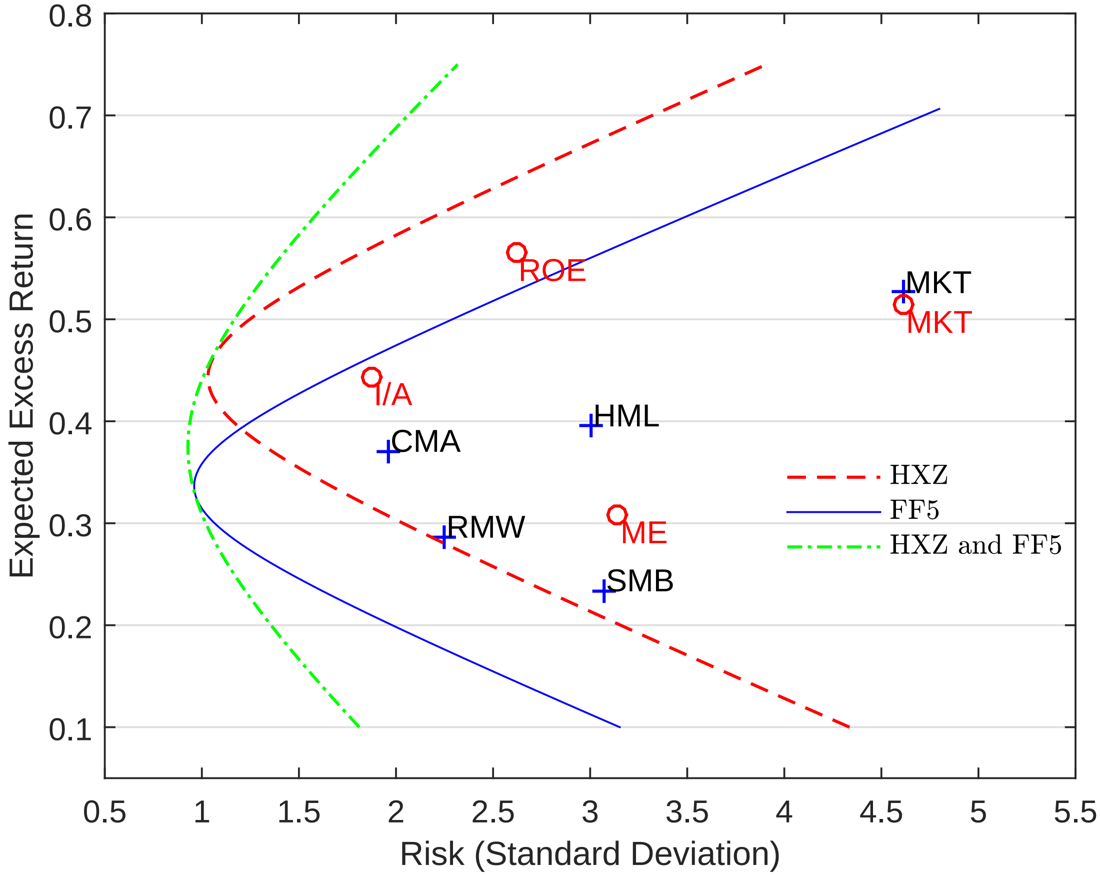

Figure 1 In-sample mean-variance frontiers (in % per month). This figure plots the mean-variance frontiers for investing in the Hou, Xue, and Zhang (HXZ, 2015) four factors, or the Fama and French (FF5, 2015a) five factors, or all the HXZ and FF5 factors. In calculating the frontiers, the HXZ model is assumed to have the same market and size factors as the FF5 model. The sample period is 1972:01–2013:12.

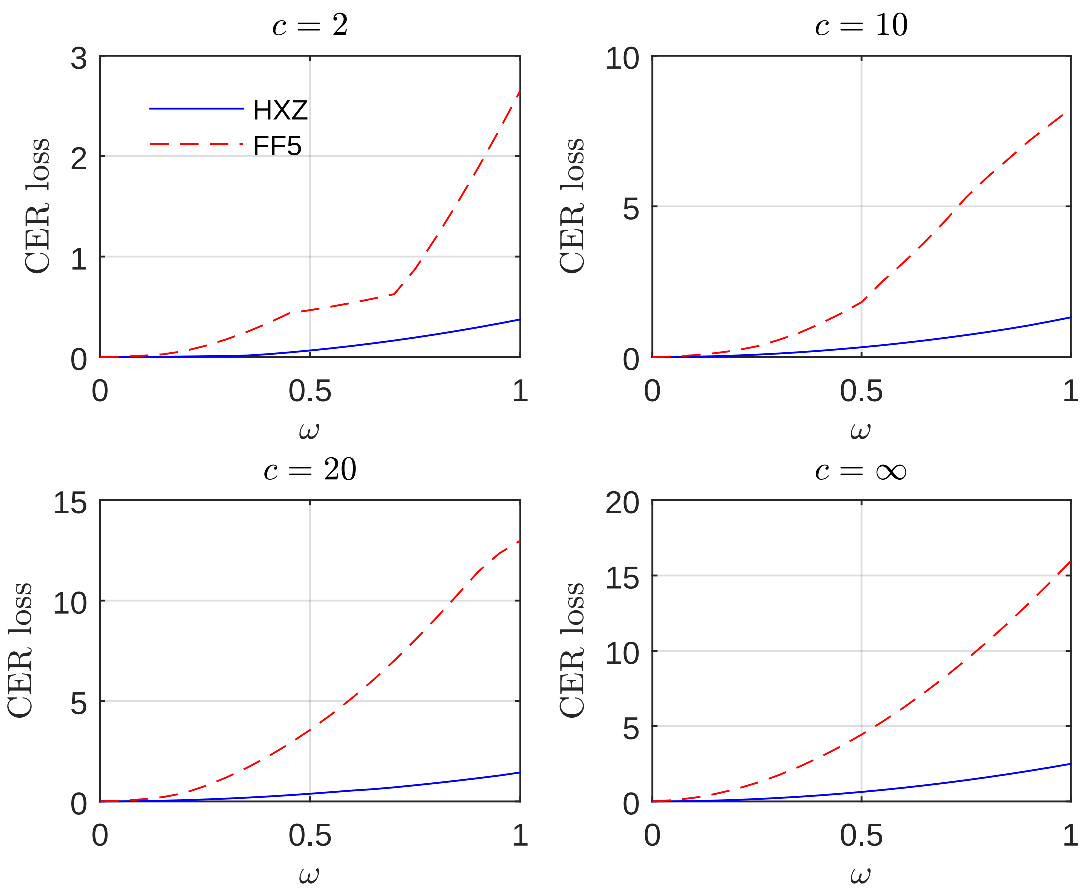

Figure 2 In-sample certainty-equivalent return (CER, in % per year) loss from the perspective of a mean-variance investor, who knows the true mean and covariance but is forced to hold the portfolio chosen by another investor who places a confidence ω in the Hou, Xue, and Zhang (HXZ, 2015) four-factor model or the Fama and French (FF5, 2015a) five-factor model. The risky assets include five anomalies from Novy-Marx and Velikov (2016) that can be explained by the HXZ model but not the FF5 model, five factors in the FF5 model, and investment and profitability factors in the HXZ model. The sample mean and covariance are assumed to be the true mean and covariance. c is the maximum value of risky positions that can be established per dollar of wealth. The investor's risk aversion is set to 3. The sample period is 1973:07–2013:12.

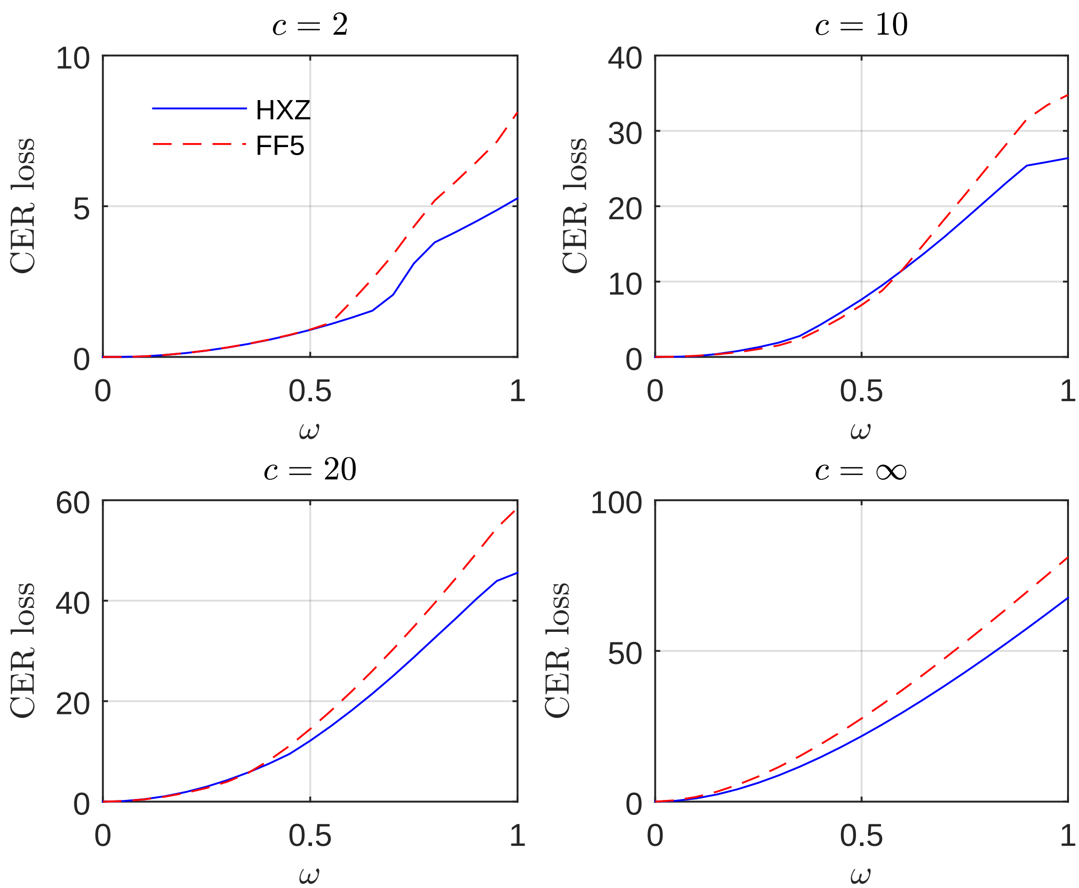

Figure 3 In-sample certainty-equivalent return (CER, in % per year) loss from the perspective of a mean-variance investor, who knows the true mean and covariance but is forced to hold the portfolio chosen by another investor who places a confidence ω in the Hou, Xue, and Zhang (HXZ, 2015) four-factor model or the Fama and French (FF5, 2015a) five-factor model. The risky assets include 10 anomalies from Novy-Marx and Velikov (2016) that cannot be explained by the HXZ or the FF5 models, five factors in the FF5 model, and investment and profitability factors in the HXZ model. c is the maximum value of risky positions that can be established per dollar of wealth. The sample mean and covariance are assumed to be the true mean and covariance. The investor's risk aversion is set to 3. The sample period is 1972:01–2013:12.

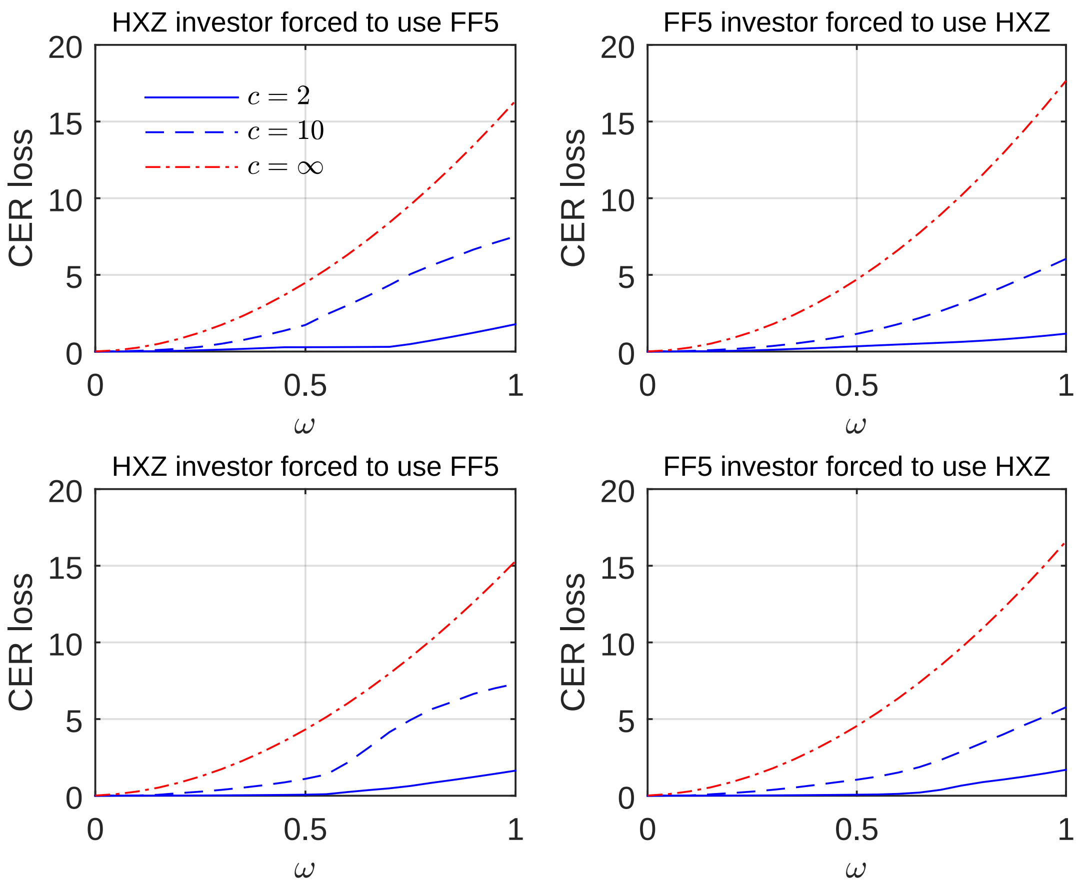

Figure 4 In-sample certainty-equivalent return (CER, in % per year) loss from the perspective of a mean-variance investor, who believes in the Hou, Xue, and Zhang (HXZ, 2015) model with a confidence ω but is forced to hold the portfolio chosen by another investor with the same degree of confidence in the Fama and French (FF5, 2015a) five-factor model (left two panels), and vice versa (right two panels). In addition to the five factors in the FF5 model and the investment and profitability factors in HXZ model, the risky assets also include five anomalies from Novy-Marx and Velikov (2016) that can be explained by the HXZ model but not the FF5 model (upper two papnels), or 10 anomalies that cannot be explained by the HXZ or the FF5 models. c is the maximum value of risky positions that can be established per dollar of wealth. The investor's risk aversion is set to 3. The sample period is 1973:07–2013:12 for upper panels and 1972:01–2013:12 for lower panels.

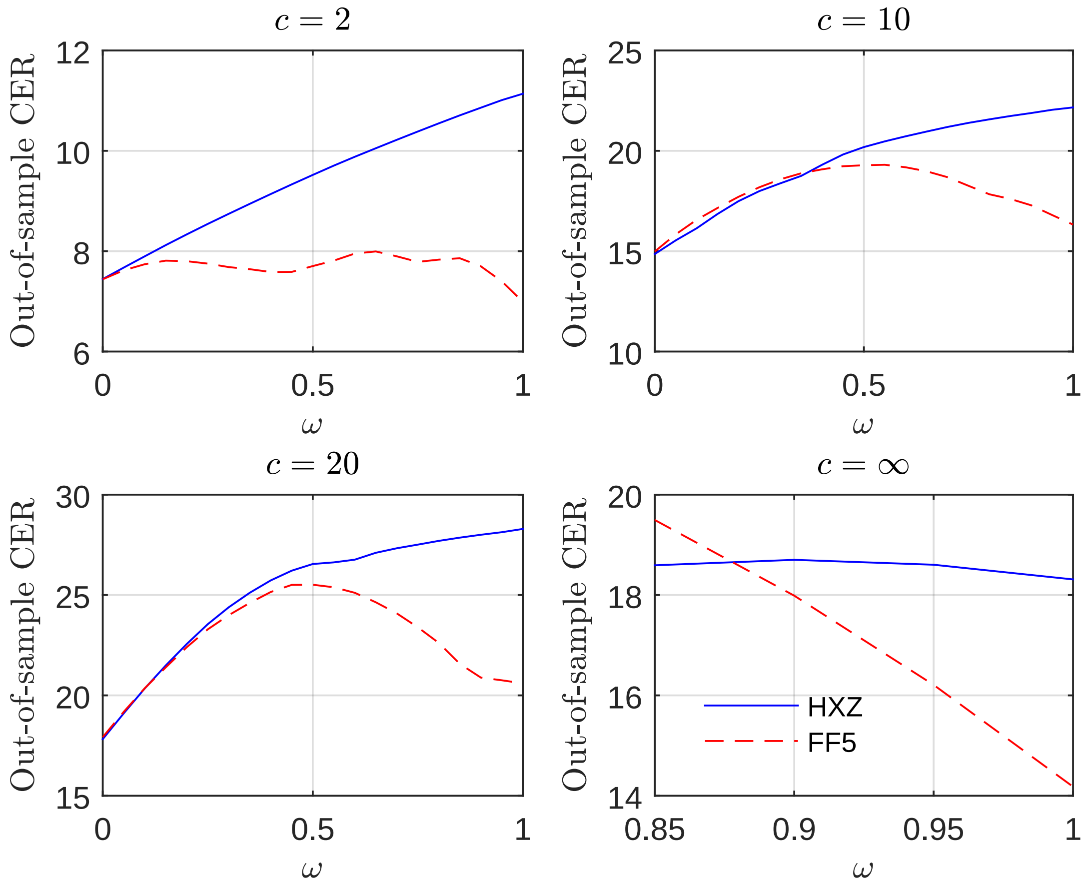

Figure 5 Out-of-sample certainty-equivalent return (CER, in % per year) for a mean-variance Bayesian investor with risk aversion equal to 3. c is the maximum value of risky positions that can be established per dollar of wealth. ω is the confidence level the investor places in the Hou, Xue, and Zhang (HXZ, 2015) four-factor model or the Fama and French (FF5, 2015a) five-factor model. The risky assets include five anomalies from Novy-Marx and Velikov (2016) that can be explained by the HXZ model but not the FF5 model, five factors in the FF5, and investment and profitability factors in the HXZ model. We use an expanding window approach in calculating the out-of-sample CER, where the initial window is 120 months. In each month t, we use data from month 1 to month t to compute the various portfolio rules, and apply them to determine the investments in the next month. For instance, let $\tilde{x}_t$ be the estimated optimal portfolio in month t and r_{t+1} be the excess return on the risky assets realized in month $t+1$. The realized excess return on the portfolio is $r_{\tilde{x},t+1} = \tilde{x}_t' r_{t+1}$. We then compute the mean and variance of the realized returns as $\hat{\mu}_{\tilde{x}}$ and $\hat{\sigma}^2_{\tilde{x}}$. The out-of-sample CER is thus given by $\text{CER}_{\text{OS}} = \hat{\mu}_{\tilde{x}} - \gamma\hat{\sigma}^2_{\tilde{x}}/2$. The sample period is 1973:07–2013:12.

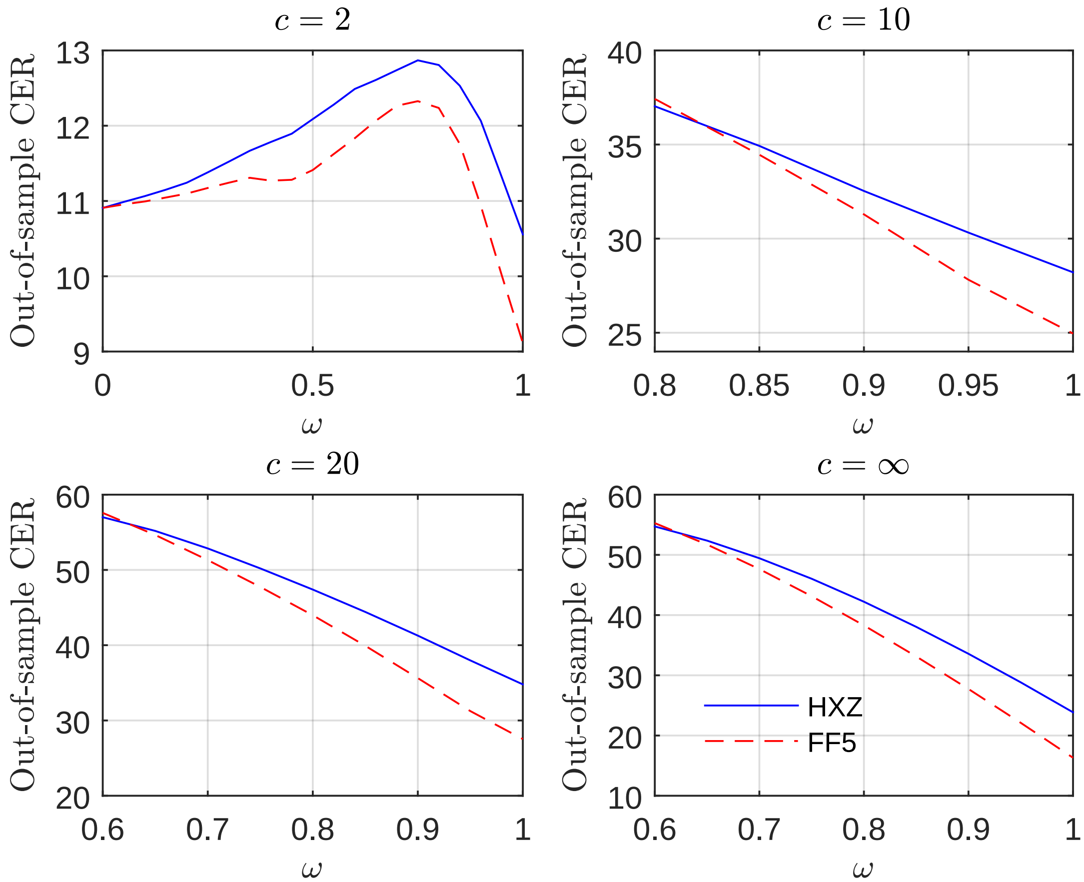

Figure 6 Out-of-sample certainty-equivalent return (CER, in % per year) for a mean-variance Bayesian investor with risk aversion equal to 3. c is the maximum value of risky positions that can be established per dollar of wealth. ω is the confidence level the investor places in the Hou, Xue, and Zhang (HXZ, 2015) four-factor model or the Fama and French (FF5, 2015a) five-factor model. The risky assets include 10 anomalies from Novy-Marx and Velikov (2016) that cannot be explained by the HXZ or the FF5 models, five factors in the FF5, and investment and profitability factors in the HXZ model. We use an expanding window approach in calculating the out-of-sample CER, where the initial window is 120 months. In each month t, we use data from month 1 to month t to compute the various portfolio rules, and apply them to determine the investments in the next month. For instance, let $\tilde{x}_t$ be the estimated optimal portfolio in month t and r_{t+1} be the excess return on the risky assets realized in month $t+1$. The realized excess return on the portfolio is $r_{\tilde{x},t+1} = \tilde{x}_t' r_{t+1}$. We then compute the mean and variance of the realized returns as $\hat{\mu}_{\tilde{x}}$ and $\hat{\sigma}^2_{\tilde{x}}$. The out-of-sample CER is thus given by $\text{CER}_{\text{OS}} = \hat{\mu}_{\tilde{x}} - \gamma\hat{\sigma}^2_{\tilde{x}}/2$. The sample period is 1972:01–2013:12.

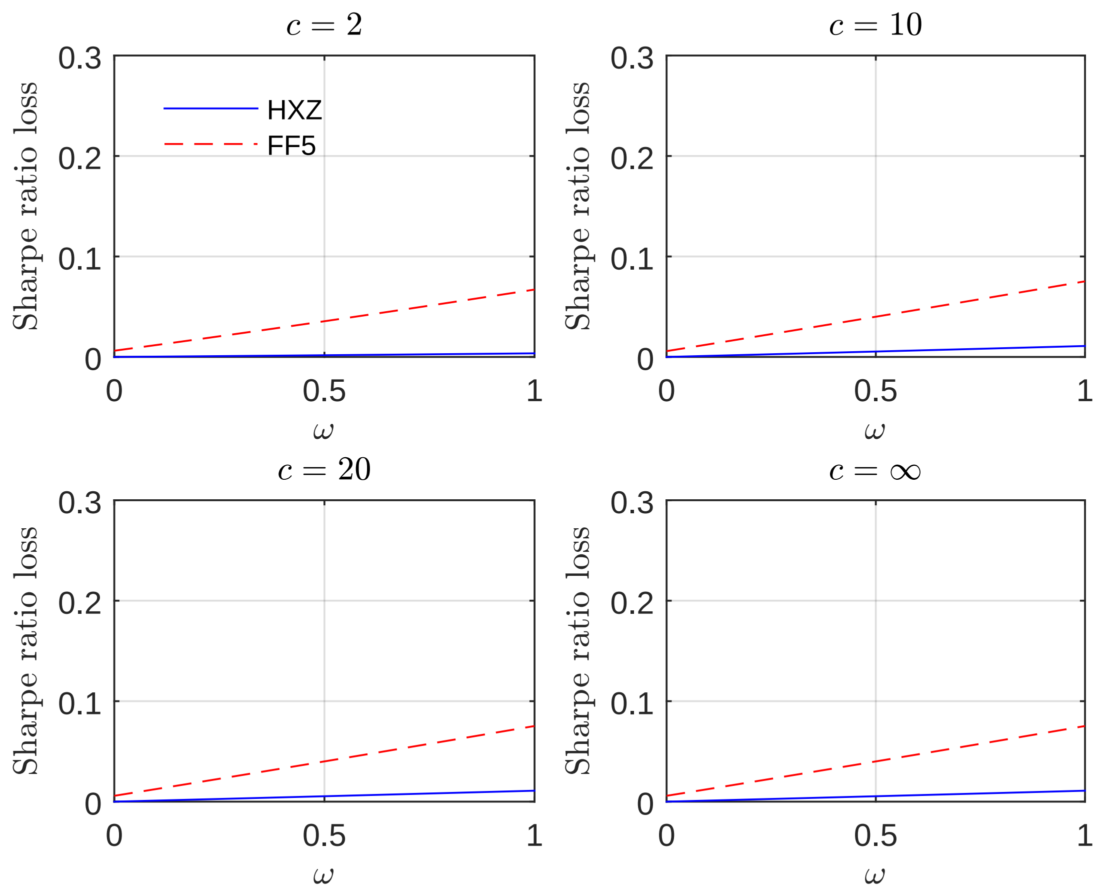

Figure 7 In-sample annualized Sharpe ratio loss from the perspective of a global-minimum-variance investor, who knows the true mean and covariance but is forced to hold the global-minimum-variance portfolio chosen by another investor who places a confidence ω in the Hou, Xue, and Zhang (HXZ, 2015) four-factor model or the Fama and French (FF5, 2015a) five-factor model. The risky assets include five anomalies from Novy-Marx and Velikov (2016) that can be explained by the HXZ but not the FF5 model, five factors in the FF5 model, and investment and profitability factors in the HXZ model. c is the maximum value of risky positions that can be established per dollar of wealth. The sample mean and covariance are assumed to be the true mean and covariance. The sample period is 1973:07–2013:12.

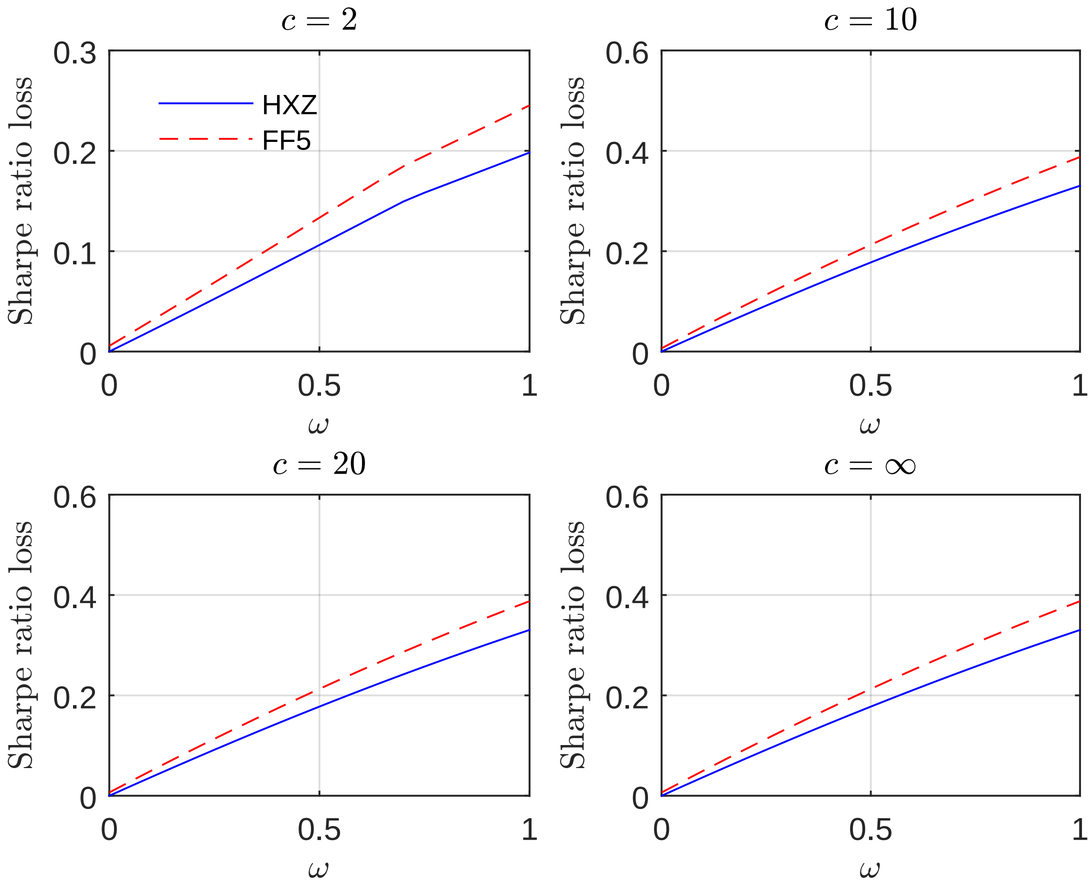

Figure 8 In-sample annualized Sharpe ratio loss from the perspective of a global-minimum-variance investor, who knows the true mean and covariance but is forced to hold global-minimum-variance portfolio chosen by another investor who places a confidence ω in the Hou, Xue, and Zhang (HXZ, 2015) four-factor model or the Fama and French (FF5, 2015a) five-factor model. The risky assets include 10 anomalies from Novy-Marx and Velikov (2016) that cannot be explained by the HXZ and the FF5 models, five factors in the FF5 model, and investment and profitability factors in the HXZ model. c is the maximum value of risky positions that can be established per dollar of wealth. The sample mean and covariance are assumed to be the true mean and covariance. The sample period is 1972:01–2013:12.

Table 1 Summary statistics of factors

Panel A of this table reports the summary statistics of asset pricing factors, where MKT^{HXZ}, ME, I/A, and ROE are the market, size, investment, and profitability factors in Hou, Xue, and Zhang (2015), and MKT, SMB, HML, CMA, and RMW are the market, size, value, investment, and profitability factors in Fama and French (2015a), respectively. t-value is from the test that the average return of the factor is zero, and is calculated using White heteroskedasticity robust standard error. AC(1) represents the first-order autocorrelation. Annualized Sharpe ratio for each individual factor is calculated as the mean return divided by its standard deviation and multiplied by $\sqrt{12}$. Panel B reports the cross-sectional correlations of the factors. The sample period is 1974:01–2016:12.

Panel A: Summary statistics

	Mean	t-value	Volatility	Skewness	Kurtosis	AC(1)	Sharpe ratio
MKT^{HXZ}	0.57	2.85	4.55	−0.57	5.19	0.07	0.43
ME	0.35	2.58	3.05	0.63	9.28	0.00	0.39
I/A	0.40	4.92	1.84	0.21	4.81	0.09	0.75
ROE	0.56	4.84	2.61	−0.71	7.85	0.11	0.74
MKT	0.59	2.96	4.55	−0.54	5.06	0.07	0.45
SMB	0.28	2.14	2.99	0.38	7.20	0.00	0.33
HML	0.36	2.78	2.94	0.08	5.12	0.16	0.42
CMA	0.34	3.91	1.98	0.39	4.89	0.12	0.60
RMW	0.30	2.88	2.36	−0.36	15.31	0.14	0.44

Panel B: Correlation

	MKT^{HXZ}	ME	I/A	ROE	MKT	SMB	HML	CMA	RMW
MKT^{HXZ}	1.00	0.23	−0.37	−0.21	1.00	0.22	−0.27	−0.39	−0.28
ME		1.00	−0.10	−0.32	0.22	0.97	0.00	−0.01	−0.40
I/A			1.00	0.08	−0.37	−0.13	0.67	0.91	0.18
ROE				1.00	−0.20	−0.39	−0.10	−0.04	0.67
MKT					1.00	0.22	−0.27	−0.40	−0.27
SMB						1.00	−0.04	−0.04	−0.40
HML							1.00	0.69	0.17
CMA								1.00	0.07
RMW									1.00

Table 2 Mean-variance spanning tests

This table reports the results of testing whether factors in Fama and French (2015a) can be spanned by the Hou, Xue, and Zhang (2015) four factors. W is the Wald test under conditional homoskedasticity, W_e is the Wald test under the i.i.d. elliptical distribution, W_a is the Wald test under the conditional heteroskedasticity, J_1 is the Bekerart-Urias test with the Errors-in-Variables (EIV) adjustment, J_2 is the Bekerart-Urias test without the EIV adjustment, and J_3 is the DeSantis test. All of the six tests have an asymptotic Chi-Squared distribution with $2m$ degrees of freedom, where m is the number of non-benchmark assets. As a comparison, the last column reports the GRS statistics of Gibbons, Ross, and Shanken (GRS, 1989). The p-values are reported in the parentheses. The sample period is 1974:01–2016:12.

Non-benchmarks	W	W_e	W_a	J_1	J_2	J_3	GRS
FF5	169.38	76.83	103.97	60.40	58.74	88.00	0.97
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.44)
CMA	19.95	10.31	13.33	10.95	10.92	12.79	0.31
	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.58)
RMW	71.30	28.78	35.33	17.38	17.26	20.90	0.19
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.67)

Table 3 Anomalies with significant HXZ and FF5 alphas

This table reports the summary statistics of 20 anomalies without and with taking account of transaction costs when constructing the long-short spread portfolios. Following DeMiguel et al. (2018), the transaction cost for stock i at time t is calculated as $\kappa_{i,t} = y_t z_{i,t}$, where y_t and $z_{i,t}$ capture the variation of the transaction cost parameter with time and firm size, respectively. Specifically, y_t decreases linearly from 3.3 in January 1974 to 1.0 in January 2002, and after that it remains at 1.0; $z_{i,t} = 0.006 - 0.0025 \times me_{i,t}$, where $me_{i,t}$ is the market capitalization of firm i at time t after being normalized cross-sectoinally so that it takes values between zero and one. The sample period is 1974:01–2016:12.

	Without transaction costs						With transaction costs					
Anomaly	Mean	t_{Mean}	α_{FF5}	t_{FF5}	α_{HXZ}	t_{HXZ}	Mean	t_{Mean}	α_{FF5}	t_{FF5}	α_{HXZ}	t_{HXZ}
Lbp	0.41	2.02	0.88	5.46	0.92	4.99	0.31	1.49	0.77	4.78	0.82	4.46
Cei	0.53	2.89	0.29	2.29	0.33	2.28	0.47	2.52	0.22	1.75	0.26	1.82
Noa	0.53	3.74	0.70	4.91	0.60	3.97	0.43	3.07	0.60	4.22	0.52	3.38
dWc	0.21	2.18	0.35	3.62	0.35	3.52	0.11	1.19	0.25	2.64	0.26	2.65
dFin	0.14	1.49	0.34	3.86	0.27	2.82	0.05	0.49	0.24	2.78	0.18	1.87
Cop	0.54	3.02	0.60	5.02	0.50	3.88	0.45	2.50	0.51	4.20	0.42	3.17
Aton	0.30	2.00	0.32	2.25	0.30	2.05	0.24	1.61	0.26	1.84	0.25	1.67
Atoq	0.61	3.95	0.63	4.09	0.51	3.28	0.34	2.19	0.36	2.33	0.25	1.58
Olaq	0.71	3.46	0.79	4.68	0.41	2.60	0.23	1.08	0.31	1.82	−0.06	−0.35
Claq	0.78	4.66	0.84	5.02	0.72	4.23	0.12	0.69	0.19	1.11	0.08	0.47
Tbiq	0.15	1.06	0.33	2.22	0.37	2.46	−0.30	−2.08	−0.12	−0.81	−0.07	−0.48
Oq	0.29	1.42	0.71	5.14	0.48	3.46	0.03	0.12	0.45	3.22	0.22	1.59
Kzq	0.22	1.15	0.50	3.07	0.38	2.27	0.00	−0.02	0.28	1.70	0.16	0.97
Acc	0.48	3.44	0.58	4.08	0.53	3.56	0.35	2.57	0.45	3.19	0.41	2.79
Cashdebt	0.15	0.85	0.38	2.96	0.40	2.79	0.08	0.43	0.31	2.38	0.33	2.30
Chinv	0.44	3.26	0.40	3.08	0.34	2.48	0.32	2.34	0.27	2.09	0.22	1.61
Gma	0.09	0.59	0.39	2.87	0.33	2.30	0.03	0.19	0.32	2.39	0.27	1.89
Pchcurrat	0.20	1.92	0.31	2.81	0.28	2.50	0.06	0.53	0.16	1.41	0.15	1.26
Cashq	0.30	1.57	0.77	5.01	0.61	3.30	0.01	0.03	0.48	3.09	0.33	1.75
Ear	0.77	5.30	0.87	5.74	0.69	4.51	0.36	2.47	0.46	3.05	0.28	1.82

Table 4 Predictive means and standard deviations of anomalies.

This table reports the predictive means (in percentage) and standard deviations of the anomaly long-short spread returns from Novy-Marx and Velikov (2016) and different benchmark factors in Hou, Xue, and Zhang (HXZ, 2015) and Fama and French (FF5, 2015a), respectively. The statistics are based on the predictive distribution using monthly returns over 1973:07–2013:12 for Panels A and C and over 1972:01–2013:12 for Panels B and D. ω is the confidence level the investor places in the HXZ or the FF5 model.

	$\omega = 1$		$\omega = 0.75$		$\omega = 0.5$		$\omega = 0$
	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
Panel A: Predictive means of anomalies that can be explained by HXZ but not FF5							
RetBE	0.70	0.25	0.70	0.36	0.70	0.48	0.71
ValMom	0.87	0.45	0.87	0.55	0.87	0.66	0.88
IVOL	0.41	0.19	0.46	0.30	0.51	0.41	0.62
Momentum	1.04	0.24	1.07	0.47	1.10	0.70	1.16
RetME	0.86	0.46	0.92	0.63	0.98	0.79	1.11
HML	0.33	0.37	0.34	0.37	0.35	0.37	0.37
CMA	0.36	0.38	0.36	0.38	0.37	0.38	0.38
RMW	0.26	0.29	0.27	0.29	0.27	0.29	0.29
I/A	0.44	0.35	0.44	0.37	0.44	0.39	0.44
ROE	0.58	0.19	0.58	0.29	0.58	0.39	0.58
SMB	0.31	0.31	0.31	0.31	0.31	0.31	0.31
MKT	0.56	0.56	0.56	0.56	0.56	0.56	0.56
Panel B: Predictive means of anomalies that cannot be explained by HXZ or FF5							
Accruals	−0.07	−0.02	0.01	0.05	0.09	0.12	0.26
Net issuance	0.40	0.41	0.49	0.49	0.58	0.58	0.76
Investment	0.31	0.26	0.37	0.33	0.43	0.40	0.55
Gross margins	−0.27	−0.26	−0.20	−0.19	−0.13	−0.12	0.02
ValMomProf	0.71	0.37	0.89	0.64	1.07	0.90	1.43
IndMom	0.18	−0.04	0.33	0.17	0.48	0.37	0.78
IndRelRev	0.10	0.21	0.30	0.38	0.50	0.55	0.90
HighFreqCom	0.35	0.21	0.63	0.52	0.90	0.83	1.45
Seasonality	0.06	0.03	0.25	0.22	0.43	0.41	0.80
IndLowVol	0.29	0.26	0.48	0.46	0.68	0.66	1.07
HML	0.33	0.39	0.35	0.39	0.36	0.39	0.39
CMA	0.36	0.37	0.36	0.37	0.36	0.37	0.37
RMW	0.26	0.29	0.27	0.29	0.27	0.29	0.29
I/A	0.44	0.34	0.44	0.37	0.44	0.39	0.44
ROE	0.57	0.18	0.57	0.28	0.57	0.37	0.57
SMB	0.23	0.23	0.23	0.23	0.23	0.23	0.23
MKT	0.53	0.53	0.53	0.53	0.53	0.53	0.53

Table 4 (continued)

	$\omega = 1$		$\omega = 0.75$		$\omega = 0.5$		$\omega = 0$
	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
Panel C: Predictive standard deviations of anomalies that can be explained by HXZ but not FF5							
RetBE	5.30	5.36	5.30	5.35	5.30	5.33	5.30
ValMom	5.01	5.07	5.01	5.06	5.01	5.04	5.00
IVOL	7.53	7.55	7.53	7.54	7.52	7.53	7.52
Momentum	7.30	7.39	7.29	7.37	7.29	7.34	7.27
RetME	5.36	5.42	5.35	5.40	5.34	5.37	5.31
HML	3.06	3.05	3.06	3.05	3.06	3.05	3.06
CMA	2.03	2.02	2.03	2.02	2.03	2.02	2.02
RMW	2.30	2.28	2.30	2.28	2.29	2.28	2.29
I/A	1.90	1.92	1.90	1.92	1.90	1.91	1.90
ROE	2.64	2.71	2.64	2.70	2.64	2.68	2.64
SMB	3.09	3.09	3.09	3.09	3.09	3.09	3.09
MKT	4.69	4.70	4.69	4.70	4.69	4.70	4.69
Panel D: Predictive standard deviations of anomalies that cannot be explained by HXZ or FF5							
Accruals	3.27	3.27	3.26	3.27	3.26	3.26	3.25
Net issuance	3.12	3.12	3.10	3.10	3.08	3.08	3.05
Investment	3.19	3.19	3.18	3.18	3.17	3.17	3.15
Gross margins	3.26	3.26	3.26	3.26	3.26	3.27	3.27
ValMomProf	5.17	5.22	5.13	5.16	5.09	5.11	5.01
IndMom	6.24	6.25	6.22	6.23	6.21	6.22	6.18
IndRelRev	4.63	4.63	4.60	4.60	4.58	4.58	4.53
HighFreqCom	4.04	4.05	3.97	3.98	3.90	3.91	3.77
Seasonality	4.21	4.21	4.18	4.19	4.16	4.17	4.12
IndLowVol	3.67	3.67	3.63	3.63	3.59	3.59	3.51
HML	3.06	3.03	3.06	3.03	3.05	3.03	3.05
CMA	2.02	2.01	2.02	2.01	2.01	2.01	2.01
RMW	2.28	2.26	2.28	2.26	2.28	2.26	2.28
I/A	1.88	1.91	1.88	1.91	1.88	1.90	1.88
ROE	2.63	2.71	2.63	2.70	2.63	2.68	2.63
SMB	3.09	3.10	3.09	3.10	3.09	3.10	3.09
MKT	4.64	4.64	4.64	4.64	4.64	4.64	4.64

Table 5 Optimal allocations (in-sample) for investing in anomalies that can be explained by the HXZ but not the FF5.

This table reports optimal allocations (position sizes) per \$100 of wealth for a mean-variance Bayesian investor with relative risk aversion equal to 3. c is the maximum value of risky positions that can be established per dollar of wealth. ω is the confidence level the investor places in the Hou, Xue, and Zhang (HXZ, 2015) four-factor model or the Fama and French (FF5, 2015a) five-factor model. Optimization is based on the predictive distribution using monthly returns over 1973:07–2013:12. The risky assets include five anomalies from Novy-Marx and Velikov (2016) that can be explained by the HXZ model but not the FF5 model, five factors in the FF5, and investment and profitability factors in the HXZ model. Also reported are the certainty-equivalent return (CER, in % per year), and annualized Sharpe ratio (SR) of the portfolio's return with respect to the given predictive distribution.

	$\omega = 1$		$\omega = 0.75$		$\omega = 0.5$		$\omega = 0$
	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
			Panel A: $c = 2$				
RetBE	19.8	0.0	18.7	0.0	17.6	6.8	3.5
ValMom	37.3	12.6	34.3	29.6	31.3	40.2	19.8
IVOL	0.0	6.5	0.0	14.5	0.0	14.4	12.1
Momentum	12.4	0.0	16.4	0.0	20.4	0.0	31.0
RetME	0.0	0.0	0.0	0.0	0.0	0.0	0.0
HML	0.0	0.0	0.0	0.0	0.0	0.0	0.0
CMA	0.0	44.7	0.0	16.6	0.0	0.0	0.0
RMW	0.0	0.0	0.0	0.0	0.0	0.0	0.0
I/A	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ROE	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SMB	0.0	0.0	0.0	0.0	0.0	0.0	0.0
MKT	61.0	72.5	61.2	78.5	61.4	77.1	67.2
CER	9.0	6.2	9.1	6.6	9.2	7.3	9.5
SR	0.93	0.75	0.93	0.75	0.92	0.79	0.91

Table 5 (continued)

	$\omega = 1$		$\omega = 0.75$		$\omega = 0.5$		$\omega = 0$
	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
			Panel B: $c = 10$				
RetBE	22.2	4.7	22.9	32.0	23.6	41.8	16.4
ValMom	54.7	7.5	49.1	18.0	43.5	16.9	28.8
IVOL	0.0	0.0	0.0	7.7	0.0	22.6	18.9
Momentum	0.5	0.0	6.4	0.8	12.2	7.2	24.9
RetME	0.0	0.0	0.0	0.0	0.0	0.0	0.0
HML	0.0	0.0	0.0	0.0	0.0	0.0	0.0
CMA	0.0	259.4	0.0	242.5	0.0	133.4	0.0
RMW	0.0	126.5	0.0	77.1	0.0	1.3	0.0
I/A	189.3	0.0	194.8	0.0	200.3	100.5	191.8
ROE	134.2	0.0	127.1	0.0	120.0	30.1	98.5
SMB	38.2	38.8	38.6	55.1	39.0	77.3	55.5
MKT	121.8	126.2	122.2	133.7	122.7	137.8	130.3
CER	26.6	19.4	26.7	19.9	26.7	21.3	27.0
SR	1.42	1.21	1.42	1.22	1.42	1.25	1.43
			Panel C: $c = \infty$				
RetBE	0.0	0.0	−16.8	−16.1	−33.7	−32.6	−67.7
ValMom	0.0	0.0	−15.4	−14.7	−30.9	−29.9	−62.1
IVOL	0.0	0.0	13.7	13.1	27.5	26.6	55.2
Momentum	0.0	0.0	10.3	9.9	20.7	20.1	41.6
RetME	0.0	0.0	17.7	16.9	35.5	34.4	71.3
HML	0.0	−70.9	12.8	−41.3	25.7	-11.2	51.7
CMA	0.0	577.8	28.7	463.7	57.5	350.3	115.6
RMW	0.0	376.7	−2.9	281.7	−5.7	186.5	−11.5
I/A	597.3	0.0	560.9	105.0	524.9	213.6	454.1
ROE	416.5	0.0	403.3	84.8	390.4	172.5	365.6
SMB	238.2	161.3	255.6	194.2	273.4	229.4	309.8
MKT	180.6	187.6	189.2	192.2	198.2	198.5	216.6
CER	40.7	27.4	40.8	28.3	41.2	31.0	42.6
SR	1.56	1.28	1.57	1.30	1.57	1.36	1.60

Table 6 Optimal allocations (in-sample) for investing in anomalies that cannot be explained by the HXZ or FF5.

This table reports optimal allocations (position sizes) per $100 of wealth for a mean-variance Bayesian investor with risk aversion equal to 3. c is the maximum value of risky positions that can be established per dollar of wealth. ω is the confidence level the investor places in the Hou, Xue, and Zhang (HXZ, 2015) four-factor model or the Fama and French (FF5, 2015a) five-factor model. Optimization is based on the predictive distribution using monthly returns over 1972:01–2013:12. The risky assets include 10 anomalies from Novy-Marx and Velikov (2016) that cannot be explained by the HXZ and the FF5 model, five factors in the FF5, and investment and profitability factors in the HXZ model. Also reported are the certainty-equivalent return (CER, in % per year), and annualized Sharpe ratio (SR) of the portfolio's return with respect to the given predictive distribution.

	$\omega = 1$		$\omega = 0.75$		$\omega = 0.5$		$\omega = 0$
	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
			Panel A: $c = 2$				
Accruals	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Net issuance	0.0	26.6	0.0	34.3	0.0	0.0	0.0
Investment	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Gross margins	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ValMomProf	33.1	3.6	54.8	30.6	51.7	42.3	33.7
IndMom	0.0	0.0	0.0	0.0	0.0	0.0	0.0
IndRelRev	0.0	0.0	0.0	0.0	0.0	0.0	0.0
HighFreqCom	0.0	0.0	4.3	3.1	30.7	37.5	66.3
Seasonality	0.0	0.0	0.0	0.0	0.0	0.0	0.0
IndLowVol	0.0	0.0	0.0	0.0	0.0	0.4	0.0
HML	0.0	25.6	0.0	1.2	0.0	0.0	0.0
CMA	0.0	11.3	0.0	0.0	0.0	0.0	0.0
RMW	0.0	0.0	0.0	0.0	0.0	0.0	0.0
I/A	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ROE	39.4	0.0	13.8	0.0	0.0	0.0	0.0
SMB	0.0	0.0	0.0	0.0	0.0	0.0	0.0
MKT	55.0	65.8	54.3	61.6	35.3	39.7	0.0
CER	7.4	5.9	8.4	6.8	10.3	9.1	15.6
SR	0.88	0.75	0.89	0.80	1.10	0.99	1.62

Table 6 (continued)

	$\omega = 1$		$\omega = 0.75$		$\omega = 0.5$		$\omega = 0$
	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
			Panel B: $c = 10$				
Accruals	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Net issuance	0.0	19.5	3.9	54.9	39.9	92.6	36.4
Investment	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Gross margins	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ValMomProf	42.2	12.4	65.5	46.8	85.5	73.9	118.6
IndMom	0.0	0.0	0.0	0.0	0.0	0.0	0.0
IndRelRev	0.0	0.0	0.0	7.7	9.5	26.2	33.5
HighFreqCom	0.0	0.0	31.3	31.4	86.8	84.0	173.2
Seasonality	0.0	0.0	0.0	0.0	0.0	11.2	27.0
IndLowVol	0.0	0.0	26.5	30.1	65.9	64.3	103.4
HML	0.0	0.0	0.0	3.3	0.0	6.1	0.0
CMA	0.0	259.7	0.0	164.4	0.0	0.0	0.0
RMW	0.0	130.1	0.0	85.8	0.0	0.0	0.0
I/A	241.4	0.0	185.6	18.4	76.3	74.2	0.0
ROE	155.3	0.0	134.1	0.0	98.0	29.4	73.0
SMB	0.0	13.5	0.0	0.0	0.0	0.0	0.0
MKT	122.2	130.5	106.3	114.6	76.1	76.2	15.6
CER	25.3	18.4	27.2	20.6	32.7	28.1	55.7
SR	1.39	1.15	1.45	1.22	1.58	1.42	2.11
			Panel C: $c = \infty$				
Accruals	0.0	0.0	23.9	22.8	50.9	49.2	117.1
Net issuance	0.0	0.0	34.9	33.3	74.4	71.9	171.1
Investment	0.0	0.0	15.4	14.7	32.8	31.6	75.3
Gross margins	0.0	0.0	20.3	19.4	43.3	41.8	99.6
ValMomProf	0.0	0.0	28.5	27.1	60.6	58.5	139.3
IndMom	0.0	0.0	6.0	5.7	12.8	12.4	29.5
IndRelRev	0.0	0.0	34.0	32.4	72.4	69.9	166.4
HighFreqCom	0.0	0.0	50.8	48.4	108.3	104.5	248.8
Seasonality	0.0	0.0	27.8	26.5	59.2	57.1	136.0
IndLowVol	0.0	0.0	44.8	42.7	95.4	92.1	219.3
HML	0.0	−39.2	3.4	−26.5	7.2	−13.2	16.5
CMA	0.0	531.3	19.7	421.1	41.9	313.8	96.3
RMW	0.0	357.2	−5.6	265.2	−11.9	172.3	−27.4
I/A	598.0	0.0	504.5	49.2	417.6	106.2	269.5
ROE	396.6	0.0	379.0	74.9	371.4	161.6	395.8
SMB	201.5	127.2	184.9	127.1	172.4	131.8	163.7
MKT	178.2	182.7	157.7	160.0	140.1	140.7	116.1
CER	37.8	24.5	41.3	28.4	52.5	41.4	105.4
SR	1.51	1.21	1.57	1.31	1.78	1.58	2.52

Table 7 Out-of-sample certainty-equivalent return (CER) for investing in anomalies that can be explained by the HXZ but not the FF5.

This table reports the out-of-sample CER (in % per year) for a mean-variance Bayesian investor with risk aversion equal to 3. c is the maximum value of risky positions that can be established per dollar of wealth. ω is the confidence level the investor places in the Hou, Xue, and Zhang (HXZ, 2015) four-factor model or the Fama and French (FF5, 2015a) five-factor model. The risky assets include five anomalies from Novy-Marx and Velikov (2016) that can be explained by the HXZ model but not the FF5 model, five factors in the FF5, and investment and profitability factors in the HXZ model over 1973:07–2013:12. We randomly sample (with replacement) $T+300$ returns of the risky assets and use the first T to calculate the portfolio weights, which are used to the remaining 300 observations for calculating the out-of-sample CER. The procedure is repeated 1,000 times; average CERs are shown.

	$\omega=1$		$\omega=0.95$		$\omega=0.90$		$\omega=0.75$		$\omega=0.50$		$\omega=0$
T	HXZ	FF5	HXZ	FF5	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
Panel A: $c=2$											
60	6.3	4.6	6.3	4.8	6.4	5.0	6.4	5.4	6.5	5.9	6.2
120	7.5	5.6	7.6	5.8	7.6	6.0	7.7	6.5	7.7	7.1	7.5
240	8.7	6.5	8.8	6.7	8.8	7.0	8.9	7.6	8.9	8.4	8.7
360	9.2	6.8	9.2	7.1	9.3	7.4	9.4	8.1	9.4	8.9	9.2
600	9.6	7.3	9.7	7.6	9.7	8.0	9.8	8.8	9.9	9.5	9.8
Panel B: $c=10$											
60	18.1	12.2	18.1	12.6	18.1	13.0	17.9	14.2	16.4	15.1	11.0
120	22.2	15.9	22.3	16.4	22.3	16.9	22.2	18.3	21.5	19.9	18.1
240	24.8	18.2	24.8	18.7	24.9	19.2	24.9	20.8	24.6	22.9	22.7
360	25.7	19.2	25.8	19.7	25.9	20.2	26.0	21.8	25.9	24.1	24.6
600	26.4	19.8	26.5	20.4	26.5	20.9	26.7	22.5	26.7	25.0	26.0
Panel C: $c=\infty$											
60	14.8	−4.0	14.7	−2.7	14.3	−1.7	10.9	−0.7	−3.6	−8.0	−90.1
120	29.3	14.0	29.3	15.3	29.2	16.5	27.9	18.9	22.5	18.9	−4.6
240	35.2	21.1	35.3	22.4	35.3	23.7	34.8	26.7	32.6	29.3	21.8
360	37.3	23.3	37.4	24.7	37.4	26.0	37.2	29.2	35.9	32.5	29.6
600	38.7	24.9	38.8	26.2	38.8	27.5	38.8	30.8	38.2	34.7	34.8

Table 8 Out-of-sample annualized Share ratio for investing in anomalies that can be explained by the HXZ but not the FF5.

This table reports the out-of-sample Sharpe ratio for a mean-variance Bayesian investor with risk aversion equal to 3. c is the maximum value of risky positions that can be established per dollar of wealth. ω is the confidence level the investor places in the Hou, Xue, and Zhang (HXZ, 2015) four-factor model or the Fama and French (FF5, 2015a) five-factor model. The risky assets include five anomalies from Novy-Marx and Velikov (2016) that can be explained by the HXZ model but not the FF5 model, five factors in the FF5, and investment and profitability factors in the HXZ model over 1973:07–2013:12. We randomly sample (with replacement) $T+300$ returns of the risky assets and use the first T to calculate the portfolio weights, which are used to the remaining 300 observations for calculating the out-of-sample Sharpe ratio. The procedure is repeated 1,000 times; average Sharpe ratios (annualized by multiplying $\sqrt{12}$) are shown.

	$\omega=1$		$\omega=0.95$		$\omega=0.90$		$\omega=0.75$		$\omega=0.50$		$\omega=0$
T	HXZ	FF5	HXZ	FF5	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
Panel A: $c=2$											
60	0.71	0.63	0.71	0.64	0.72	0.65	0.72	0.67	0.71	0.69	0.68
120	0.79	0.72	0.79	0.73	0.80	0.73	0.80	0.76	0.80	0.77	0.77
240	0.88	0.79	0.88	0.81	0.88	0.82	0.89	0.85	0.88	0.87	0.86
360	0.92	0.84	0.93	0.86	0.93	0.87	0.93	0.90	0.93	0.92	0.90
600	0.97	0.88	0.97	0.90	0.98	0.92	0.98	0.95	0.98	0.97	0.96
Panel B: $c=10$											
60	1.12	0.92	1.12	0.93	1.12	0.95	1.11	0.99	1.08	1.02	0.99
120	1.23	1.03	1.23	1.05	1.23	1.07	1.22	1.11	1.20	1.15	1.12
240	1.33	1.14	1.33	1.16	1.33	1.17	1.32	1.22	1.30	1.27	1.23
360	1.37	1.19	1.37	1.21	1.37	1.22	1.37	1.27	1.35	1.32	1.29
600	1.40	1.23	1.40	1.25	1.40	1.27	1.40	1.31	1.38	1.36	1.33
Panel C: $c=\infty$											
60	1.34	0.98	1.34	1.00	1.34	1.02	1.32	1.07	1.24	1.08	1.06
120	1.45	1.11	1.45	1.14	1.45	1.16	1.44	1.22	1.38	1.26	1.24
240	1.51	1.19	1.52	1.22	1.52	1.25	1.51	1.32	1.48	1.38	1.38
360	1.54	1.22	1.54	1.25	1.54	1.28	1.54	1.36	1.52	1.43	1.44
600	1.55	1.25	1.56	1.28	1.56	1.31	1.56	1.39	1.55	1.47	1.50

Table 9 Out-of-sample certainty-equivalent return (CER) for investing in anomalies that cannot be explained by the HXZ or the FF5.

This table reports the out-of-sample CER (in % per year) for a mean-variance Bayesian investor with risk aversion equal to 3. c is the maximum value of risky positions that can be established per dollar of wealth. ω is the confidence level the investor places in the Hou, Xue, and Zhang (HXZ, 2015) four-factor model or the Fama and French (FF5, 2015a) five-factor model. The risky assets include 10 anomalies from Novy-Marx and Velikov (2016) that cannot be explained by the HXZ or the FF5 models, five factors in the FF5, and investment and profitability factors in the HXZ model over 1972:01–2013:12. We randomly sample (with replacement) $T+300$ returns of the risky assets and use the first T to calculate the portfolio weights, which are used to the remaining 300 observations for calculating the out-of-sample CER. The procedure is repeated 1,000 times; average CERs are shown.

	$\omega=1$		$\omega=0.95$		$\omega=0.90$		$\omega=0.75$		$\omega=0.50$		$\omega=0$
T	HXZ	FF5	HXZ	FF5	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
					Panel A: $c=2$						
60	6.7	5.1	7.0	5.5	7.4	6.0	8.6	7.6	10.4	10.0	11.7
120	7.9	5.7	8.3	6.2	8.7	6.8	10.1	9.0	12.0	11.6	13.1
240	8.7	6.3	9.1	6.8	9.5	7.4	11.0	9.8	13.0	12.6	14.0
360	9.0	6.5	9.4	7.1	9.8	7.8	11.4	10.2	13.4	13.0	14.4
600	9.3	6.7	9.8	7.4	10.2	8.1	11.7	10.2	13.8	13.5	14.9
					Panel B: $c=10$						
60	20.6	12.6	21.7	14.2	23.1	16.2	28.0	23.5	35.9	34.2	41.2
120	23.9	15.7	25.0	17.3	26.5	19.6	32.2	28.1	41.3	40.1	47.7
240	26.1	17.8	27.1	19.4	28.4	21.7	34.7	31.1	44.6	44.1	51.4
360	27.0	18.6	27.9	20.1	29.2	22.3	35.6	32.1	45.9	45.6	52.8
600	27.8	19.4	28.5	20.8	29.7	22.9	36.5	33.1	46.9	46.9	53.7
					Panel C: $c=\infty$						
60	11.3	−7.1	16.0	−1.5	20.4	3.8	31.3	17.8	35.7	29.2	−146
120	25.9	10.8	31.1	16.8	36.1	22.6	49.5	38.8	63.8	58.5	13.8
240	31.7	17.7	37.0	23.8	42.2	29.7	56.6	46.6	75.1	69.8	66.3
360	33.9	20.0	39.2	26.0	44.4	32.0	58.9	48.9	78.3	72.9	80.7
600	35.4	21.6	40.7	27.6	45.9	33.5	60.5	50.7	81.0	75.5	91.6

Table 10 Out-of-sample annualized Share ratio for investing in anomalies that cannot be explained by the HXZ or the FF5.

This table reports the out-of-sample Sharpe ratio for a mean-variance Bayesian investor with risk aversion equal to 3. c is the maximum value of risky positions that can be established per dollar of wealth. ω is the confidence level the investor places in the Hou, Xue, and Zhang (HXZ, 2015) four-factor model or the Fama and French (FF5, 2015a) five-factor model. The risky assets include 10 anomalies from Novy-Marx and Velikov (2016) that cannot be explained by the HXZ or FF5 models, five factors in the FF5, and investment and profitability factors in the HXZ model over 1972:01–2013:12. We randomly sample (with replacement) $T+300$ returns of the risky assets and use the first T to calculate the portfolio weights, which are used to the remaining 300 observations for calculating the out-of-sample Sharpe ratio. The procedure is repeated 1,000 times; average Sharpe ratios (annualized by multiplying $\sqrt{12}$) are shown.

	$\omega=1$		$\omega=0.95$		$\omega=0.90$		$\omega=0.75$		$\omega=0.50$		$\omega=0$
T	HXZ	FF5	HXZ	FF5	HXZ	FF5	HXZ	FF5	HXZ	FF5	All
Panel A: $c=2$											
60	0.85	0.72	0.87	0.75	0.90	0.79	0.99	0.94	1.13	1.11	1.21
120	0.97	0.79	0.99	0.84	1.02	0.89	1.11	1.07	1.25	1.26	1.35
240	1.04	0.84	1.06	0.89	1.09	0.95	1.18	1.15	1.33	1.35	1.46
360	1.08	0.87	1.10	0.93	1.12	0.99	1.21	1.19	1.38	1.41	1.52
600	1.12	0.89	1.14	0.95	1.15	1.01	1.23	1.22	1.43	1.48	1.58
Panel B: $c=10$											
60	1.21	0.93	1.25	0.99	1.30	1.07	1.47	1.33	1.69	1.65	1.73
120	1.33	1.03	1.37	1.10	1.42	1.18	1.63	1.50	1.87	1.85	1.90
240	1.43	1.12	1.46	1.18	1.51	1.27	1.76	1.65	2.03	2.03	2.01
360	1.48	1.16	1.50	1.22	1.56	1.31	1.81	1.72	2.09	2.10	2.04
600	1.52	1.20	1.54	1.26	1.59	1.35	1.87	1.78	2.15	2.17	2.07
Panel C: $c=\infty$											
60	1.28	0.91	1.36	1.01	1.43	1.11	1.62	1.38	1.78	1.65	1.76
120	1.39	1.04	1.48	1.16	1.57	1.29	1.81	1.62	2.03	1.94	2.06
240	1.45	1.11	1.55	1.25	1.64	1.39	1.91	1.76	2.18	2.13	2.25
360	1.47	1.14	1.57	1.29	1.67	1.43	1.94	1.81	2.24	2.20	2.33
600	1.49	1.17	1.59	1.31	1.69	1.46	1.98	1.85	2.29	2.26	2.41

Online Appendix

Table A1 Average returns and alphas of 121 anomalies

This table reports the average returns and alphas of high minus low spread portfolios of 121 anomalies, where the alphas are calculated with the FF3, HXZ, and FF5 models, respectively. All portfolios are value-weighted and rebalanced annually or quarterly, where the first 66 are based on Hou, Xue, and Zhang (2019) and the rest 55 on Green, Hand, and Zhang (2017). An anomaly is included if it is available over the sample period 1974:01–2016:12.

id	Anomaly	Mean	t_{Mean}	α_{FF3}	t_{FF3}	α_{HXZ}	t_{HXZ}	α_{FF5}	t_{FF5}
1	Bm	0.58	2.70	−0.09	−0.67	0.24	1.40	−0.10	−0.71
2	Mv	−0.42	−1.88	0.05	0.53	−0.11	−1.10	−0.08	−0.85
3	Dvp	0.30	1.30	0.18	1.04	0.33	1.75	0.02	0.14
4	Top	0.44	2.40	0.24	1.72	0.26	1.62	0.11	0.80
5	Nop	0.52	2.49	0.48	3.42	0.27	1.63	0.09	0.67
6	Ame	0.32	1.51	−0.42	−3.10	−0.27	−1.56	−0.49	−3.55
7	Em	−0.27	−1.41	0.00	−0.02	−0.65	−3.94	−0.45	−2.80
8	Ssgrow	−0.33	−2.07	−0.19	−1.31	0.02	0.17	0.04	0.34
9	Ebp	0.43	2.28	−0.09	−0.64	0.33	2.04	0.04	0.32
10	Lbp	−0.41	−2.02	−0.42	−2.32	−0.92	−4.99	−0.88	−5.46
11	Ndp	0.63	2.76	0.31	1.52	0.38	1.80	0.45	2.18
12	Dur	−0.65	−3.35	−0.09	−0.71	−0.30	−1.76	−0.03	−0.26
13	Cdi	−0.01	−0.05	−0.01	−0.05	0.14	1.03	0.09	0.71
14	Ndf	−0.29	−2.23	−0.27	−2.11	0.02	0.18	−0.06	−0.44
15	Nxf	−0.30	−1.62	−0.56	−3.91	−0.12	−0.86	−0.17	−1.30
16	Cei	−0.53	−2.89	−0.58	−4.43	−0.33	−2.28	−0.29	−2.29
17	Aci	−0.34	−2.47	−0.35	−2.54	−0.13	−0.91	−0.32	−2.23
18	Noa	−0.53	−3.74	−0.65	−4.60	−0.60	−3.97	−0.70	−4.91
19	Tacc	−0.21	−1.47	−0.07	−0.54	−0.05	−0.38	0.00	−0.02
20	Pta	−0.25	−1.64	−0.22	−1.62	−0.17	−1.20	−0.03	−0.19
21	dWc	−0.21	−2.18	−0.25	−2.66	−0.35	−3.52	−0.35	−3.62
22	dCoa	−0.22	−1.50	−0.02	−0.15	0.17	1.37	0.17	1.44
23	dNco	−0.25	−2.43	−0.16	−1.51	0.16	1.59	0.03	0.34
24	dNca	−0.48	−3.53	−0.36	−2.68	−0.12	−0.96	−0.20	−1.61
25	dNcl	−0.11	−0.94	0.03	0.29	0.16	1.29	0.17	1.39
26	dFin	0.14	1.49	0.21	2.42	0.27	2.82	0.34	3.86
27	dSti	0.17	1.00	0.33	2.01	0.27	1.60	0.34	2.12
28	dLti	−0.01	−0.10	0.09	0.56	0.07	0.42	0.16	1.02
29	dFnl	−0.33	−2.78	−0.34	−2.83	−0.04	−0.32	−0.15	−1.26
30	Cop	0.54	3.02	0.98	7.25	0.50	3.88	0.60	5.02
31	Rna	−0.02	−0.14	0.25	1.69	−0.01	−0.08	0.02	0.16
32	Pm	0.11	0.50	0.50	3.25	0.03	0.23	0.11	0.93
33	Aton	0.30	2.00	0.34	2.42	0.30	2.05	0.32	2.25
34	F_g7	0.24	1.66	0.40	2.87	0.16	1.12	0.28	2.09
35	Oscore	0.07	0.35	−0.41	−2.70	−0.19	−1.29	−0.23	−1.63

Table A1 (continued)

id	Anomaly	Mean	t_{Mean}	α_{FF3}	t_{FF3}	α_{HXZ}	t_{HXZ}	α_{FF5}	t_{FF5}
36	Zscore	−0.23	−1.44	0.18	1.52	0.06	0.39	0.16	1.30
37	Ol	0.35	2.22	0.34	2.17	0.00	0.03	0.06	0.38
38	gAd	0.06	0.31	0.13	0.71	0.40	2.08	0.41	2.19
39	Rdm	0.69	2.86	0.23	1.11	0.71	3.36	0.45	2.15
40	Etr	−0.17	−1.51	−0.14	−1.21	−0.03	−0.26	−0.11	−0.91
41	Lfe	0.18	1.30	0.31	2.28	0.20	1.42	0.38	2.71
42	Kz	0.28	1.65	−0.05	−0.38	−0.10	−0.68	−0.14	−1.08
43	Sa	0.09	0.35	−0.17	−1.51	0.19	1.50	0.27	3.20
44	Ala	−0.04	−0.21	0.00	0.03	0.30	2.15	0.34	2.74
45	Ww	0.05	0.22	−0.32	−2.41	0.02	0.17	0.02	0.18
46	Adm	0.62	2.58	0.08	0.42	0.00	0.01	−0.17	−0.87
47	Bca	0.24	1.07	0.44	2.18	−0.14	−0.71	−0.08	−0.42
48	Oca_ia	0.60	4.53	0.63	4.74	0.17	1.32	0.35	2.71
49	Iaq	−0.29	−1.44	0.11	0.67	−0.04	−0.23	0.31	1.97
50	Rnaq	0.48	2.24	0.82	4.39	0.01	0.08	0.36	2.33
51	Pmq	0.47	2.05	0.86	4.67	−0.02	−0.16	0.39	2.51
52	Atoq	0.61	3.95	0.76	4.98	0.51	3.28	0.63	4.09
53	Ctoq	0.48	3.01	0.56	3.42	0.03	0.23	0.18	1.24
54	Glaq	0.45	2.49	0.70	4.02	0.25	1.44	0.47	2.73
55	Oleq	0.66	2.79	0.93	4.50	−0.19	−1.23	0.23	1.46
56	Olaq	0.71	3.46	1.15	6.34	0.41	2.60	0.79	4.68
57	Claq	0.78	4.66	0.98	6.01	0.72	4.23	0.84	5.02
58	Zq	−0.13	−0.69	0.31	2.08	−0.26	−1.63	0.15	0.97
59	Tbiq	0.15	1.06	0.31	2.13	0.37	2.46	0.33	2.22
60	Blq	0.20	1.06	−0.13	−0.83	−0.52	−3.02	−0.48	−3.26
61	Oq	−0.29	−1.42	−0.85	−6.30	−0.48	−3.46	−0.71	−5.14
62	Sgq	0.30	1.50	0.59	3.31	0.05	0.28	0.57	3.10
63	Olq	0.62	3.58	0.69	4.00	0.17	1.02	0.35	2.15
64	Tanq	−0.02	−0.11	−0.08	−0.46	−0.01	−0.06	−0.07	−0.37
65	Kzq	−0.22	−1.15	−0.63	−3.93	−0.38	−2.27	−0.50	−3.07
66	Alaq	0.22	0.94	0.22	1.10	0.53	2.49	0.55	2.68
67	Acc	−0.48	−3.44	−0.47	−3.31	−0.53	−3.56	−0.58	−4.08
68	Age	−0.02	−0.09	0.15	1.30	−0.24	−2.06	−0.27	−2.81
69	Agr	−0.48	−3.00	−0.24	−1.73	0.09	0.70	0.07	0.55
70	Bm_ia	0.52	2.81	0.06	0.38	0.26	1.55	0.10	0.67
71	Cashdebt	0.15	0.85	0.54	3.66	0.40	2.79	0.38	2.96
72	Cfp	0.57	2.52	0.48	2.60	0.00	0.00	0.01	0.05
73	Cfp_ia	0.34	2.31	0.25	1.73	0.21	1.33	0.16	1.08
74	Chatoia	0.10	0.77	0.07	0.52	−0.06	−0.41	0.03	0.23
75	Chcsho	−0.55	−3.75	−0.43	−3.55	−0.18	−1.43	−0.15	−1.32
76	Chempia	0.00	0.02	0.01	0.08	0.18	1.17	0.19	1.35
77	Chinv	−0.44	−3.26	−0.38	−2.78	−0.34	−2.48	−0.40	−3.08
78	Chpmia	−0.19	−1.35	−0.17	−1.15	−0.13	−0.83	−0.11	−0.73
79	Currat	−0.04	−0.18	−0.01	−0.10	0.29	1.87	0.37	3.20
80	Depr	0.04	0.26	0.04	0.27	0.40	2.67	0.32	2.40

Table A1 (continued)

id	Anomaly	Mean	t_{Mean}	α_{FF3}	t_{FF3}	α_{HXZ}	t_{HXZ}	α_{FF5}	t_{FF5}
81	Egr	−0.43	−2.69	−0.31	−2.21	−0.10	−0.74	−0.09	−0.71
82	Ep	0.47	1.89	0.47	2.20	0.11	0.48	0.07	0.38
83	Gma	0.09	0.59	0.42	3.07	0.33	2.30	0.39	2.87
84	gCapx	−0.36	−2.44	−0.24	−1.71	0.14	1.01	−0.03	−0.23
85	gLtnoa	−0.45	−3.13	−0.32	−2.24	−0.04	−0.25	−0.15	−1.07
86	Herf	−0.03	−0.26	−0.14	−1.14	−0.29	−2.20	−0.32	−2.66
87	Hire	−0.27	−1.72	−0.10	−0.77	0.32	2.68	0.21	1.85
88	Invest	−0.51	−3.68	−0.43	−3.17	−0.23	−1.73	−0.32	−2.56
89	Lev	0.30	1.39	−0.40	−2.76	−0.36	−1.90	−0.53	−3.62
90	Lgr	−0.21	−1.62	−0.05	−0.45	0.21	1.92	0.17	1.53
91	Mve_ia	−0.22	−1.53	0.04	0.39	0.02	0.23	−0.03	−0.30
92	Operprof	0.22	1.38	0.41	2.67	0.14	0.90	0.15	1.08
93	Orgcap	0.41	1.80	0.60	2.68	−0.06	−0.28	0.18	0.80
94	Pchcapx_ia	−0.04	−0.24	−0.02	−0.11	0.22	1.43	0.23	1.54
95	Pchcurrat	−0.20	−1.92	−0.25	−2.34	−0.28	−2.50	−0.31	−2.81
96	Pchdepr	0.15	1.15	0.16	1.17	0.33	2.39	0.22	1.62
97	Pchgm_Pchsale	0.17	1.33	0.25	2.05	0.13	1.07	0.21	1.68
98	Pchquick	−0.08	−0.69	−0.08	−0.67	−0.19	−1.53	−0.15	−1.27
99	Pchsale_Pchinvt	0.33	2.44	0.34	2.47	0.11	0.78	0.25	1.74
100	Pchsale_Pchrect	−0.01	−0.09	0.07	0.53	0.17	1.30	0.13	1.03
101	Pchsale_Pchxsga	−0.06	−0.40	0.04	0.26	−0.09	−0.66	0.03	0.19
102	Pchsaleinv	0.30	2.09	0.35	2.40	0.21	1.37	0.31	2.06
103	Pctacc	−0.18	−1.21	−0.20	−1.43	0.10	0.69	−0.02	−0.11
104	Quick	−0.07	−0.37	0.05	0.38	0.42	2.62	0.46	3.70
105	Roic	0.18	0.92	0.55	3.47	0.11	0.75	0.16	1.42
106	Salecash	0.12	0.68	−0.08	−0.53	−0.44	−2.68	−0.51	−4.03
107	Saleinv	0.23	1.65	0.40	3.32	0.24	1.92	0.24	1.94
108	Salerec	0.41	2.54	0.49	3.20	0.18	1.16	0.21	1.45
109	Sgr	−0.04	−0.22	0.19	1.30	0.40	3.03	0.44	3.53
110	Sp	0.58	2.86	−0.02	−0.16	−0.03	−0.19	−0.20	−1.52
111	Tang	0.07	0.41	0.17	1.20	0.40	2.54	0.46	3.26
112	Tb	0.20	1.40	0.23	1.65	0.15	1.02	0.15	1.01
113	Aeavol	0.15	1.29	0.14	1.28	0.16	1.32	0.20	1.75
114	Cashq	0.30	1.57	0.43	2.67	0.61	3.30	0.77	5.01
115	Chtxq	0.53	2.79	0.74	4.14	0.13	0.74	0.68	3.70
116	Cinvq	−0.14	−1.07	−0.09	−0.68	−0.21	−1.43	−0.07	−0.51
117	Ear	0.77	5.30	0.90	6.09	0.69	4.51	0.87	5.74
118	Roaq	0.57	2.65	1.00	5.48	0.08	0.60	0.52	3.59
119	Roavol	−0.10	−0.54	−0.27	−1.78	0.23	1.50	0.07	0.46
120	Roeq	0.60	2.53	1.04	5.14	−0.11	−0.86	0.43	2.71
121	Rsup	0.29	1.74	0.41	2.42	−0.20	−1.34	0.17	1.02