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Climate Risk Management: the Case of Forecasting Tropical Cyclones

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ABSTRACT

Global warming has induced an increasing number of deadly tropical cyclones with a continuing trend. Developing high-functional climate risk management tools in forecasting, catastrophe modeling, pricing and hedging is thus crucial. By using transactional price changes of traded hurricane derivatives as the predictor in a doubly-binomial pricing framework, we develop a dynamic market-consensus hurricane forecasting model. Our model can forecast when and how a hurricane will make landfall, and how these forecasts will update themselves upon trading arrival.

JEL classification: G13
Keywords: Tropical cyclones; Climate risk management; Forecasting; Doubly-binomial Tree; Stochastic intensity arrival; Random time steps; Option pricing

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Introduction

How tropical cyclone activity will respond to human-induced global warming is a topic of much popular interest and scientific debate. This is especially true since Hurricane Katrina, a powerful category 5 storm, devastated the gulf coast of the United States in 2005 as it passed through. Two frequently asked questions on global warming and Atlantic hurricanes are: i) Have humans already caused a discernible increase in Atlantic hurricane activity? ii) What changes in Atlantic hurricane activity are expected for the late 21st century, given the pronounced global warming scenarios from current IPCC (Intergovernmental Panel on Climate Change) models?

A consensus[1] developed by the global community of tropical cyclone researchers and forecasters in November 2006, indicating that it is likely that greenhouse warming will cause hurricanes in the coming decades to be more destructive by being more intense and having higher rainfall rates than present-day hurricanes. Among the evidence provided is a comprehensive idealized hurricane intensity modeling study by Knutson and Tuleya (2004).[2] According to this study, an 80 year build-up of atmospheric CO_2 at 1% per year compounding would induce roughly a one-half category increase in potential hurricane intensity on the Saffir-Simpson scale and an 18% increase in precipitation near the hurricane core. A 1% per year CO_2 increase is a realistic scenario of future climate forcing. An implication is that if the frequency of tropical cyclones remains the same over the coming century, a greenhouse-gas induced warming may lead to an increasing risk in the occurrence of highly destructive category-5 storms. This finding has been shared by many other recent studies, e.g. Emanuel (2005). As noted by IPCC, however, there is considerable uncertainty in projections of future radiative forcing of earth's climate.

[1] As represented at the 6th International Workshop on Tropical Cyclones of the WMO (World Meteorological Organization) with the summary statement "The surfaces of most tropical oceans have warmed by 0.25-0.5 degree Celsius during the past several decades. The IPCC considers that the likely primary cause of the rise in global mean surface temperature in the past 50 years is the increase in greenhouse gas concentrations.......Some recent scientific articles have reported a large increase in tropical cyclone energy, numbers, and wind-speeds in some regions during the last few decades in association with warmer sea surface temperatures. Other studies report that changes in observational techniques and instrumentation are responsible for these increases."

[2] Knutson and Tuleya (2004) use future climate projections from nine different global climate models and four different versions of the GFDL (Geophysical Fluid Dynamics Lab) hurricane model. The GFDL hurricane model used is an enhanced resolution version of the model used to predict hurricanes operationally at NOAA's National Centers for Environmental Prediction.

In light of the significant economic impact of global warming in general and to the insurance and energy industries in particular, the global investment community has participated in the debate by developing new catastrophe risk management tools. New 10-day hurricane forecasting tools have been developed by global weather risk specialists like WSI and Guy Carpenter, and new catastrophe simulation models have been developed by highly skillful, multi-disciplinary-based specialist vendors like AIR, RMS and EqeCAT. In an effort to mitigate the costs of extreme weather events, i.e. creating building codes, setting insurance premiums and planning for evacuations and relief efforts, federal agencies have also increased funding to finance weather research programs, aiming at improving the accuracy of hurricane tracking and intensity forecasts by developing high-resolution dynamic numerical simulation and prediction, statistical, and hybrid models to enhance risk assessment.

Prevailing individual meteorological and statistical hurricane forecasting models however have not been successful in forecasting hurricane intensities and often diverse and inconsistent results are reported, e.g. Emanuel et al. (2004), although they have been increasing skillful in tracking hurricanes. This discrepancy stems from that hurricanes are complex dynamical systems whose intensities at any given time are affected by a variety of physical processes many of which are poorly understood. In this research, we develop a new hurricane futures and futures options pricing model to implement a novel market-consensus forecast of hurricane intensities by calibration. Since from operational forecasting we know that model consensus is usually superior to any individual model, our market consensus forecast could provide a functional alternative to prevailing individual models.

Since Hurricane Katrina, several new hurricane futures and options contracts have also been developed for trading on exchanges for related parties to mitigate their extreme weather exposures. These include hurricane futures and options contracts listed on CME (Chicago Mercantile Exchange) since 2007, hurricane futures contracts listed on IEM (Iowa Electronic Markets) since 2006, and the newly launched (from June 29, 2009) Eurex hurricane futures contracts. Since traders of these contracts employ all available forecasting models, public or proprietary, to forecast hurricanes in order to make their pricing and trading decisions, by using the transactional price levels of these contracts as the predictor and with calibration through the developments of pricing models, one can gain a market consensus on future hurricane activities out of all of the individual models employed, and thus produce a consistent aggregated forecast.

In the finance literature there has been only one research that deals with this topic. Kelly et al. (2009) use the IEM futures data to predict whether a hurricane will or will not make landfall in a given area. They find that futures price changes are more accurate than the NHC for storms more than five days from landfall (69% to 54%), but less accurate for storms two days or less from landfall (90% versus 100%). Our investigation will be based on the more comprehensive CHI contracts[3] to predict 1) how destructive a hurricane will be when it makes landfall in a given area by using CHI futures data, and 2) when this landfall will occur and how the predicted destructive power will change over time from inception to landfall, by using the CME futures option data.[3]

The rest of our paper is organized as follows: In Section 2, we briefly introduce current hurricane forecasting models and the CME CHI futures and options contracts. In Section 3, we analyze current hurricane futures and futures options pricing methodologies to determine which method is most appropriate to employ for our purpose and then develop our pricing model. In Section 4, we discuss how to extract out the market consensus view about hurricane activities from transactional CHI futures and futures options prices by using calibration, and then illustrate the implementation of a dynamic forecast. In section 6, we conclude the paper and discuss future research directions.

2. Hurricane Forecasting and the CME CHI Futures and Futures Options Contracts

2.1. Hurricane Forecasting

Prevailing hurricane forecast models vary widely in structure and complexity. Dynamical numerical and simulation models, using high-speed computers to solve the physical equations of motion governing the atmosphere, are the most complex. Statistical models, in contrast, use historical relationships between storm behavior and storm-specific details such as location and date to forecast and are simple to implement. Statistical-dynamical models blend both dynamical and statistical techniques by making a forecast based on established historical relationships between storm behavior and atmospheric variables provided by dynamical models. Trajectory models move a tropical cyclone along based on the prevailing flow obtained from a separate dynamical model. Finally, consensus models are created by combining the forecasts from a collection of other models. A collection of existing models can be found from the website of NHC (National Hurricane Center).

While prevailing models have been increasingly successful in the forecasts of hurricane tracks, hurricane intensity forecasts are very difficult tasks. Hurricanes are complex dynamical

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[3] We focus on the CME contracts since their underlying CME Hurricane Index (CHI) measures the destructive potential of a hurricane calculated using its intensity and radius, but the Eurex contracts are settled based on actual losses as compiled by ISO's Property Claim Services (PCS) unit, while the IEM contracts are based on tracking - where a given hurricane makes its first landfall.

systems whose intensities at any given time are affected by a variety of physical processes, some of which are internal and others involve interactions between the storms and their environments. Since many of these processes are poorly understood, the forecasts of the intensity change of individual storms can not be precise. Existing computing powers are also limited in horizontal resolutions to compute hurricane eyes and eye-walls properly. It is generally agreed however that there exist thermodynamic limits to intensity that apply in the absence of significant interaction between storms and their environment (Emanuel, 1987, 1988). While there remains some uncertainty about how to calculate such limits, they do appear to provide reasonable upper bounds on the intensities of observed storms. One particular advantage of limit calculations is they depend only on sea surface temperature and the vertical temperature structure of the atmosphere, so they are easily calculable from standard data sets.[4]

2.2. The CME CHI Futures and Futures Options Contracts

Following the devastating 2005 hurricane season, the CME Group developed hurricane futures and options contracts for insurers and other related parties to lay off their hurricane exposures. The underlying index for various hurricane futures and options on futures contracts is called the CME Hurricane Index (CHI). It is maintained and calculated by EqeCAT, a leading authority on extreme-risk modeling. CHI determines a numerical measure of the potential for damage from a hurricane, using publicly available data from NHC of the National Weather Service. The CHI incorporates maximum wind velocity and size (radius) of hurricane force winds and is a continuous measurement. The commonly used Saffir-Simpson Hurricane Scale (SSHS) classifies hurricanes in categories from 1 to 5 by considering the velocity but not the radius of a hurricane, and thus can not be used to measure the actual physical impact, making it less than optimal for use by the insurance community and the public at large. For example, Hurricane Katrina in 2005 was described as a weak category-4 storm at the time of its landfall but exerted significantly more physical damage than Hurricane Wilma, which at one point in its life was mentioned as the strongest storm on record.

There are two types of event-driven CHI futures contracts – the Eastern USA contract, and the CHI-Cat-In-A-Box contract that covers the major oil & gas production in the Gulf of

[4] Emanuel et al. (2004) have developed an atmospheric hurricane intensity forecast model that is a simple axisymmetric-balance model coupled to an equally simple one-dimensional ocean model, phrased in angular momentum coordinates. Emanuel et al. (2006) graft the above model to a statistical track generation model to simulate hurricane intensity movement along generated hurricane tracks.

Mexico. They trade as follows: at the beginning of each season, storm names are used from a list, starting with A and ending with Z, maintained by the World Meteorological Organization. In the event that more than 21 named events occur in a season, additional storms will take names from the Greek alphabet: Alpha, Beta, Gamma, Delta, and so on. Named hurricanes must make landfall in the Eastern U.S. (Brownsville, TX to Eastport, ME) for the Eastern USA contract and Galveston-Mobile area (95°30'0"W on the West, 87°30'0"W on the East, 27°30'0"N on the South, and the corresponding segment of the U.S. coastline on the North) for the CHI-Cat-In-A-Box contract, respectively. Trading shall terminate at 9:00 A.M. on the first Exchange business day that is at least two calendar days following the dissipation or exit from the designated area of a named storm.

All futures contracts remaining open at the termination of trading shall be settled using the reported respective CHI final value and CHI-Cat-In-A-Box final value (for the latter the maximum calculated CHI value while the hurricane is within the Box) by EqeCAT. As an initial attempt to focus on the fundamental issues, in this research we concentrate on the Eastern USA contract. CME has also offered seasonal, seasonal maximum and second event seasonal types of contracts. Since these contracts are not straight event-driven but based on the total, maximum or second event in an entire season, they are irrelevant to our study. Two types of American-style call options are traded on these futures contracts – plain vanilla and binary. Payoff for the former is the in-the-money amount but $10,000 for the latter.

3. Pricing Hurricane Futures and Futures Options by No-Arbitrage

We are interested in using the CHI futures data to predict the expected destructive power of a hurricane when it makes landfall, and the futures options data to predict when this date will be, and how this power will evolve over time since the hurricane's inception. The latter investigation is three-fold: how likely news regarding the velocity, size and location of the hurricane that may affect this prediction will arrive in the next period, and when news does arrive, will the hurricane's expected power accentuate or attenuate, and to what extent? In other words, we would need data on transaction arrival as a proxy to news arrival and data on price changes per transaction arrival to update the prediction. Existent approaches for pricing catastrophe (CAT hereafter) derivatives include Aase (2001), Cummins and Geman (1995), and Chang, Chang, and Yu (1996 and CCY hereafter) for pricing CBOT CAT futures and futures call spreads; Bakshi and Madan (2002), Aase (1999) and Geman and Yor (1997) for pricing CBOT PCS-type cash options; Lee and Yu (2002) and Loubergé, Kellezi, and Gilli (1999]) for pricing

CAT bonds; and Jaimungal and Wang (2006) for pricing CatEPut. However among these studies, only CCY is based on both transaction arrival and price changes as explain below.

CCY have proposed a unique "randomized operational time" approach to price CAT futures options. This "randomized operational time" concept, originated in probability theory (Feller, 1971), is widely applied in systems and engineering fields. It dictates that a simple change of time scale will frequently reduce a general nonstationary process in the usual calendar-time scale to its stationary operational counterpart in a new time scale dictated by the nature of things. In the finance literature, Clark (1973) first applied this concept to subordinate stock returns to news arrivals with transaction arrivals being a market proxy, while in the insurance literature CCY first applied this concept to subordinate CAT futures return to CAT futures transaction arrivals. This time-change transforms a calendar-time CAT option with stochastic volatility to an isomorphic transaction-time CAT option with random maturity (to reflect the randomness of transaction arrivals), which leads to a transaction-time option pricing formula as a risk-neutral Poisson sum of Black's (1976) prices over the option's maturity domain. It is parsimonious in requiring only two unobservable variables – the transaction arrival intensity and the per-transaction futures volatility. Therefore as Geman and Yor (1997) have suggested, unlike other pricing models that are developed in calendar time and do not incorporate information conveyed in transaction arrivals, CCY is unique in having the merit of illuminating the information conveyed by transactions. The CCY methodology is thus the only one that would be relevant to our purpose of calibrating hurricane activities.

To apply the setup however we encounter four problems: 1) unlike usual transaction arrivals in financial markets that can be approximately continuous, hurricane news arrivals are sporadic, discrete and random, 2) CCY only price European-type of options but the CHI futures options are of the American-type with the early exercise feature, 3) the intensity of catastrophe arrival in CCY is assumed to be constant, however hurricane arrivals is event-specific and often exhibits time-varying arrival intensity with mean-reversion (see Levi and Partrat (1991) for a discussion on arrival processes of different types of natural disasters in the U.S.A.), and 4) in CCY the futures' expiration date is preset but for the CHI futures it is random, i.e. when the hurricane will make landfall is uncertain. Chang et al. (2008) have discretized CCY by employing a doubly-binomial framework consistent with the compound binomial models of Gerber (1984, 1988) and others in the actuarial risk theory to simultaneously capture a hurricane's arrival and severity (or intensity in the hurricane forecasting literature) uncertainties in discrete time to circumvent the first two problems. In this paper, we would further generalize this model by incorporating a discretized mean-reverting stochastic arrival intensity process to circumvent the last two problems.

3.1. The Hurricane Futures Process

CME contracts are settled based on the CHI value, which measures the destructive potential of a hurricane. Since this index is physical in nature, its arrival uncertainty should exhibit no correlation with changes in financial prices, and thus should be non-systematic (see Hoyt and McCullough, 1999, for empirical evidence and why this benefit of diversification is one major motivation for portfolio managers to invest in catastrophe products). This implies 1) one can price futures and futures options based on the expected landfall date, and 2) today's futures price embeds no risk premiums and thus should be a forwarding-looking market prediction of the expected CHI value on the expected landfall date.

To model how the futures price changes over time since a hurricane's inception, next we set up a doubly-binomial model as in Gerber (1984, 1988) and others in the actuarial risk theory to model futures price changes. The first binomial variable is to determine if a transaction will arrive and the second to determine if the corresponding futures price jumps up or down. Thus constructed, the total price change over *n* calendar- time steps is a random sum of *k* price changes, where $k \in [0,n]$ is the number of transaction arrivals. Transaction arrival is defined as any news that will impact on the stochastic change of the CHI value. This construction essentially defines a subordinated binomial process[2] where the sequence of transaction arrivals serves as the directing process and the subsequent futures price changes from transaction to transaction form the parent process. In other words, we subordinate binomial futures price changes to random transaction arrivals such that the futures price will only change when a transaction arrives, irrespective of the passage of calendar time. The parent process, or the price change from transaction to transaction, is a stationary recombining binomial tree that will be used later to develop the transaction-time option pricing model.

Subordination collapses the two binomial processes onto to the following trinomial futures price changes:

/	*uF*	with probability *gh* one transaction arrives and the futures price jumps up at a gross rate *u*,
F –	*F*	with probability 1-*g* no transaction arrives and the futures price stays the same,
\	*dF*	with probability *g(1-h)* one transaction arrives and the futures price jumps down at a gross rate *d*,

where *F* denotes the futures price at the beginning of the period, *u* and *d* denote the respective constant up and down gross jump sizes, *g* and *1-g* denote the respective transaction arrival and no arrival probabilities, and *h* and *1-h* denote the respective jump up and down probabilities upon an arrival. We let *R* denote one plus the riskless rate over one period with the usual regularity condition that $u>R>d$ to prevent riskless arbitrage.

Next we model the changes of hurricane arrival intensity *j* as a mean-reverting Ornstein-Uhlenbeck process:

(1) $$dj = \kappa_j(m_j - j)dt + \sigma_j dZ_j,$$

where κ_j denotes the speed of adjustment, m_j the long-run mean rate, σ_j^2 the instantaneous variance, and Z_j the standard Wiener process. This process exhibits mean-reversion and clustering. Surrounding the time of news about a hurricane arrival, the intensity takes on higher values to reflect the lumpiness of news arrival, but it reverts back to the long-run mean level m_j after the arrival, and conversely, after a lull, the intensity can revert to a higher state. While *j* itself governs the intensity of arrival, the speed of adjustment parameter κ_j governs the level of persistence of the intensity process. Higher values for κ_j imply that the intensity process leaves the high state sooner, and vice-versa. Judiciously combining these parameter values leads to different specifications of hurricane arrivals.

The solution of Eq. (1) for the time-varying intensity is known to be

(2) $$j(v) = m_j + (j(t) - m_j)e^{-\kappa_j(v-t)} + \sigma_j e^{-\kappa_j v}\int_t^v e^{\kappa_j s} dZ_j(s), \quad v \in [t, T],$$

where *j(t)* denotes the current level of intensity. The expected intensity over a time period *T-t* is determined via integration as

(3) $$E(j(T-t)) = m_j \times (T-t) + [j(t) - m_j]H_j(T-t),$$

where *H* denotes a function of *(T-t)* or $H_j(T-t) = \frac{1-e^{-\kappa_j(T-t)}}{\kappa_j}$.

Assuming we choose *n* calendar-time steps over an expected maturity of *T-t*, then the time-varying probability that news (transaction) will arrive in the next period is

(4) $$g_t = E_t\left(j\left(\frac{T-t}{n}\right)\right) = \left(m_j(\frac{T-t}{n}) + [j(t) - m_j]H_j(\frac{T-t}{n})\right).$$

subject to the parameters m_j and *j(t)* being appropriately chosen so that $g_t \leq 1$. This physical probability should also be equal to the transaction-arrival martingale probability over the next time period, m_t, since the CHI embeds no systematic risks as we have discussed before. This risk-neutral claim-arrival probability is driven by the intensity arrival process as a function of the long-run mean rate, the deviation of the current level of intensity from the long-run mean rate, and how this level persists. We examine this persistence effect in Figure 1 below. With the long-run claim arrival intensity at 80 in an event quarter and the number of time steps at 30, we have m_j = 80 and Δt = 0.0083. We then consider two scenarios: when the initial intensity is low at 60 and high at 100. In each scenario, we vary κ_j, the speed of adjustment toward the mean, from 2 to 30, and then compute the corresponding risk-neutral claim-arrival probability per period. The latter is based on the average or integrated intensity over the quarter. The results show that 1) in the low initial intensity case, as the speed of adjustment increases, clustering weakens and mean-reversion toward the higher mean strengthens, leading to increasing claim-arrival probability, but 2) in the high initial intensity case, as the speed of adjustment increases, clustering weakens and mean-reversion toward the lower mean strengthens, leading to decreasing claim-arrival probability.

Insert Figure 1 About Here

Note the probability that news (transaction) will arrive in the second period is then

(5)
$$\begin{aligned} m_{t+1} &= E_t\left(j\left(2\left(\frac{T-t}{n}\right)\right)\right) - E_t\left(j\left(\frac{T-t}{n}\right)\right) \\ &= \left(m_j \times 2(\frac{T-t}{n}) + [\,j(t) - m_j\,]H_j\left(2\left(\frac{T-t}{n}\right)\right)\right) - \left(m_j \times (\frac{T-t}{n}) + [\,j(t) - m_j\,]H_j\left(\frac{T-t}{n}\right)\right). \end{aligned}$$

The future forward-looking m_{t+i}, *i=2,3,…,* and so on, can thus be similarly calculated as $E_t\{j(i([T-t]/n))\}- E_t\{j([i-1]([T-t]/n))\}$.

3.2. Risk-Neutralization

Next we apply the discrete-time no-arbitrage martingale pricing methodology to determine the price-change martingale probability and then develop the risk-neutral tree. No-arbitrage dictates the following one-period martingale representation for the futures price:

(6) $$F = mpuF + (1-m)F + m(1-p)dF,$$

where p and $1\text{-}p$ are the respective equivalent martingale probability measures over one step for the asset price to move up and down; and m and $1\text{-}m$ are the respective equivalent martingale probability measures over one step for transaction arrival and non-arrival. As we have shown before, $m = g$.

Solving and simplifying Eq. (6), we obtain the price-change martingale probability:

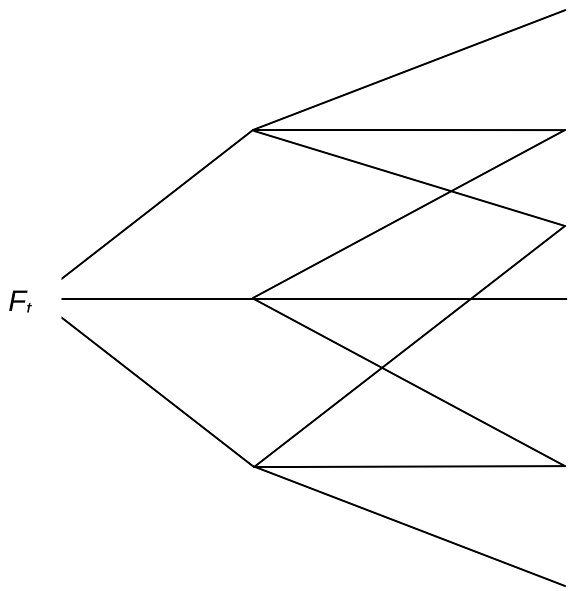
(7) $$p = \frac{1-d}{u-d},$$

where u $(=exp(\sigma_1))$ and d $(= 1/u)$ are the gross up/down movements of the futures price. This probability measure resembles closely in format to the standard binomial measure with one difference - u and d here are determined by the per-transaction volatility σ_1 only. This is because in our model the futures price will only jump when a transaction arrives, irrespective of the passage of calendar time.

To summarize, the risk-neutral trinomial tree is obtained by superimposing a jump process on a standard binomial process such that the movement of the underlying asset at calendar time t over a generic calendar-time period is modeled by the following trinomial setup:

```
           uF_t          with probability p_1 = mp,
        /
F_t   -    F_t           with probability p_2 = 1-m,
        \
           dF_t          with probability p_3 = m(1-p),
```

where in each period, a transaction arrives with probability m and upon its arrival futures price either jumps up to uF_t with probability p or jumps down to dF_t with probability $1\text{-}p$. However, because the transaction-arrival probability is time-varying, the futures price should evolve over two time periods as follows.



u^2F_t with probability $m_t m_{t+1} p^2$

uF_t with probability $[m_t\,(1-m_{t+1})+(1-m_t)\,m_{t+1}]p$

udF_t with probability $2m_t m_{t+1} p(1-p)$

F_t with probability $(1-m_t)\,(1-m_{t+1})$

dF_t with probability $[(1-m_t)\,m_{t+1}+m_t\,(1-m_{t+1})]\,(1-p)$

d^2F_t with probability $m_t m_{t+1}(1-p)^2$,

where per Eq. (4) a transaction arrives with probability m_t in the first period but with probability m_{t+1} in the second period, allowing for varying news arrival probabilities.

3.3. Stochastic Time Change and Transaction-Time Option Pricing

In the spirit of SP modeling, a stochastic time change from calendar time to transaction time restores the stationary binomial parent process. However, since the number of transaction arrivals in an *n* calendar time-step trinomial tree (where *n* x intervals equal *T*) may vary from a minimum of zero to a maximum of *n*, assuming *n* is chosen to be sufficiently large, the restored binomial tree has a random number of time step, *k*, where $k \in [0, n]$. For example over two calendar time periods we would have:

1) when $k = 0$ with probability $(1-m_t)\,(1-m_{t+1})$, F_t does not change because no news arrives over two time periods,

2) when $K = 1$ with one news arrival and probability $[(1-m_t)\,m_{t+1}+m_t(1-m_{t+1})]$,

$$F_t < \begin{cases} uF_t & \text{with probability } p, \\ dF_t & \text{with probability } 1-p, \end{cases}$$

3) when $K = 2$ with two consecutive news arrivals and probability $m_t\,m_{t+1}$,

$$
F_t < \begin{array}{l} uF_t < \begin{array}{ll} u^2F_t & \textit{with probability } m_t m_{t+1} p^2 \\ udF_t & \textit{with probability } 2m_t m_{t+1} p(1-p) \end{array} \\ dF_t < \begin{array}{ll} udF_t & \\ d^2F_t & \textit{with probability } m_t m_{t+1} (1-p)^2. \end{array} \end{array}
$$

In other words, our task now is to price an isomorphic option with random maturity in transaction time. We solve this problem by using the Euler equation as a conditional expectation over the transaction arrival uncertainty. More specifically, the normalized price of an *n*-period call option can be solved as a random sum of the arrival-probability-weighted normalized prices of *n+1 k*-transaction fixed-maturity options (denoted as C_k):

(8) $$\frac{C(n)}{B_T} = \sum_{k=0}^{n} M_k \frac{C_k}{B_T},$$ which simplifies to

(9) $$C(n) = \sum_{k=0}^{n} M_k C_k,$$

where B_T is the price of the matching bond, M_k is the transaction-arrival martingale probability measure of *k* claim arrivals in *n* periods, and C_k is the transaction-time American binomial futures call price with *k* transactions in maturity. In the case of European options with an *N* transaction-time-step setup, C_k is

(10) $$C_k = B_T \sum_{i=0}^{N} P_k(i)\left(u_k^i d_k^{N-i} F - X\right)^+,$$

where $P_k(i) = \frac{N!}{i!(N-i)!} p_k^i (1-p_k)^{N-i}$ is the *N*-step martingale probability that the ending futures price level is $u_k^i d_k^{N-i} F$ with $u_k = e^{\sigma_1 \sqrt{k/N}}$, $d_k = \frac{1}{u_k}$, and $p_k = \frac{1-d_k}{u_k - d_k}$. Since C_k is now priced by the standard binomial model defined over transaction time but not calendar time, the size of the gross up/down rate now depends on the transaction-time interval $\Delta k = k/N$.

As in CCY, this pricing model links an option value to the expected intensity of transaction arrival and the per transaction futures volatility (σ_1). As the intensity increases, the tree grows faster, and thus the option price increases to reflect the larger expected total price volatility, and vice versa. For forecasting however we will need at least three transactional option prices to simultaneously track three parameters: 1) the per transaction volatility (σ_1); 2) the speed of intensity adjustment (κ_j); and 3) the long-run mean intensity level (m_j). By equation (7), it is seen that σ_1 also determines u as well as p, the probability the destructive power will go up. By equation (5), it is seen that, given the current intensity $j(t)$ and κ_j and m_j, we can forecast the probability of news arrival in the future periods, m_{t+i}. Collectively, these values offer a market consensus view as to how likely a hurricane will change activity level in the next time period, and if it does happen, how likely the change would be to accentuate and to what new level.

4. Model Comparative Statics, Calibration and Forecasting

We shall construct the hurricane derivatives based on a named storm with 90 days to expected landfall, or expected maturity at $T=1/4$. Conditional on T, and supposing the number of transactional events that impact the CHI is not more than 30 per quarter, we set $n=30$. The random arrivals of transactions during $[t,T]$ imply that total number of transactions in this forward period is $k \in [0,n]$. Suppose current intensity is observed to be $j(t)=100$. We shall first construct the intensity process in equation (1) and show how the probability of transaction events, m_{t+i}, occurring in the future period $[t+i\Delta t,\ t+(i+1)\Delta t]$ for $i=0,1,2,\ldots,29$, is computed using values $\kappa_j = 2, 15, 30$, and $m_j = 80$. We also include the case of $\kappa_j = 15$ and $j(t) = 60$ for comparison as the latter is a case of upward adjustment instead.

This transaction arrival probability forecast under stochastic intensity is shown in Figure 2 where each period is 3 days. In practice, the number of intervals may be increased in this discrete framework to improve on the estimates. The limitation of finite discretized intervals is that it imposes an arbitrary assumption that the transactions arrive either once or none during these regularly spaced intervals.

Insert Figure 2 About Here

Figure 2 shows that m_{t+i} reduces over time if $j(t)>m_j$, but increases over time if $j(t)<m$. The rate of increase or decrease is higher or lower depending directly on the value of κ_j. The sequence of values $\{m_t,\ m_{t+1},\ m_{t+2},\ldots\}$ for a particular parameterization $\{\kappa_j\ ,\ m_j\}$ is then employed to find the probability of number of transactions M_k shown in Figure 3.

Insert Figure 3 About Here

This probability is different from existing models in that we accommodate a stochastic intensity specification as in (1). These probabilities form the risk-neutral probabilities M_k for pricing in (9) under transaction time-scale k. We vary κ_j to examine the probability distribution M_k and find that for a given m_j and $j(t)>m_j$, as κ_j increases, the mode of the distribution tends to decrease and likewise its probability. This is because under downward adjustment since $j(t)>m_j$, increasing κ_j implies reduction in future probability m_{t+i} of transaction arrival, and hence lower probabilities for total number of arrivals. The figure also shows that for $j(t)<m_j$, the lower intensities typically produces a probability distribution that is lower in number of arrivals and its attendant probabilities. A comparison with a constant intensity specification $m_j \Delta t$ = 80 x 0.0083 = 0.6667 as in the dotted curve shows that for an upward adjustment $j(t)>m_j$, the probability distribution dominates that from using an averaged constant intensity. The situation is converse for the case of downward adjustment where $j(t)<m_j$.

Next, we employ N_k time-steps to compute the discretized binomial American option prices with random maturity at transaction times $k=1,2,\ldots,30$. For parsimonious reasons, let N_k be the same $N \geq n \geq k$. We choose $N=n$ in this case. The transaction-time volatility is fixed at $\sigma_1=0.2\sqrt{k/N}$ and separately $\sigma_1=0.4\sqrt{k/N}$ in order to evaluate the hurricane futures option no-arbitrage prices. The latter prices are computed for options with different strike prices at $K=6$ (in-the-money), $K=8$ (at-the-money), and $K=10$ (out-of-the-money) for a named storm with a traded CHI futures at value $F_t=8$. For option pricing, an annual riskfree rate of 2% is assumed. The price results for different sets of parameterizations are shown in Table 1 below. N in principle should be as large as is computationally feasible, and as N increases, the binomial trees under transaction maturity should converge to their counterparts under lognormal diffusion. For computational tractability, we demonstrate the methods here using $N=n=30$ for all k.

Insert Table 1 About Here

From Table 1, it is seen that the American-styled hurricane futures option prices increase significantly with increase in transaction-time volatility σ_1, with moneyness, and with decrease in

κ_j (since *j(t)-*m_j*=20* here indicates an adjustment downward toward the long-run mean). We also compared the American futures option prices with European ones without early exercise and for cases of low transaction volatilities, the early exercise premium becomes more significant as American-styled futures options are worth more than European-styled futures options when the upward price potential becomes less and immediate exercise for positive profit becomes more valuable. Comparing with the case of long-run constant intensity by setting $j(t)=m_j=80$, it is seen that whenever $j(t)>m_j$, the American futures option prices will be higher than in the case of constant intensity.

In Figure 4 below, we plot in 3-dimension the hurricane futures option price as a function of κ_j taking the range 2 to 30, and of σ_1 under unit transaction time taking the range *0.1* to *0.9*. Current futures price is $F_0=8$, and the strike price is $K=8$. Maturity is $T=1/4$. Riskfree interest rate is assumed to be 2% p.a. It is seen that the price surface increases in σ_1 and decreases slowly in κ_j (in the case $j(t)>m_j$) for any given option with strike K. The price surface corresponds to a particular value of m_j. Here $m_j=80$ and $j(t)=100$.

Insert Figure 4 About Here

Any given futures option price level forms a three-dimensional surface in the m_j -σ_j –κ_j space. Two such surfaces from two derivatives would form an intersection of a curve at points equivalent to the observed market prices of the two derivatives. Three derivatives would be able to provide an intersection equivalent to a point in the m_j -σ_j –κ_j space, and hence providing the implied values of $\hat{m}_j$,$\hat{\sigma}_j$, and $\hat{\kappa}_j$. Once the three parameters are implied at any trading time t before landfall, they can be used to form a risk-neutral or similarly physical probability distribution of the CHI values at the expected landfall or maturity time. This is done using the random variable $\tilde{F}_T = u_k^i d_k^{N-i} F_t$ for different k= *1* to n, and for each k, i = *1* to N, where as seen earlier, $u_k = e^{\sigma_1\sqrt{k/N}}$ and $d_k = \frac{1}{u_k}$. We employ all the binomial trees for each k to construct the implied risk-neutral distribution of $\tilde{F}_T$ for our forecasting and risk management purposes. For each k, we have N nodal values of F_T at T, and thus N probability values. Conditional on k, these probability values sum to one. Since the probability of observing k transactions in T is M_k, we have the unconditional probability of nodal value $\tilde{F}_T = u_k^i d_k^{N-i} F_t$ as $\Pr(\tilde{F}_T) = M_k \times P_k(i) = M_k \times \frac{N!}{i!(N-i)!} p_k^i (1-p_k)^{N-i}$.

Next suppose three traded futures options with strikes at *K=6*, *K=8*, and *K=10*, are priced at *2.73*, *1.92*, and *1.38* in terms of CHI units respectively in the market. Using the above theoretical model, we can imply out the parameters $\hat{m}_j$,$\hat{\sigma}_j$, and $\hat{\kappa}_j$ to be *80*, *0.15*, and *20* respectively, given *j(t)=100* which is assumed to be observed. Then the risk-neutral distribution of CHI values at expected landfall is shown in Figure 5. This implied probability distribution provides a forecast of how destructive it will be when it makes landfall with time-varying arrival probabilities. As the implied probability distribution is tracked over time, it also provides information on how its expected destructive power will behave over time from inception to landfall. Figure 7 shows that the mean and also mode of the distribution is *8*, the current future value, with a probability of about *30%*. The distribution is skewed to the right.

Insert Figure 5 About Here

The distribution also provides a way of measuring the risk or probability of hurricane devastation when the CHI value is expected to exceed certain thresholds. Hurricane Katrina for example made landfall with a CHI value of *19.0*, a considerably destructive storm. In contrast, Florida's Hurricane Dennis had only a CHI value of 6.9, a mild to medium-sized storm. From the distribution, we can infer that the probability of exceeding CHI value of *20* is about *4.95%* or close to *5%*. Hence there is a *5%* chance of a serious hurricane hit within *90* days in this example.

Finally, In Table 2 below we illustrate the implementation of a dynamic market-consensus hurricane forecasting model as to how the expected destructive power of a hurricane would evolve from news arrival to news arrival as a multi-period transaction-time binomial tree. We use the following implied parameters: *m=80, k=20,* σ*=0.15; u=1.1618, d=0.8607, p=0.4625*, given an expected number of transactional arrival of *30* over *90* days. As shown in the Table, the value in the cell of each node of the binomial tree denotes the expected destructive power in that period with an initial CHI value of *8.00* and after *30* arrivals this value will involve into a range from a low of *0.09* with probability of *0.15529* to a high of *720.14* with probability of *0.08982*. The probability values at the bottom rows denoted as "prob" represents the probability news will arrive in the next 3 days, and the term “period” labels the transaction count with a total of *30* expected transaction arrivals in *90* days.

Insert Table 2 About Here

6. Concluding Remarks and Futures Research directions

By using transactional price changes of traded hurricane derivative contracts as the predictor in a doubly-binomial hurricane futures and futures-option pricing framework, we have developed a dynamic market-consensus hurricane forecasting model. Our model can forecast when a hurricane will make landfall and how destructive it will be, and how this destructive power will evolve from inception to landfall. Since from operational forecasting we know that model consensus is usually superior to any individual physical model, our market consensus forecast could provide a functional alternative to prevailing individual physical models. Since tropical cyclones arrivals are also modeled in the climatic science domain, one possible extension is to incorporate additional stylized effects of measurable physical variables into the financial futures price process. Empirical verifications to examine the performance and robustness of our market-based predictor would be important and provide exciting possibilities to enhance the realm of physical science in hurricane forecasting.

References

Ané, T. and H. Geman, 2000: Order Flow, Transaction Clock, and Normality of Asset Returns, *Journal of Finance,* **55**, 2259-2284.

Black, F., 1976: The Pricing of Commodity Contracts, *Journal of Financial Economics* **3**, 167-179.

Chang, C.W.. J.S.K. Chang, and M.T. Yu, 1996: Pricing catastrophe insurance futures call spreads: a randomized operational time approach, *The Journal of Risk and Insurance*, **63**, 599-617.

Chang, C.W.. J.S.K. Chang, and W. Lu, 2008: Pricing catastrophe options in discrete operational time, *Insurance: Mathematics and Economics*, **43**, 422-430.

Chang, C.W.. J.S.K. Chang, and W. Lu, 2010: Pricing catastrophe options with stochastic claim arrival intensity in claim time, *Journal of Banking & Finance*, **34**, 24-32.

Cox, J.C., S.A. Ross, and M. Rubinstein, 1979: Option Pricing: A Simplified Approach, *Journal of Financial Economics* **7**, 229-263

Emanuel, K.A., 1987: The dependence of hurricane intensity on climate. *Nature*, 326, 483-485.

Emanuel, K.A., 1988: The maximum intensity of hurricanes. *J. Atmos. Sci.*, 45, 1143-1155.

Emanuel, K., C. DesAutels, C. Holloway, and R. Korty, 2004: Environmental control of tropical cyclone intensity. *J. Atmos. Sci*., **61**, 843-858

Emanuel, K. A., 2005: Increasing destructiveness of tropical cyclones over the past 30 years. *Nature*, **436**, 686-688. Online supplement to this paper.

Emanuel, K., S. Ravela, E. Vivant, and C. Risi. 2006: A Statistical-Deterministic Approach to Hurricane Risk Assessment**.** *Bull. Amer. Meteor. Soc.,* **87**, 299–314. Online Supplement

Geman, H., 2005: From Measure Changes to Time Changes in Asset Pricing, *Journal of Banking & Finance* **29**, 2701-2722.

Gerber, H.U.,1984: Error Bounds for the Compound Poisson Approximation, *Insurance: Mathematics and Economics* **3,** 191-194.

Gerber, H.U., 1988: Mathematical Fun with the Compound Binomial Process, *ASTIN Bulletin* **18**,161-168.

Huang, C.F., and R.H. Litzenberger, 1988, *Foundations for Financial Economics*, New York: Elsevier Science Publishing.

Kelly D.L., D. Letson, F. Nelson, and D. Nolan, 2009: Evolution of Subjective Hurricane Risk Perception: A Bayesian Approach, Working Paper, University of Miami Abess Center for Ecosystems Science and Policy.

Jaimungal S., and T. Wang, 2006: Catastrophe Options with Stochastic Interest Rates and Compound Poisson Losses, *Insurance: Mathematics and Economics*, **38**, 469-483.

Knutson T.R., and R.E. Tuleya, 2004: Impact of CO2-Induced Warming on Simulated Hurricane Intensity and Precipitation: Sensitivity of the Choice of Climate Control and Convective Parameterization, *Journal of Climate*, **17**, 3477-3495.

Lee, J.P., and M.T. Yu, 2002: Pricing Default-Risky CAT Bonds with Moral Hazard and Basis Risk, *Journal of Risk and Insurance* **69**, 25-44.

Levi, C., and C. Partrat, 1991: Statistical Analysis of Natural Events in the United States, *ASTIN Bulletin* **21**, 253-276.

Luo, Z., G. L. Stephens, K. A. Emanuel, D. G. Vane, N. Tourville, and J. M. Haynes, 2008: On the use of CloudSat and MODIS data for estimating hurricane intensity. *IEEE Geoscience Remote Sensing Lett*., **5**, 13-16.

Figure 1: Risk-Neutral Claim Arrival Probability per period under Constant Averaged Intensity

We assume that the claim arrival intensity follows a mean-reverting Ornstein-Uhlenbeck process and then we examine how the risk-neutral transaction claim-arrival probability, m_t, is affected by the deviation of the current level of arrival intensity from the long-run mean rate, $j(t)$-m_j, and the speed of adjustment, κ_j. With the long-run arrival intensity set at 80 and the number of time steps at 30 in an event quarter, we compute m_t as a function of κ_j ranging from 2 to 30 in two scenarios: when the initial intensity is low at 60 and high at 100.

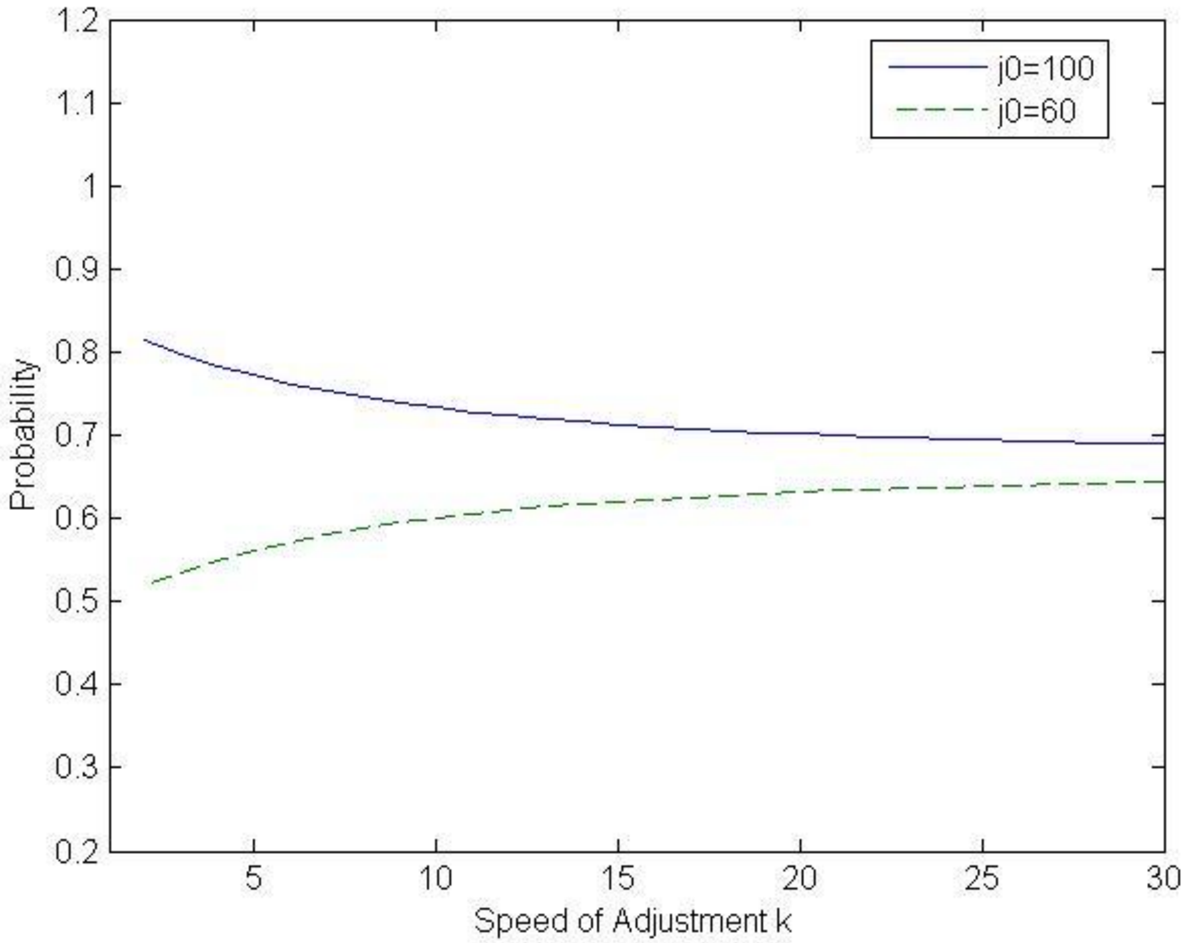


Figure 2: Transaction Arrival Probability Forecast

We assume that the claim arrival intensity follows a mean-reverting Ornstein-Uhlenbeck process. T=1/4, n=30, m_j=80. Different values of k and initial intensity j_0 are used to forecast the transaction arrival probability m_{t+i} at future period [t+iΔt,t+(i+1) Δt] for i=0,1,2,….,29. Each period is 3 days in this current setup.

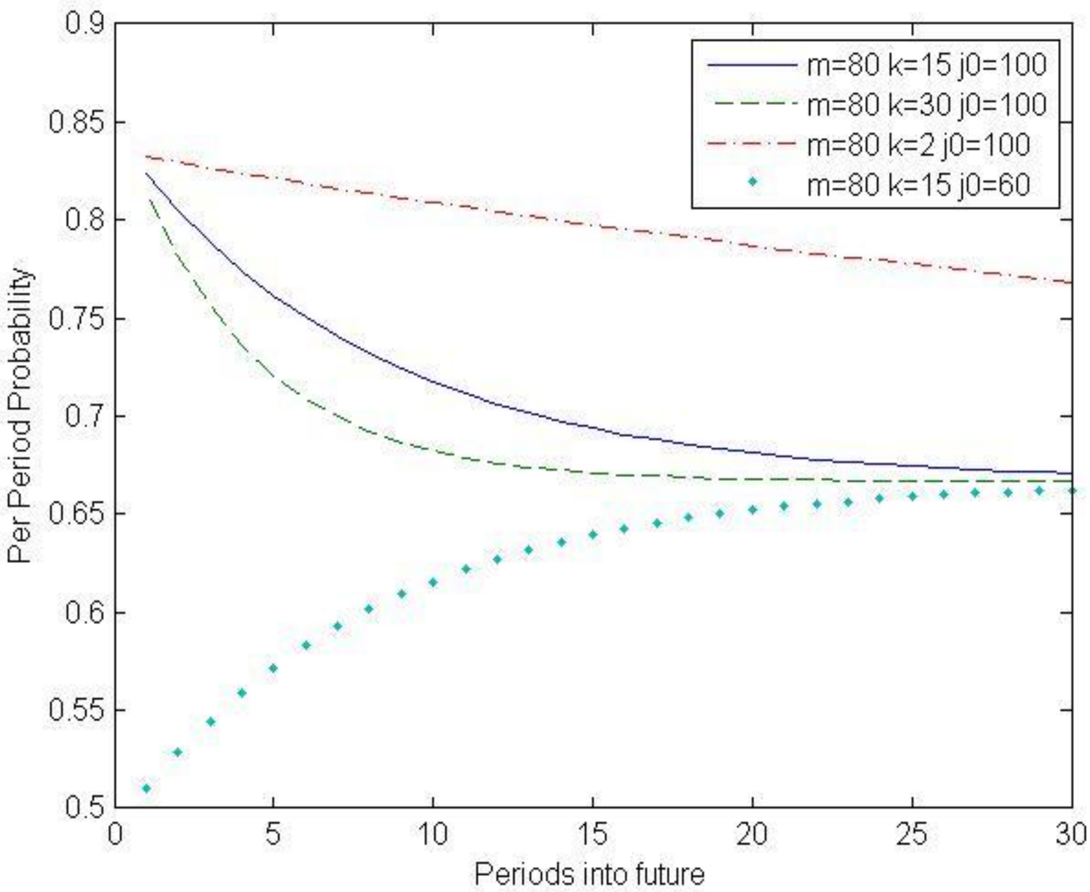


Figure 3: Probability of Number of periods of Transaction Arrivals

We assume that the claim arrival intensity follows a mean-reverting Ornstein-Uhlenbeck process. T=1/4, n=30, m_j=80. Different values of k and initial intensity j_0 are used to forecast the transaction arrival probability m_{t+i} at future period [t+iΔt,t+(i+1) Δt] for i=0,1,2,….,29. The sequence of values {m_t, m_{t+1}, m_{t+2},…} for a particular parameterization {κ_j , m_j} is then employed to find the probability of number of transactions M_k shown below. The case of constant intensity m is derived using $m_j\Delta t$ = 80 x 0.0083 = 0.6667 as per period probability.

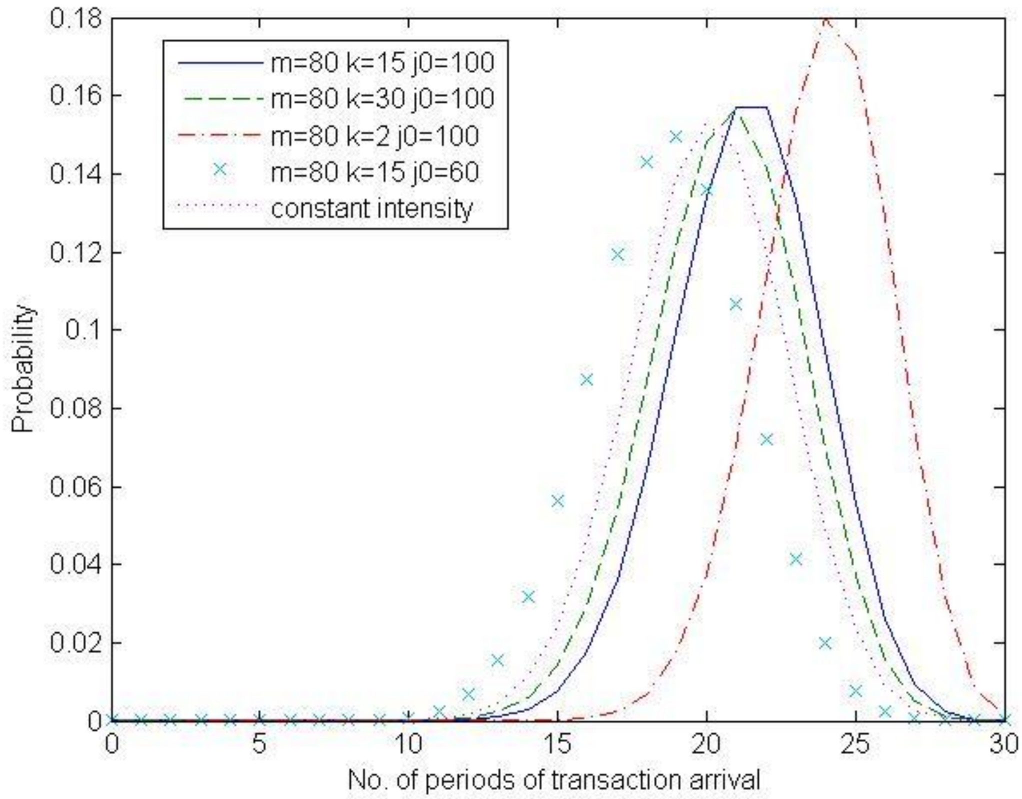


Figure 4: Futures Call Price Surface

The price surface corresponds to m_j=80, j(t)=100, and varying levels of κ_j taking the range 2 to 30, and of σ_1 under unit transaction time taking the range 0.1 to 0.9. Current futures price is F_0=8, and the strike price is K=8. Maturity is T=1/4. Riskfree interest rate is 2% p.a.

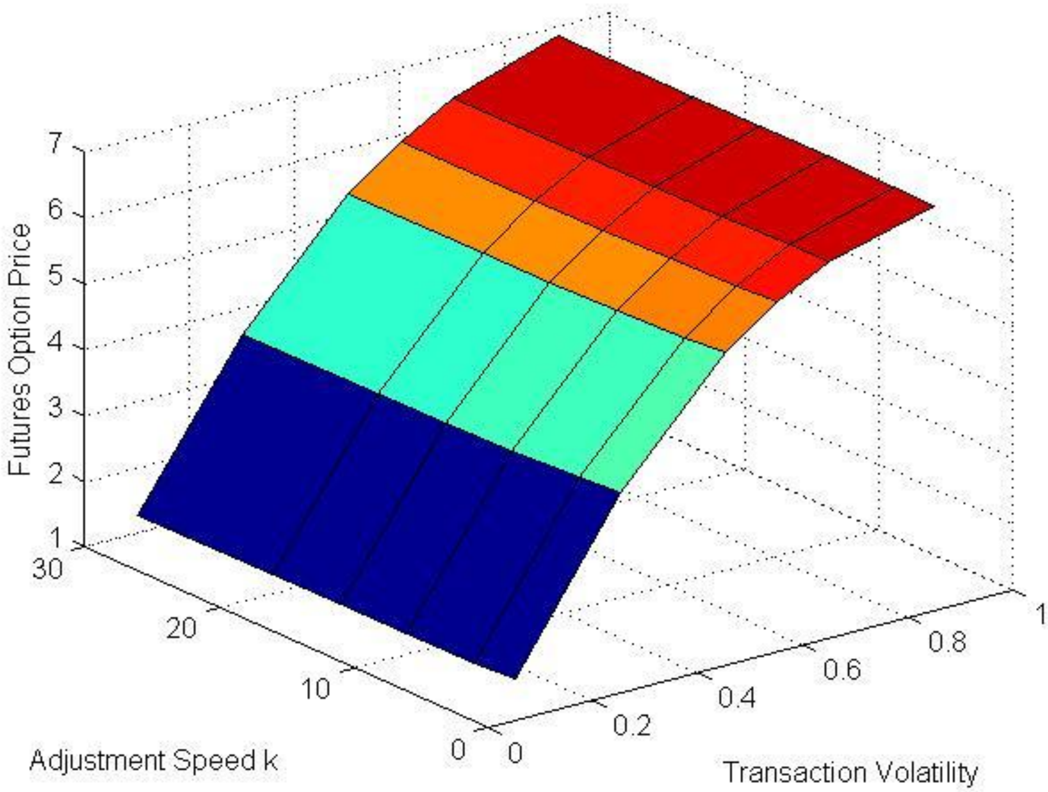


Figure 5: Discrete Risk-Neutral Probability of CHI at Maturity

We suppose 3 traded futures options with strikes at K=6, K=8, and K=10, are priced at 2.73, 1.92, and 1.38 in terms of CHI units respectively in the market. Current futures price is F_0=8, maturity is T=1/4, and j(t)=100. Riskfree interest rate is 2% p.a. Using these prices, we employ our theoretical model to imply out the parameters $\hat{m}_j$,$\hat{\sigma}_j$, and $\hat{\kappa}_j$ as 80, 0.15, and 20 respectively. These values are used to find the probability of occurrences of number of transactions over T based on the mean-reverting Ornstein-Uhlenbeck process. The unconditional risk-neutral distribution of CHI values at expected landfall can be obtained via the binomial trees. The histogram is smoothed as follows.

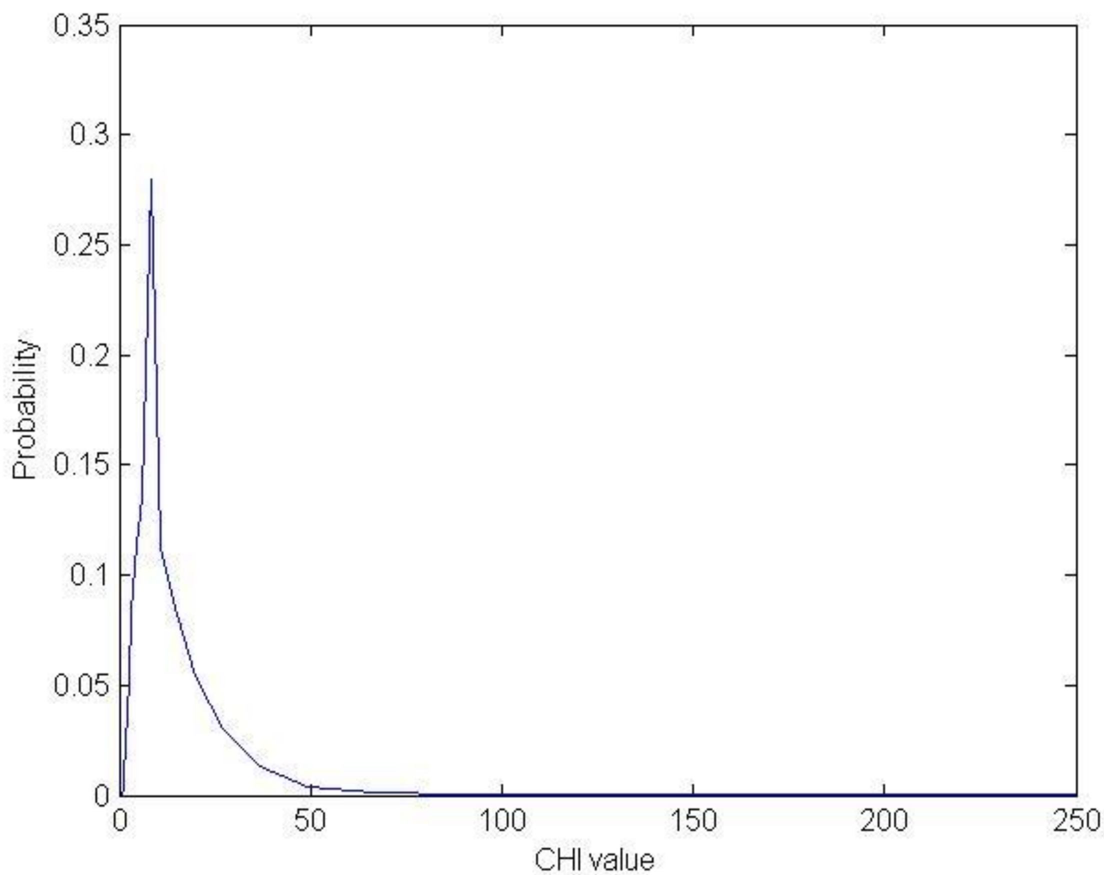


Table 1

Hurricane Futures Option Prices based on expected maturity of T=0.25, Current futures price of F_t=8, and discretization scheme of N=n=30.

Prices in CHI value	σ_1=0.2			σ_1=0.4		
Different parameterizations	K=6	K=8	K=10	K=6	K=8	K=10
m_j =80, κ_j =15, j(t)=100	3.23	2.53	2.06	4.99	4.58	4.34
m_j =80, κ_j =30, j(t)=100	3.21	2.50	2.03	4.96	4.54	4.30
m_j =80, κ_j =2, j(t)=100	3.31	2.64	2.19	5.10	4.72	4.51
m_j =80, κ_j =15, j(t)=60	3.14	2.41	1.92	4.86	4.41	4.14
constantintensity $m_j \Delta t$=0.6667	3.19	2.47	1.99	4.93	4.50	4.25

Table 2

The market-consensus forward-looking forecast as to how the expected destructive power of a hurricane would evolve from news arrival to news arrival as a multi-period transaction-time binomial tree using implied parameters m=80, k=20, σ=0.15; u=1.1618, d=0.8607, p=0.4625. Parameters are conditioned on 30 transaction arrivals over 90 days, and each period in the binomial tree denotes 3 days. The term “prob” denotes the probability news will arrive in the next 3 days, and the term “period” labels the transaction count with a total of 30 expected transaction arrivals in 90 days. The initial CHI value is 8.00 but after 30 arrivals the value will range from a low of 0.09 with probability of 0.15529 to a high of 720.14 with probability of 0.08982.

																75.90
															65.33	56.23
														56.23	48.40	41.66
													48.40	41.66	35.85	30.86
												41.66	35.85	30.86	26.56	22.86
											35.85	30.86	26.56	22.86	19.68	16.94
										30.86	26.56	22.86	19.68	16.94	14.58	12.55
									26.56	22.86	19.68	16.94	14.58	12.55	10.80	9.29
								22.86	19.68	16.94	14.58	12.55	10.80	9.29	8.00	6.89
							19.68	16.94	14.58	12.55	10.80	9.29	8.00	6.89	5.93	5.10
						16.94	14.58	12.55	10.80	9.29	8.00	6.89	5.93	5.10	4.39	3.78
					14.58	12.55	10.80	9.29	8.00	6.89	5.93	5.10	4.39	3.78	3.25	2.80
				12.55	10.80	9.29	8.00	6.89	5.93	5.10	4.39	3.78	3.25	2.80	2.41	2.07
			10.80	9.29	8.00	6.89	5.93	5.10	4.39	3.78	3.25	2.80	2.41	2.07	1.79	1.54
		9.29	8.00	6.89	5.93	5.10	4.39	3.78	3.25	2.80	2.41	2.07	1.79	1.54	1.32	1.14
	8.00	6.89	5.93	5.10	4.39	3.78	3.25	2.80	2.41	2.07	1.79	1.54	1.32	1.14	0.98	0.84
prob	0.82	0.80	0.78	0.76	0.75	0.73	0.72	0.71	0.71	0.70	0.70	0.69	0.69	0.68	0.68	0.68
period	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

														720.14
													619.83	533.49
												533.49	459.18	395.22
											459.18	395.22	340.17	292.79
										395.22	340.17	292.79	252.00	216.90
									340.17	292.79	252.00	216.90	186.69	160.68
								292.79	252.00	216.90	186.69	160.68	138.30	119.04
							252.00	216.90	186.69	160.68	138.30	119.04	102.46	88.19
						216.90	186.69	160.68	138.30	119.04	102.46	88.19	75.90	65.33
					186.69	160.68	138.30	119.04	102.46	88.19	75.90	65.33	56.23	48.40
				160.68	138.30	119.04	102.46	88.19	75.90	65.33	56.23	48.40	41.66	35.85
			138.30	119.04	102.46	88.19	75.90	65.33	56.23	48.40	41.66	35.85	30.86	26.56
		119.04	102.46	88.19	75.90	65.33	56.23	48.40	41.66	35.85	30.86	26.56	22.86	19.68
	102.46	88.19	75.90	65.33	56.23	48.40	41.66	35.85	30.86	26.56	22.86	19.68	16.94	14.58
88.19	75.90	65.33	56.23	48.40	41.66	35.85	30.86	26.56	22.86	19.68	16.94	14.58	12.55	10.80
65.33	56.23	48.40	41.66	35.85	30.86	26.56	22.86	19.68	16.94	14.58	12.55	10.80	9.29	8.00
48.40	41.66	35.85	30.86	26.56	22.86	19.68	16.94	14.58	12.55	10.80	9.29	8.00	6.89	5.93
35.85	30.86	26.56	22.86	19.68	16.94	14.58	12.55	10.80	9.29	8.00	6.89	5.93	5.10	4.39
26.56	22.86	19.68	16.94	14.58	12.55	10.80	9.29	8.00	6.89	5.93	5.10	4.39	3.78	3.25
19.68	16.94	14.58	12.55	10.80	9.29	8.00	6.89	5.93	5.10	4.39	3.78	3.25	2.80	2.41
14.58	12.55	10.80	9.29	8.00	6.89	5.93	5.10	4.39	3.78	3.25	2.80	2.41	2.07	1.79
10.80	9.29	8.00	6.89	5.93	5.10	4.39	3.78	3.25	2.80	2.41	2.07	1.79	1.54	1.32
8.00	6.89	5.93	5.10	4.39	3.78	3.25	2.80	2.41	2.07	1.79	1.54	1.32	1.14	0.98
5.93	5.10	4.39	3.78	3.25	2.80	2.41	2.07	1.79	1.54	1.32	1.14	0.98	0.84	0.73
4.39	3.78	3.25	2.80	2.41	2.07	1.79	1.54	1.32	1.14	0.98	0.84	0.73	0.62	0.54
3.25	2.80	2.41	2.07	1.79	1.54	1.32	1.14	0.98	0.84	0.73	0.62	0.54	0.46	0.40
2.41	2.07	1.79	1.54	1.32	1.14	0.98	0.84	0.73	0.62	0.54	0.46	0.40	0.34	0.30
1.79	1.54	1.32	1.14	0.98	0.84	0.73	0.62	0.54	0.46	0.40	0.34	0.30	0.25	0.22
1.32	1.14	0.98	0.84	0.73	0.62	0.54	0.46	0.40	0.34	0.30	0.25	0.22	0.19	0.16
0.98	0.84	0.73	0.62	0.54	0.46	0.40	0.34	0.30	0.25	0.22	0.19	0.16	0.14	0.12
0.73	0.62	0.54	0.46	0.40	0.34	0.30	0.25	0.22	0.19	0.16	0.14	0.12	0.10	0.09
0.68	0.68	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30