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Twin Momentum: Fundamental Trends Matter

Abstract

Using time-series trends of a set of firms' major fundamentals, we find that there is a fundamental momentum in the stock market. Buying stocks in the top quintile of fundamental trends and selling stocks in the bottom quintile earns a monthly average return of 0.88%, whose magnitude is comparable to price momentum. Combining price momentum and fundamental momentum produces a twin momentum, earning an average return that exceeds the sum of the two momentum returns. Our results show that firm fundamental trends play an economically much more important role than previously thought. Theoretically, we show that investors can learn from fundamental trends about future stock returns in an equilibrium model, providing an economic rationale for fundamental momentum.

Keywords: Price momentum; Fundamental momentum; Twin momentum; Information asymmetry

JEL Classification: G12, G14

1 Introduction

In his presidential address, Cochrane (2011) highlights that "asset prices should equal expected discounted cashflows." If this principle holds, expected fundamentals should be the most powerful predictors of future stock returns. Prior studies, however, find that the predictive power of fundamentals is usually eclipsed by that of price momentum, which is based on stock prices alone. For example, among the 452 anomalies in Hou, Xue, and Zhang (2018), price momentum earns the highest value-weighted average return (1.16%) over the 1967:01–2016:12 sample period, outperforming all fundamental-based trading strategies.[1] While there are top traders who believe in either fundamental analysis or technical analysis or both (Schwager, 1989), academic research and education are almost entirely fundamentals. Therefore, from the latter point of view, it is important to show that fundamentals matter in predicting the cross section of stock returns.

In this paper, we provide strong evidence that fundamentals matter. We discover that the underperformance of fundamental analysis in existing studies is driven primarily by not making full use of the available fundamental information. In contrast, we incorporate not only the lagged values of fundamental variables, but also their trends. As it turns out, the inclusion of the trends are critical to uncover the significantly predictive power of the fundamentals. Theoretically, we show in an equilibrium model, in the presence of information asymmetry and Bayesian learning, that both price and fundamental trends can predict future stock returns, justifying both fundamental momentum and twin momentum proposed by this paper.

Specifically, we construct a measure of fundamental implied return (hereafter, FIR) for each stock based on the lagged values of a set of seven firm fundamental variables and their trends.[2] Similar to price momentum, a fundamental momentum trading strategy is to buy stocks in the top FIR quintile and sells stocks in the bottom FIR quintile. With value-weighting, the spread portfolio earns an average return of 0.88% per month over the sample period from April 1976 to September 2015, which is comparable to the average return from price momentum (0.93% per month) over the same sample period.[3] Moreover,

[1] The best fundamental-based trading strategy is the quarterly R&D-to-market anomaly, which has a 1.12% average return in Hou, Xue, and Zhang (2018); however, the statistical evidence on this strategy is limited because the sample period only starts in January 1990.

[2] The idea of screening stocks based on multiple fundamentals goes back at least to Graham and Dodd (1934) which are followed widely by value investors such as Warren Buffett.

[3] With decile sorting, the fundamental and price momentum strategies earn 1.28% and 1.30%, respectively. Results with equal-weighting and gross-return-weighting (Asparouhova, Bessembinder, and Kalcheva, 2013) are similar and reported in the online appendix.

their Sharpe ratios are close too, 0.14 and 0.16. Like price momentum, fundamental momentum cannot be explained by existing factor models. In short, our paper shows, for the first time, that fundamentals matter, which can generate a fundamental momentum as important as price momentum in terms of Sharpe ratio.

Econometrically, the FIRs are based on standard multivariate regression models. Since there are many variables included in the multivariate regressions, some of them may be highly correlated, and therefore there may be concerns on model overfitting and regressor multicollinearity.[4] To mitigate these concerns, we also use an alternative forecast combination approach to construct FIR. This approach simply runs a univariate regression for each predictor to obtain an FIR, and then uses the average of all FIRs as the final FIR. Timmermann (2006) provides an extensive survey of this approach, while Rapach, Strauss, and Zhou (2010) seem the first to apply it to time series forecasting in finance. Our paper appears one of the first to apply it to cross-sectional prediction. Based on the combined FIR, the new fundamental momentum earns an average return of 0.92% per month, slightly better but little changed from 0.88%. This finding shows that our results are robust econometrically. In the sequel, for brevity, we focus only on results based on multivariate regressions because they are the standard tool in finance.

Fundamental momentum and price momentum are different, and the former does not suffer from short-term reversal.[5] According to Campbell, Giglio, Polk, and Turley (2018), if firm profits suffer from cash flow shocks, they are likely to be permanent in the sense that rational investors have no reasons to expect the stock price to rebound to previous levels. On the other hand, if the profits are due to discount rate shocks or variance shocks, the declines are likely to be temporary, since rational investors expect the stock price to rebound to previous levels. Indeed, over our sample period, price momentum reverts four months after portfolio formation, whereas fundamental momentum reaches its maximum at the 12-month mark, due to the fact that price momentum has greater exposures to both discount rate shocks and variance shocks. Moreover, price momentum is known to have crash risk (Daniel and Moskowitz, 2016). For example, its two worst monthly returns are -42.78% and -41.66%. In contrast, the fundamental momentum has much less crash risk, with two worst monthly returns -29.2% and -24.31%, respectively.

Our paper also exploits the use of fundamental information further by combining fundamental

[4]As argued by Lewellen (2015), the potential multicollinearity in the regressions is not a significant issue because our main focus is the overall predictive power of all the predictors, not the slopes on individual predictors.

[5]The correlation between fundamental momentum and price momentum is 0.14.

momentum with price momentum, yielding a *twin momentum*. To do this, we take long positions in stocks in the top past return and FIR quintiles and short positions in stocks in the bottom past return and FIR quintiles, and find that the resulting twin momentum portfolio earns an amazing 2.16% monthly average return, more than doubling the returns from both price and fundamental momentum (0.93% and 0.88%, respectively). This result also suggests that fundamental momentum and price momentum are largely complimentary, rather than overlapping. Moreover, twin momentum has a monthly Sharpe ratio of 0.26, exceeding by far those of price momentum (0.16) and fundamental momentum (0.14). As is the case with fundamental momentum, the superior performance of twin momentum cannot be explained by extant factor models.

From an investment perspective, an important question is whether twin momentum adds value beyond the well-known factors, such as those in Fama and French (2015) and Hou, Xue, and Zhang (2015). To address this question, we run six mean-variance spanning tests under various distributional assumptions of the data, and find strong evidence that twin momentum cannot be spanned by any portfolio of the extant factors, suggesting that it can significantly improve the mean-variance frontier for investors.

Stambaugh, Yu, and Yuan (2012, 2014) find that most anomalies are mainly from mispricing in the short-leg due to arbitrage asymmetry. An interesting question is whether this is also true for twin momentum. As it turns out, twin momentum has an alpha from the long-leg ranging from 0.80% (t-value = 3.61) to 0.37% (t-value = 2.34). Hence, the risk-adjusted return from the long-leg, though smaller in magnitude than that from the short-leg, is both statistically significant and economically sizeable. Therefore, twin momentum is unique among all anomalies due to the economic importance of its long-leg.

We conduct a battery of robustness checks on the findings. First, unlike most anomalies, twin momentum is not driven by firm size. Following Stambaugh, Yu, and Yuan (2015), we construct twin momentum portfolios by eliminating firms with market capitalization falling in the bottom 20th, 40th, 60th, and 80th percentiles of the stock universe, respectively, and find that the results remain significant across all size groups. For example, in the mega-cap group (excluding firms below the 80th percentile), twin momentum earns a monthly average return of 1.42% and its monthly alphas are at least 0.63% across factor models. Second, twin momentum is not attributable to transaction costs. Its turnover ratio is about 87.89% per month, comparable to that of price momentum (Grundy and Martin, 2001; Novy-Marx and Velikov, 2016), and the break-even transaction cost that offsets the twin momentum profit is 2.46% (Barroso and Santa-

Clara, 2015). Third, twin momentum is not driven by investor sentiment. Its monthly average return is 2.29% in high-sentiment periods and 2.03% in low-sentiment periods. Finally, we show that mutual funds whose firm holdings scores are high in FIR deliver better performance.

Why do we obtain so much stronger results than existing studies? There are two reasons. First, existing studies often focus on one fundamental variable at a time. In contrast, we utilize the information of seven major fundamentals *jointly*, in the spirit of "big-data". Due to reasons such as structural breaks or accounting rule changes, a single fundamental variable may not always predict returns and may not predict in the same way. A collection of them can better capture the entire economic outlook of a firm, resulting in greater predictive power overall. Second, we use not only fundamentals, but also their *trends* that contain incremental information. For example, strong earnings in one quarter do not necessarily suggest that the firm's fundamental is strong, but consistent strong earnings do (Loh and Warachka, 2012). Incorporating more variables and their trends makes the difference, an idea that can have wide applications.

Our paper is closely related to studies on profitability trend. To the best of our knowledge, Akbas, Jiang, and Kock (2017) are the first to address explicitly the importance of profitability trend. They focus on quarterly gross profit and define the trend as the regression slope of gross profit on a time trend. Empirically, over our sample period, the decile spread portfolio based on the profitability trend earns and average return of only 0.42%, about half of the fundamental momentum. In addition, its Fama and French (2015) alpha is 0.07%, and its Hou, Xue, and Zhang (2015) alpha is even negative, -0.36%. In contrast, we use moving averages and let the data weigh the relative importance of short- and long-term trends. Moreover, we use the information of seven variables instead of just one.

Our paper is also related to studies on earnings momentum. Chan, Jegadeesh, and Lakonishok (1996) find that earnings surprises can generate momentum and contain distinct information from price momentum. However, the magnitude of earnings momentum is small, typically less than half that of price momentum. Griffin, Ji, and Martin (2005) and many others observe similarly weak performance in international markets. Because of this, later studies of earnings momentum focus on the interaction between price momentum and earnings momentum. For example, Chordia and Shivakumar (2006) find that price momentum is captured by the systematic component of earnings momentum, especially for small stocks. In contrast to this strand of literature, our study is the first to uncover fundamentals' strong predictive power over future returns.

This paper has implications to the recent growing literature on the cross section of stock returns and multiple firm characteristics, such as Lewellen (2015), Cosemans, Frehen, Schotman, and Bauer (2016), Stambaugh and Yuan (2017), Green, Hand, and Zhang (2017), Light, Maslov, and Rytchkov (2017), Yan and Zheng (2017), and Bartram and Grinblatt (2018), among others. While these studies explore the use of "big-data" of both fundamental and technical predictors including price trends, none of them exploits the use of fundamental trends. We contribute to the literature by highlighting the importance of using trend signals in a cross sectional forecasting framework.

The rest of the paper is organized as follows. Section 2 provides the model and methodology for estimating expected stock returns using fundamentals and their trends. Section 3 shows that fundamental momentum is comparable to and different from price momentum. Section 4 proposes the twin momentum. Section 5 provides robustness checks. Section 6 concludes.

2 Methodology and Data

2.1 A model of twin momentum

In this section, we propose an equilibrium model to show the predictive power of fundamental trends and price trends, providing a theoretical justification for fundamental momentum and twin momentum. The predictability of fundamental variables can be theoretically explained in light of investors' irrational behaviors. For example, Bouchaud, Krueger, Landier, and Thesmar (2018) argue that investors' sticky expectations can result in predictability by using fundamental variables. Barberis, Greenwood, Jin, and Shleifer (2018) show that investors' extrapolative expectations can yield fundamental predictability too and can even lead potentially to bubbles. However, there is an absence of a model that allows for both predictability of fundamentals beyond prices and the predictability of fundamental trends. We provide the first such a rational model, to show that both price and fundamental trends have predictive power for future returns in an economy with asymmetric information and investor learning.

Our model is an extension of Wang (1993). Consider a market for a risky stock that pays out a random stream of dividendsp. The market is populated with two types of investors: informed rational investors (arbitrageurs), and uninformed investors, who use observable variables, both technical and fundamental, to infer unobservable information. In addition, the market is populated with noise traders whose demand for the stock is random and exogenous. Wang (1993) seems the first to provide such a setting to study asymmetric information with optimal Bayesian leaning, while Han, Zhou, and Zhu (2016) use it to study market impact of technical investors.

To formalize the model, we make the following seven assumptions.

Assumption 1. The market is endowed with a certain amount of one risky stock, each unit of which provides a dividend flow given by

$$dD_t = (\pi_t - \alpha_D D_t)dt + \sigma_D dB_{1t}, \tag{1}$$

where π_t measures the long-term mean growth rate of dividend, given by another stochastic process

$$d\pi_t = \alpha_\pi(\bar{\pi} - \pi_t)dt + \sigma_\pi dB_{2t}, \tag{2}$$

where B_{1t} and B_{2t} are independent innovations.

Assumption 2. The supply of the stock is $1+\theta_t$ with

$$d\theta_t = -\alpha_\theta \theta_t dt + \sigma_\theta dB_{3t}, \tag{3}$$

where B_{3t} is another Brownian Motion independent from both B_{1t} and B_{2t}. This assumption normalizes the long-run stationary level of the supply of the risky asset to 1, whereas θ_t represents shocks away from that level representing noise trades' risk.

Assumption 3. The market is competitive with no transaction cost. The stock is the only security traded in the market. Let P_t be the equilibrium price of the stock.

Assumption 4. There is a risk-free bond to all investors with a constant rate of return $1+r$ $(r>0)$.

Assumption 5. There are two types of investors: informed investors and uninformed investors. Informed investors observe the dividend D_t, mean growth rate of dividend π_t, the price as well as all histories of the variables, while they do not directly observe the supply of the stock. Uninformed investors only observe dividend and price processes, and do not directly observe π_t. Instead of optimally learning the unobservables, they infer information from both price and fundamental information. Specifically, they use the following updating rule to infer π_t,

$$\pi_t^u = \bar{\pi} + \beta_D(D_t - \alpha_{D_L} A_{D_t}) + \beta_p(P_t - \alpha_{p_L} A_t) + \sigma_u u_t, \tag{4}$$

where parameters β_D, β_p and σ_u are constants reflecting uninformed investors' belief, while A_{D_t} and A_t are defined as the exponential moving averages of dividend D_t and price P_t, *resp*, as

$$A_{D_t} \equiv \int_{-\infty}^{t} \exp[-\alpha_{D_L}(t-s)] D_s ds, \tag{5}$$

$$A_t \equiv \int_{-\infty}^{t} \exp[-\alpha_{p_L}(t-s)] P_s ds, \tag{6}$$

where α_{D_L} and α_{p_L} are the inverses of moving average lag windows of D_t and P_t. Note that $D_t - \alpha_{D_L} A_{D_t}$ and $P_t - \alpha_{p_L} A_t$ are the slopes of A_{D_t} and A_t, i.e.,

$$dA_{D_t} = (D_t - \alpha_{D_L} A_{D_t}) dt, \tag{7}$$

$$dA_t = (P_t - \alpha_{p_L} A_t) dt. \tag{8}$$

Our assumption on the updating rule by uninformed investors is based on the observation that these variables are equivalent to the technical indicator MACD (moving average convergence and divergence), defined as the difference between moving averages with short- and long-lags of moving averages of examined variables, such as dividend and price in our paper. These are widely used technical indicators in the investment industry. In addition, the constant term in Equation (4) is $\bar{\pi}$, the unconditional expectation of π_t, due to the fact that the unconditional expectation of $(D_t - \alpha_{D_L} A_{D_t})$ and $(P_t - \alpha_{p_L} A_t)$ are both zeroes.

Assumption 6. The structure of the market is common knowledge.

Assumption 7. Both informed and uninformed investors have expected additive utility with constant absolute risk aversion (CARA) conditional on their respective information set, $E[\int u(c(\tau), \tau) d\tau | \cdot]$, with

$$u(c(t), t) = -e^{-\rho t - c(t)}, \tag{9}$$

where ρ is the discount parameter, and $c(t)$ is the consumption rate at time t.

Based on the above assumptions 1-7, we prove that there exists an equilibrium price in the market, characterized in the following proposition.

Proposition 1 *In an economy defined by Assumptions 1-7, there exists a stationary rational expectations equilibrium. The equilibrium price function has the following linear form:*

$$P_t = p_0 + p_1 D_t + p_2 \pi_t + p_3 \theta_t + p_4 A_t + p_5 A_{Dt}, \tag{10}$$

where p_0, p_1, p_2, p_3, p_4, and p_5 are constants determined only by model parameters. A_{D_t} and A_t are given in Equations (5) and (6).

The proof and detailed computation method of parameters p_0, p_1, p_2, p_3, p_4, and p_5 are given in Appendix.

Proposition 1 indicates that uninformed investors can survive in the market equilibrium, and the equilibrium price is a linear function based on the union of information sets of all investors. Furthermore, define the stock return R_{t+1} as

$$R_{t+1} \equiv \frac{P_{t+\Delta t} - P_t}{\Delta t},$$

which can be derived by differentiating (10) to obtain

$$R_{t+1} = \gamma_0 + \gamma_1 D_t + \gamma_2 \pi_t + \gamma_3 \theta_t + \gamma_4 A_t + \gamma_5 A_{Dt} + \sigma_P \varepsilon_P, \tag{11}$$

where

$$\begin{aligned} \gamma_1 &= (p_4 - \alpha_D) p_1, \;\; \gamma_2 = (p_4 - \alpha_\pi) p_2, \\ \gamma_3 &= (p_4 - \alpha_\theta) p_3, \;\; \gamma_4 = (p_4 - \alpha_{p_L}) p_4, \;\; \gamma_5 = (p_4 - \alpha_{D_L}) p_5. \end{aligned} \tag{12}$$

There are a few implications of the model. The first and foremost is that future stock returns can be predicted by the fundamental trend A_{Dt} and the price trend A_t. In the model, uninformed investors do not observe π_t, but they can measure it with both A_t and A_{Dt}. Equation (11) shows that both A_t and A_{Dt} have predictive power to future stock returns in equilibrium. In fact, Equation (10) can be extended to include variables other than A_{Dt}, as long as it follows a process similar to that of D_t, and can be used to learn the long-term dividend growth rate (Kozak, Nagel, and Santosh, 2018). In our empirical study, we use seven fundamental variables to capture the information of a firm's long-term dividend growth rate.

Second, the model is still valid if investors use multiple moving average windows to measure fundamental trends. In the model, we assume a single moving average lag window to measure A_{Dt}, while in reality, investors may not know which moving average window is optimal, and may simply use multiple windows to make inference. Hence, in our empirical study, we use multiple moving averages with different windows to construct the fundamental implied return in Equation (16).

Third, due to the existence of noise traders, the informed investors in our model face noise trader risk and cannot fully arbitrage away the uninformed investors. Hence, the uninformed investors survive in the long run, and in turn they have impact on the equilibrium price. In contrast to a market without noise traders and without information asymmetry, informed investors with full information will make risk-free arbitrage to drive out uninformed investors, so that the latter have no effect on the equilibrium price.

Fourth and finally, it is of theoretical interest to see how model parameters affect γ_5. Intuitively, out of all the parameters, the population of uninformed investors w, and parameters β_D and β_p in Equation (4) are the most important ones. This fact is illustrated in Table A1 in the Appendix, where we provide the numerical values of γ_5 for a range of possible β_D, β_p, and w. The results suggest that the sign of γ_5 depends on β_D, with positive β_D corresponding to positive γ_5. The magnitude of γ_5 increases with w, the population of uninformed investors. The impact of β_p on γ_5 is of second order importance.

In summary, the model provides a theoretical explanation on why both fundamental and price trends can jointly predict future stock returns, yielding fundamental and twin momentum of this paper.

2.2 Measures of fundamental trends

The central idea of this paper is to make use of information on fundamental trends in constructing investment portfolios. To explore the econometric intuition, consider a general decomposition of the expected stock return of firm i as:

$$\mathrm{E}_t[R_{i,t+1}] = f_{i,t} + \beta \mathrm{E}_t[F_{i,t+1}], \tag{13}$$

where $f_{i,t}$ is the required return based on the current fundamental (e.g., a constant or book-to-market ratio in Fama and French (2015)), $\mathrm{E}_t[F_{i,t+1}]$ is the required return based on future fundamental, and β is the sensitivity coefficient. This decomposition is usually accomplished with log linearization and is widely used in the literature. For example, $F_{i,t+1}$ represents future return on equity (ROE) in Pastor and Veronesi (2003), future growth opportunities in Kogan and Papanikolaou (2014), and future investment and profitability in Fama and French (2015) and Hou, Xue, and Zhang (2015), respectively.

Since the expected fundamental $\mathrm{E}_t[F_{i,t+1}]$ is unobservable, it is often estimated by its current value to test (13):

$$\mathrm{E}_t[F_{i,t+1}] = F_{i,t}. \tag{14}$$

If $F_{i,t}$ follows a random walk or AR(1) process, this estimation works well, especially in a portfolio sort setting that focuses on the rank of a firm's expected fundamental. However, if the AR(1) assumption is violated, $F_{i,t}$ will not be sufficient to capture the expected fundamental. For example, Akbas, Jiang, and Kock (2017) find that the trend in gross profitability has power to predict future stock returns and is not subsumed by current gross profitability. Hou, Xue, and Zhang (2018) show that the ROE from four quarters ago has incremental forecasting power relative to current quarter ROE. This paper explores whether a simply refined estimate for $\mathrm{E}_t[F_{i,t+1}]$ has incremental power. Particularly, we measure $\mathrm{E}_t[F_{i,t+1}]$ using the trend of $F_{i,t}$ as

$$\mathrm{E}_t[F_{i,t+1}] = \mathrm{MA}_{i,t,L} \;\text{ with }\; \mathrm{MA}_{i,t,L} = \frac{F_{i,t} + F_{i,t-1} + \cdots + F_{i,t-L+1}}{L}, \tag{15}$$

where $F_{i,t-j}$ denotes the realized value of firm i's fundamental in the jth lagged quarter, and $\mathrm{MA}_{i,t,L}$ is the moving average, a popular trend measure. One can interpret (15) as using the most recent L observations to estimate the expected value. If only the past value is used as often done in the literature, it implicitly assumes a random walk or AR(1) process for the fundamental.

Since it is clearly difficult to know which L to use in practice, we consider $L = 1, 2, 4$, and 8 in what follows, but do examine the robustness with longer lags (which forces to exclude a large number of firms). By allowing L to vary, we let data determine the weights on how information in different time horizons affects the expected return.

In addition to trend, we also consider a sizable number, K, of firm fundamentals. While most studies focus on one fundamental variable, Hou, Xue, and Zhang (2015) and Fama and French (2015) do allow the expected return to be related to both expected profitability and expected investment. In contrast, we generalize this and utilize $K = 7$ major fundamentals below. Hence, the cross-sectional predictors we use will be $\text{MA}^k_{i,t-1,L}$, where $k = 1, \cdots, K$ and $L = 1, 2, 4$, and 8 (i.e., 28 predictor in total). Note that the usual choice of past observations corresponds to the case of $L = 1$.

2.3 Multivariate regression approach

Our goal is to estimate expected stock returns based on all the predictors. Following Haugen and Baker (1996) and many others, we take a two-step procedure. First, we run a cross-sectional regression of each stock return $R_{i,t}$ on all the predictors,

$$R_{i,t} = \alpha_t + \sum_{k=1}^{K} \sum_{L=1,2,4,8} \beta^k_{L,t} \text{MA}^k_{i,t-1,L} + \varepsilon_{i,t}, \tag{16}$$

where α_t is the intercept, K is the number of fundamental variables, $\beta^k_{L,t}$ is the regression coefficient of $\text{MA}^k_{i,t-1,L}$, and $\varepsilon_{i,t}$ is the residual of stock i in month t. In implementation, $\text{MA}^k_{i,t-1,L}$ is used in the months immediately following the most recent public quarterly announcement dates.[6] For a firm to be included in regression (16), we require the end of the fiscal quarter that corresponds to its announcement of various fundamental variables' dates to be within six months prior to the regression month.

In the second step, we construct firm i's fundamental implied return, FIR, in month t by using the forecasted return for month $t + 1$,

$$\text{FIR}_{i,t} = \sum_{k=1}^{K} \sum_{L=1,2,4,8} \text{E}_t[\beta^k_{L,t+1}] \text{MA}^k_{i,t,L}, \tag{17}$$

[6] The results using a four-month lag are similar. For example, the average return of the fundamental momentum strategy is 0.82% per month, and its HXZ and FF5 alphas are 0.69% (t-value $= 2.29$) and 0.78% (t-value $= 3.07$), respectively.

where $E_t[\beta^k_{L,t+1}]$ is the expected coefficient on the kth fundamental variable with L lags and is defined as $\mathrm{E}_t[\beta^k_{L,t+1}] = \beta^k_{L,t}$. Note that we do not include the intercept in (16) because it is the same for all stocks and does not affect the ranking of stocks. It is also worth pointing out that, because only information in or prior to month t is used, the forecasted expected returns for month $t+1$, FIR, is a real-time predictor of stock returns and does not suffer from looking-forward biases. Moreover, unlike the usual sorting procedure, the cross-sectional regression (16) allows us to use multiple variables simultaneously, echoing Cochrane (2011) that "we will end up running multivariate regressions" to address the question in the zoo of new variables.

2.4 Forecast combination approach

Since the fundamentals of a firm are likely correlated, and trends of different time horizons are not independent, some of our predictors can have high correlations with each other. Econometrically, this can raise the degree of multicollinearity in multivariate regression (16), causing over-fitting.

To resolve the issue, we consider an alternative forecast combination approach. This approach is strikingly simple. Let $\{x^m_{i,t-1}\}^M_{m=1}$ be all the predictors in (16). Instead of running the multivariate regression, we now run a univariate regression on $x^m_{i,t-1}$ for each m and estimate an $\mathrm{FIR}^m_{i,t}$, which is similar to (17). However, since different $x^m_{i,t-1}$s may imply different intercepts, $\mathrm{FIR}^m_{i,t}$ should have the intercept added in. Doing so for all $m = 1, \cdots, M$, we then obtain

$$\mathrm{FIR}_{i,t} = \frac{1}{M}\sum_{m=1}^{M} \mathrm{FIR}^m_{i,t}, \tag{18}$$

which combines the univariate forecasts into an aggregated forecast of the expected return on firm i at time t.

Timmermann (2006) provides an extensive survey of the forecast combination approach and its various applications in economics. The approach has a number of robustness properties. In particular, it is robust to correlated regressors. For example, adding an additional predictor that is perfectly correlated with an existing one changes little the outcome. Rapach, Strauss, and Zhou (2010) seem the first to apply it to time series forecasting in finance, and this paper appears the first to apply it for cross-sectional prediction.

2.5 Data

Accounting data are collected from the quarterly Compustat database and market data come from the monthly CRSP database. We merge the two datasets through the CCM link table. The Compustat sample period is from January 1973 to August 2015 and the CRSP sample period is from April 1976 to September 2015. To construct the sorting variable for fundamental momentum, we use seven fundamental variables: ROE, return on assets (ROA), earnings per share (EARN), accrual-based operating profitability to equity (APE), cash-based operating profitability to assets (CPA), gross profitability to assets (GPA), and net payout ratio (NPY). These fundamental variables are earnings- and profitability-related, and are relevant for investors to use for valuation. Because our approach is flexible, any other fundamental variable can be easily included in the analyses.

Specifically, ROE is defined as income before extraordinary items (IBQ in Compustat) divided by one-quarter-lagged book equity as in Haugen and Baker (1996). Book equity is shareholders' equity, plus balance sheet deferred taxes and investment tax credit (item TXDITCQ) if available, minus the book value of preferred stocks. Depending on availability, we use stockholders' equity (item SEQQ), or common equity (item CEQQ) plus the carrying value of preferred stocks (item PSTKQ), or total assets (item ATQ) minus total liabilities (item LTQ), in this order, to calculate shareholders' equity. For the book value of preferred stocks, we use redemption value (item PSTKRQ), or, if that is not available, carrying value. ROA is income before extraordinary items (IBQ) divided by one-quarter-lagged total assets (ATQ) as in Balakrishnan, Bartov, and Faurel (2010). EARN is income before extraordinary items (IBQ), plus deferred taxes (TXDIY), minus preferred dividends (DVPY), and divided by one-quarter-lagged common outstanding shares (CSHO) as in Fama and French (1992). APE is revenue (REVTQ) minus cost of goods sold (COGSQ), minus selling, general, and administrative expenses (XSGAQ), minus interest expenses (TIEQ), and divided by one-quarter-lagged book equity as in Fama and French (2015). CPA is accrual-based operating profitability on assets minus change in accounts receivable (RECTQ) and change in inventory (INVTQ), plus deferred revenue (DRCQ+DRLTQ), trade accounts payable (APQ) and change in accrued expenses (XACCQ), and scaled by one-quarter lagged total assets as in Ball, Gerakos, Linnainmaa, and Nikolaev (2016). GPA is total revenue (REVTQ) minus cost of goods sold (COGSQ) divided by total assets (ATQ) as in Novy-Marx (2013). Finally, NPY is net payout divided by one-quarter-lagged market capitalization of common shares as in Boudoukh, Michaely, Richardson, and Roberts (2007). Net payout is measured as dividends per share (DVPSPQ) multiplied by one-quarter-lagged common shares outstanding (CSHOQ) plus total expenditures on the repurchase of common and preferred shares (PRSTKCY) minus the sale of common and preferred stocks (SSTKY). To remove the outlier effect, all of the variables are cross-sectionally winsorized at the 1% and 99% percentiles each quarter.

Although standard unexpected earnings (SUE) is also widely used in the literature as the sorting variable for earnings or fundamental momentum (Chan, Jegadeesh, and Lakonishok, 1996; Chordia and Shivakumar, 2006), we do not include it for two reasons. First, we already include earnings per share (EARN), which contains a firm's expected and unexpected earnings. Second, Hou, Mo, Xue, and Zhang (2018) show that ROE subsumes SUE in forecasting future stock returns. Indeed, the major results of our paper continue to hold when choosing different sets of earnings or profitability variables to construct a fundamental momentum portfolio, as will be discussed in Section 5.

3 Fundamental Momentum

In this section, we provide empirical evidence of the existence of fundamental momentum using conventional portfolio approach based on firm's FIR proposed in Section 2.2. We further provide evidence in next section the coexistence of both fundamental and price momentum.

3.1 Performance of fundamental momentum

In this section, we show that the theoretical implications of Proposition 1 are supported by real data and our FIR measure can effectively approximate firm's fundamental trend. Table 1 reports the monthly average and risk-adjusted returns of the spread portfolios based on different sorting variables, where the benchmark asset pricing models include the capital asset pricing model (CAPM), Fama and French (1993) three-factor model (FF3), FF3 plus a price momentum factor model (FF3M), Hou, Xue, and Zhang (2015) four-factor model (HXZ), and Fama and French (2015) five-factor model (FF5). For comparison, we consider three specifications in this table. Panel A forms quintile portfolios based on a single variable, which has been the benchmark approach since Fama and French (1993). Consistent with the literature, the average returns of the spread portfolios based on these fundamental variables are two standard deviations away from zero, with NPY as an exception. The risk-adjusted returns are also significant relative to the most widely used FF3 and FF3M models. However, none of the risk-adjusted returns is significant at the 5% level when the benchmark model is HXZ, and only two are significant at the 5% level when the benchmark model is FF5, suggesting that the forecasting power of these fundamental variables seems substantially subsumed, if not all, by the profitability and investment factors.

Panel B is based on a single fundamental variable but incorporates its trend information when constructing the spread portfolios. Specifically, at the end of each month, we run a cross-sectional regression (16), restricting it to a single fundamental variable (i.e., $K = 1$) to estimate FIR as (17). In so doing, we explicitly show that fundamental trends matter. With one exception (CPA), the average returns of the spread portfolios constructed on the seven fundamental variables are higher than those in Panel A, where the trend information is not used. In terms of abnormal returns, the improvement with the FF5 model is pronounced, and five variables are significant at the 5% level.

Panel C represents this paper's main result, where FIR is estimated by using all fundamental variables and their trends. At the end of each month, all stocks are sorted into five quintiles by FIR, where the bottom quintile (quintile 1) contains stocks with the lowest FIR and the top quintile (quintile 5) contains stocks with the highest FIR. Value-weighted portfolios are constructed within each quintile and held for one month.[7] Fundamental momentum is the zero-investment strategy that buys the top FIR quintile portfolio and sells the bottom FIR quintile portfolio. Over the sample period 1976–2015, fundamental momentum performs better than strategies that do not make full use of the information provided by multiple variables and their trends. The average return is 0.88% per month, almost the same as that of price momentum (0.93%), and its alpha is significant at the 5% level for any of the asset pricing models we consider. Table 2 shows that the monthly average and risk-adjusted returns of FIR quintile portfolios, and suggests that FIR is indeed aligned with expected returns in the cross section: the average and risk-adjusted returns monotonically increase from quintile 1 to quintile 5.

Note that, unlike most studies that attempt to explain average stock returns by sorting on a few (usually less than three) firm characteristics or fundamental variables, we construct fundamental momentum from

[7] The results with equal-weighting and gross-return weighting as Asparouhova, Bessembinder, and Kalcheva (2013) are reported in the online appendix.

cross-sectional regressions that allow us to incorporate multiple fundamentals. Including or excluding one or more variables is easy in our framework. In the literature, there is no consensus on the best proxy for a firm's fundamental value, and we do not take a stance as to which are the most important variables in constructing fundamental momentum. No matter how variables are chosen, a set of common fundamentals collectively have stronger power in predicting stock returns.

3.2 Performance with an alternative forecast combination approach

Empirically, although fundamentals are highly correlated with each other, the potential multicollinearity in the cross-sectional regression (16) is not a concern in this paper, because we are primarily interested in the overall predictive power of the model, not the slopes on individual variables. To see this, we employ the alternative forecast combination approach introduced in Section 2.4.

Table 3 reports the average and risk-adjusted returns. The results are quantitatively similar to previous ones. For example, the average return of the fundamental momentum strategy and its HXZ and FF5 alphas are 0.92% (t-value = 4.18), 0.79% (t-value = 2.75), and 0.86% (t-value = 3.35), which are surprisingly close to the corresponding values in Table 2, 0.88%, 0.72%, and 0.85%, respectively. Therefore, consistent with Lewellen (2015), the multicollinearity is not an issue if our focus is the expected returns, rather than a specific return predictor. For this reason, we focus on the results based on multivariate regressions hereafter.

3.3 Comparison with price momentum

We conduct three analyses, portfolio sort, Fama-MacBeth regression, and Campbell, Giglio, Polk, and Turley (2018) decomposition, to show that fundamental momentum and price momentum exploit different sources of information and complement each other.

3.3.1 Bivariate portfolio analysis

In this section, we investigate the performance of fundamental momentum controlling for price momentum. In the literature, there is no consensus as to whether price momentum and fundamental momentum are two separate anomalies, where the latter is usually constructed based on standard unexpected earnings or cumulative three-day abnormal returns (e.g., Chan, Jegadeesh, and Lakonishok, 1996; Chordia and Shivakumar, 2006; Hou, Peng, and Xiong, 2009).

At the end of each month t, we independently sort stocks into five groups based on their past returns (11-month cumulative return from month $t-12$ to month $t-2$) and five groups based on their FIRs. The intersections of this double sort produce 25 value-weighted portfolios. Table 4 reports the average return of each portfolio in excess of the risk-free rate (the one-month U.S. Treasury-bill rate) and the associated Newey-West t-value. The results suggest that fundamental momentum is different from price momentum (e.g., Hou, Peng, and Xiong, 2009). Within each past return quintile, the average return of the fundamental momentum portfolio monotonically increases in FIR, and within each FIR quintile, the average return of the price momentum portfolio increases in past return. The higher the average FIR, the higher the average return earned by fundamental momentum. For example, in the lowest past return quintile, the average return of the FIR portfolios increases from -0.83% (t-value $= -2.07$) per month for quintile 1 to 0.33% (t-value $= 0.71$) per month for quintile 5, suggesting that the fundamental momentum strategy, measured by the spread return, earns a significant average return of 1.16% (t-value $= 4.02$) per month. Similarly, in the highest past return quintile, the average return of the FIR portfolios increases from 0.72% (t-value $= 2.11$) per month for quintile 1 to 1.33% (t-value $= 4.13$) per month for quintile 5, suggesting that fundamental momentum earns a significant average return of 0.61% (t-value $= 2.26$) per month.

One interesting finding in the last column of Table 4 is that the fundamental momentum profit generally decreases in the past return quintile rank, which suggests that fundamental momentum exists not only in the past winner stocks but also in the past loser stocks with even stronger performance. The last row of Table 4 shows that price momentum exists in each FIR quintile, and its performance is even stronger than that of the fundamental momentum with the same sorting rank. The average monthly return of price momentum is 1.55% (t-value $= 4.43$) in the lowest FIR quintile and 1.00% (t-value $= 2.58$) in the highest FIR quintile. To summarize Table 4, fundamental momentum exists in stock returns and is not simply a manifestation of price momentum.

3.3.2 Fama-MacBeth regression

A complementary approach to the portfolio sort is to run a Fama-MacBeth regression of stock return in month t on past return (i.e., 11-month cumulative return from month $t-12$ to month $t-2$) and FIR in month $t-1$ directly. The advantage of this cross-sectional regression is that one can control for other firm characteristics, which may contain information on the variables of interest. As such, we choose five firm characteristics that have been commonly used in the literature, including short-term reversal (stock return in month $t-1$), long-term reversal (cumulative stock return between month $t-60$ and month $t-13$), log market capitalization (log size), book-to-market (B/M), and idiosyncratic volatility (IVOL) estimated from the FF3 model using the past month's daily returns, with a requirement of at least 16 observations.

The regression results are reported in Table 5. The first two columns are the results of regressing stock returns on past return or past FIR, and the regression slopes are positive and statistically significant, suggesting that both past return and FIR have forecasting power for future stock returns. Column 3 is the result of regressing stock returns on past return and FIR simultaneously, and the regression slopes are also positive and statistically significant, consistent with Table 4 that fundamental momentum exists in stock returns. When we include the other five firm characteristics as controls in Columns 4 to 6, the regression slopes on past return and FIR are quantitatively unchanged and remain statistically significant. Overall, Tables 4 and 5 suggest that fundamental momentum and price momentum coexist in the stock market and neither subsumes the other.

3.3.3 Cash flow, discount rate, and variance betas

The previous results suggest that fundamental momentum and price momentum are different and complementary. In this section, we investigate the difference in more depth and attempt to uncover its sources.

Campbell and Vuolteenaho (2004) show that unexpected returns on the market portfolio can be decomposed into two components: shocks relating to future cash flows and shocks relating to discount rates that investors apply to these cash flows. Campbell, Giglio, Polk, and Turley (2018) extend this model by assuming that the variance of stock returns is stochastic and that unexpected returns also respond to variance shocks. As such, investment opportunities may deteriorate because the expected return on the stock market declines (discount rate shocks) or because the variance of the stock market increases (variance shocks), and a conservative long-term investor has to hedge against two types of shocks to the investment opportunity set. This means that the single CAPM beta can be decomposed into three betas: one reflecting the covariance with news about future cash flows, one with news about discount rates, and one with news about variance.

Following Campbell, Giglio, Polk, and Turley (2018), we run time-series regressions of price momentum and fundamental momentum returns on shocks about future cash flows (N_{cf}), discount rate ($-N_{dr}$), and variance (N_v) of the aggregate market,[8]

$$R_i = \alpha + \beta_{cf} N_{cf} + \beta_{dr}(-N_{dr}) + \beta_v N_v + \varepsilon_i,$$

where R_i is the quarterly price momentum or fundamental momentum returns over the sample period 1976Q2–2011Q4.

Table 6 reports the regression results, which reveal two interesting facts. First, price momentum and fundamental momentum are related. Specifically, both price momentum and fundamental momentum have negative cash flow betas and discount rate betas, but positive variance betas, which is consistent with the signs of the UMD and RMW factors in Campbell, Giglio, Polk, and Turley (2018). Moreover, their cash flow betas are insignificant, whereas the discount rate betas and variance betas are significant, suggesting that the profits from price momentum and fundamental momentum do not stem from cash flow shocks. According to Campbell and Shiller (1988) and Campbell and Vuolteenaho (2004), the significant discount rate beta suggests that the effect of discount rate shocks is temporary, that is, rational investors expect the stock price to rebound to previous levels. As a result, both price momentum and fundamental momentum profits should revert sometime after portfolio formation. Second, price momentum and fundamental momentum are different. Their discount rate betas are -0.35 (t-value $= -2.00$) and -0.16 (t-value $= -1.97$), and their variance betas are 1.06 (t-value $= 3.09$) and 0.37 (t-value $= 2.00$), respectively. In magnitude, the effect of discount rate shocks on price momentum is about two times as large as that on fundamental momentum and the effect of variance shocks is about three times as large. The relative effect of variance shocks vs. discount rate shocks is stronger for price momentum too. Moreover, the three shocks in Campbell, Giglio, Polk, and Turley (2018) describe more variation in price momentum than in fundamental momentum in terms of the

[8]We thank Christopher Polk for providing the data on his web page.

R^2 (8.61% vs. 2.71%).

To further explain the difference between price momentum and fundamental momentum, Figure 1 plots their cumulative returns over time after portfolio formation, calculated by adding monthly returns from formation month t to month $t+i$. As expected, the price momentum profit reaches its maximum four months after portfolio formation and reverts to a negative value after eight months. In contrast, the fundamental momentum profit reaches its maximum after 12 months and reverts to a negative value after 18 months.

4 Twin Momentum

In this section, we explore whether we can combine the fundamental momentum and price momentum strategies, the twin momentum strategy, to improve investment performance. We construct the twin momentum by buying stocks in the intersection of the top past return and FIR quintiles and selling stocks in the intersection of the bottom past return and FIR quintiles. This approach is intuitive and commonly used in the literature such as Da and Warachka (2011). In the end, we show that the return delivered by twin momentum portfolio is higher than the simple sum of the returns of price and fundamental momentum portfolios.

4.1 Average returns

Panel A of Table 7 reports summary statistics of the portfolio returns for the price momentum, fundamental momentum, and twin momentum strategies, which include the monthly average return, t-value (from the test of whether average return is equal to zero), volatility (standard deviation), skewness, kurtosis, and correlations between these three momentum profits. Over the sample period, price momentum has a skewness of -1.21, which implies a higher probability of experiencing large crashes than would be the case under a normal distribution. In contrast, fundamental momentum has a 0.73 skewness and therefore has a higher chance of generating extremely positive returns.

The average return of twin momentum is an impressive 2.16% (t-value $= 5.64$) per month, which is higher than the simple sum of the average returns of price momentum and fundamental momentum. The skewness of twin momentum is only 0.04, between the values for price momentum and fundamental momentum. One reason may be that the strategy chooses stocks with positive skewness from fundamental momentum and negative skewness from price momentum, and as a result, their aggregate skewness is neutralized. Another reason may be that twin momentum does not choose stocks with positive or negative skewness at all, bringing its overall skewness close to zero. In either case, twin momentum is more likely to follow a normal distribution.

The last two columns of Table 7 shows that the correlation between price momentum and fundamental momentum is as low as 0.14. This value sheds light on the success of twin momentum. Indeed, if price momentum and fundamental momentum are strongly positively correlated, combining them would not generate significant improvements. In addition, twin momentum has a correlation of 0.69 with price momentum and 0.61 with fundamental momentum, implying that price momentum and fundamental momentum are equally important to twin momentum and neither strategy dominates the other.

To explore whether our results are driven by specific time periods, we split the sample period into four subperiods. The summary statistics for each subperiod are reported in Panel B of Table 7, and lead us to make three observations. First, twin momentum exists in each subperiod. In contrast to the recent literature, such as Chordia, Subrahmanyam, and Tong (2014) and McLean and Pontiff (2016), which argues that anomaly returns decay over time, we show that twin momentum does not display a declining pattern. Its monthly average return is 1.64% (t-value $= 2.66$) in the period 1976–1985, 2.10% (t-value $= 4.44$) in 1986–1995, 2.82% (t-value $= 2.62$) in 1996–2005, and 2.06% (t-value $= 2.70$) in 2006–2015, respectively. Second, price momentum does decay over time, with significant average returns in the first two subperiods but insignificant returns in the last two subperiods. However, the declining price momentum performance does not reduce the twin momentum profit as price momentum and fundamental momentum intensify each other. That is, in the latter periods, the twin momentum profit is much larger than the sum of price momentum and fundamental momentum profits. For example, the twin momentum's average return is 2.06% per month over the period 2006–2015, 0.44% higher than the sum of the returns from price momentum (0.73%) and fundamental momentum (0.79%).

Third and finally, while the skewness of price momentum is always negative, it is mainly from the last subperiod, 2006–2015, over which, the value is -2.06. This pattern is consistent with Daniel and Moskowitz (2016) who show that three of the ten top momentum crash months occur in 2009 (March, April, and August). In contrast, the skewness of fundamental momentum is always positive with one exception in 1986–1995, over which the value is negative but close to zero, -0.05.

To visualize the outperformance of the twin momentum strategy, Figure 2 plots the time series returns (upper panel) and the log cumulative wealth (lower panel) from investing in price momentum, fundamental momentum, and twin momentum, respectively. To iron out idiosyncratic returns, we smooth portfolio returns in the upper panel with 12-month moving average values. It is easy to conclude that twin momentum generates much better performance than price momentum and fundamental momentum. Table 7 also shows that the volatility of twin momentum is relatively high, and Figure 2 provides the reason: twin momentum has a higher chance of generating large positive profits and therefore, the high volatility likely stems from these upside movements.

4.2 Alphas

This section examines whether the outperformance of twin momentum can be explained by existing risk factor models. If an asset pricing model completely captures the average return of twin momentum, the intercept should be indistinguishable from zero in a regression of the twin momentum portfolio return on the model's factor returns. Table 8 reports the results, where the first row is alpha (abnormal return) in percentage points from each risk factor model, the last row is the associated R^2, and other rows are the corresponding loadings that measure how much risk twin momentum exposes to the factors. The Newey-West t-values are reported in parentheses.

Table 8 reveals several facts about twin momentum. First, the abnormal returns from the five factor models are economically and statistically significant, varying from 1.37% (t-value $= 2.45$) per month with the HXZ model to 2.50% (t-value $= 6.69$) per month with the FF3 model, suggesting that the high return delivered by twin momentum cannot be fully explained by these extant risk factors. Second, the FF3M, HXZ, and FF5 models have some power to explain the twin momentum profit (i.e., twin momentum's alphas are lower than its average return). Twin momentum has a positive exposure to the price momentum factor in the FF3M model, to the ROE factor in the HXZ model, and to the CMA factor in the FF5 model. In contrast, the risk-adjusted returns with the CAPM and FF3 models are higher than the average return, as they push twin momentum from 2.16% per month to 2.31% (t-value $= 6.34$) and 2.50% (t-value $= 6.69$)

per month, respectively. The reason is that twin momentum has a negative exposure of -0.26 (t-value $= -1.94$) to MKT in the CAPM model, and negative exposures of -0.38 (t-value $= -2.63$) and -0.51 (t-value $= -1.87$) to MKT and HML in the FF3 model, which move the adjusted return away from zero.

Third, since twin momentum has a negative exposure to MKT in all the five factor models and has a correlation of -0.14 with MKT, twin momentum can serve as a hedge for the market portfolio. Fourth, the size factor in the FF3, FF3M, HXZ, and FF5 models has no power to explain the variation in twin momentum and none of the loadings is significant, which previews the result in Section 5.1: the size effect cannot explain twin momentum. Finally, the risk factor models explain a small fraction of the twin momentum variation and their regression R^2s are less than 15%. The only exception is the FF3M model, which augments FF3 with the price momentum factor (obtained from Ken French's website), giving it the ability to explain the variation in price momentum. As a result, the regression R^2 is 45.8%, suggesting that at least half of the variation in twin momentum stems from fundamental momentum, which is different from price momentum.

4.3 Mean-variance spanning tests

This subsection explores whether twin momentum adds any investment value from the perspective of an investor who holds a well-diversified portfolio, such as the market portfolio or a portfolio spanned by the FF5 factors. The mean-variance spanning test originally proposed by Huberman and Kandel (1987) provides the answer to this question.

The key idea of this test is to show whether twin momentum lies outside the mean-variance frontier spanned by an asset pricing model's factor returns. As such, we run a time-series regression of the twin momentum portfolio return on the factor returns in each asset pricing model over the whole sample period as follows:

$$R_t = \alpha + \sum_{j=1}^{J} \beta_j f_{j,t} + \varepsilon_t, \tag{19}$$

where $f_{j,t}$ is the return of factor j in month t and J is the number of risk factors in the asset pricing model, such as $J = 5$ in the FF5. Huberman and Kandel (1987) show that the spanning test is equivalent to the test of the following restrictions:

$$H_0 : \alpha = 0 \text{ and } \sum_{j=1}^{J} \beta_j = 1. \tag{20}$$

We follow Kan and Zhou (2012) and carry out six tests, which include a Wald test under conditional homoskedasticity, a Wald test under independent and identically distributed (IID) elliptical distribution, a Wald test under conditional heteroscedasticity, a Bekerart-Urias spanning test with errors-in-variables (EIV) adjustment, a Bekerart-Urias spanning test without the EIV adjustment, and a DeSantis spanning test. All these tests have asymptotic chi-squared distributions with 2 degrees of freedom.

The results in Table 9 suggest a strong rejection of the hypothesis that twin momentum is inside the mean-variance frontier of any of the five factor models considered in this paper. Therefore, twin momentum is clearly a unique trading strategy that describes the cross section of stock returns unexplained by extant risk factor models, and provides incremental investing value.

4.4 Long- and short-leg portfolios

Stambaugh, Yu, and Yuan (2012, 2014, 2015) show that investors face both arbitrage risk and arbitrage asymmetry, where the former refers to risk that deters arbitrage and the latter refers to the greater ability or willingness of an investor to take a long position as opposed to a short position when perceiving mispricing in a stock. Combining these two concepts, they find that most of the anomaly returns in the finance literature is from the short-leg portfolios as overpricing is more prevalent than underpricing, due to short-sale impediments.[9]

Table 10 reports the average and risk-adjusted returns of the long- and short-leg portfolios of price momentum, fundamental momentum, and twin momentum, respectively. Price momentum has a high average return, 1.35% (t-value $= 5.01$) per month, in the long-leg and a low average return, 0.42% (t-value $= 1.13$) per month, in the short-leg. However, its risk-adjusted return (in absolute value) is much smaller in the long-leg than that in the short-leg with the CAPM, FF3, and FF5 models. For example, the FF3 alpha is 0.35% (t-value $= 3.10$) per month in the long-leg and -0.78% (t-value $= -3.52$) per month in the short-leg. As our price momentum is only a finer sort relative to the Fama-French price momentum factor, our price

[9] Nagel (2005) also finds that overpricing is more pronounced in short-leg portfolios, especially those with low institutional ownership.

momentum portfolio is fully explained by the FF3M. It is also explained by the HXZ model, consistent with Hou, Xue, and Zhang (2015) that the HXZ four factors can explain price momentum. Generally, this result is consistent with Daniel and Moskowitz (2016), who show that the price momentum winner decile portfolio has a much larger average return (15.3% vs. -2.5% per year) but a much smaller CAPM alpha (7.5% vs. -14.7% per year) relative to the loser decile portfolio.

Fundamental momentum has a similar average return to price momentum: 1.39% (t-value = 5.15) per month in the long-leg and 0.52% (t-value = 1.81) per month in the short-leg. However, the risk-adjusted returns of the long- and short-legs are large and have the same magnitude. In the long-leg, the alpha ranges from 0.36% (t-value = 2.63) per month with the CAPM model to 0.53% (t-value = 3.87) per month with the FF5 model. In the short-leg, the alpha ranges from -0.23% (t-value = -1.31) per month with the HXZ model to -0.58% (t-value = 4.31) per month with the CAPM model.

As a combination strategy, twin momentum reveals several interesting patterns. The average return is 1.72% (t-value = 5.34) per month in the long-leg and -0.44% (t-value = -1.06) in the short-leg. This substantial spread explains why twin momentum has a much higher average return than price momentum and fundamental momentum. In the long-leg, the alpha is significant for all the five asset pricing models, ranging from 0.51% (t-value = 1.89) per month with the HXZ model to 0.80% (t-value = 3.61) per month with the FF5 model. This outperformance suggests that twin momentum is unlikely to be driven by short-selling constraints and other market frictions. In the short-leg, the alpha is also significant for all models and ranges from -0.86% (t-value = -2.40) per month with the HXZ model to -1.75% (t-value = -6.42) per month with the FF3 model.

To further understand the long- and short-legs in each strategy, we examine the difference in firms' characteristics between the two legs and report the results in Table 11. The characteristics we consider include past return (the sorting variable for price momentum), FIR (the sorting variable for fundamental momentum defined as (17)), and the seven fundamental variables used in constructing FIR. In each month we calculate the cross-sectional mean for each characteristic across all firms within each leg. Table 11 reports the time series average of the cross-sectional means of each characteristic. By construction, price momentum effectively buys high past return stocks and sells low past return stocks, while fundamental momentum effectively buys high FIR stocks and sells low FIR stocks. The average past return of the price momentum portfolio is 0.76% in the long-leg and −0.30% in the short-leg, resulting in a spread of 1.06%; the average FIR of the fundamental portfolio is 3.96% in the long-leg and −3.36% in the short-leg, a spread of 7.32%. However, the price momentum strategy is not effective in choosing high and low FIR stocks, while the fundamental momentum strategy cannot pick up stocks with high and low past returns. For example, when sorting on FIR for fundamental momentum, the average past return is 0.28% in the long-leg and 0.21% in the short-leg, generating a negligible spread. When sorting on past return for price momentum, the average FIR is 0.56% in the long-leg and 0.02% in the short-leg, which also generates a negligible spread of 0.54%, compared with the 7.32% spread from sorting on FIR .

Compared with price momentum and fundamental momentum, twin momentum is effective in distinguishing stocks with both high past return and high FIR from stocks with both low past return and low FIR. Its long-short spread is 0.51% in past return and 7.67% in FIR, half of the spread of price momentum (1.06%) and similar to fundamental momentum (7.32%), respectively. This result indicates that twin momentum does choose stocks with high past return and FIR in the long-leg and stocks with low past return and FIR in the short-leg. The seven fundamental variables used to construct the FIR lend further support to twin momentum, among them, six generate much larger long-short spreads (last column of Table 11) than those of price momentum (third to last column) and fundamental momentum (second to last column).

Summarizing Tables 10 and 11, we conclude that twin momentum is unlikely to be fully explained by market frictions or limits to arbitrage, as its profit partially stemps from the long-leg portfolios. Rather, the strategy's outperformance is likely due to its ability to exploit information in both price and fundamental trends.

5 Robustness

In this section, we conduct a number of analyses to show the robustness of twin momentum.

5.1 Size effect

It is well known that smaller firms tend to exhibit stronger mispricing, which plagues the existing factor models (Fama and French, 2015). This fact raises the question of whether twin momentum is heavily concentrated in small firms. While the use of value-weighted portfolios in the previous sections reduces this possibility, this subsection further explores the sensitivity of our results to size by excluding firms below a given size threshold.

Following Stambaugh, Yu, and Yuan (2015), before performing the double sort on past return and FIR, we eliminate all firms whose market capitalizations fall in the bottom p percentile of the stock universe for various choices of p. Specifically, we sequentially exclude the bottom 20%, 40%, 60%, and 80% of firms and compute the average and risk-adjusted returns of price momentum, fundamental momentum, and twin momentum, respectively. The results are reported in Panels A, B, C, and D of Table 12.

Table 12 provides two interesting observations. First, fundamental momentum and twin momentum exist in all size groups. Although their performance weakens as the size threshold increases, even among the largest quintile of stocks (Panel D), the average returns of fundamental momentum and twin momentum are still as large as 0.62% (t-value = 3.70) and 1.42% (t-value = 4.21) per month, respectively. Second, the profits from fundamental momentum and twin momentum cannot be explained away by the five factor models. This evidence lends further support to Section 4.4 that our findings in this paper are not simply attributable to limits to arbitrage, as large firms (megacaps) suffer from fewer market frictions and behavioral biases from irrational investors.

5.2 Transaction costs

In the literature, a price momentum portfolio usually has a higher turnover ratio than the market portfolio. In this section, we examine whether the outperformance of the twin momentum strategy is fully offset by high turnover-related transaction costs. Following Brandt, Santa-Clara, and Valkanov (2009), we calculate the turnover ratio in month t as the summation of the absolute values of the weight changes of all securities in the corresponding portfolio between month $t-1$ and month t. Instead of computing the transaction costs directly, we follow Grundy and Martin (2001) and Barroso and Santa-Clara (2015) and calculate the break-even costs for the average returns of the price momentum, fundamental momentum, and twin momentum portfolios. We consider two types of break-even transaction costs: zero-return cost, defined as the percentage cost per dollar paid to make the strategy deliver an exactly zero return, and 5% significance cost, defined as the cost per dollar paid to render the return statistically insignificant at the 5% level.

Table 13 shows that the turnover ratios of the price and fundamental momentum portfolios are 46.36% and 63.50% per month, respectively. These values are comparable with the ratio of 74% per month in Barroso and Santa-Clara (2015), and are smaller than that in Grundy and Martin (2001), which exceeds 100% per month. The turnover ratio of twin momentum is 87.89% per month. Although this ratio is higher than those of price momentum and fundamental momentum, it is still moderate given the fact that both price momentum and fundamental momentum are constructed on a quintile sort whereas twin momentum is based a 5×5 double sort, which is a finer sort and naturally has a higher turnover ratio. Another reason is that as twin momentum is so profitable that, at the end of each month, we have to rebalance the weights of stocks to the optimal ones.

In general, it takes, on average, 2.00%, 1.38%, and 2.46% of transaction costs to achieve a zero return in the price, fundamental, and twin momentum strategies. The necessary transaction costs to render the returns of these three strategies insignificant at the 5% level are 0.63%, 0.72%, and 1.60%, respectively. Overall, the findings in Table 13 suggest that the profitability of the twin momentum strategy is unlikely to be fully driven by transaction costs.

5.3 Impact of investor sentiment

In the spirit of Stambaugh, Yu, and Yuan (2012), this section examines whether the profits of twin momentum is related to investor sentiment. We use Baker and Wurgler (2006) investor sentiment as the proxy for the aggregate investor sentiment in the stock market and define a month as a high-sentiment period if the Baker-Wurgler sentiment index over the previous month is above the median of the whole sample period and a low-sentiment period otherwise. We then evaluate the profitability of the price, fundamental, and twin momentum strategies over high- and low-sentiment periods, respectively.

Table 14 shows that price momentum, fundamental momentum, and twin momentum are associated with investor sentiment, and that their profits are higher in high-sentiment periods than in low-sentiment periods. During high-sentiment periods, the most optimistic views tend to be overly optimistic and stocks are more likely to be overpriced. In contrast, during low-sentiment periods, the most optimistic views tend to be closer to those of rational investors and stocks are more likely to be correctly priced. As a result, mispricing is more likely to occur during high-sentiment periods. In Table 14, the monthly average return of price momentum is 0.90% (t-value = 2.08) in high-sentiment periods, and 0.95% (t-value = 2.12) in low-sentiment periods. Among the five asset pricing models, HXZ explains price momentum in both periods, whereas FF5 can only explain it in high sentiment periods.

Similarly, fundamental momentum yields an average return of 0.76% (t-value = 1.89) in high-sentiment periods and 0.99% (t-value = 3.56) in low-sentiment periods. Its alphas with all factor models except for the HXZ model are significant in both high- and low-sentiment periods. The HXZ alpha is insignificant in high-sentiment periods and significant in low-sentiment periods, suggesting that fundamental momentum is different from price momentum.

The effect of investor sentiment on twin momentum is negligible. In high-sentiment periods, the average return is 2.29% (t-value = 4.15) and the risk-adjusted return varies from 1.37% (t-value = 0.96) with the HXZ model to 2.80% (t-value = 4.80) with the CAPM model. In low-sentiment periods, the average return is 2.03% (t-value = 3.82) and the risk-adjusted return varies from 1.34% (t-value = 3.64) with the FF3M model to 2.23% (t-value = 4.27) with the CAPM model. Since twin momentum exists in both high- and low-sentiment periods and the difference between the two periods is negligible, one can conclude that investor sentiment does not drive twin momentum. This result is consistent with Stambaugh, Yu, and Yuan (2012) who show that the long-short decile price momentum return averages 1.09% per month in low-sentiment periods and 2.03% per month in high-sentiment periods, and that the difference between these values is not statistically significant.

5.4 Estimate FIR with alternative formation periods

To construct FIR for fundamental momentum, we use fundamental information from the prior two years for the main results. In this section, we show that our results continue to hold when using fundamental information from the most recent three or five years. We use the same procedure to construct fundamental momentum and twin momentum and report the results in Table 15. We also include the results for price momentum, in that when longer historical information is used to construct fundamental momentum, there are more younger or smaller firms excluded in the double sort for twin momentum and therefore, the performance of price momentum changes accordingly.

Panel A of Table 15 shows that the average and risk-adjusted returns of fundamental momentum and twin momentum are quantitatively unchanged when FIR is constructed based on fundamental information from the most recent three years. For example, their average monthly returns are 0.82% (t-value = 4.12) and 2.01% (t-value = 4.90), respectively, compared to average monthly returns of 0.88% and 2.16% when using two years of fundamental information (Table 7).

When we calculate fundamental momentum using data from the past five years, the performance of twin momentum weakens (see Panel B of Table 15). Two potential reasons explain this decline. First, when incorporating longer historical information into our calculations, we include more stale information, which may weaken the fundamental momentum's predictive ability. Second, requiring five years of data inevitably excludes younger and smaller firms in constructing the strategy. Panel B appears to support the second explanation, since the performance of fundamental momentum remains largely unchanged when using five years of data, while the performance of price momentum delcines.

In untabulated results, we find that our findings do not change qualitatively and quantitatively when we include more fundamental variables, such as the cash-based profitability to assets scaled by total assets in the current quarter and the accrual-based operating profitability to assets (Hou, Xue, and Zhang, 2015). Our findings also hold when we exclude APE and GPA in constructing fundamental momentum. However, the risk-adjusted return of twin momentum may become statistically insignificant when we consider too few fundamental variables. For example, a twin momentum strategy based only on ROE, OPP, EARN, and NPY delivers an FF3M alpha of 0.44% (t-value = 1.78) and an HXZ alpha of 0.45% (t-value = 0.52), while a twin momentum strategy based on ROA, GPA, and COP produces an average return of 0.81% per month with a t-value of 1.47. To sum up, combining multiple fundamental variables is essential to construct robust fundamental momentum and twin momentum.

5.5 Twin momentum and mutual fund performance

Carhart (1997) and Daniel, Grinblatt, Titman, and Wermers (1997) show that mutual funds that trade on price momentum significantly outperform their peers. This section examines whether mutual funds that trade on fundamental or twin momentum can also generate better performance. For this purpose, we collect data on funds' quarterly stock holdings from the Thomson Reuters mutual fund holdings database and monthly fund returns from the CRSP survivor-bias-free US mutual fund database. We merge the two databases and focus on open-end US domestic equity mutual funds. We clean fund data following the same process as Kacperczyk, Sialm, and Zheng (2008) and end up with 3,840 mutual funds over the period January 1980–September 2015.

Based on Section 3.3.1, for each fund, we construct a fund level TNA-weighted FIR across all stocks held by the fund and a fund level past return. At the end of each quarter, we independently sort all mutual funds into five groups based on the fund-level FIR and past return, and then compute TNA-weighted cumulative net returns in the next quarter for each group. In so doing, we investigate whether funds with high fund FIR outperform other funds

Table 16 reports the average and risk-adjusted returns for each fund group, as well as the associated Newey-West t-values (in parentheses). As a comparison, the first three columns show that mutual funds that actively trade on price momentum outperform those that do not by 0.39% per quarter (t-value = 1.47). While this outperformance is not explained by the Fama-French three factors, it is fully described by the FF3M model, lowering alpha to be 0.02% (t-value = 0.10). This is consistent with the findings in Carhart (1997) and Sapp and Tiwari (2004), which show that the price momentum factor is able to explain mutual fund performance.

Columns 4-6 show that funds that actively trade on fundamental momentum deliver an average return of 1.98% (t-value = 2.72) per quarter, 0.73% (t-value = 2.02) higher than those that do not do so. Moreover, the risk-adjusted return is 0.89% (t-value = 2.17) and 0.64% (t-value = 2.16) after adjusted by the factors in the FF3 and FF3M models, respectively. The last three columns of Table 16 show that funds that trade on twin momentum generate even better performance than those trading on fundamental momentum alone. The average return is 0.88% per quarter (t-value = 2.64), and the corresponding FF3 and FF3M alphas are 1.13% (t-value = 2.86) and 0.60% (t-value = 2.07), respectively. In short, Table 16 shows that fundamental momentum and twin momentum generate sizeable economic value to investors.

6 Conclusion

Using time-series trends of a set of seven major firm fundamentals, this paper provides strong evidence for the presence of fundamental momentum in the stock market. The top 20% of stocks sorted by FIR outperform the bottom 20% by 0.88% per month, yielding a significant fundamental momentum. It has little correlation with the widely analyzed price momentum, and its performance is comparable with the latter. Similar results are also obtained if an alternative and more robust forecast combination approach is used instead of multivariate regressions. We also propose an equilibrium model for understanding the predictive power of fundamental trends from investors's Bayesian learning perspective.

Our results provide strong support for the argument that fundamentals matter in asset pricing, in contrast with prior studies that find only weak results due to the insufficient use of fundamental information. Our results vindicate fundamental analysis in both academic research and practical investing.

Moreover, by exploiting information in both price momentum and fundamental momentum, we also provide a twin momentum trading strategy, which offers a monthly average return more than twice of that from price momentum without taking any additional risk. This twin momentum portfolio cannot be spanned by existing factors, nor can it be explained by short-sale impediments, size effect, or investor sentiment.

There are a number of issues that are of interest for future research. First, it is an open question whether there exists twin momentum in other markets, such as bond, commodity, and currency markets. Second, it is also unknown to what degree a twin momentum factor can explain individual stocks, stock anomalies, and mutual funds. Third, while we use simple regression and forecast combination approaches, more sophisticated nonlinear econometric models may be developed to fully exploit the information of fundamental trends, uncovering perhaps an even greater role of fundamentals in asset pricing.

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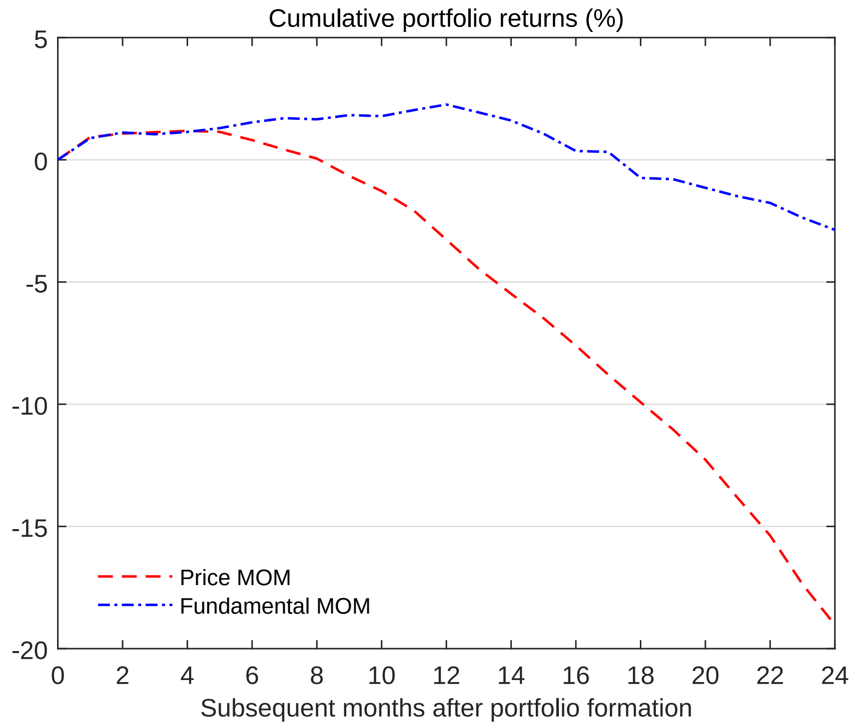

Figure 1 Cumulative portfolio returns after formation

This figure plots the cumulative returns of price and fundamental momentum portfolios after portfolio formation. At the end of each month t, we independently sort stocks into five groups based on past return and FIR, and construct price (fundamental) momentum by buying the highest past return (FIR) quintile portfolio and selling the lowest past return (FIR) quintile portfolio. Cumulative portfolio return in month $t+i$ is defined as the compounding monthly return from month $t+1$ through month $t+i$. The sample period is 1976:04–2015:09.

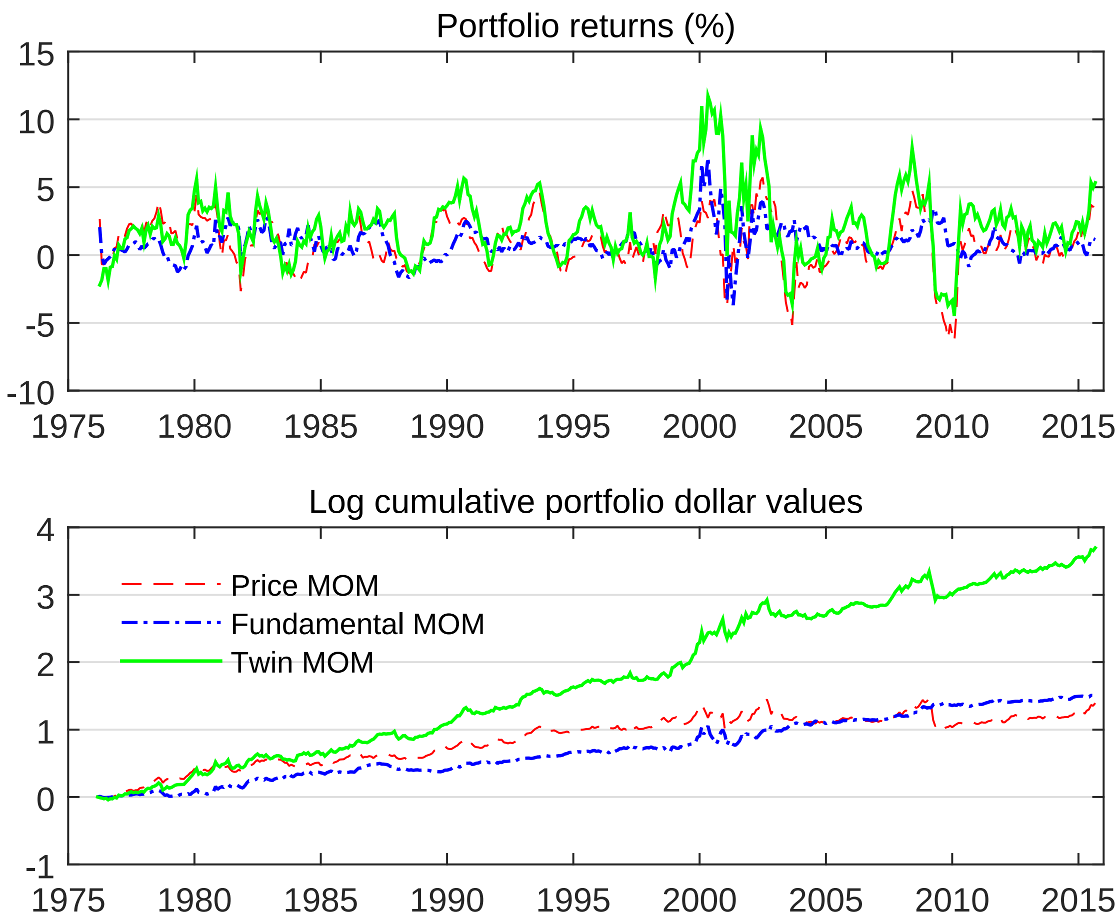

Figure 2 Price momentum vs. fundamental momentum vs. twin momentum

This figure plots the time series of the 12-month moving average portfolio returns (upper panel) and the log cumulative wealth (lower panel) for investing in price, fundamental, and twin momentums over the sample period 1976:04–2015:09, respectively.

Table 1 Average returns and alphas of spread portfolios formed by fundamentals

This table reports the average returns and alphas of the long-short spread portfolios formed by fundamentals, as well as Newey-West t-values over 1976:04–2015:09, where all portfolios are value-weighted and monthly rebalanced. Panel A forms portfolios based on the most recent quarterly value of a fundamental variable, say, ROE, Panel B is based on the most recent quarterly ROE and its trends, and Panel C is based on the seven fundamental variables and their trends jointly. The long-short spread portfolio in Panel C is the fundamental momentum. The benchmark asset pricing models include the CAPM, Fama and French (FF3, 1993) three-factor model, FF3 plus a price momentum factor model (FF3M), Hou, Xue, and Zhang (HXZ, 2015) four-factor model, and Fama and French (FF5, 2015) five-factor model.

		Alphas					t-values					
	Mean	CAPM	FF3	FF3M	HXZ	FF5	Mean	CAPM	FF3	FF3M	HXZ	FF5
Panel A: Sort on a single variable without its trends												
ROE	0.51	0.57	0.61	0.62	0.40	0.54	2.54	2.69	2.93	3.01	1.60	2.44
ROA	0.41	0.45	0.49	0.51	0.31	0.41	2.03	2.12	2.34	2.48	1.27	1.87
EARN	0.42	0.49	0.50	0.43	0.25	0.34	2.60	2.87	2.84	2.46	1.14	1.67
APE	0.51	0.60	0.65	0.59	0.35	0.45	2.27	2.64	2.70	2.62	1.10	1.66
CPA	0.27	0.33	0.36	0.28	0.21	0.31	2.17	2.38	1.56	1.99	1.30	2.07
GPA	0.30	0.33	0.36	0.37	0.27	0.27	2.23	2.45	2.45	2.50	1.77	1.89
NPY	0.25	0.36	0.37	0.31	0.26	0.28	1.39	2.06	1.94	1.72	1.02	1.27
Panel B: Sort on a single variable with its trends												
ROE	0.61	0.66	0.70	0.56	0.42	0.58	3.06	3.18	3.30	2.88	1.61	2.44
ROA	0.56	0.62	0.65	0.54	0.39	0.52	2.75	2.90	3.11	2.60	1.49	2.23
EARN	0.67	0.73	0.73	0.63	0.49	0.59	3.74	3.79	3.67	3.25	1.81	2.49
APE	0.52	0.58	0.61	0.55	0.33	0.47	2.37	2.61	2.61	2.49	1.07	1.75
CPA	0.16	0.22	0.24	0.20	0.22	0.21	1.37	1.72	1.77	1.42	1.41	1.56
GPA	0.46	0.47	0.48	0.39	0.20	0.34	3.36	3.34	3.15	2.69	1.29	2.30
NPY	0.61	0.66	0.70	0.56	0.42	0.58	3.06	3.18	3.30	2.88	1.61	2.44
Panel C: Sort on multiple variables with their trends (i.e., fundamental momentum)												
	0.88	0.95	0.98	0.81	0.72	0.85	4.09	4.24	4.15	3.79	2.22	3.00

Table 2 Fundamental momentum: portfolios formed by fundamental implied return

At the end of each month t, we sort stocks into five groups based on their fundamental implied returns (FIRs) and construct the fundamental momentum by buying the highest FIR quintile portfolio and selling the lowest FIR quintile portfolio, where all portfolios are value-weighted and monthly rebalanced. This table reports the average returns and alphas from the CAPM, Fama and French (FF3, 1993) three-factor model, FF3 plus a price momentum factor model (FF3M), Hou, Xue, and Zhang (HXZ, 2015) four-factor model, and Fama and French (FF5, 2015) five-factor model, respectively. Newey-West t-values are in parentheses. The sample period is 1976:04–2015:09. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	Low FIR	2	3	4	High FIR	High-Low
Average return	0.14	0.39*	0.52**	0.70***	1.02***	0.88***
	(0.45)	(1.75)	(2.40)	(3.09)	(3.73)	(4.09)
CAPM alpha	−0.58***	−0.25***	−0.10	0.10	0.38***	0.95***
	(−4.16)	(−2.98)	(−1.57)	(1.29)	(2.73)	(4.24)
FF3 alpha	−0.55***	−0.24***	−0.08	0.12	0.43***	0.98***
	(−3.73)	(−2.70)	(−1.25)	(1.55)	(3.28)	(4.15)
FF3M alpha	−0.41***	−0.16*	−0.04	0.13*	0.41***	0.81***
	(−3.07)	(−1.87)	(−0.69)	(1.71)	(3.23)	(3.79)
HXZ alpha	−0.21	−0.10	−0.01	0.14	0.52***	0.72**
	(−1.19)	(−0.98)	(−0.18)	(1.59)	(2.89)	(2.22)
FF5 alpha	−0.31*	−0.18*	−0.04	0.15*	0.54***	0.85***
	(−1.95)	(−1.88)	(−0.66)	(1.87)	(3.76)	(3.00)

Table 3 Fundamental momentum: portfolio are based on a forecast combination method

This table reports the average returns of the fundamental momentum portfolio formed on the average of 28 single FIRs, and its alphas from the CAPM, Fama and French (FF3, 1993) three-factor model, FF3 plus a price momentum factor model (FF3M), Hou, Xue, and Zhang (HXZ, 2015) four-factor model, and Fama and French (FF5, 2015) five-factor model, respectively. Newey-West t-values are reported in parentheses. The sample period is 1976:04–2015:09. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	Low FIR	2	3	4	High FIR	High-Low
Average return	0.02	0.42*	0.55**	0.60**	0.94***	0.92***
	(0.08)	(1.78)	(2.39)	(2.49)	(3.37)	(4.18)
CAPM alpha	−0.71***	−0.24***	−0.08	−0.03	0.29**	1.00***
	(−4.67)	(−2.79)	(−1.11)	(−0.40)	(2.09)	(4.46)
FF3 alpha	−0.67***	−0.27***	−0.11	−0.04	0.34***	1.01***
	(−4.19)	(−2.92)	(−1.53)	(−0.44)	(2.57)	(4.34)
FF3M alpha	−0.56***	−0.16*	−0.06	0.03	0.41***	0.97***
	(−3.66)	(−1.74)	(−0.77)	(0.38)	(2.90)	(4.11)
HXZ alpha	−0.27	−0.11	0.00	0.11	0.53***	0.79***
	(−1.61)	(−1.10)	(0.00)	(1.02)	(3.19)	(2.75)
FF5 alpha	−0.36**	−0.17*	−0.06	0.04	0.50***	0.88***
	(−2.33)	(−1.79)	(−0.73)	(0.49)	(3.91)	(3.35)

Table 4 Double sort on past return and fundamental implied return

This table reports the average returns of portfolios sorted by past return and fundamental implied return (FIR), where past return is the cumulative return from month $t-12$ to month $t-1$. All portfolios are value-weighted and monthly rebalanced. Newey-West t-values are reported in parentheses. The sample period is 1976:04–2015:09. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	Fundamental MOM					
Price MOM	Low FIR	2	3	4	High FIR	High-Low
Low past return	-0.83^{**}	-0.11	0.11	0.27	0.33	1.16^{***}
	(-2.07)	(-0.28)	(0.28)	(0.73)	(0.71)	(4.02)
2	-0.25	0.30	0.23	0.77^{***}	0.58^{**}	0.83^{***}
	(-0.76)	(1.09)	(0.87)	(2.87)	(2.08)	(3.86)
3	-0.01	0.37^{*}	0.46^{**}	0.76^{***}	0.75^{***}	0.76^{***}
	(-0.02)	(1.69)	(2.05)	(3.57)	(2.93)	(2.94)
4	0.56^{**}	0.52^{**}	0.70^{***}	0.64^{***}	0.92^{***}	0.36^{*}
	(2.12)	(2.29)	(3.19)	(2.85)	(3.77)	(1.76)
High past return	0.72^{**}	0.76^{***}	1.02^{***}	1.13^{***}	1.33^{***}	0.61^{**}
	(2.11)	(2.66)	(3.66)	(3.87)	(4.13)	(2.36)
High-Low	1.55^{***}	0.87^{**}	0.91^{***}	0.85^{**}	1.00^{**}	
	(4.43)	(2.48)	(2.73)	(2.59)	(2.58)	

Table 5 Fama-MacBeth regressions

This table reports the Fama-MacBeth results of regressing stock returns in month t on past 12-month cumulative return and FIR, as well as other firm characteristics. Short-term return reversal is defined as the return in month $t-1$ and long-term reversal is the cumulative return from month $t-60$ to month $t-13$. B/M is the one-quarter-lagged book value of equity divided by market value of common equity in month $t-1$. IVOL is the idiosyncratic volatility calculated by applying the FF3 to the past month's daily returns. Newey-West t-values are reported in parentheses. The sample period is 1976:04–2015:09. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	Dependent variable: one-month-ahead stock excess returns					
	1	2	3	4	5	6
Intercept	0.008***	0.009***	0.008***	0.014***	0.014***	0.013***
	(3.37)	(3.40)	(3.02)	(4.30)	(4.30)	(4.19)
Past return	0.005**		0.004**	0.006***		0.005***
	(2.13)		(2.05)	(3.25)		(3.07)
FIR		0.158**	0.144***		0.119***	0.108***
		(2.13)	(7.46)		(7.14)	(6.58)
Short-term reversal				−0.007**	−0.001	−0.007**
				(−2.18)	(−0.37)	(−2.34)
Long-term reversal				−0.001***	−0.001***	−0.001***
				(−2.86)	(−3.16)	(−3.05)
Log size				−0.125***	−0.120***	−0.130***
				(−3.96)	(−4.08)	(−4.59)
B/M				0.054*	0.004	0.057**
				(1.93)	(0.15)	(2.20)
IVOL				0.017	0.020	0.023
				(0.47)	(0.59)	(0.69)
R^2 (%)	1.26	1.11	2.14	4.09	3.86	4.56

Table 6 Cash flow, discount rate, and variance betas

This table presents estimates from time-series regressions of price momentum and fundamental momentum returns on shocks about future cash flows (N_{cf}), discount rate ($-N_{dr}$), and variance (N_v), $r_i = \alpha + \beta_{cf}N_{cf} + \beta_{dr}(-N_{dr}) + \beta_v N_v + \varepsilon_i$, where r_i is the quarterly price momentum or fundamental momentum returns. Newey-West t-values are reported in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	α	β_{cf}	β_{dr}	β_v	R^2 (%)
Price MOM	2.06**	−0.16	−0.35**	1.06***	8.61
	(2.54)	(−0.85)	(−2.00)	(3.09)	
Fundamental MOM	2.38***	−0.03	−0.16**	0.37**	2.71
	(7.23)	(−0.12)	(−1.97)	(2.00)	

Table 7 Summary statistics of price, fundamental, and twin momentum returns

This table reports summary statistics of the price, fundamental, and twin momentum portfolio returns over the full sample period 1976:04–2015:09 (Panel A) and four sub-sample periods (Panel B). The White Heteroscedasticity robust t-value tests whether the average return is zero. At the end of each month t, we independently sort stocks into quintile portfolios by past return and FIR. Price, fundamental, and twin momentum portfolios are respectively formed by buying stocks in the top quintile of past return or FIR or both, and selling stocks in the corresponding bottom quintile. All portfolios are value-weighted and monthly rebalanced. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

						Correlation	
	Average return	t-value	Volatility	Skewness	Kurtosis	P-MOM	F-MOM
Panel A: Full sample period (1976–2015)							
Price MOM	0.93	2.97	6.78	−1.20	10.39		0.14***
Fundamental MOM	0.88	3.57	5.34	0.73	13.22		
Twin MOM	2.16	5.64	8.34	0.04	7.26	0.69***	0.61***
Panel B: Sub-sample periods							
			1976–1985				
Price MOM	1.24	2.58	5.21	−0.11	2.28		0.47***
Fundamental MOM	0.83	2.06	4.37	0.59	5.80		
Twin MOM	1.64	2.66	6.68	−0.05	4.64	0.81***	0.72***
			1986–1995				
Price MOM	1.00	2.58	4.25	−0.35	3.41		0.11***
Fundamental MOM	0.62	2.63	2.57	−0.05	3.62		
Twin MOM	2.10	4.44	5.18	−0.16	4.42	0.70***	0.43***
			1996–2005				
Price MOM	0.73	0.85	9.48	−0.83	6.20		0.08***
Fundamental MOM	1.25	1.54	8.88	0.42	6.38		
Twin MOM	2.82	2.62	11.78	0.27	5.13	0.57***	0.72***
			2006–2015				
Price MOM	0.73	1.12	7.04	−2.06	13.61		0.08***
Fundamental MOM	0.79	2.84	3.02	0.74	5.80		
Twin MOM	2.06	2.70	8.24	−0.92	6.05	0.85***	0.30***

Table 8 Alphas of twin momentum

This table reports the value-weighted twin momentum alphas from the CAPM, Fama and French (FF3, 1993) three-factor model, FF3 plus a price momentum factor model (FF3M), Hou, Xue, and Zhang (HXZ, 2015) four-factor model, and Fama and French (FF5, 2015) five-factor model, respectively. Newey-West t-values are reported in parentheses. The sample period is 1976:04–2015:09. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	CAPM	FF3	FF3M	HXZ	FF5
Alpha (%)	2.31***	2.50***	1.41***	1.37**	2.13***
	(6.34)	(6.69)	(5.11)	(2.45)	(4.59)
MKT	−0.26*	−0.38***	−0.16*	−0.20	−0.26*
	(−1.94)	(−2.63)	(−1.70)	(−1.31)	(−1.79)
SMB		0.10	−0.01	0.47	0.12
		(0.36)	(−0.06)	(1.30)	(0.52)
HML		−0.51*	−0.06		−1.05***
		(−1.87)	(−0.35)		(−2.97)
MOM			1.24***		
			(11.92)		
I/A				0.05	
				(0.12)	
ROE				1.14***	
				(3.87)	
RMW					0.34
					(0.89)
CMA					1.16**
					(2.24)
R^2 (%)	1.86	4.98	45.8	14.0	8.62

Table 9 Mean-variance spanning tests

This table reports the testing results of whether the twin momentum portfolio can be spanned by factors in the CAPM, Fama and French (FF3, 1993) three-factor model, FF3 plus a price momentum factor model (FF3M), Hou, Xue, and Zhang (HXZ, 2015) four-factor model, and Fama and French (FF5, 2015) five-factor model, respectively. W is the Wald test under conditional homoskedasticity, W_e is the Wald test under IID elliptical, W_a is the Wald test under the conditional heteroscedasticity, J_1 is the Bekerart-Urias test with the Errors-in-Variables (EIV) adjustment, J_2 is the Bekerart-Urias test without the Errors-in-Variables (EIV) adjustment, J_3 is the DeSantis test. The associated p-values are in parentheses. The sample period is 1976:04–2015:09. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	W	W_e	W_a	J_1	J_2	J_3
CAPM	235.36***	128.59***	127.98***	46.09***	47.79***	118.24***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
FF3	97.01***	60.61***	51.89***	29.70***	33.30***	49.85***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
FF3M	25.35***	24.82***	24.26***	19.42***	21.12***	19.44***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
HXZ	20.71***	19.01***	20.04***	15.88***	16.73***	16.30***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
FF5	29.45***	26.99***	25.65***	18.30***	20.15***	18.96***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Table 10 Long- and short-leg portfolio returns

This table report the average returns of the long- and short-leg portfolios of price, fundamental, and twin momentums, and their alphas from the CAPM, Fama and French (FF3, 1993) three-factor model, FF3 plus a price momentum factor model (FF3M), Hou, Xue, and Zhang (HXZ, 2015) four-factor model, and Fama and French (FF5, 2015) five-factor model. Newey-West t-values are reported in parentheses. The sample period is 1976:04–2015:09. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	Average	Alphas				
	return	CAPM	FF3	FF3M	HXZ	FF5
Panel A: Long-leg						
Price MOM	1.35***	0.29**	0.35***	−0.06	−0.01	0.29***
	(5.01)	(2.16)	(3.10)	(−0.72)	(−0.08)	(2.14)
Fundamental MOM	1.39***	0.36***	0.41***	0.40***	0.51***	0.53***
	(5.15)	(2.63)	(3.29)	(3.40)	(2.88)	(3.87)
Twin MOM	1.72***	0.65***	0.76***	0.37**	0.51*	0.80***
	(5.34)	(3.13)	(3.92)	(2.34)	(1.89)	(3.61)
Panel B: Short-leg						
Price MOM	0.42	−0.78***	−0.89***	−0.06	0.14	−0.52**
	(1.13)	(−3.52)	(−4.11)	(−0.42)	(0.52)	(−2.05)
Fundamental MOM	0.52*	−0.58***	−0.55***	−0.42***	−0.23	−0.32**
	(1.81)	(−4.31)	(−3.88)	(−3.24)	(−1.31)	(−2.03)
Twin MOM	−0.44	−1.66***	−1.75***	−1.05***	−0.86**	−1.33***
	(−1.06)	(−6.42)	(−6.42)	(−4.79)	(−2.40)	(−4.42)

Table 11 Characteristics of price, fundamental, and twin momentum portfolios

This table reports summary statistics of main firm characteristics in the long- and short-legs of price, fundamental, and twin momentum portfolios, as well as the difference between the long and short leg portfolios. The numbers are time series averages of the cross-sectional means of each characteristic. The sample period is 1976:04–2015:09.

	Long-leg			Short-leg			Long-short		
	P-MOM	F-MOM	T-MOM	P-MOM	F-MOM	T-MOM	P-MOM	F-MOM	T-MOM
Past return (%)	0.76	0.28	0.84	−0.30	0.21	0.33	1.06	0.07	0.51
FIR (%)	0.56	3.96	4.08	0.02	−3.36	−3.59	0.54	7.32	7.67
ROE (%)	2.45	0.68	2.32	−3.08	−1.41	−4.96	5.53	2.09	7.28
ROA (%)	1.15	0.20	1.01	−1.56	−0.79	−2.44	2.71	0.99	3.45
GPA (%)	9.99	9.48	10.78	7.52	7.34	6.59	2.47	2.14	4.19
COP (%)	3.39	3.42	4.02	1.68	1.60	0.71	1.71	1.82	3.31
OPE (%)	8.32	7.03	9.12	2.48	4.14	1.06	5.84	2.89	8.06
EARN ($)	0.40	0.37	0.44	0.04	0.23	0.24	0.36	0.14	0.20
NPY (%)	−0.57	−0.35	−0.66	−0.70	−0.91	−1.24	0.13	0.56	0.58

Table 12 Size effect

This table reports the average returns and alphas of the price, fundamental, and twin momentum portfolios formed on a sample after excluding a given percentile of smallest stocks. All portfolios are value-weighted and monthly rebalanced. Newey-West t-values are reported in parentheses. The sample period is 1976:04–2015:09. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	Average	Alphas				
	return	CAPM	FF3	FF3M	HXZ	FF5
Panel A: Excluding smallest 20% stocks						
Price MOM	0.83***	0.90***	1.06***	−0.12	−0.16	0.71***
	(2.88)	(3.22)	(4.16)	(−0.92)	(−0.44)	(2.19)
Fundamental MOM	0.81***	0.87***	0.90***	0.74***	0.63**	0.76***
	(3.96)	(4.11)	(4.10)	(0.60)	(2.15)	(2.95)
Twin MOM	1.82***	1.94***	2.16***	1.17***	1.14**	1.82***
	(5.42)	(5.86)	(6.66)	(5.05)	(2.42)	(4.70)
Panel B: Excluding smallest 40% stocks						
Price MOM	0.77***	0.81***	0.96***	−0.20*	−0.13	0.69**
	(2.71)	(2.89)	(3.76)	(−1.80)	(−0.35)	(2.13)
Fundamental MOM	0.76***	0.82***	0.85***	0.70***	0.59**	0.70***
	(3.96)	(4.12)	(4.06)	(3.77)	(2.10)	(2.95)
Twin MOM	1.67***	1.78***	1.93***	0.88***	1.79*	1.56***
	(5.14)	(5.63)	(6.08)	(4.33)	(1.92)	(4.00)
Panel C: Excluding smallest 60% stocks						
Price MOM	0.76***	0.79***	0.93***	−0.22*	−0.03	0.72**
	(2.63)	(2.76)	(3.56)	(−1.73)	(−0.07)	(2.18)
Fundamental MOM	0.63***	0.67***	0.72***	0.52***	0.48**	0.63***
	(3.63)	(3.69)	(3.92)	(3.01)	(1.96)	(2.89)
Twin MOM	1.54***	1.62***	1.80***	0.69***	0.83*	1.53***
	(4.66)	(5.02)	(5.65)	(3.52)	(1.75)	(3.93)
Panel D: Excluding smallest 80% stocks						
Price MOM	0.61**	0.62***	0.77***	−0.25*	−0.07	0.62*
	(2.10)	(2.92)	(2.92)	(−1.80)	(−0.19)	(1.90)
Fundamental MOM	0.62***	0.66***	0.72***	0.49***	0.46*	0.60***
	(3.70)	(3.58)	(3.86)	(2.81)	(1.90)	(2.92)
Twin MOM	1.42***	1.48***	1.62***	0.63***	0.67*	1.34***
	(4.21)	(4.42)	(4.88)	(2.62)	(1.65)	(3.24)

Table 13 Turnover ratio and break-even transaction cost

This table reports the turnover ratio of the price, fundamental, and twin momentum strategies and the corresponding break-even transaction costs. Zero return refers to the transaction costs that would completely offset the return to implement the strategy, and 5% insignificance refers to the transaction costs that make the return insignificant at the 5% level. The sample period is 1976:04–2015:09.

	Turnover ratio	Break-even costs (in % per month)	
	(in % per month)	Zero return	5% insignificance
Price MOM	46.36	2.00	0.63
Fundamental MOM	63.50	1.38	0.72
Twin MOM	87.89	2.46	1.60

Table 14 Momentums in high and low investor sentiment periods

This table reports the average returns and alphas of price, fundamental, and twin momentums across high and low sentiment periods, which are calculate from $R_{i,t} = a_H d_{H,t} + a_L d_{L,t} + BF_t + \varepsilon_t$, where $d_{H,t}$ and $d_{L,t}$ are dummy variables indicating high and low sentiment periods, F_t is the returns of one factor model (e.g., Fama and French (2015) five-factor returns), and $R_{i,t}$ is the excess return in month t. A month is defined as a high (low) sentiment month if the Baker and Wurgler (2006) sentiment index in the previous month is above (below) the median of the sample period 1976:04–2015:09. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	Sentiment	Average	Alphas				
		return	CAPM	FF3	FF3M	HXZ	FF5
Price MOM	High	0.90**	1.00**	1.37***	−0.15	−0.37	0.80
		(2.08)	(2.29)	(2.87)	(−0.82)	(−0.76)	(1.55)
	Low	0.95**	1.14***	1.12***	0.12	0.06	0.81**
		(2.12)	(2.68)	(2.71)	(0.77)	(0.16)	(1.94)
Fundamental MOM	High	0.76*	0.80**	0.89**	0.71*	0.57	0.76*
		(1.89)	(1.97)	(2.09)	(1.83)	(1.26)	(1.66)
	Low	0.99***	1.06**	1.04***	0.93***	0.90**	0.93***
		(3.56)	(3.77)	(3.59)	(2.97)	(2.51)	(3.17)
Twin MOM	High	2.29***	2.39***	2.80***	1.49***	1.37**	2.35***
		(4.15)	(4.39)	(4.80)	(3.38)	(2.01)	(3.64)
	Low	2.03***	2.23***	2.21***	1.34***	1.37**	1.93***
		(3.82)	(4.27)	(4.22)	(3.64)	(2.35)	(3.59)

Table 15 Fundamental momentum with alternative estimation periods

We use fundamental information over the most recent three (Panel A) or five (Panel B) years to estimate the fundamental implied return (FIR) and construct alternative fundamental and twin momentum strategies. This table reports the average returns of the corresponding price, fundamental, and twin momentum portfolios, and their alphas from the CAPM, Fama and French (FF3, 1993) three-factor model, FF3 plus a price momentum factor model (FF3M), Hou, Xue, and Zhang (HXZ, 2015) four-factor model, and Fama and French (FF5, 2015) five-factor model, respectively. Newey-West t-values are reported in parentheses. The sample period is 1976:04–2015:09. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	Average	Alphas				
	return	CAPM	FF3	FF3M	HXZ	FF5
Panel A: Construct FIR with the past three year fundamental information						
Price MOM	0.91***	1.05***	1.23***	−0.02	−0.15	0.80**
	(2.81)	(3.37)	(4.28)	(−0.11)	(−0.38)	(2.24)
Fundamental MOM	0.82***	0.90***	0.93***	0.76***	0.70**	0.85***
	(4.12)	(4.29)	(4.31)	(3.87)	(2.35)	(3.28)
Twin MOM	2.01***	2.27***	2.45***	1.32***	1.21**	2.06***
	(4.90)	(5.83)	(6.25)	(5.05)	(2.29)	(4.43)
Panel B: Construct FIR with the past five year fundamental information						
Price MOM	0.74**	0.87***	1.06***	−0.15	−0.27	0.62*
	(2.33)	(2.84)	(3.79)	(−1.06)	(−0.68)	(1.77)
Fundamental MOM	0.80***	0.87***	0.92***	0.76***	0.78**	0.89***
	(3.57)	(3.87)	(3.72)	(3.52)	(2.16)	(2.90)
Twin MOM	1.60***	1.81***	2.07***	0.94***	0.87	1.71***
	(3.60)	(4.62)	(5.34)	(3.70)	(1.49)	(3.58)

Table 16 Twin momentum and mutual fund performance

At the end of each quarter, we independently sort mutual funds by fund level past return and FIR, and construct price, fundamental, and twin momentums accordingly. This table reports the time series mean of cross-sectional average quarterly returns of funds in each price, fundamental, or twin quintile, and the corresponding alphas from the Fama and French (FF3, 1993) three-factor model and the FF3 plus a price momentum factor model (FF3M), respectively. The sample period is 1980:01–2015:09. Newey-West t-values are reported in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	Price MOM			Fundamental MOM			Twin MOM		
Fund FIR	Average return	FF3 alpha	FF3M alpha	Average return	FF3 alpha	FF3M alpha	Average return	FF3 alpha	FF3M alpha
Low	1.48^{**}	-0.63^{***}	-0.29^{*}	1.25^{*}	-0.78^{***}	-0.73^{***}	1.08	-1.01^{***}	-0.76^{***}
	(2.22)	(−3.34)	(−1.82)	(1.68)	(−3.16)	(−3.38)	(1.54)	(−4.11)	(−3.55)
2	1.73^{***}	-0.24^{***}	-0.19^{*}	1.72^{**}	-0.24^{**}	-0.22^{*}	1.81^{***}	−0.120	−0.17
	(2.61)	(−2.73)	(−1.74)	(2.52)	(−2.36)	(−1.78)	(2.66)	(−1.25)	(−1.54)
3	1.80^{***}	−0.02	-0.28^{**}	1.87^{***}	−0.05	−0.14	1.81^{**}	−0.19	-0.48^{**}
	(2.59)	(−0.15)	(−2.12)	(2.68)	(−0.26)	(−0.84)	(2.37)	(−0.74)	(−2.20)
4	1.94^{***}	0.01	−0.14	2.02^{***}	0.07	0.01	2.02^{***}	0.03	−0.05
	(2.74)	(0.09)	(−0.98)	(2.87)	(0.47)	(0.04)	(2.87)	(0.10)	(−0.23)
High	1.87^{**}	−0.03	−0.28	1.98^{***}	0.11	−0.10	1.97^{***}	0.12	−0.16
	(2.54)	(−0.18)	(−1.60)	(2.72)	(0.46)	(−0.48)	(2.79)	(0.53)	(−0.78)
High-Low	0.39	0.61^{**}	0.02	0.73^{**}	0.89^{**}	0.64^{**}	0.88^{***}	1.13^{***}	0.60^{**}
	(1.47)	(2.49)	(0.10)	(2.02)	(2.17)	(2.16)	(2.64)	(2.86)	(2.07)

Appendix: Proof of Proposition 1

We first describe the investment problem faced by investors given the price process in Equation (10). The investment opportunity or the excess stock return can be defined as

$$dQ = (D - rP)dt + dP. \tag{A1}$$

This equation holds for both informed and uninformed investors. Due to different information sets faced by different types of investors, their perceived opportunity sets are different, and hence they trade with different investment strategies. The information sets and the investment opportunities for both types of investors can be characterized in the following lemma.

Lemma 1. For both informed and uninformed investors, the information set can be given by an N-dimensional state variable $\Psi = (1, \Psi_1, ..., \Psi_{N-1})^T$ that satisfies a linear Stochastic Differential Equation (SDE):

$$d\Psi = e_\Psi \Psi dt + \sigma_\Psi dB_t, \tag{A2}$$

where B_t is an N-dimensional Brownian Motion, and e_Ψ and $\sigma_\Psi \in R^{N\times N}$ are constant matrices. Further, the investment opportunity for investors satisfies the linear SDE:

$$dQ = e_Q \Psi dt + \sigma_Q dB_t, \tag{A3}$$

with $e_Q \in R^{N\times 1}$ and $\sigma_Q \in R^{N\times 1}$.

The proof of this lemma is provided in Appendix A.2, where the detailed characterization of Equations (A2) and (A3) is specified separately for the two types of investors.

The investors' optimization problem is given by the following utility optimization problem

$$\max_{\eta,c} E\left[-\int_t^\infty e^{-\rho s - c(s)} ds | \mathscr{F}_t\right] \; s.t. \; dW(t) = (rW(t) - c(t))dt + \eta dQ. \tag{A4}$$

Let $J(W, \Psi; t)$ be the value function given wealth W and information set Ψ, and then it satisfies the following HJB equation

$$\begin{aligned} 0 \;\; = \;\; & \max_{c,\eta} \Big[-e^{-\rho t - c} + J_W(rW - c + \eta e_Q \Psi) + \frac{1}{2}\sigma_Q \sigma_Q^T \eta^2 J_{WW} + \eta \sigma_Q \sigma_\Psi^T J_{W\Psi} \\ & -\rho J + (e_\Psi \Psi)^T J_\Psi + \frac{1}{2}\sigma_\Psi J_{\Psi\Psi} \sigma_\Psi^T \Big]. \end{aligned} \tag{A5}$$

The solution to the optimization problem is provided in the following lemma.

Lemma 2. Given an investor's wealth W and N-dimensional information set Ψ which satisfies the SDE

(A2), the HJB Equation (A5) has a solution of the form:

$$J(W,\Psi;t) = -e^{-\rho t - rW - \frac{1}{2}\Psi^T V \Psi}, \tag{A6}$$

with $V \in R^{N\times N}$ a positive definite symmetric matrix. The optimal demand for stock is given by

$$\eta = f\Psi, \tag{A7}$$

where $f \in R^{1\times N}$, a constant N-dimensional vector.

Proof of Lemma 2. We conjecture a solution for the value function in the form of Equation (A6) and a trading rule of the form (A7). Substituting the two equations into the HJB Equation (A5), we obtain

$$f = \frac{1}{r}(\sigma_Q \sigma_Q^T)^{-1}(e_Q - \sigma_Q \sigma_\Psi^T V), \tag{A8}$$

with V a symmetric positive matrix satisfying

$$V\sigma_\Psi \sigma_\Psi^T V^T - (\sigma_Q \sigma_Q^T)^{-1}(e_Q - \sigma_Q \sigma_\Psi^T V)^T(e_Q - \sigma_Q \sigma_\Psi^T V) + rV - (e_\Psi^T V + V e_\Psi) + 2k\delta_{11}^{(N)} = 0, \tag{A9}$$

$k \equiv [(r-\rho) - r\ln r] - \frac{1}{2}Tr(\sigma_\Psi^T \sigma_\Psi V)$ and

$$[\delta_{(11)}^{(N)}]_{ij} = \begin{cases} 1, \; i=j=1 \\ 0, \; \text{otherwise.} \end{cases} \tag{A10}$$

This proves Lemma 2. Q.E.D.

Lemma 2 says that, given the model assumptions, both informed and uninformed investors' demands for the stock are a linear function of their respective information sets, and hence are also a linear function of their joint information sets. What is left to prove Proposition 1 is to characterize investors' demand function, using the market clearing condition to derive the price. The details are provided in Appendix A.1.

A.1 Demand Functions and Market Clearing

Informed investor. The state variables for the informed investors are given as

$$\Psi^i = (1, D_t, \pi_t, P_t, A_t, A_{Dt})^T,$$

which can be written as

$$d\Psi^i = e_\Psi^i \Psi^i dt + \sigma_\Psi^i dB_t^i. \tag{A11}$$

The investment opportunity is then

$$dQ = (D_t - rP_t)dt + dP_t \equiv e_Q^i \Psi dt + \sigma_Q^i dB_t, \tag{A12}$$

with more details given in Appendix A.2. Now, by differentiating Equation (10), and by using Lemma 2, we have

$$\eta^i = f^i \Psi^i = f_0^i + f_1^i D_t + f_2^i \pi_t + f_3^i \theta_t + f_4^i A_t + f_5^i A_{Dt}. \tag{A13}$$

Uninformed investor. The investment opportunity set can be given as

$$\Psi^u = (1, D_t, P_t, A_t, A_{Dt})^T. \tag{A14}$$

Since uninformed investors do not observe π_t, the updating rule is given by Equation (4).

Given the price in Equation (10), uninformed investors infer their estimation of the state variable θ_t, θ_t^u, from the price via

$$\theta_t^u = \frac{1}{p_3}[P_t - (p_0 + p_1 D_t + p_2 \pi_t^u + p_4 A_t + p_5 A_{Dt})]. \tag{A15}$$

The state variable dynamics will be

$$d\Psi^u = e_\Psi^u \Psi^u dt + \sigma_\Psi^u dB_t^u, \tag{A16}$$

The investment opportunity is given as

$$dQ = (D_t - rP_t)dt + dP_t \equiv e_Q^u \Psi dt + \sigma_Q^u dB_t, \tag{A17}$$

with more details given in Appendix A.3. Now by differentiating Equation (10) and by using Lemma 2, we have

$$\eta^u = f^u \Psi^u = f_0^u + f_1^u D_t + f_2^u P_t + f_3^u A_t + f_4^u A_{Dt}. \tag{A18}$$

Market clearing. Given Equations (A13) and (A18) for the demands of stock by informed and uninformed investors, the market clearing condition requires

$$\eta^i + \eta^u = 1 + \theta_t,$$

or equivalently,

$$\begin{aligned} & (1-w)[f_0^i + f_1^i D_t + f_2^i \pi_t + f_3^i P_t + f_4^i A_t + f_5^i A_{Dt}] \\ + \quad & w[f_0^u + f_1^u D_t + f_2^u P_t + f_3^u A_t + f_4^u A_{Dt}] \\ = \quad & 1 - \frac{p_0}{p_3} - \frac{1}{p_3}[p_1 D_t + p_2 \pi_t - P_t + p_4 A_t + p_5 A_{Dt}]. \end{aligned}$$

where in the right hand side, we have substituted θ_t as function of P_t in (10). By matching coefficients of the state variables, we obtain the coefficients p_0, p_1, p_2, p_3, p_4, and p_5 for the price function of (10). This implies that Proposition 1 holds.

A.2 Proof of Lemma 1

Given SDEs (1), (2) and (3), and applying the conjectured price for P_t in Equation (10), we obtain

$$dA_t = (P_t - \alpha A_t)dt = (p_0 + p_1 D_t + p_2 \pi_t + p_3 \theta_t + (p_4 - \alpha)A_t + p_5 A_{D_t})dt. \tag{A19}$$

In addition, we have the following dynamics for A_{Dt},

$$dA_{D_t} = (D_t - \alpha_{D_L} A_{D_t})dt. \tag{A20}$$

So the SDE for Ψ^i is given by

$$d\Psi^i = e^i_\Psi \Psi^i dt + \sigma^i_\Psi dB^i_t,$$

where e^i_Ψ and σ^i_Ψ are given as

$$e^i_\Psi = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\alpha_D & 1 & 0 & 0 & 0 \\ \alpha_\pi \bar{\pi} & 0 & -\alpha_\pi & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha_\theta & 0 & 0 \\ p_0 & p_1 & p_2 & p_3 & p_4 - \alpha_{p_L} & p_5 \\ 0 & 1 & 0 & 0 & 0 & -\alpha_{D_L} \end{pmatrix},$$

and

$$\sigma^i_\Psi = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_D & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_\pi & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\theta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Therefore, the investment opportunity can be computed as

$$dQ = (D_t - rP_t)dt + dP_t = e^i_Q \Psi dt + \sigma^i_Q dB_t,$$

where

$$dP_t = p_1 dD_t + p_2 d\pi_t + p_3 d\theta_t + p_4 dA_t + p_5 dA_{D_t}.$$

Using Equations (A19) and (A20), we obtain

$$e^i_Q = \Big(\begin{matrix} -rp_0 + p_2\alpha_\pi\bar{\pi} + p_4 p_0 & 1 - rp_1 - p_1\alpha_D + p_4 p_1 + p_5 & -rp_2 + p_1 - p_2\alpha_\pi + p_4 p_2 \end{matrix}$$
$$\begin{matrix} -rp_3 - p_3\alpha_\theta + p_4 p_3 & -rp_4 + p_4^2 - p_4\alpha_{p_L} & -rp_5 + p_4 p_5 - p_5\alpha_{D_L} \end{matrix} \Big),$$

and

$$\sigma_Q^i = \begin{pmatrix} 0 & p_1\sigma_D & p_2\sigma_\pi & p_3\sigma_\theta & 0 & 0 \end{pmatrix}.$$

A.3 Details for Equations (A16) and (A17)

For uninformed investors, θ_t can be inferred using the following linear equation

$$\theta_t^u = \eta_0 + \eta_1 D_t + \eta_2 P_t + \eta_3 A_t + \eta_4 A_{D_t}, \tag{A21}$$

where $\eta' s$ can be given by matching

$$p_2\pi_t^u + p_3\theta_t^u = P_t - p_0 - p_1 D_t - p_4 A_t - p_5 A_{D_t}$$

to obtain

$$\eta_0 = \frac{-p_0 - p_2\bar{\pi}}{p_3}, \eta_1 = \frac{-p_1 - p_2\beta_1}{p_3}, \eta_2 = \frac{1 - p_2\beta_2}{p_3}, \eta_3 = \frac{-p_4 + p_2\alpha\beta_2}{p_3}, \eta_4 = \frac{-p_5 + p_2\alpha_2\beta_1}{p_3}.$$

To derive the dynamics of dD_t, dP_t, dA_t, and dA_{D_t} for uninformed investors, we note first that dA_t and dA_{D_t} can be characterized as

$$dA_t = (P_t - \alpha_{pL} A_t)dt,$$

and

$$dA_{D_t} = (D_t - \alpha_{D_L} A_{D_t})dt.$$

To derive the SDE for D_t, we apply Equation (4) to obtain

$$\begin{aligned} dD_t &= (\pi_t^u + \sigma_u u_t - \alpha_D D_t)dt + \sigma_D dB_{1t} \\ &= (\bar{\pi} + \beta_D(D_t - \alpha_2 A_{D_t}) + \beta_p(P_t - \alpha A_t) - \alpha_D D_t)dt + \hat{\sigma}_D dB_{1t}^u, \end{aligned}$$

where

$$\hat{\sigma}_D dB_{1t}^u = \sigma_D dB_{1t} + \sigma_u dZ_t, \tag{A22}$$

with

$$\hat{\sigma}_D^2 = \sigma_D^2 + \sigma_u^2. \tag{A23}$$

In the above, Z_t is defined as $Z_t = \int_0^t u_s ds$, which is another independent Brownian motion with u_t the white noise in the updating rule of Equation (4).

Define

$$\Lambda_t \equiv p_2\pi_t + p_3\theta_t, \tag{A24}$$

which is observable by uninformed investors who can infer it from the price Equation (10), along with other observable variables D_t, A_t and A_{D_t}, through

$$\Lambda_t = P_t - (p_0 + p_1 D_t + p_4 A_t + p_5 A_{D_t}).$$

Hence, from uninformed investor's point of view, $d\Lambda_t$ can be given as

$$\begin{aligned} d\Lambda_t &= p_2 d\pi_t + p_3 d\theta_t \\ &= (p_2 \alpha_\pi (\bar{\pi} - \beta_D (D_t - \alpha_2 A_{D_t}) + \beta_p (P_t - \alpha A_t)) - p_3 \alpha_\theta {\theta_t^u}) dt + \hat{\sigma}_\Lambda dB_{2t}^u. \end{aligned}$$

with

$$\begin{aligned} \hat{\sigma}_\Lambda dB_{2t}^u &= p_2(\sigma_\pi dB_{2t} - \alpha_\pi \sigma_u dZ_t) + p_3(\sigma_\theta dB_{3t} + \alpha_\theta \frac{p_2}{p_3} \sigma_u dZ_t) \\ &= p_2 \sigma_\pi dB_{2t} + p_3 \sigma_\theta dB_{3t} + (\alpha_\theta - \alpha_\pi) p_2 \sigma_u dZ_t, \end{aligned} \tag{A25}$$

and

$$\hat{\sigma}_\Lambda^2 = (p_2 \sigma_\pi)^2 + (p_3 \sigma_\theta)^2 + (\alpha_\theta - \alpha_\pi)^2 p_2^2 \sigma_u^2. \tag{A26}$$

Based on Equations (A22) and (A25), the correlation between dB_{1t}^u and dB_{2t}^u, defined as

$$\mathrm{Var}(dB_{1t}^u, dB_{2t}^u) \equiv \rho dt,$$

can be written as

$$\rho = \frac{p_2 \sigma_u^2 (\alpha_\theta - \alpha_\pi)}{\hat{\sigma}_D \hat{\sigma}_\Lambda}. \tag{A27}$$

Substituting Equations (A21) into (6), we obtain the formula for $d\Lambda_t$ in terms of Ψ^u as given in (A14).

To summarize, we have

$$d\Psi^u = e_\Psi^u \Psi^u dt + \sigma_\Psi^u dB_t^u,$$

e_Ψ^u and σ_Ψ^u are

$$e_\Psi^u = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ \bar{\pi} & \beta_1 - \alpha_D & \beta_2 & -\alpha\beta_2 & -\alpha_2\beta_1 \\ q_0 & q_1 & q_2 & q_3 & q_4 \\ 0 & 0 & 1 & -\alpha & 0 \\ 0 & 1 & 0 & 0 & -\alpha_2 \end{pmatrix},$$

and

$$\sigma_{\Psi}^{u} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \hat{\sigma}_D & 0 & 0 & 0 \\ 0 & p_1\hat{\sigma}_D + \rho\hat{\sigma}_\Lambda & \sqrt{1-\rho^2}\hat{\sigma}_\Lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

where

$$\begin{aligned} q_0 &= p_1\bar{\pi} - p_3\alpha_\theta\gamma_0, \\ q_1 &= p_1(\beta_1 - \alpha_D) - p_2\alpha_\pi\beta_1 - p_3\alpha_\theta\gamma_1 + p_5, \\ q_2 &= p_1\beta_2 - p_2\alpha_\pi\beta_2 - p_3\alpha_\theta\gamma_2 + p_4, \\ q_3 &= -p_1\alpha\beta_2 + p_2\alpha_\pi\alpha\beta_2 - p_3\alpha_\theta\gamma_3 - p_4\alpha, \\ q_4 &= -p_1\alpha_2\beta_1 + p_2\alpha_\pi\alpha_2\beta_1 - p_3\alpha_\theta\gamma_4 - p_5\alpha_2, \end{aligned}$$

and

$$\begin{aligned} \rho &= \frac{p_2\sigma_u^2(\alpha_\theta - \alpha_\pi)}{\hat{\sigma}_D\hat{\sigma}_\Lambda}, \\ \hat{\sigma}_D^2 &= \sigma_D^2 + \sigma_u^2, \\ \hat{\sigma}_\Lambda^2 &= (p_2\sigma_\pi)^2 + (p_3\sigma_\theta)^2 + (\alpha_\theta - \alpha_\pi)^2 p_2^2\sigma_u^2. \end{aligned}$$

The investment opportunity for uninformed investor is then

$$dQ = (D_t - rP_t)dt + dP_t = e_Q^u\Psi dt + \sigma_Q^u dB_t,$$

where

$$e_Q^u = \begin{pmatrix} q_0 & 1+q_1 & q_2 - r & q_3 & q_4 \end{pmatrix},$$

and

$$e_Q^u = \begin{pmatrix} 0 & p_1\hat{\sigma}_D + \rho\hat{\sigma}_\Lambda & \sqrt{1-\rho^2}\hat{\sigma}_\Lambda & 0 & 0 \end{pmatrix}.$$

Q.E.D.

A.4 Model Parameters and γ_5

Although the empirical results of the paper do not depend on the model which aims at understanding the role of fundamental trends, it is of interest to examine the relation between model parameters and γ_5. Table A1 provides the results that are discussed in Section 2.1.

Table A1   Predictability of fundamental trends

The table presents the stock return predictability of fundamental trends proxied by the moving average of dividend payments, i.e., the parameter γ_5 in Equation (11)

$$R_{t+1} \equiv \frac{P_{t+\Delta t} - P_t}{\Delta t} = \gamma_0 + \gamma_1 D_t + \gamma_2 \pi_t + \gamma 3 \theta_t + \gamma_4 A_t + \gamma_5 A_{Dt} + \sigma_P \varepsilon_P.$$

The parameters for the model are set as follows: $r = 0.05, \rho = 0.2, \bar{\pi} = 0.85, \sigma_D = 1.0, \sigma_\pi = 0.6, \sigma_\theta = 3.0, \alpha_\theta = 0.4$, and $\alpha_D = 1.0$. The moving average windows for price and dividend are $\alpha_{pL} = 1$ and $\alpha_{DL} = 0.9$, respectively. The two panels present the numerical results of γ_5 for various w, the fraction of uninformed investors.

$\beta_D \backslash \beta_p$	−0.4	−0.3	−0.2	−0.1	0	0.1	0.2	0.3	0.4
Panel A: γ_5 for $w = 0.1$									
−0.4	−0.0595	−0.0583	−0.0570	−0.0555	−0.0538	−0.0521	−0.0503	−0.0485	−0.0466
−0.3	−0.0445	−0.0435	−0.0425	−0.0413	−0.0401	−0.0387	−0.0374	−0.0359	−0.0345
−0.2	−0.0295	−0.0289	−0.0282	−0.0274	−0.0265	−0.0256	−0.0246	−0.0237	−0.0226
−0.1	−0.0147	−0.0144	−0.0140	−0.0136	−0.0131	−0.0127	−0.0122	−0.0117	−0.0111
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	0.0146	0.0142	0.0138	0.0134	0.0129	0.0124	0.0119	0.0113	0.0108
0.2	0.0290	0.0283	0.0274	0.0265	0.0255	0.0245	0.0234	0.0223	0.0211
0.3	0.0433	0.0421	0.0408	0.0394	0.0379	0.0362	0.0345	0.0328	0.0309
0.4	0.0575	0.0558	0.0540	0.0520	0.0499	0.0476	0.0453	0.0428	0.0403
Panel B: γ_5 for $w = 0.9$									
−0.4	−0.1819	−0.2079	−0.2347	−0.2622	−0.2901	−0.3184	−0.3468	−0.3754	−0.4040
−0.3	−0.1367	−0.1563	−0.1765	−0.1972	−0.2181	−0.2394	−0.2607	−0.2821	−0.3035
−0.2	−0.0913	−0.1044	−0.1180	−0.1318	−0.1458	−0.1599	−0.1741	−0.1884	−0.2026
−0.1	−0.0458	−0.0523	−0.0591	−0.0661	−0.0731	−0.0801	−0.0872	−0.0943	−0.1014
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	0.0459	0.0526	0.0594	0.0664	0.0734	0.0804	0.0875	0.0946	0.1016
0.2	0.0920	0.1054	0.1191	0.1330	0.1471	0.1611	0.1752	0.1893	0.2032
0.3	0.1383	0.1584	0.1790	0.1999	0.2210	0.2420	0.2631	0.2840	0.3048
0.4	0.1847	0.2117	0.2392	0.2671	0.2951	0.3231	0.3510	0.3787	0.4062