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Abstract. Mining trajectory datasets has many important applications. Real trajectory data often involve uncertainty due to inadequate sampling rates and measurement errors. For some trajectories, their precise positions cannot be recovered and the exact routes that vehicles traveled cannot be accurately reconstructed. In this paper, we investigate the uncertainty problem in trajectory data and present a visual analytics system to reveal, analyze, and solve the uncertainties associated with trajectory samples. We first propose two novel visual encoding schemes called the *road map analyzer* and the *uncertainty lens* for discovering road map errors and visually analyzing the uncertainty in trajectory data respectively. Then, we conduct three case studies to discover the map errors, to address the ambiguity problem in map-matching, and to reconstruct the trajectories with historical data. These case studies demonstrate the capability and effectiveness of our system.

Keywords: Uncertainty, trajectory, visual analysis.

1 Introduction

In recent years, there has been a dramatic increase in GPS-embedded devices used for navigation and tracking, which enables the collection of large volumes of GPS trajectories [8]. Trajectory data play important roles in urban planning, route recommendation, traffic analysis, and transportation management. Usually, trajectories are presented in two styles: curves (parameterized by time) in a 2D plane or trajectory samples that are discrete spatial-temporal points. The latter style is widely adopted in trajectory datasets since the cost of capturing and maintaining data is relatively low. However, the trajectories represented by samples often involve *uncertainty* which might appear as data imprecision due to sampling/measurement errors or fuzziness caused by pre-processing for preserving anonymity. Uncertainty in trajectories poses challenges for enhancing, reconstructing, and mining trajectories.

Uncertainty that appears as measurement errors has been studied for enhancing historical trajectories [9]. Limited by the technology used, the trajectory data are not precise due to measurement and sampling errors. Therefore, the recorded GPS positions often need to be matched with the given road network (referred to as road map). This process is called *map-matching*. Map-matching is integrated in many trajectory-based applications as a pre-processing module, which aligns trajectory samples with the given road networks. Quddus *et al.* [14] summarized the existing map-matching approaches and compared their performances. They addressed the measurement error problem and focused on the sampling errors. While map-matching methods are effective in solving the problem of sampling errors, few of them address the issues of inaccurate road maps and ambiguous selections of roads for trajectory samples.

Road maps, considered as the vital input to map-matching, are not always reliable due to two reasons. First, the update of road maps is not as frequent as the collection of trajectory data. Usually, a city digital map is only updated monthly or even less frequently, but the actual road changes happen everyday or even every hour in large cities. Thus, some road changes might not be incorporated in the road maps used for map-matching. Second, although the semi-automatic or fully-automatic methods used for road-map extraction are often effective, they might still fail to obtain correct road positions for various reasons, such as the low resolution of images, and the overlapping of roads. Therefore, road maps should be checked for errors before they are used for map-matching. Since road maps are complex and large, an effective visual analysis tool for revealing and fixing road map errors is needed.

Another kind of uncertainty is the ambiguity in the selection of roads to match samples. An essential step of map-matching methods is selecting an appropriate road for an off-road sample to align with. For that, the map-matching algorithms first select several road candidates with loose conditions, and then score the road candidates with specific cost functions, and finally choose the road with the highest score as the target. However, when the scores of the road candidates are similar, *e.g.*, the sample positions fall in the middle of multiple roads, all map-matchin algorithms will encounter the ambiguity problem as it is no longer clear which road should be chosen.

Uncertainty involving low sampling rates is also a serious issue for reconstructing trajectories. Some applications may need to reconstruct a vehicle's continuous route from its discrete trajectory samples. Due to low sampling rates, the collected trajectory samples can be very sparse. Between two consecutive sparse samples, there may exist several routes. Thus, additional information is needed to help choose routes to complete the reconstruction. Historical data could be very helpful. For example, we can investigate the historical data and check whether there are other relevant trajectories with denser sampling rates in the region of interest. With the help of relevant trajectories, we may have a better chance to find out the correct routes. In addition, by investigating the uncertainty patterns in the data, we may find ways to reduce the uncertainty and improve the data quality. For example, for areas dense with poor trajectory samples, we can add some road-side-units to improve the position accuracy and increase the sampling rates.

To solve the uncertainty problem, it is vital to keep humans in the loop and present all the relevant information to the users in an intuitive manner, especially for some fuzzy patterns and tricky cases. In this paper, we present our visual analytics solutions to the uncertainty problem in the trajectory data. Specifically, we propose two novel visual designs, *i.e.*, a *road map analyzer* for discovering potential errors in road maps, and an *uncertainty lens* for resolving the uncertainty in trajectory data. We demonstrate how to visually reveal the road map errors, resolve the ambiguities in map-matching, and reconstruct trajectories from sparse consecutive samples. We further test the effectiveness and usefulness of our approach with three case studies on real trajectory data.

The major contributions of our work are as follows:

- A visual design called road map analyzer to reveal errors in road maps based on observed trajectory samples. Our visual design is able to discover map errors such as road shifting and road missing.
- A visual design called uncertainty lens to reveal and resolve the uncertainty in the trajectory data. Our method integrates multiple factors (*e.g.,* speed, time, sparseness, direction) related to uncertainty into a coherent analytical framework.
- Case studies with real trajectories and digital maps to demonstrate the effectiveness and usefulness of the approach.

2 Related Work

Uncertainty Modeling Kraak [7] proposed an interactive system to explore and visualize space-time data under a space time cube. Pfoser and Jensen[12] described the notion of the uncertainty in sampling error and the error across time. Later, a cylindrical model was presented by Trajcevski [16] to represent and capture the uncertainty for efficient querying. Most of the works solve the uncertainty problem in trajectory datasets, but few of them address the uncertainty in the context of specific trajectory-based applications. In our work, we deal with the uncertainty issue in the context of map-matching approaches, and address not only the uncertainty problem in trajectory datasets but also the uncertainty issue in map-matching approaches.

Uncertainty Visualization Visualization techniques have also been developed for uncertainty, *e.g.,* glyphs, error bars, scale modulation, and ambulation. Pang *et al.* [11] proposed an uncertainty classification, studied its representation, and presented various approaches for its visualization. Fisher [3] surveyed the literature on uncertainty visualizations for bounded errors. Color and texture were considered to be the best choices for visualizing uncertainty [2][5]. The traditional representations of uncertainty usually consider uncertainty as a one-dimensional variable, but the uncertainty we want to address is more complicated. Traditional methods are inadequate for solving our problem.

Map-Matching To deal with the errors in trajectory datasets, various map-matching approaches have been proposed and they can be categorized into three groups: geometric-based methods[15], topological-based methods [4][10], and statistical methods [6]. Geometric-based methods are effective in finding local matches, but sensitive to map errors. Extended from geometric-based methods, topological methods aim at matching the entire trajectory to road maps by using the topologies of road networks. Although topological methods are more robust than geometric-based methods, they still suffer from various errors associated with navigation sensors and road maps. Statistical models are employed in map-matching include Kalman Filter [6], and Bayesian classifier [13]. All map-matching approaches will encounter the ambiguity problem when the trajectory samples fall into the middle of multiple roads. Besides, few of the existing works take the map errors into consideration, which has a big impact on the map-matching results. In this paper, we present a novel visualization method to identify and fix the map errors and propose a visual-guided approach to resolve the ambiguities.

3 Visualization Design

3.1 Design Principles

We identify a few key principles to follow during the development of our designs: (1) To address the uncertainty in trajectories, maps should be used to facilitate the analysis but their errors should be identified and fixed first. The visualization schemes should help reveal and differentiate different map errors.

(2) To resolve the uncertainty in samples, multiple factors (the maps, other trajectories, and the traffic) should be taken into account and these factors should be put into a coherent analytical framework.

(3) The visualization mantra, "overview first, zoom and filter, then details-on-demand", will be followed.

3.2 Road Map Analyzer

As road maps are the critical reference for map-matching, road maps should be as accurate as possible and the errors in road maps need to be identified and corrected. After studying the map error problem and checking with domain experts, we focus on two specific cases are commonly seen in applications: (a) *Road Shifting*. In this case, the distribution of the samples forms a road shape, but the samples are not symmetric with respect to the road. (b) *Road Missing*. In this case, samples are roughly separated into multiple sets (usually two). The distribution of the samples forms two road shapes, and one of them is different from the observed road. It is likely that the road near the observed road is missing. Since the trajectory samples are sparse, we try to establish the relations among the samples. Given the set of the samples, we first establish the relations by building a tree considering the samples as the nodes and the distances between samples as the weights of the edges. The goal is to obtain a set of edges that minimize the sum of the weights. After establishing the relations, we get a set of edges that link the samples, called virtual edge. The virtual edges assign each pixel (on the display) they covered a density value by weights. Then, we generate a density map with the samples and their virtual edges. Finally we bound the density map by the bubble sets algorithm [1]. Fig. 1 is an example of the visualization of the boundary, called bubble view. Using the bubble view, we can easily detect the road error cases. For road shifting case, the bubble doesn't cover the observed road. For road missing case, the bubble not only covers the observed road, but also forms a trace linking with other roads. We also provide a statistical view that review statistical information of trajectory samples on road. In this view, we encode the distribution of the samples by plotting the samples into a tranform space. Each line represents a sample. It starts from a dot where indicates the average distance of the samples to the road, passes through its relative position to the road (left side or right side, by clockwise on the map), and ends on its projection on the road. Fig. 6 gives an example of the statistical view.

3.3 Uncertainty Lens

Identifying and resolving the uncertainty in trajectory data are our major goals. To achieve these goals, we design a novel visualization scheme, uncertainty lens, which

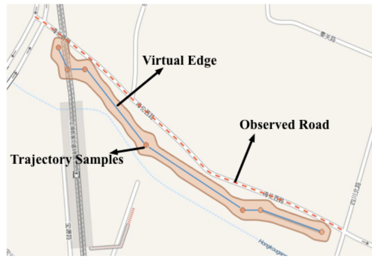

Fig. 1. Bubble view. The bubble view helps the users to identify the map errors, and it is generated from the trajectory samples around the observed road and the virtual edges linking the trajectory samples.

integrates multiple visualizations into one display to present all the important information that may help users resolve the uncertainty.

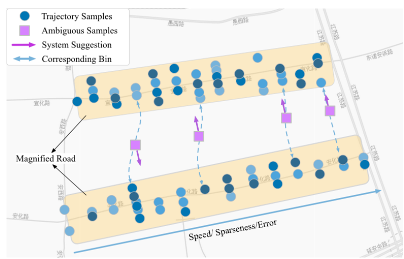

Fig. 2. Uncertainty lens. The uncertainty lens magnifies relative roads and sorts relative trajectory samples by their information (speed, error or sparseness). Based on the ordered information, the uncertainty lens provides suggestions for resolving ambiguities.

Fig.2 shows the design of the uncertainty lens which consists of relevant magnified roads with the trajectory samples, and a set of system suggestions. The uncertainty lens is overlaid on the geographical map. One straightforward way to observe the trajectory samples is plotting the samples on the geographical map. In this way, users can quickly find out the geographical information for the trajectory data and whether the samples fall on roads. However, as the number of trajectory samples is usually very large, the visualization becomes very cluttered. Therefore, we magnify the roads and re-order the trajectory samples in a certain way to get a better view. Additionally, users often need to consider other information besides maps to perform the analytical tasks, so this information should also be presented. After discussing with domain experts, we identify three features that are most important for analysis, including speed, error and sparseness.

The uncertainty lens magnifies relevant roads and displays trajectory samples on the relevant roads. For better analysis, the uncertainty lens encodes the speed, error, and sparseness by users' preferences. The encoding scheme is as follows.

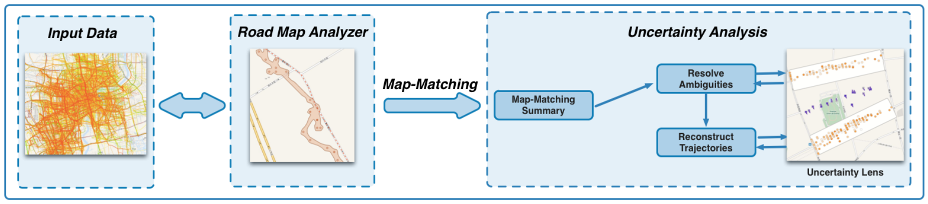

Fig. 3. System overview. Our system consists of two primary components. The input of the system is a set of trajectory samples and a digital map. The digital map is firstly processed by the module *road map analyzer*. The road map analyzer refines the digital map with the bubble views. Then a map-matching process is applied on the trajectory data with the refined map. The *uncertainty analysis* module analyzes the uncertainty of the trajectory data, resolves ambiguity, and reconstructs continuous trajectories.

- **Position:** We sort the trajectory sample by speed, error, or sparseness along the relevant road direction. At the same time, we keep the distances of the samples to the center of the road. In this way, we encode the error information implicitly. But users still can show the error information in an explicit way. The system uses speed to sort by default.
- **Color:** Users can set color saturation to encode one measurement that is different from the one used in the position visual channel.

4 System Overview and Workflow

Fig. 3 shows the overview of our system which consists of two primary components for three major tasks (*i.e.,* analyzing map errors, resolving ambiguities, and reconstructing trajectories). The input to the system is a set of trajectory samples and a digital map. The digital map is first processed by the module *road map analyzer*. The road map analyzer refines the digital map with the help of the bubble views. With the refined map, the map-matching process adjusts the observed positions of the trajectory samples, which partially solves the uncertainty problem in the data. After summarizing the results of the map-matching, the *uncertainty analysis* module identifies the ambiguous samples, analyzes the uncertainty of the ambiguous samples by using the uncertainty lens, and resolves the ambiguities. Then, to reconstruct trajectories, the system extracts the sparse consecutive samples with multiple route candidates. For selecting the best candidate route as a partial trajectory, the uncertainty lens provides historical trajectory information as hints for users to make decision.

Analyze map errors Since the road network is large and complex, it is infeasible to review every road segment or display the visualizations of the roads in a size-limited screen. Therefore, our system computes the uncertainties of the roads and provides a list of the uncertainties. The list is shown in Fig. 4(b). The texts and the colors of the list cells refer to the identifications of the roads and their uncertainties respectively. The light color indicates the high uncertainty while the dark color indicates the low uncertainty (usually dark color suggests the correct road). The users can double click the list to select the roads. Once a road is selected to observe, the main viewer (Fig. 4(c)) automatically zooms to fit the display to the road and shows the bubble view of the observed road. If the users find any road errors, they can refine the roads by drawing the estimated roads manually.

Resolve ambiguities In our system, we leverage a map-matching approach [15] to adjust the observed position of the trajectory samples with the refined map. The core step in map-matching is to select an appropriate road for an off-road sample to align with by utilizing a specific cost function. In some cases, the selections are quite certain since the samples are close to a road. When the samples are reported in the middle of multiple roads, the selections become ambiguous (the samples in this case are called *ambiguous samples*). To resolve the ambiguities, we extract the ambiguous samples from the map-matching results. To locate the ambiguous samples, our system provides different levels of the map views. In low-level views, the system generates heatmaps that describe the distribution of the ambiguous samples. In high-level views, the system shows the ambiguous samples directly (using rectangles by default).

Once a set of ambiguous samples is focused, the uncertainty lens first auto-detects the relevant roads according to the directions of the ambiguous samples. Then, the uncertainty lens magnifies the relevant roads and displays the trajectory samples on the relevant roads. The users can sort the samples by their preferences. For example, they can sort the samples by speed and set the color of the samples by sparseness. After that, the system sorts the ambiguous samples accordingly and shows the suggestions for resolving the ambiguities. The users can resolve the ambiguities by clicking the ambiguous samples to follow the system suggestions or dragging the ambiguous samples to the roads depending on their own decisions.

Reconstruct trajectories Once the users load a trajectory, the system reconstructs the trajectory from the most sparse sample pair. To find out the routes between a sample pair, the system employs the time-space prism model [12]. By using this model, the possible movement region (of the trajectory) between two neighboring samples can be found. The moving object (such as vehicles) can only move within this region under a given speed. Therefore, the possible routes are confined within this region. For any sample pair, the system detects the possible routes. If there exists only one route, it continues to the next pair. If there exist multiple routes, the system shows the uncertainty lens for choosing a route as a partial trajectory. In the uncertainty lens, every possible route refers to a relevant route. Similar to resolving ambiguities, the users can sort the trajectory samples on the relevant routes. The system treats the trajectory between the neighboring samples as an ambiguous sample with information from this pair of samples. For example, the trajectory speed can be considered as the average speed of its adjacent samples. In this way, the system can make the suggestion for the users to choose a route.

5 Experiment and Discussion

The entire system was developed using Java, DaVinci and JOSM. We tested our system on a 1.8 GHz Intel Core i7 laptop with 4 GB DDR3. Raw map data were collected from OpenStreetMap.org. Trajectory data were collected from 4,000 taxis in a big city over an eight-month period. Each GPS record contains longitude, latitude, timestamp, and logs of other activities. The preprocessing of the data includes removing erroneous trajectories with impossible speeds and storing the valid trajectories in binary files to reduce the storage space and processing time.

Fig. 4 shows the system user interface. The system user interface consists of three components, including a tool bar that enables users to perform operations such as selecting and zooming, a road list that uses color to show the uncertainties of the roads, and a main viewer that provides the views of the geographical map, the road map analyzer, and the uncertainty lens.

Fig. 4. System user interface. The system user interface consists of three components, including (a) a tool bar that enables users to perform operations such as selecting and zooming, (b) a road list that uses color to show the uncertainties of the roads, and (c) a main viewer that provides the views of the geographical map, the road map analyzer, and the uncertainty lens. In the road list, the light color indicates the roads with high uncertainty while the dark color suggests the roads with low uncertainty.

5.1 Experiment

To show the system usability, two transportation system researchers were invited to use our system to investigate the uncertainty in trajectory data. We have consulted the researchers when we design our system, so they are familiar with our design. We first gave a tutorial for about 15 minutes, then the researchers played with our system and did a test run to get familiar with the system user interface. After that, they were asked to perform three tasks: (1) analyzing map errors, (2) resolving ambiguities, and (3) reconstructing trajectories. The process for each researcher was about 40 minutes.

Analyzing Map Errors In this task, the participants were asked to identify errors on the map. They first investigated some roads by clicking the road list. Fig. 5 shows three of the roads that the participant checked. The roads with different uncertainty showed different patterns in the bubble views. For example, the uncertainty of the road in Fig. 5

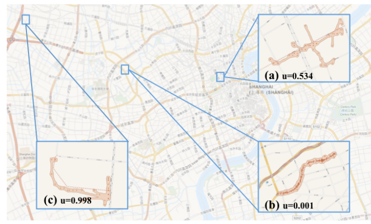

Fig. 5. Roads with different uncertainties. (a) Road uncertainty=0.534, the road is probably with errors. (b) Road uncertainty=0.001, the road is correct. (c) Road uncertainty=0.998, the road is with errors.

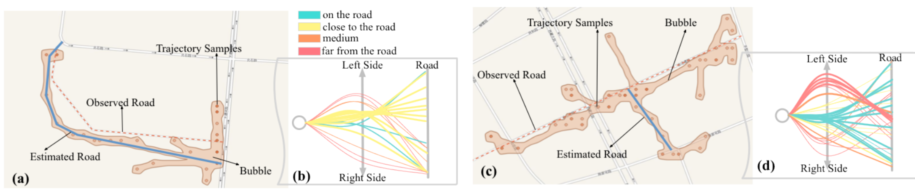

Fig. 6. Map error cases. (a) (b) Road shifting case. Most of the trajectory samples are reported slightly away from the observed road (red dashed lines). The actual road position (blue lines) is estimated using the bubble generated from the trajectory samples. (c) (d) Road missing case. The bubble forms a trace from the observed road to another road, which means that a road is absent from the map.

(b) was very low. The bubble aligned this road very well. Therefore, the participants considered this road was correct. For the case a participant found in Fig. 5 (c), it was with high uncertainty. He observed the bubble view (a zoom view shown in Fig. 6 (a)) and recognized the road as a road shifting case. Because most of the trajectory samples were not falling on the road and the bubble didn't cover the road. He estimated the road position according to the bubble (Fig. 6 (a)). For an interesting error another participant found on Fig. 5 (a), the uncertainty of this road was medium. The uncertainty of this road was medium. The bubble covered the observed road, which confirmed the observed road is correct. However, the bubble formed a trace (without covering any road) connecting to another road. He considered that a road was absent from the map. Then he manually added the road on the map (Fig. 6(c)). The bubble view is effective at revealing road missing and estimating the actual positions of road with errors.

Resolving Ambiguities To resolve the ambiguities, a participant located the ambiguous samples by using multiple level map views. He first selected a set of ambiguous samples in the region in Fig. 7(a). Then he opened the uncertainty lens. The uncertainty lens suggested two relevant roads (shown in Fig. 7(b)) and magnified them with the trajectory samples (see Fig. 7(c)). After that, he chose to sort the samples by speed and use the color to encode error. Following the setting by the participant, the system provided the suggestions for the ambiguous samples. Fig. 7(d) shows the system suggestions on the ambiguous samples by arrows. The arrows pointed to the road that the system recommended for the ambiguous samples. The participant resolved the ambiguities by

Fig. 7. Resolving ambiguities. (a) User selects a region to resolve ambiguous samples. (b) System suggests two candidate roads according to the directions of the ambiguous samples. (c) The system magnifies the candidate roads and shows the trajectory samples positioning on the roads. (d) Trajectory samples and ambiguous samples are sorted according to their speeds. And the system provides suggestions for the ambiguous samples. (e) User makes the decision according to the system suggestions. (f) Ambiguous samples are resolved and placed on the roads.

clicking on the ambiguous samples (shown in Fig. 7(e)). Finally the ambiguous samples moved to the road (see Fig. 7(f)). From the task, the system's capability of resolving the ambiguities in map-matching was confirmed.

Reconstructing Trajectories A participant started the task with loading a sparse trajectory. The system focused on the most sparse sample pair at the beginning. He reconstructed the trajectory pair by pair. Fig.8 shows a process of the participant reconstructing one trajectory between a pair of samples. The system focused on a partial trajectory that needed to be reconstructed. In Fig. 8 (a), this trajectory didn't pass any roads. The system computed the possible movement region for the sample pair and found out two possible routes (Route 1 and Route 2). Then, the participant opened the uncertainty lens. The uncertainty lens combined the roads in the routes and magnified the routes (Fig.8 (b)). After that, the participant sorted the trajectory samples on the routes, and obtained system suggestion (Fig.8 (c)). The system suggested the trajectory should pass Route 1. Finally, he reconstructed the trajectory by accepting system suggestion. Fig. 8 (d) depicts reconstruction result. From the task, the usefulness of our system for reconstructing the continuous trajectories was confirmed.

5.2 User Feedback

We consulted two experts who have expertise in intelligent transportation system research for comments after internally testing our system. They both have worked with trajectory data and are familiar with map-matching.

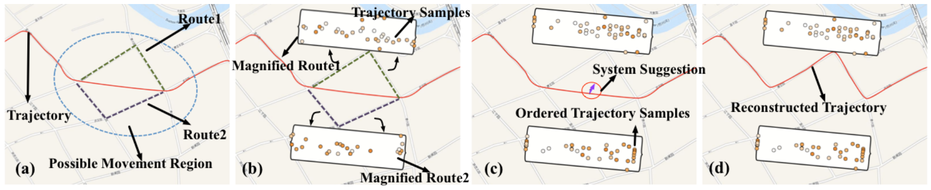

Fig. 8. Reconstructing a trajectory. (a) A sparse trajectory doesn't pass any roads. System computes the possible movement region for its adjacent samples. Within the possible movement region, the system finds out two possible routes. (b) Uncertainty lens combines the roads on the routes and magnifies the routes. It shows the trajectory samples on the routes at the same time. (c) Uncertainty lens re-orders the trajectory samples and the system provides a suggestion accordingly. (d) Reconstruction result.

The well-designed visual encoding schemes to resolve uncertainty in trajectory data and the highly supported interactions with users are greatly appreciated by the experts. Especially the powerful encoding scheme to reveal errors in road maps as well as the method to reveal the uncertainty in the trajectory data received very positive comments. These features largely improved the user experiences in traffic data exploration. Compared with their current map-matching analysis tools, which only plots the trajectory samples on the map to reconstruct the trajectory, our system makes the whole exploration process much more accurate and free of map errors. According to the feedback, our system is also good for demonstration. The visualization provides intuitive interface for understanding the trajectory data with many unique attributes. Utilizing the uncertainty lenses, users can gain insight into the speed, time, and direction attributes of the data which are very useful to detect the anomaly of the traffic, study drivers' driving patterns, and investigate the passengers' distribution. Our visual analytics cases are very convincing for domain experts. The analysis of the traffic patterns and trajectory reconstruction is considered very valuable to guide the road traffic capacity design and driving pattern study. The discovery of the map errors in Fig. 6 greatly attracts the experts' attention. The detected map errors are later confirmed by their manual inspection on and comparison of different versions of the maps. The experts also feel excited that the ambiguity analysis and demonstration of our system shows a data exploration process that is more promising than traditional methods to retrieve driving patterns.

6 Conclusion

In this paper we have presented a comprehensive visual analytics system for revealing and resolving the uncertainty in trajectory data. Two novel encoding schemes are designed. The experiments with real data, three case studies, and the feedback from the domain experts, have demonstrated the effectiveness of our system.

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References

1. C. Collins, G. Penn, and S. Carpendale. Bubble sets: Revealing set relations with isocontours over existing visualizations. *IEEE Transactions on Visualization and Computer Graphics*, 15(6):1009–1015, 2009.
2. T. J. Davis and C. P. Keller. Modelling and visualizing multiple spatial uncertainties. *Computers and Geosciences*, 23(4):397–408, 1997.
3. D. Fisher. Incremental, approximate database queries and uncertainty for exploratory visualization. In *IEEE Symposium on Large Data Analysis and Visualization*, pages 73–80, 2011.
4. J. Greenfeld and S. Joshua. Matching gps observations to locations on a digital map. *Environmental Engineering*, 1(3):1–13, 2002.
5. T. Hengl and N. Toomanian. Maps are not what they seem: representing uncertainty in soil-property maps. In *International Symposium on Spatial Accuracy Assessment in Natural Resources and Environmental Sciences*, pages 805–813, 2006.
6. B. Hummel and K. Tischler. Robust, gps-only map matching: exploiting vehicle position history, driving restriction information and road network topology in a statistical framework. In *the GIS Research UK Conference*, pages 68–77, 2005.
7. M. J. Kraak. The space-time cube revisited from a geovisualization perspective. In *International Cartographic Conference*, pages 1988–1995, 2003.
8. S. Liu, Y. Liu, L. M. Ni, J. Fan, and M. Li. Towards mobility-based clustering. In *ACM SIGKDD Conference on Knowledge Discovery and Data Mining*, pages 919–928, 2010.
9. S. Liu, J. Pu, Q. Luo, H. Qu, L. M. Ni, and R. Krishnan. Vait: A visual analytics system for metropolitan transportation. pages 1586–1596, 2013.
10. Y. Lou, C. Zhang, Y. Zheng, X. Xie, W. Wang, and Y. Huang. Map-matching for low-sampling-rate gps trajectories. In *Geographic Information Systems*, pages 352–361, 2009.
11. A. T. Pang, C. M. Wittenbrink, and S. K. Lodha. Approaches to uncertainty visualization. *The Visual Computer*, 13(8):370–390, 1997.
12. D. Pfoser and C. S. Jensen. Capturing the uncertainty of moving-object representations. In *Advances in Spatial Databases*, pages 111–131, 1999.
13. O. Pink and B. Hummel. A statistical approach to map matching using road network geometry, topology and vehicular motion constraints. In *Intelligent Transportation Systems*, pages 862–867, 2008.
14. M. A. Quddus, W. Y. Ochieng, and R. B. Noland. Current map-matching algorithms for transport applications: state-of-the art and future research directions. *Transportation Research*, 15(5):312–328, 2007.
15. G. Taylor, C. Brunsdon, J. Li, A. Olden, D. Steup, and M. Winter. Gps accuracy estimation using map matching techniques: Applied to vehicle positioning and odometer calibration. *Computers, Environment and Urban Systems*, 30(6):757–772, 2006.
16. G. Trajcevski, O. Wolfson, K. Hinrichs, and S. Chamberlain. Managing uncertainty in moving objects databases. *ACM Transaction Database System*, 29(3):463–507, 2004.