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Abstract: In this paper we adopt a nonparametric genetic programming approach to identify the structural changes in the Nikkei spot index and futures price. Due to the dominance of the "normal" period in sample size, the lead-lag relationship identified in the spot-futures system based on conventional methods such as test for Granger causality pertains to the normal period and may not be applicable in the "extreme" period. Using genetic programming we identify the lead-lag relationship based on the chronological ordering of the structural changes in the spot and futures markets. Our results show that in recent periods, *major* market changes originated from the spot market and then spread over to the futures market.

Key words: structural change, operating mechanism, genetic programming.

1 Introduction

The stock market crash of 1987 brought up the debate over the lead-lag relationship between stock spot index and futures price. A popular conjecture is that the stock index futures market leads the spot market. The implication of this conjecture is that the crash started off with disorders in the futures market. To examine this conjecture, the Granger causality test and Geweke feedback measure were adopted in many studies to identify the lead-lag relationship. These methods provide useful information about the relationship pertaining to the whole sample, inclusive of both "normal" and "extreme" periods. Recent studies, however, suggest that market behavior is different over normal and extreme periods.[1] Applying the same statistical model to the whole sample may provide misleading conclusions concerning the lead-lag relationship during the extreme period, as clearly the data points pertaining to the normal period exceed those of the extreme period overwhelmingly. Applying statistical methods such as the Markovian regime-switching model to separate the data into two sub-samples may help identify the differences over the two periods. However, these models are parametric, and the validity of their conclusion may critically depend on the particular structure assumed. In this paper, we adopt a nonparametric approach and consider the extreme period as the result of a structural change. We use the genetic programming method to identify the structural changes over the whole sample. As a result, we obtain the lead-lag relationship between the spot and futures prices during the extreme period based on the chronological ordering of the structural changes in the spot and futures markets.

The conventional approach in modelling structural change begins with constructing a parametric specification for the data generation process (DGP). A structural change is said to occur when a parameter value changes. A list of possible break points (at which structural change may have occurred) are short-listed and tests for shifts in the parameter values at the possible break points are performed.[2] The theoretical underpinning of this approach is the assumption that the parametric specification correctly describes the DGP of the sample data. When this assumption fails (as may well be the case), the approach is reliable only when the structural change to be identified is robust to the misspecification.

Chen and Yeh (1997, 2001) provided an alternative, nonparametric approach to identify structural changes for univariate time series. This approach treats structural change as a "loss of familiar pattern learned from the experience". Learning is characterized by

[1] See, e.g., Longin and Solnik (2001), Silvapulle and Granger (2001), and Ang and Chen (2002).

[2] Some test procedures (see, e.g., Bai and Perron (1998)) do not assume knowledge of possible break points. Instead, search methods are used over the whole sample.

genetic programming. Zhang, Tse and Chan (2000) extended this method to multivariate time series. The approach helps identify structural changes in the relationship between variables. In this paper we adopt the above method to identify structural changes in the Nikkei spot index and the Nikkei futures price, as well as the structural changes over the two-market system. If a structural change occurs in the spot index and in the bivariate system before it occurs in the futures price, we conclude that the spot market leads the futures market (as the change originated in the former market). Similarly, we say that the futures market leads the spot market if a structural change occurs in the former market and in the bivariate system before it occurs in the latter.

Overall, in the period from September 1995 through December 1999, we find three clusters of structural changes common to the spot and futures price series, as well as in the bivariate system. In the first cluster, the structural change originated in the futures market whereas in the other two, the spot market had the leading effect. We do not observe a structural change that initiated on the same date for both prices. Thus, extreme market situations are created by a single market and dissipated to the other subsequently. There is also an additional structural change found in the spot price series. The impact of this break was, however, minimal and did not spread over to the futures market.

For comparison purposes, we also consider both linear and nonlinear tests for Granger causality. Similar to other studies in the literature, we find that the futures market leads the spot market in the linear model. Extending to a nonlinear model, however, we find no lead-lag relationship between the two prices. In other words, although the futures price has a leading effect on daily market information flows, the spot market is more likely to prompt the turbulent period.

This paper is organized as follows. Section 2 discusses the notion of model-free structural changes in a dynamic system. In Section 3 we briefly describe the concepts of genetic programming and summarize the procedure for the detection of structural changes using genetic programming. Section 4 presents the empirical results for the Nikkei spot index and futures price. We report the results of the Granger linear causality test, the identification of the lead-lag relationship between the spot index and futures price using the genetic programming approach, as well as the Granger nonlinear causality test. Finally, Section 5 concludes the paper.

2 Model-Free Structural Change

In this section we discuss the notion of model-free structural change. This notion is different from structural change defined by shifts of parameter values in a statistical model.

The genetic programming approach will be used to identify model-free structural change. As the concepts have been covered elsewhere,[3] our discussion in this section will be brief.

Suppose the realization of a system can be represented by a multivariate time series which are the outcomes of the activities of the participants. The participants of the system are called individuals, and the collection of all individuals is called the population. Suppose that the system has n_t individuals at time t and the multivariate time series is denoted by $X_t = (x_{t1}, x_{t2}, \cdots, x_{tm})'$ for $t = 1, 2, \cdots, T$. The activities of the individuals are assumed to be described by the activity functions $f_i(X_t)$, for $i = 1, 2, \cdots, n_t$. Let the operating mechanism of the system be recognized as

$$\hat{g}\left(f_1(X_t), f_2(X_t), \cdots, f_{n_t}(X_t)\right),$$

which is based on the activity function of each individual.

The individuals are participants of the competition in the learning process of the system, and the success or failure of each individual is subject to its activity function $f_i(X_t)$, for $i = 1, 2, \cdots, n_t$. An evaluation function is also defined to estimate the competition ability of each individual. During the intelligent learning process, the individuals accumulate the experiences of their own and those of others. Accordingly, they modify their activity functions. Thus, the activity of each individual is intelligent and adaptive. Some individuals will be driven out of the system due to failures, while some new participants will be included into the system as new entrants.

Structural changes in a system are the "surprises" or "shocks" which have significantly changed the operating mechanism of the system but cannot be dealt with by some intelligent cognition methods such as adaptive and self-organization training. Thus, the system $\{X_t\}$ is said to have undergone a structural change at time t if given a tolerance level $\delta > 0$,

$$d\left(X_t, \hat{g}(.)\right) > \delta,$$

where $d(.)$ is a distance measure.

The main difference between model-free structural change and model-specific structural change is the sensitivity to the perturbation of the existing pattern. It is clear that model-free structural change is less sensitive to perturbations than the model-specific one, since the cognitive process of the existing pattern through a model-free approach must be adaptive. Any small perturbation to the existing pattern cannot be detected as a structural change due to the adaptive and self-organization of the underlying data generation process.

[3] See, e.g., Zhang, Tse and Chan (2000).

From the view point of model-free approach, the notion of structural change is equivalent to “surprise”, “shock”, or “breakdown” in the operating mechanism of a dynamic system.

3 Detection of Model-Free Structural Change

In the natural evolutionary process, individuals in a population compete and mate with each other in order to produce increasingly stronger individuals. The genetic algorithm developed by Holland (1992) mimics the evolutionary process and works as a general optimization technique, which requires some genetic-styled operations in order to produce solutions that minimize an objective function. Successful applications in many optimization problems show the effectiveness of this algorithm.

Genetic programming is an extension of the conventional genetic algorithm, in which the structures undergoing adaptation are hierarchical computer programs of dynamically varying sizes and shapes. In applying genetic programming to a problem, there are five major preparatory steps, which involve (i) determining the set of terminals, (ii) the set of primitive functions, (iii) the fitness measure, (iv) the parameters for controlling the run, and (v) the method for designating a result and the criterion for terminating a run. Each run of genetic programming requires the specification of a termination criterion for deciding when to terminate a run and a method of result designation. We usually designate the best-so-far individual as the result of a run. Once these steps for preparing to run the genetic programming have been established, a run can be made.

For a given data set, genetic programming can be implemented recursively. Suppose that $\{X_t : t = 1, 2, \cdots, T\}$ is the observed multivariate time series. Let the window width be n_1 and the moving step be n_2. The first sub-sample S_1 is the first n_1 observations of $\{X_t\}$, while the second sub-sample is the modification of S_1 by pushing it forward by n_2 steps. In general, the j-th sub-sample is

$$S_j = \{X_t\}_{t=(j-1)n_2+1}^{n_1+(j-1)n_2}, \qquad j = 1, 2, \cdots, L,$$

where $L = [(T - n_1)/n_2] + 1$ with $[x]$ denoting the largest integer smaller than x. Given the sequence of sub-samples $S = \{S_1, S_2, \cdots, S_L\}$, genetic programming can be implemented sequentially to each of the sub-samples. Chen and Yeh (1997) used the so-called recursive genetic programming (RGP) to detect structural changes in a univariate time series. Zhang, Tse and Chan (2000) made an extension of the algorithm, which is applicable to the detection of structural changes in multivariate time series.

The genetic programming algorithm in the first sub-sample begins with a randomly selected initial generation denoted by $GP_1^{[0]}$. When the training process is over, the last generation, $GP_1^{[n]}$, is so far the best generation. The fitness of each GP-tree in $GP_1^{[n]}$ is computed through a fitness function $fit(.)$, which is defined as the sum of the squared residuals in this paper. Then the fitness of each GP-tree in $GP_1^{[n]}$ is sorted in increasing order[4]

$$fit_1(.) \leq fit_2(.) \leq \cdots \leq fit_n(.).$$

The best q GP-trees with the smallest fitness values are chosen to act as the representative GP-trees for $GP_1^{[n]}$. The fitness of the last generation, denoted as $\overline{fit}_1$, is the average fitness of the representative GP-trees.

When the algorithm is implemented to the second sub-sample S_2, the initial generation $GP_2^{[n]}$ is set to $GP_1^{[n]}$, which accumulates knowledge obtained through the training process in the preceding sub-sample. By using the same procedure, we can obtain the last generation $GP_2^{[n]}$ and the representative GP-trees. The fitness of the last generation, denoted as $\overline{fit}_2$, is the average fitness of the representative GP-trees.

When implementing the algorithm sequentially to each sub-sample, we obtain a sequence of fitness functions, $\{\overline{fit}_k : k = 1, 2, \cdots, L\}$, corresponding to every sub-sample, respectively. Then we define a statistic

$$D_k = \overline{fit}_k / \overline{fit}_{k-1}, \qquad k = 1, 2, \cdots, L, \tag{1}$$

with initial value $\overline{fit}_0 = \overline{fit}_1$. D_k reflects the relative change in average fitness between two adjacent sub-samples.

The cognition of the operating pattern in a sub-sample is based on the knowledge accumulated in the preceding sub-samples, since the initial generation in the current sub-sample is assigned to the last generation in its preceding sub-sample. Suppose that there is a structural change in the k^*-th sub-sample, then the recognized operating pattern in the preceding sub-sample is unable to provide an appropriate description of the current

[4]The fitness of each GP-tree is defined as the sum of the squared residuals. When computing the fitness of each GP-tree, the algorithm calculates the fitness of the last univariate time series in the underlying sub-sample. It might be more reasonable to compute the sum of the squared errors for each univariate time series and take the mean over all the univariate time series. However, in our experience, it makes no significant difference in choosing which univariate time series to compute the fitness of a GP-tree, since the training process is focused on the development or evolution of the multivariate time series. We also alternated the order of the components of a bivariate time series, but found both outputs to be quite similar.

sub-sample. Consequently, the average fitness $\overline{fit}_{k^*}$ will be much larger than $\overline{fit}_{k^*-1}$, so that D_{k^*} is much larger than one. The knowledge of the current pattern will be cumulated while the training process is going on in the k^*-th sub-sample. If the pattern in (k^*+1)-th sub-sample is similar to that in its preceding sub-sample, its average fitness will be smaller than or similar to that of its preceding sub-sample. Then the statistic D_k will be close to one.

We now present the details of the genetic programming method, which is developed according to the algorithms in Koza (1994).

Step 1: Define the function set for the configuration of the activity functions associated with the individuals in the bivariate time series:

$$F = \{+, -, \times, \div, \sin, \cos, \log, \exp, \cdots\}.$$

The largest lagged order in the individual activity function is denoted as h. Let the terminal set for the leaves in the genetic process be ($m = 2$ for bivariate process)

$$\kappa = \{x_{t-i,1}, x_{t-i,2} : i = 1, 2, \cdots, h\}.$$

The probabilities of the selection, crossover, mutation, and reproduction of each GP-tree are denoted as p_s, p_c, p_m, and p_r, respectively. Denote the number of generations as $MaxGen$,the maximum length or depth of each GP-tree as $MaxLen$, the number of the GP-trees in the last generation as n, and the number the representative GP-trees for the last generation as q ($q \leq n$).

Step 2: Generate an initial generation for the first sub-sample. At the beginning we randomly choose a function $f(.)$ from F and denote it as the root terminal. The number of input variables for the selected function is denoted as $N(f)$. For example, the "+" operation is a two-variable operation, then its $N(f)$ is two. The selected terminal will be connected with the $N(f)$ terminal in the next layer. Then we choose an element from the set $B = F \cup \kappa$ as the final terminal. If the selected element is a function from F, we should select an element for the second time so that the GP-tree in this branch keeps on growing. If the selected element is a terminal from κ, it is regarded as a final terminal, and the GP-tree at this branch terminates. This selection process continues until n GP-trees are generated.

Step 3: Consider the j-th generation in the k-th sub-sample. The operations of selection, crossover, mutation and reproduction are implemented to each GP-tree in the j-

th generation, and then we calculate the fitness of each GP-tree. Suppose the i-th GP-tree in the j-th generation is represented as

$$f_i^{[j]}(x_{t1}, x_{t2}; x_{t-1,1}, x_{t-1,2}; \cdots; x_{t-h,1}, x_{t-h,2}),$$

for $i = 1, 2, \cdots, n$. Then the fitness of the i-th GP-tree is the sum of the squared residuals of this GP-tree,

$$fit_i = \sum_{t=n_2(k-1)+1}^{n_2(k-1)+n_1} \left(x_{tm} - f_i^{[j]}(.) \right)^2,$$

for $j = 1, 2, \cdots, MaxGen$ and $k = 1, 2, \cdots, L$, where $MaxGen$ denotes the number of generations. The fitness of the last generation $GP_k^{[MaxGen]}$ is the average fitness of the best q GP-trees with the smallest fitness values.

In the next section we apply the genetic programming algorithm to the identification of structural changes in the Nikkei spot and futures prices.

4 The Empirical Results

Our data consist of the Nikkei 225 spot index and the futures price of the index traded on the SGX.[5] The sample period is from September 26, 1995 through December 30, 1999. There are totally 1035 daily observations. Figure 1 presents the two price series. In what follows, we discuss the results of the Granger linear causality test using the error correction model approach. Then we report our findings on structural changes using the GP approach. Finally, we examine the Granger nonlinear causality test.

Following the conventional approach, we first test for the possibility of unit root in each series. The Dickey-Fuller statistics indicate that the hypothesis of a unit root cannot be rejected for both series. Moreover, applying the Engle-Granger method, we confirm a cointegration relationship between the spot and futures prices, as the efficient market hypothesis would have predicted. Thus, we adopt a vector autoregression-error correction (VAR-EC) model to describe the behavior of the two price series. Akaike information criterion (AIC) and Schwarz criterion are applied to select the optimal lags, and they lead to the same order selection.

Let S_t and F_t denote the spot and futures prices in logarithm, respectively. Let z_t denote the cointegration residual constructed from the Engle-Granger method by regressing the spot price on the futures price. Also, let Δ denote the difference operator such that

[5] Formerly the futures was traded on the SIMEX.

$\Delta S_t = S_t - S_{t-1}$ and $\Delta F_t = F_t - F_{t-1}$. These two variables are the (continuously compounded) spot and futures returns, respectively. The estimated equations are as follows (figures in parentheses are t ratios):

$$\begin{aligned}\Delta S_t &= \underset{(0.6154)}{-0.0017} - \underset{(3.8044)}{0.1877 z_t} - \underset{(3.8352)}{0.2229 \Delta S_{t-1}} + \underset{(3.8272)}{0.2166 \Delta F_{t-1}} \\ \Delta F_t &= \underset{(0.6434)}{-0.0019} + \underset{(1.3384)}{0.0694 z_t} + \underset{(0.7224)}{0.0441 \Delta S_{t-1}} - \underset{(1.4916)}{0.0887 \Delta F_{t-1}}\end{aligned}$$

The spot return is affected by the cointegration error, the lagged spot return and the lagged futures return. We observe mean reversion such that the previous spot return has a negative impact on the current spot return. A large previous futures return increases the current spot return. On the other hand, the negative coefficient for the error-correction term indicates that when the spot price is too high relative to the futures price the current spot price will be adjusted downward to meet the long-run equilibrium relationship. The futures return, however, behaves like a random walk. None of the explanatory variables produce statistically significant effect on the futures return. The results indicate that there is a lead-lag relationship between spot and futures prices such that the futures market leads the spot market. Similar conclusions are obtained if we replace the error correction variable by the basis, $S_t - F_t$, a popular approach in the futures market literature.

We now examine the model-free structural change in the two price series. The window width (n_1) is taken to be 35, the moving step (n_2) is 5, and the number of windows (L) is 200. The number of generations ($MaxGen$) is 10, and the maximum length of GP-trees ($MaxLen$) is 26. If there is no structural change in the underlying time series, the test statistic should be around 1. If the test statistic is larger than 1 by 20% or more, we say that there is structural change in the associated moving window. Figures 2-4 display the values of the average fitness over the moving windows. By examining the portions of the graph lying above the horizontal line corresponding to 1.0, we observe from Figure 2 that a structural break occurred in the spot price series before it appeared in the bivariate series system. Figure 3 indicates that a structural break is likely to have occurred in the bivariate system before it appeared in the futures series. Similarly, Figure 4 compares the timing of the structural break occurrence between the spot and futures price series and demonstrates that the former tends to appear earlier than the latter.

The above observations are summarized in Tables 1-3. Table 1 presents the test results for the bivariate price series whereas Tables 2 and 3 provide results for the spot and futures prices, respectively. There are three clusters of structural changes in the bivariate time series. The first occurred at moving windows 58-62. Meanwhile, the spot price had a structural change at moving windows 58-63 and the futures price at 57-63. It indicates that the futures price initiated the structural change. As the spot price responded to the change, the relationship between the spot and futures prices also incurred a structural change. At window 63, both spot and futures prices change in a systematic pattern such that their relationship is maintained. This cluster highlights the leading effect of the futures market.

The second cluster of structural changes is in the moving windows 91-103. Herein we detect the structural change in the bivariate time series earlier than in the individual time series. That is, both spot and futures markets seem to maintain their own properties (within a reasonable bound) but deviations in the relationships between the two markets have occurred. In terms of detectable structural changes in each price series, it was more likely to occur in the spot prices before it appeared in the futures prices. Thus, we conclude that the spot market has the leading effect.

A third cluster of structural change is in the moving windows 137-145 for the bivariate time series. The corresponding windows for the spot and futures markets are 137-143 and 139-145, respectively. This is a very clear-cut case: A structural change in the spot market occurred first. Because the futures market did not respond to it quickly enough (as evidenced by no structural change in the futures price), it caused a structural change in the bivariate time series system as the relationship between spot and futures prices had changed. At the end of window 143, the spot market had settled. However, the futures market continued to adjust to the spot price and caused structural changes in both futures price and the relationships between spot and futures prices. Overall, changes in the time series of spot prices had leading effect.

For the time series of spot price, there was structural change in the moving windows 124-126. This patch of structural change had no detectable effect on the time series of futures prices and the bivariate time series. Indeed, transaction costs and other market frictions generated an arbitrage bound within which an individual price (either spot or futures) may change its behavior without affecting the other price. It is expected that such changes are more likely to occur in the spot market than in the futures market.

The above analysis points to two cases in which the spot market has the leading effect and one case in which the futures price leads the spot price. It is interesting to note that the latter case occurs in the early period of the Nikkei stock index futures market and is short lived as compared to the other two cases. In more recent periods, structural changes have been initiated in the spot market and tended to have longer lasting effects. Following our analysis, we conclude that stock market breakdowns are more likely to originate from the spot market instead of the futures market. In contrast, the VAR-EC model suggests that the futures market leads the spot market. Thus, during the normal period market information flows from the futures market to the spot market. However, extreme price movements are likely to begin at the spot market.

Table 1: Clusters of Diagnostics: Bivariate Time Series

Cluster	Window number	Time period
1	58 – 62	20/11/96 – 13/02/97
2	91, 92, 96 – 98, 100	24/07/97 – 19/11/97
3	137, 139, 141 – 145	02/07/98 – 19/10/98

Table 2: Clusters of Diagnostics: Univariate Series of Spot Price

Cluster	Window number	Time period
1	58 – 63	20/11/96 – 20/02/97
2	91, 93, 97, 98, 100	24/07/97 – 19/11/97
3	124 – 126	30/03/98 – 03/06/98
4	137 – 143	02/07/98 – 05/10/98

Table 3: Clusters of Diagnostics: Univariate Series of Future Price

Cluster	Window number	Time period
1	57 – 63	13/11/96 – 20/02/97
2	92, 98, 100, 103	31/07/97 – 11/12/97
3	139 – 145	16/07/98 – 19/10/98

While the results of the linear causality analysis contradict those of the GP approach, it is possible that the lead-lag relationship identified by the latter method can be captured by a nonlinear causality analysis. To investigate this possibility we apply the Granger nonlinear causality test as discussed in Hiemstra and Jones (1994). A summary of their methodology can be found in the Appendix.

First, we estimate the VAR residuals of $\{F_t\}$ and $\{S_t\}$ that are denoted, respectively, by $\{u_t\}$ and $\{v_t\}$, where the AIC is used to determine the optimal lagged order. Second, we apply the nonlinear causality test to $\{u_t\}$ and $\{v_t\}$. Consider the null that S_t (spot price) does not nonlinearly Granger cause F_t (future price). Under this null, the test statistic given in Hiemstra and Jones (1994) is distributed as a standard normal variable. The results of the test statistic are presented in Table 4, which shows that there is no evidence of unidirectional nonlinear Granger causality from the logarithmic spot price to the logarithmic futures price. Indeed, the results show that the test statistic is robust with respect to the choice of e and the lag length.

Next, consider the null that F_t (futures price) does not nonlinearly Granger cause S_t (spot price). The results, presented in Table 5, again show that there is no evidence of unidirectional nonlinear Granger causality from the logarithmic futures price to the logarithmic spot price. Similar conclusion can be obtained when the nonlinear causality test is applied to the daily return time series. That is, we find no evidence of any nonlinear causality. Thus, we conclude that the lead-lag relationship identified by the structural change analysis cannot be captured by the causality test methods.

Table 4: Testing for Unidirectional Nonlinear Causality from S_t to F_t.

Lag Length	$e = 0.005$	$e = 0.008$	$e = 0.012$	$e = 0.015$
1	0.0832	0.1323	0.1389	0.1277
2	0.0610	0.1227	0.1473	0.1534
3	0.0352	0.1331	0.1865	0.1986
4	0.0060	0.0900	0.1896	0.2342
5	-0.0017	0.0471	0.1288	0.1734
6	0.0037	0.0383	0.0950	0.1354
7	-0.0059	0.0354	0.0807	0.1134
8	-0.0424	0.0135	0.0706	0.0949

Table 5: Testing for Unidirectional Nonlinear Causality from F_t to S_t.

Lag Length	$e = 0.005$	$e = 0.008$	$e = 0.012$	$e = 0.015$
1	-0.0006	0.0000	0.0434	0.1231
2	-0.0169	-0.0064	-0.0131	0.0427
3	0.0005	0.0305	0.0504	0.0738
4	0.0010	0.0180	0.0513	0.0763
5	0.0192	0.0309	0.0429	0.0575
6	0.0238	0.0346	0.0659	0.0911
7	-0.0089	0.0007	0.0397	0.0753
8	-0.0474	-0.0034	0.0476	0.0732

5 Conclusions

In this paper we adopt a model-free approach to identify structural changes in univariate and multivariate time series via recursive genetic programming methods. The approach is applied to examine the behavior of the Nikkei stock index and Nikkei futures price, and to explore the relationship between the two time series. Based upon the timing of structural changes, we conclude that in more recent periods, major market changes originated from the spot market and then spread over to the futures market. The directions of the flow of information suggest a lead-lag relationship from the spot market to the futures market. In contrast, the conventional linear causality test concludes that the futures market leads the spot market, while the Granger nonlinear causality test concludes that there is no nonlinear lead-lag relationship. Thus, the relationship identified from the structural-change events is not captured by the conventional methods.

Because our method considers only the major market shifts, the results are direct testimony of the lead-lag relationship during such periods. On the other hand, conventional causality tests assume that the relationship pertains to the whole sample period. Due to the dominance of the "normal" period in sample size, the relationship identified refer to the data points in the normal period and may not be applicable in the "extreme" period. The genetic programming approach and the conventional causality test, therefore, are complementary to each other in exploring the lead-lag relationship between two time series.

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Appendix

Consider two strictly stationary and weakly dependent time series $\{X_t\}$ and $\{Y_t\}$. Denote the m-length lead vector of X_t by X_t^m, and the Lx-length and Ly-length lag vectors of X_t and Y_t, respectively, by X_{t-Lx}^{Lx} and Y_{t-Ly}^{Ly}. That is,

$$\begin{aligned}
X_t^m &= (X_t, X_{t+1}, \cdots, X_{t+m-1}), \qquad m = 1, 2, \cdots, \qquad t = 1, 2, \cdots, \\
X_{t-Lx}^{Lx} &= (X_{t-Lx}, X_{t-Lx+1}, \cdots, X_{t-1}), \qquad Lx = 1, 2, \cdots, \qquad t = Lx+1, Lx+2, \cdots, \\
Y_{t-Ly}^{Ly} &= (Y_{t-Ly}, Y_{t-Ly+1}, \cdots, Y_{t-1}), \qquad Ly = 1, 2, \cdots, \qquad t = Ly+1, Ly+2, \cdots.
\end{aligned}$$

For given values of m, Lx, and $Ly \geq 1$ and for $e > 0$, Y does not strictly Granger cause X if

$$\begin{aligned}
& Pr\{\|X_t^m - X_s^m\| < e \mid \|X_{t-Lx}^{Lx} - X_{s-Lx}^{Lx}\| < e, \|Y_{t-Ly}^{Ly} - Y_{s-Ly}^{Ly}\| < e\} \\
= \; & Pr\{\|X_t^m - X_s^m\| < e \mid \|X_{t-Lx}^{Lx} - X_{s-Lx}^{Lx}\| < e\},
\end{aligned}$$

where $Pr(.)$ denotes probability and $\|.\|$ denotes the maximum norm.

The strict Granger noncausality condition can be tested by using the joint probability estimates in the above-mentioned equations. For given values of m, Lx, and $Ly \geq 1$ and for $e > 0$, we adopt the assumptions that $\{X_t\}$ and $\{Y_t\}$ are strictly stationary, weakly dependent, and satisfy the mixing condition. Under the null hypothesis that Y does not strictly Granger cause X the following result applies

$$\sqrt{n}\left[\frac{C1(m+Lx, Ly, e, n)}{C2(Lx, Ly, e, n)} - \frac{C3(m+Lx, Ly, e, n)}{C4(Lx, e, n)}\right] \sim N\left(0, \sigma^2(m, Lx, Ly, e)\right),$$

where $C1(.)$ to $C4(.)$ and $\sigma^2(.)$ are defined in Hiemstra and Jones (1994). Currently, there is no literature on the appropriate way to specify optimal values for the lag lengths L and m, and the scale parameter e. Based on the Monte Carlo studies of Hiemstra and Jones (1994), we set lead length $m = 1$ and $Lx = Ly$, using common lag lengths of 1 to 8 lags.

Figure 1: Time Series of Spot and Futures Prices of Nikkei 225

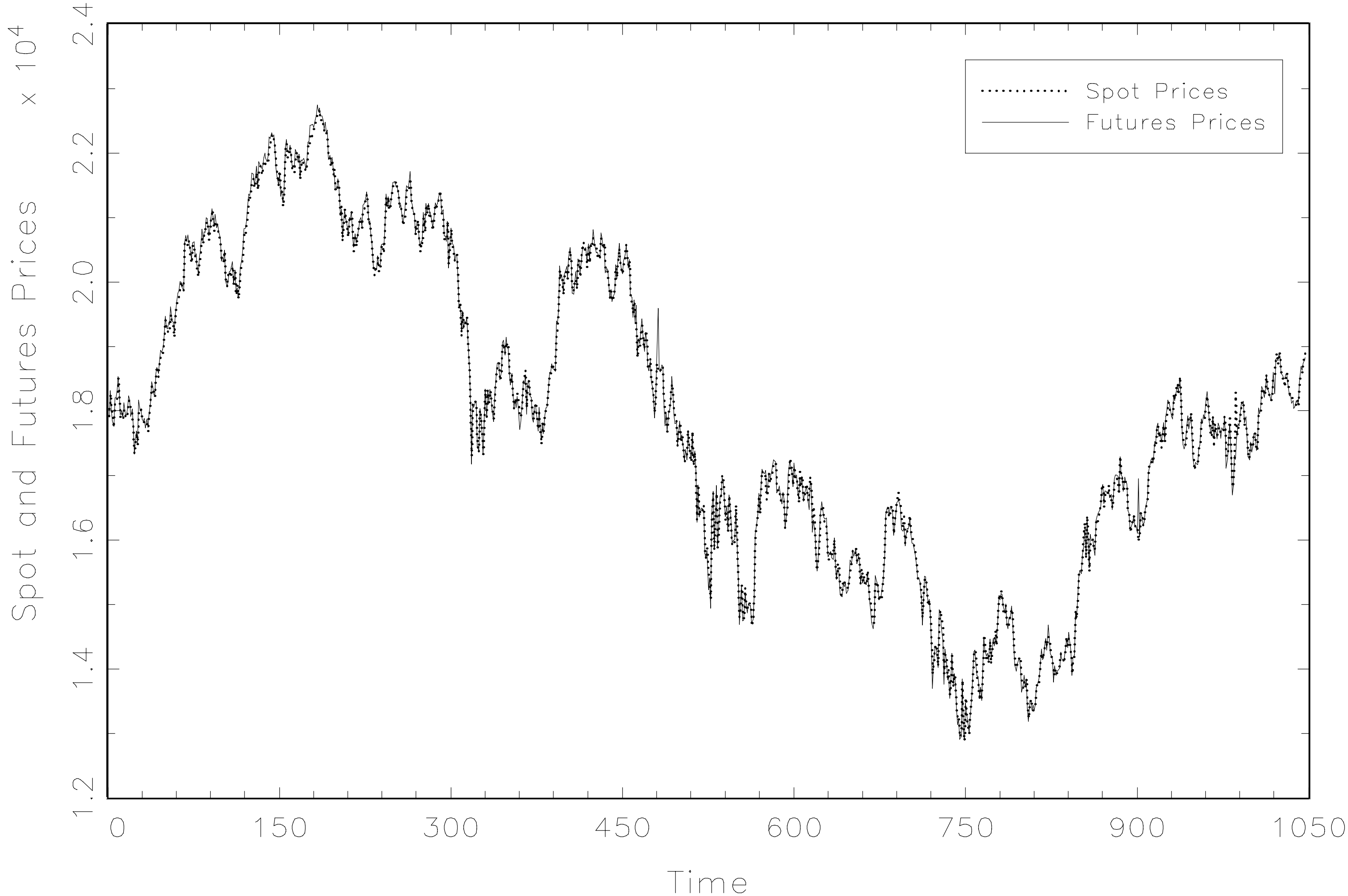

Figure 2: Index Plot: Spot–Price Series and Bivariate Time Series

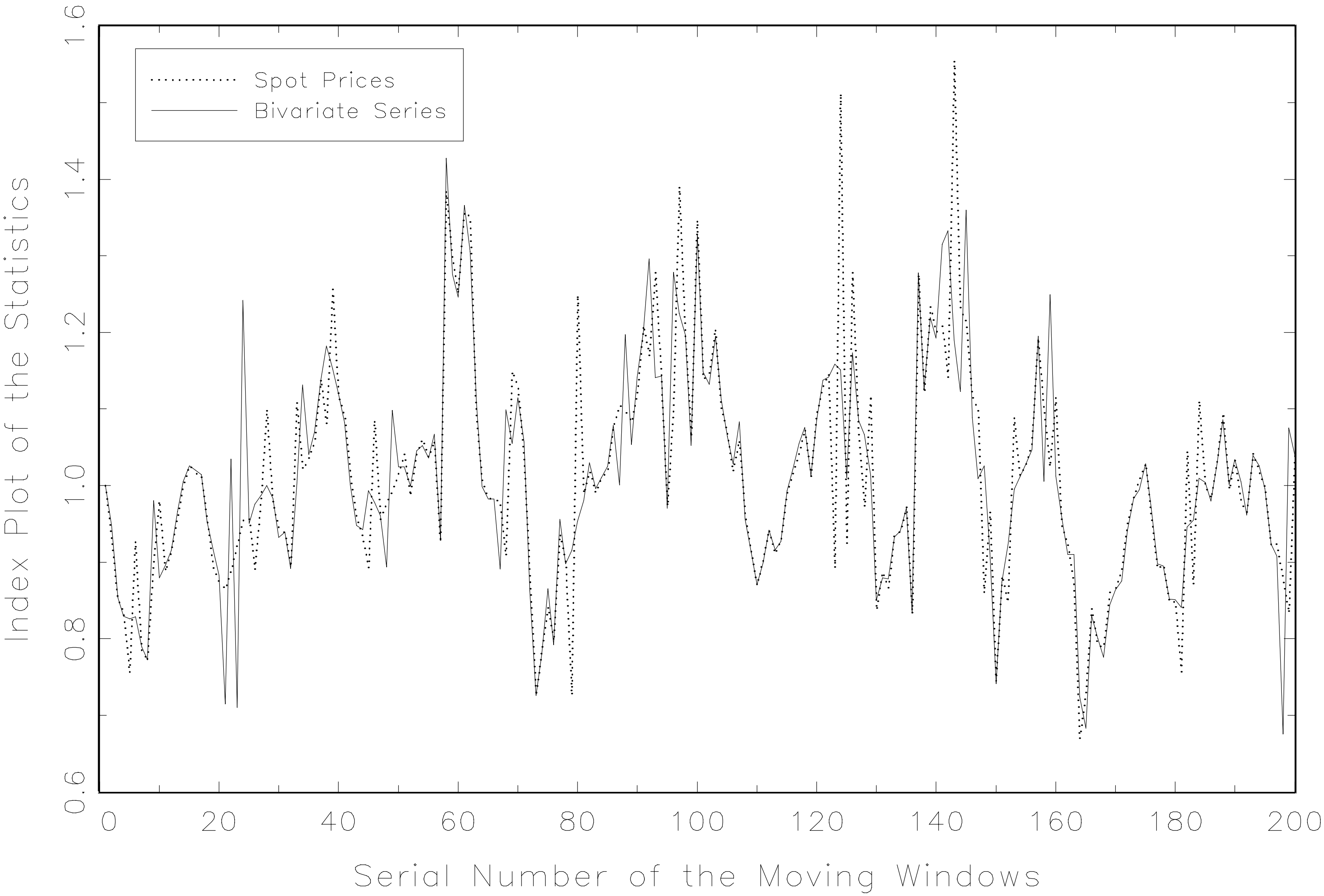

Figure 3: Index Plot: Futures-Price Series and Bivariate Time Series

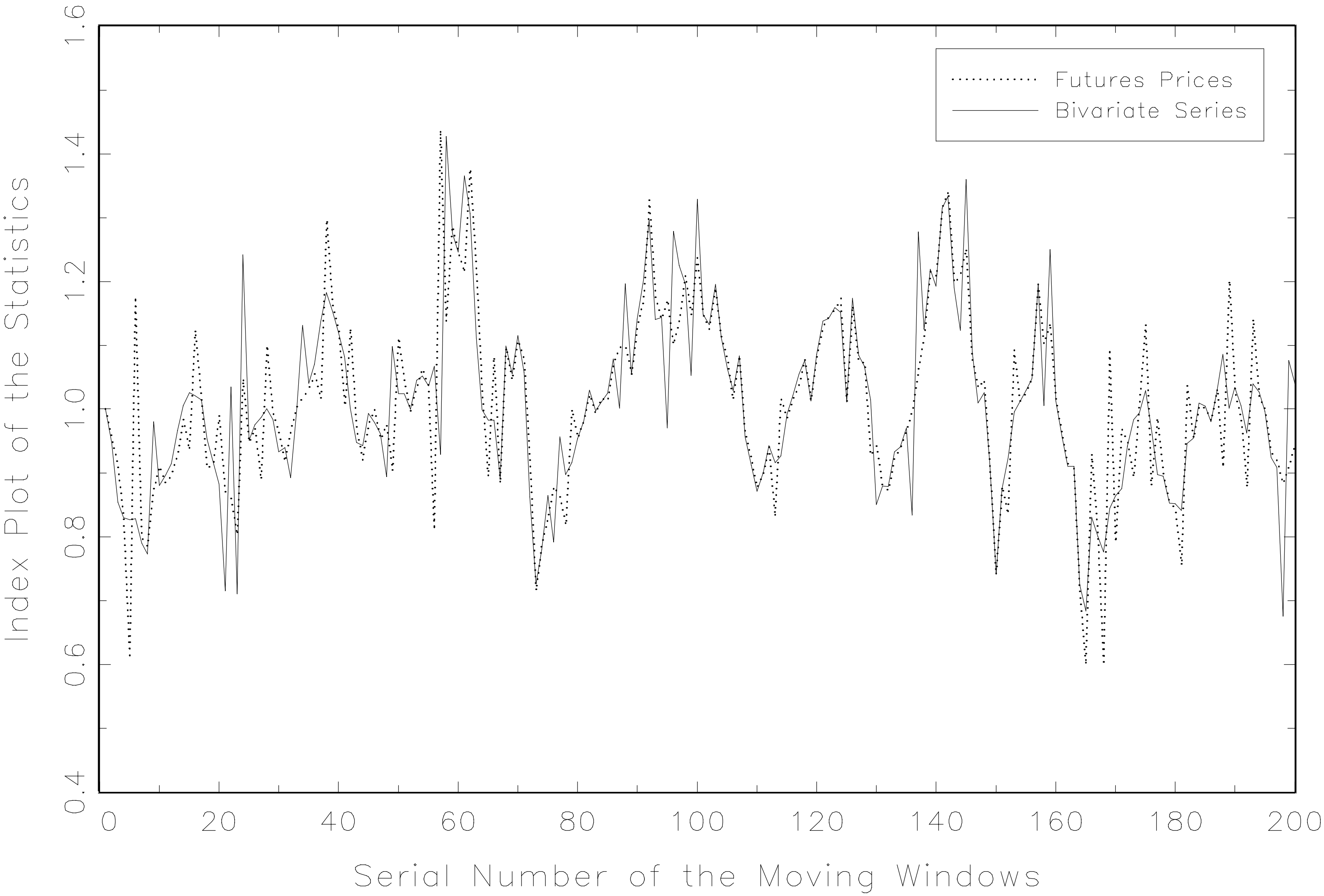

Figure 4: Index Plot: Time Series of Spot and Futures Prices