ANALISI MATEMATICA L-B (L-Z) (C.d.L. Ing. Gestionale)

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Esercizi sugli integrali generalizzati

Discutere la convergenza dei seguenti integrali generalizzati (se appare α, discutere al variare di $\alpha \in \mathbb{R}$):

$\int_0^{+\infty} xe^{-x^2}\,dx$ [C]; $\int_1^{+\infty} x^\alpha e^x\,dx$ [$+\infty$ $\forall\alpha$]; $\int_1^{+\infty} x^\alpha e^{-x}\,dx$ [C $\forall\alpha$];

$\int_0^{\frac{\pi}{2}} \tan x\,dx$ [$+\infty$]; $\int_{-\infty}^0 x^2 e^x\,dx$ [C]; $\int_0^{+\infty} x^2 e^x\,dx$ [$+\infty$];

$\int_0^1 x^2 e^{1/x}\,dx$ [$+\infty$]; $\int_{-1}^0 x^2 e^{1/x}\,dx$ [C]; $\int_{-\infty}^0 x^2 e^{1/x}\,dx$ [$+\infty$];

$\int_0^1 \frac{e^{1/x}}{x^2}\,dx$ [$+\infty$]; $\int_{-1}^0 \frac{e^{1/x}}{x^2}\,dx$ [C]; $\int_{-\infty}^{-1} \frac{e^{1/x}}{x^2}\,dx$ [C];

$\int_0^1 \left(\frac{1}{\sqrt[3]{x}} - \frac{1}{\sqrt{x}}\right) dx$ [C]; $\int_1^{+\infty} \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$ [$+\infty$]; $\int_1^{+\infty} \left(\frac{1}{x} - \frac{1}{\sqrt{x}}\right) dx$ [$+\infty$];

$\int_0^1 \left(\frac{1}{x} - \frac{1}{\sqrt{x}}\right) dx$ [$+\infty$]; $\int_0^1 \left(\frac{1}{x^2} - \frac{1}{x^3}\right) dx$ [$-\infty$]; $\int_1^{+\infty} \left(\frac{1}{x^2} - \frac{1}{x^3}\right) dx$ [C];

$\int_0^1 \frac{\sin^2(3x)}{x^2}\,dx$ [C]; $\int_1^{+\infty} \left(1 - \cos\frac{1}{x}\right) dx$ [C]; $\int_1^{+\infty} \frac{1}{1+x^\alpha}\,dx$ [C sse $\alpha > 1$];

$\int_0^{+\infty} \frac{x^{100}}{2^x}\,dx$ [C]; $\int_1^{+\infty} \frac{2^x+3}{2-3^x}\,dx$ [C]; $\int_2^{+\infty} \frac{-x^6}{x^8-2}\,dx$ [C];

$\int_0^1 \log x\,dx$ [-1]; $\int_2^{+\infty} \frac{\sqrt[3]{x}}{\log x}\,dx$ [$+\infty$]; $\int_1^{+\infty} \frac{\log x}{x^\alpha}\,dx$ [C sse $\alpha > 1$];

$\int_0^{+\infty} \frac{1}{\sqrt{x}(1+x)}\,dx$ [C]; $\int_0^{+\infty} \frac{-x^2}{2x^2+2}\,dx$ [$-\infty$]; $\int_2^{+\infty} \frac{x^5}{x^2-x^6}\,dx$ [$-\infty$];

$\int_1^{+\infty} \frac{1}{x^2-x^4}\,dx$ [$-\infty$]; $\int_0^{1/2} \frac{1}{x-x^2}\,dx$ [$+\infty$]; $\int_{1/2}^1 \frac{1}{x^2-x^3}\,dx$ [$+\infty$];

$\int_1^{+\infty} \frac{x+\sin(x^3 e^x)}{x^3}\,dx$ [CA]; $\int_0^1 \frac{x+\sin(x^3 e^x)}{x^3}\,dx$ [$+\infty$]; $\int_1^{+\infty} \frac{\sin x}{x^2}\,dx$ [CA];

$$\int_1^{+\infty} \left(1 - \cos\frac{1}{x^\alpha}\right) dx \quad [\text{C sse } \alpha > \frac{1}{2}]; \quad \int_2^{+\infty} \frac{1}{x\log^\alpha x}\, dx \quad [\text{C sse } \alpha > 1]; \quad \int_1^{+\infty} \frac{1}{x\log(1+\frac{1}{x^2})}\, dx \quad [+\infty];$$

$$\int_1^{+\infty} \frac{1}{x\log(1+\frac{1}{x^3})}\, dx \quad [+\infty]; \quad \int_1^{+\infty} \frac{1}{x\log\left(1+\frac{1}{x}\right)}\, dx \quad [+\infty]; \quad \int_0^{1} \frac{1}{x\log x}\, dx \quad [-\infty];$$

$$\int_{-10}^{+\infty} x^3 e^{-x}\, dx \quad [\text{C}]; \quad \int_0^{+\infty} \frac{1}{\sqrt{x(1+2x^2)}}\, dx \quad [\text{C}]; \quad \int_0^{+\infty} \frac{1}{\sqrt{x(1+2x)}}\, dx \quad [+\infty];$$

$$\int_1^{+\infty} \frac{\log x}{x-1}\, dx \quad [+\infty]; \quad \int_1^{+\infty} \log\left(1+\frac{1}{x^{2\alpha}}\right) dx \quad [\text{C sse } \alpha > \frac{1}{2}]; \quad \int_1^{+\infty} \frac{3^x x^4}{2^{2x}}\, dx \quad [\text{C}].$$

[C]=converge, [CA]= convergenza assoluta, [no C]= non converge, [no CA]= non converge assolutamente, sse=se e solo se.

Grazie agli studenti del corso che comunicheranno eventuali errori.