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5. Physical properties of clusters

5.1 Physical properties of the hot gas in clusters

5.1.1 Mean free path

[S:5.4.1]

→ The mean free path for electron-electron Coulomb collisions is

$$\lambda_e \propto \frac{T_e^2}{n_e \ln \Lambda}, \quad (5.2)$$

where $\ln \Lambda$ is the Coulomb logarithm (ln of the ratio between largest and smallest impact parameter), weakly dependent of T_e and n_e :

$$\ln \Lambda = 37.8 \ln \left[\left(\frac{T_e}{10^8 K} \right) \left(\frac{n_e}{10^{-3} cm^{-3}} \right)^{-1/2} \right]. \quad (5.3)$$

→ For typical T_e and n_e of Intracluster Medium (ICM):

$$\lambda_e \simeq 23 \left(\frac{T_e}{10^8 K} \right)^2 \left(\frac{n_e}{10^{-3} cm^{-3}} \right)^{-1} kpc \quad (5.4)$$

$\implies \lambda_e \ll$ cluster size \implies ICM is a collisional fluid satisfying the hydrodynamic equations.

→ Electrons achieve an isotropic Maxwellian velocity distribution in a timescale

$$t_{ee} \equiv \frac{\lambda_e}{\sigma_e} \simeq 3 \times 10^5 \left(\frac{T_e}{10^8 K} \right)^{3/2} \left(\frac{n_e}{10^{-3} cm^{-3}} \right)^{-1} yr, \quad (5.5)$$

where σ_e is the r.m.s. electron velocity, given by

$$\frac{1}{2} m_e \sigma_e^2 = \frac{3}{2} k_B T_e. \quad (5.6)$$

→ For protons $t_{pp} \simeq \sqrt{m_p/m_e} t_{ee}$, and for equipartition $t_{ep} \simeq (m_p/m_e) t_{ee} \simeq 6 \times 10^8 yr \ll$ cluster age \implies ICM is a plasma at $T_{gas} \sim T_e \sim T_p$.

5.1.2 Hydrostatic models of the ICM

[S:5.5, S:5.5.1, S:5.5.2, V.II.B.2]

→ The sound crossing time $t_s = D/c_s \propto D/\sqrt{T_e}$ (where c_s is sound speed and D is the cluster size) is shorter than the cluster age and cooling time \implies hydrostatic equilibrium

$$\nabla P = -\rho_{gas} \nabla \Phi, \quad (5.7)$$

where Φ is the gravitational potential.

If spherically symmetric

$$\frac{dP}{dr} = -\rho_{gas} \frac{d\Phi}{dr} = -\rho_{gas} \frac{GM(r)}{r^2}, \quad (5.8)$$

where $M(r)$ is the total mass (DM+gas+galaxies) within r .

Isothermal distributions

→ Consider isothermal gas at temperature T_e in equilibrium \implies using $P = \rho_{gas} k_B T_e / \mu m_p = n_{gas} k_B T_e$ we get

$$\frac{d \ln n_{gas}}{d \ln r} = -\frac{\mu m_p r}{k_B T_e} \frac{d\Phi}{dr} = -\frac{\mu m_p}{k_B T_e} \frac{GM(r)}{r}. \quad (5.9)$$

→ For example if the total mass distribution is a singular isothermal sphere (SIS) $\rho(r) = \sigma^2/2\pi G r^2$, $M(r) = 2\sigma^2 r/G$ ($\sigma = const$ is 1D velocity dispersion), the density distribution of an isothermal gas in equilibrium is a power law $n_{gas} \propto r^{-\alpha}$ with

$$\alpha = \frac{d \ln n_{gas}}{d \ln r} = -\frac{2\mu m_p \sigma^2}{k_B T_e}, \quad (5.10)$$

dependent on the gas temperature.

→ Introducing the virial temperature of the cluster (SIS) potential $T_{vir} \equiv \mu m_p \sigma^2 / k_B$, the density slope is $\alpha = 2T_{vir}/T_e$.

→ Observed cluster temperature profiles are almost (but not exactly) isothermal. However, typically the temperature slightly increases with radius in the central regions and decreases with radius in the outer regions.

Polytropic distributions

→ Isothermal distributions are special cases of a polytropic distributions, in which $p \propto \rho^{\gamma_{\text{pol}}}$, where γ_{pol} is the polytropic index. If $\gamma_{\text{pol}} = 1 \implies$ isothermal distribution; if $\gamma_{\text{pol}} = \gamma = 5/3 \implies$ adiabatic distribution. We will consider $1 \leq \gamma_{\text{pol}} \leq 5/3$.

→ In the adiabatic case $\gamma_{\text{pol}} = \gamma = 5/3$ the entropy K of the gas is constant throughout the cluster \implies distributions with $\gamma_{\text{pol}} > 5/3$ are convectively unstable. If $\gamma_{\text{pol}} < 5/3$ entropy increases outwards.

→ In polytropic models $p/p_0 = (\rho/\rho_0)^{\gamma_{\text{pol}}}$, $T_e/T_{e,0} = (\rho/\rho_0)^{\gamma_{\text{pol}}-1}$, where subscript 0 indicates quantities evaluated at a reference radius $r_0 \implies$

$$\frac{1}{\rho} \nabla P = \frac{\gamma_{\text{pol}}}{\gamma_{\text{pol}} - 1} \frac{k_B}{\mu m_p} \nabla T_e. \quad (5.11)$$

→ From the hydrostatic equation we get the solution

$$\frac{T_e}{T_{e,0}} = 1 - \frac{\mu m_p}{k_B T_{e,0}} \frac{\gamma_{\text{pol}} - 1}{\gamma_{\text{pol}}} (\Phi - \Phi_0) \quad (5.12)$$

and

$$\frac{\rho}{\rho_0} = \left(\frac{T_e}{T_{e,0}} \right)^{\frac{1}{\gamma_{\text{pol}}-1}}. \quad (5.13)$$

The gas distribution can be truncated or not depending on the value of $T_{e,0}$.

5.1.3 Intracluster entropy

[V:IV.A.1]

→ A quantity often used in the study of the ICM is the “entropy”

$$K \equiv \frac{k_B T}{\mu m_p \rho_{gas}^{2/3}}, \quad (5.14)$$

in units of $erg\ cm^2\ g^{-5/3}$ This quantity is related to the thermodynamic entropy per particle by

$$s = k_B \ln K^{3/2} + s_0. \quad (5.15)$$

→ Sometimes also the following quantity is called “entropy”

$$K_e = k_B T n_e^{-2/3} \quad (5.16)$$

in units of $keV\ cm^2$. K_e is often indicated also as S .

→ Independent of the details of the definition entropy is a quantity such that it is conserved in adiabatic transformations $p = K\rho^{5/3} \implies$ in a polytropic distribution with $\gamma_{pol} = \gamma = 5/3$ (= adiabatic distribution) all gas particles have the same entropy.

→ Heating $\implies K$ increases; cooling $\implies K$ decreases.

→ Observed ISM density, temperature, and entropy profiles: entropy increases outwards.

5.1.4 Intracluster metallicity

→ Metallicity or metal abundance is the fraction of metals with respect to H and HE (Metals are all elements other than H and He). Metallicity Z usually measured in units of solar metallicity Z_\odot

→ X, Y, Z are the H, He and metal mass fraction. For the sun: $X \simeq 0.74$, $Y \simeq 0.24$ and $Z \simeq 0.02$.

- ICM abundances are relatively easy to measure from intensity of emission lines of X-ray spectra. In particular 7 keV Fe line. \implies on average $Z \sim 0.3Z_{\odot}$.
- Metallicity gradients: metallicity decreases outward in clusters (with central dominant galaxy)
- No evidence of variation of ISM metallicity with redshift.
- Iron in ICM $>$ iron in all stars in the cluster galaxies \implies ICM enriched by supernovae (not enough?)

5.2 Mass of galaxy clusters

5.2.1 Baryons and dark matter in clusters

- Total mass with different methods, using galaxies, gas, gravitational lensing. Not always agreement among estimates with different methods, but improving.
- Typically total mass profile not far from isothermal $M(r) \propto r$. Total masses $\sim 10^{14} - 10^{15} M_{\odot}$.
- Mass fractions (in rich clusters): dark matter 80-87%, baryons (13-20%) [hot gas 11-15%, galaxies (stars) 2-5%]
- Galaxies and dark matter distribution more concentrated than gas distribution

5.2.2 Virial theorem

[S:2.8]

- Zeroth-order estimate of cluster mass M_{tot} (Zwicky 1937)

→ Assume cluster is in equilibrium $\implies 2T + W = 0$ (virial theorem),
where the kinetic energy is

$$T = \frac{1}{2} M_{tot} \sigma_V^2, \quad (5.17)$$

where σ_V is the virial velocity dispersion and the gravitational potential energy is

$$W = -\frac{GM_{tot}^2}{r_g}, \quad (5.18)$$

where r_g is the gravitational radius (dependent on the mass distribution)

→ If one knew σ_V and $r_g \implies M_{tot}$ because

$$M_{tot} = \frac{r_g \sigma_V}{G} \quad (5.19)$$

→ Assuming spherical symmetry and isotropic velocity distribution $\implies \sigma_V^2 = 3\sigma_{los}^2$, where σ_{los} is the line-of-sight velocity dispersion of cluster galaxies.

→ Assuming also that galaxy distribution traces mass distribution

$$r_g = 2 \left(\sum_i m_i \right)^2 \left(\sum_{i \neq j} \frac{m_i m_j}{r_{ij}} \right)^{-1}, \quad (5.20)$$

where m_i is galaxy mass and r_{ij} galaxy-galaxy separation. In terms of the projected galaxy-galaxy separation R_{ij} , $r_g = (\pi/2)R_g$ (derive) where

$$R_g = 2 \left(\sum_i m_i \right)^2 \left(\sum_{i \neq j} \frac{m_i m_j}{R_{ij}} \right)^{-1}. \quad (5.21)$$

→ Thus

$$M_{tot} = \frac{3r_g \sigma_{los}^2}{G} = 7 \times 10^{14} \left(\frac{\sigma_{los}}{1000 \text{ km/s}} \right)^2 \left(\frac{r_g}{\text{Mpc}} \right) \quad (5.22)$$

→ OK order of magnitude, but assumptions not necessarily justified.

5.2.3 Jeans modeling

[Binney & Tremaine (Galactic dynamics):4.8.1]

→ More accurate mass determination using galaxy velocity and density profiles \implies Jeans modeling \implies determine cluster mass profile.

→ For spherical stationary cluster the radial component $\sigma_r(r)$ of the velocity dispersion tensor is given by solving the Jeans equation

$$\frac{dn_{gal}\sigma_r^2}{dr} + \frac{2\beta n_{gal}\sigma_r^2}{r} = -n_{gal}g_r, \quad (5.23)$$

where $g_r(r) = d\Phi(r)/dr = GM(r)/r^2$, $\Phi(r)$ is the total gravitational potential, $M(r)$ is the total mass profile and

$$\beta(r) \equiv 1 - \frac{\sigma_\vartheta^2 + \sigma_\varphi^2}{2\sigma_r^2} \quad (5.24)$$

is the anisotropy parameter (σ_ϑ and σ_φ are, respectively, the ϑ and φ components of the velocity-dispersion tensor).

The line-of-sight velocity dispersion is

$$\sigma_{los}^2(R) = \frac{2}{\Sigma_{gal}(R)} \int_R^\infty \left[1 - \beta(r) \frac{R^2}{r^2} \right] \frac{n_{gal}(r)\sigma_r^2 r dr}{\sqrt{r^2 - R^2}}, \quad (5.25)$$

where

$$\Sigma_{gal}(R) = 2 \int_R^\infty \frac{n_{gal}(r)r dr}{\sqrt{r^2 - R^2}}. \quad (5.26)$$

n_{gal} and Σ_{gal} are the intrinsic and projected number of galaxy density. σ_r and σ_{los} are the intrinsic and projected velocity dispersions of galaxies.

→ It is possible to build model of given $\beta(r)$ and $M(r)$, compute $\Sigma_{gal}(R)$ and $\sigma_{los}(R)$ and compare with observed quantities.

→ Viceversa, assuming $\beta(r)$ it is possible to derive $M(r)$ if we know $n_{gal}(r)$ and $\sigma_r(r)$. By deprojecting Σ_{gal} one obtains the corresponding intrinsic density distribution (Abel inversion)

$$n_{gal}(r) = -\frac{1}{\pi} \int_r^\infty \frac{d\Sigma_{gal}}{dR} \frac{dR}{\sqrt{R^2 - r^2}} \quad (5.27)$$

Similarly, deprojecting σ_{los} one can obtain $\sigma_r(r)$.

5.2.4 Hydrostatic equilibrium of hot gas

[S:5.5.5,V:II.B.1,V:II.B.2]

→ Assume hot gas in hydrostatic equilibrium in the cluster gravitational potential:

$$\nabla P = -\rho_{gas} \nabla \Phi. \quad (5.28)$$

If spherically symmetric

$$\frac{dP}{dr} = -\rho_{gas} \frac{d\Phi}{dr} = -\rho_{gas} \frac{GM(r)}{r^2}. \quad (5.29)$$

→ Using $P = \rho_{gas} k_B T_e / \mu m_p$ we get

$$M(r) = -\frac{k_B r T_e(r)}{\mu m_p G} \left(\frac{d \ln \rho_{gas}}{d \ln r} + \frac{d \ln T_e}{d \ln r} \right) \quad (5.30)$$

→ If we know ρ_{gas} and $T_e \implies M(r)$. But the observables are X-ray SB profiles and spectrum. When spectra and intensity at different annuli are available it is possible to obtain T_e and n_e profiles by deprojection, assuming spherical symmetry (not easy! different techniques).

→ Recall that X-ray emissivity depends on temperature as well as on density $\epsilon \propto n_e^2 \Lambda(T)$

→ If gas is isothermal at temperature T_e :

$$M(r) = -\frac{k_B r T_e}{\mu m_p G} \frac{d \ln \rho_{gas}}{d \ln r} \quad (5.31)$$

If β model (derive)

$$M(r) = \frac{3\beta r k_B T_e}{\mu m_p G} \frac{r^2}{r^2 + r_c^2}. \quad (5.32)$$

5.2.5 Gravitational lensing

[V:II.A.3]

- Gravitational lensing is sensitive to mass within a projected radius R , which deflects light from background galaxies. Large deflection angle \implies strong lensing; small deflection angle \implies weak lensing
- Deflection angle (for circularly symmetric lens)

$$\alpha = \frac{4GM(R)}{c^2 R}. \quad (5.33)$$

For a singular isothermal sphere with $M(r) = 2\sigma^2 r/G$ (σ is 1D velocity dispersion)

$$\alpha = \frac{4\pi\sigma^2}{c^2} \quad (5.34)$$

independent of radius.

- Strong lensing produces gravitational arcs (typically tangential). Needs high surface mass density \implies central regions of clusters \implies measures of the central mass distribution
- Weak lensing produces small distortion of shape and orientation of background galaxies \implies background galaxies result tangentially stretched w.r.t. cluster mass distribution. Intrinsic orientation of galaxies uncorrelated. Weak lensing does not need high surface density \implies mass distribution also in outer regions of cluster

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6. Gravitational lensing

6.1 Bending of light by a point-like deflector

[MR:2.1.6, 2.2.1]

→ Gravitational lensing is an effect predicted by general relativity. Consider spacetime interval $ds^2 = g_{ij}dx^i dx^j$, where g_{ij} is the metric tensor.

→ Schwarzschild metric (solution of Einstein equations outside a spherical distribution of total mass M):

$$ds^2 = \left(1 - \frac{r_S}{r}\right) c^2 dt^2 - \left(1 - \frac{r_S}{r}\right)^{-1} dr^2 - r^2(\sin^2 \theta d\phi^2 + d\theta^2) \quad (6.2)$$

where

$$r_S \equiv \frac{2GM}{c^2} \quad (6.3)$$

is the Schwarzschild radius.

→ Consider trajectory of photon close to a point-like object. Trajectory is in a plane (say $\theta = \pi/2$) \implies

$$ds^2 = \left(1 - \frac{r_S}{r}\right) c^2 dt^2 - \left(1 - \frac{r_S}{r}\right)^{-1} dr^2 - r^2 d\phi^2 \quad (6.4)$$

Photons follow null geodesics

$$g_{ij} \frac{dx^i}{d\lambda} \frac{dx^j}{d\lambda} = 0, \quad (6.5)$$

where λ is a parameter used to parameterize the trajectory. \implies

$$d\phi = \frac{J}{r^2} \frac{dr}{\sqrt{1 - \frac{J^2}{r^2} \left(1 - \frac{r_s}{r}\right)}}. \quad (6.6)$$

where $J = r^2(d\phi/d\lambda) = \text{const}$ is an integral of motion.

\rightarrow At closest approach $r = r_m$, $dr/d\phi = 0 \implies$

$$J = \frac{r_m}{\sqrt{1 - \frac{r_s}{r_m}}}. \quad (6.7)$$

\rightarrow Combining the two equations above and integrating we get

$$\phi_m - \phi_\infty = \int_0^1 \frac{dx}{\sqrt{1 - x^2 - \frac{r_s}{r_m}(1 - x^3)}}, \quad (6.8)$$

where $x \equiv r_m/r$. If $r_m \gg r_s \implies$

$$\phi_m - \phi_\infty = \frac{\pi}{2} + \frac{r_s}{r_m}, \quad (6.9)$$

\implies deflection angle

$$\hat{\alpha} = 2(\phi_m - \phi_\infty) - \pi = 2\frac{r_s}{r_m} = \frac{4GM}{r_m c^2} \quad (6.10)$$

6.2 Bending of light by extended mass distribution

[MR:2.2.3]

\rightarrow Consider mass density distribution $\rho(\mathbf{x}) \implies$ surface mass density

$$\Sigma(\boldsymbol{\xi}) = \int dz \rho(\mathbf{x}), \quad (6.11)$$

where $\mathbf{x} = (x, y, z)$, $\boldsymbol{\xi} = (x, y)$ and z is the line of sight.

\rightarrow Thin-lens approximation: all mass distributed in the lens plane

→ In weak field limit we can sum contributions from mass elements \implies

$$\hat{\alpha}(\boldsymbol{\xi}) = \frac{4G}{c^2} \int d^2\xi' \Sigma(\xi') \frac{\boldsymbol{\xi} - \boldsymbol{\xi}'}{|\boldsymbol{\xi} - \boldsymbol{\xi}'|^2} \quad (6.12)$$

→ Define projected gravitational potential:

$$\psi(\boldsymbol{\xi}) = \int dz \Phi(\mathbf{x}), \quad (6.13)$$

\implies from Poisson equation

$$\nabla_{\xi}^2 \psi(\boldsymbol{\xi}) = 4\pi G \Sigma(\boldsymbol{\xi}) \quad (6.14)$$

or

$$\psi(\boldsymbol{\xi}) = 2G \int \Sigma(\xi') \ln |\boldsymbol{\xi} - \boldsymbol{\xi}'| d^2\xi'. \quad (6.15)$$

\implies

$$\hat{\alpha}(\boldsymbol{\xi}) = \frac{2}{c^2} \nabla_{\xi} \psi(\boldsymbol{\xi}) \quad (6.16)$$

→ Consider spherical mass distribution. From Gauss theorem

$$\int \nabla \cdot \hat{\alpha} dA = \oint \hat{\alpha} \cdot \frac{\boldsymbol{\xi}}{\xi} dl; \quad (6.17)$$

but $\nabla \cdot \hat{\alpha} = 8\pi G \Sigma / c^2$ and $\oint \hat{\alpha} \cdot \frac{\boldsymbol{\xi}}{\xi} dl = 2\pi \xi \hat{\alpha} \implies$

$$\hat{\alpha}(\xi) = \frac{4GM(\xi)}{c^2 \xi}, \quad (6.18)$$

where

$$M(\xi) = 2\pi \int_0^{\xi} d\xi' \xi' \Sigma(\xi') \quad (6.19)$$

6.3 The lens equation for a point-mass lens

[MR:3.1.1]

→ Consider point-mass lens of mass M where D_{OS} , D_{OL} and D_{LS} are the angular diameter distances from the observer to the source, from

the observer to the lens and from the lens to the source, respectively.

\implies

$$\theta D_{\text{OS}} = \beta D_{\text{OS}} + \hat{\alpha} D_{\text{LS}}, \quad (6.20)$$

where θ is the angular position of the image, β is the angular position of the source and $\hat{\alpha}$ is the deflection angle. \implies

$$\beta = \theta - \alpha \quad (\text{lens equation}), \quad (6.21)$$

where

$$\alpha \equiv \hat{\alpha} \frac{D_{\text{LS}}}{D_{\text{OS}}} \quad (6.22)$$

is the reduced deflection angle.

\rightarrow The closest approach distance $r_m = \theta D_{\text{OL}} \implies$

$$\alpha = \frac{4GM}{c^2 \theta} \frac{D_{\text{LS}}}{D_{\text{OL}} D_{\text{OS}}} \quad (6.23)$$

\rightarrow Lens equation can be written as

$$\theta^2 - \beta\theta - \theta_E^2 = 0, \quad (6.24)$$

where

$$\theta_E \equiv \sqrt{\frac{4GM}{c^2} \frac{D_{\text{LS}}}{D_{\text{OL}} D_{\text{OS}}}} \quad (\text{Einstein angle}). \quad (6.25)$$

\rightarrow When $\beta = 0$ (observer, lens and source on a straight line) \implies the image is a ring of angular radius θ_E (Einstein ring). Einstein radius $r_E \equiv D_{\text{OL}} \theta_E$.

6.4 The lens equation for an extended lens

[MR:3.1.2; MR:3.3; MR:3.4]

\rightarrow Thin-lens approximation \implies

$$\beta = \theta - \alpha, \quad (6.26)$$

where $\boldsymbol{\alpha} = \hat{\boldsymbol{\alpha}}D_{\text{LS}}/D_{\text{OS}}$ and $\boldsymbol{\theta} = \boldsymbol{\xi}/D_{\text{OL}}$. \implies

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \nabla_{\boldsymbol{\theta}}\Psi(\boldsymbol{\theta}), \quad (6.27)$$

where

$$\Psi(\boldsymbol{\theta}) \equiv \frac{2}{c^2} \frac{D_{\text{LS}}}{D_{\text{OS}}D_{\text{OL}}} \psi(\boldsymbol{\xi}). \quad (6.28)$$

→ For given $\boldsymbol{\beta}$ we can have different $\boldsymbol{\theta}$ \implies possible multiple images (strong lensing)

→ The lens equation can be seen as a 2D mapping between the positions of the images $\boldsymbol{\theta}$ and the positions of the sources $\boldsymbol{\beta}$. Consider the Jacobian of the transformation

$$J = \det \frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}}. \quad (6.29)$$

→ Let us define the matrix of the mapping

$$\mathcal{T}_{ij} \equiv \frac{\partial \beta_i}{\partial \theta_j}, \quad (6.30)$$

which can be written as

$$\mathcal{T}_{ij} = \delta_{ij} - \frac{\partial^2 \Psi}{\partial \theta_i \partial \theta_j}. \quad (6.31)$$

\implies

$$\text{Tr} \frac{\partial^2 \Psi}{\partial \theta_i \partial \theta_j} = \nabla_{\boldsymbol{\theta}}^2 \Psi = 2 \frac{\Sigma(\boldsymbol{\theta})}{\Sigma_{cr}} \equiv 2\kappa(\boldsymbol{\theta}), \quad (6.32)$$

where

$$\Sigma_{cr} \equiv \frac{c^2 D_{\text{OS}}}{4\pi G D_{\text{OL}} D_{\text{LS}}} \quad (6.33)$$

is the critical surface density and

$$\kappa(\boldsymbol{\theta}) \equiv \frac{\Sigma(\boldsymbol{\theta})}{\Sigma_{cr}} \quad (6.34)$$

is the convergence.

→ The matrix of the mapping can be written as

$$\mathcal{T} = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}, \quad (6.35)$$

where

$$\gamma_1 \equiv \frac{1}{2} \left(\frac{\partial^2 \Psi}{\partial \theta_1^2} - \frac{\partial^2 \Psi}{\partial \theta_2^2} \right) \quad (6.36)$$

and

$$\gamma_2 \equiv \frac{\partial^2 \Psi}{\partial \theta_1 \partial \theta_2} \quad (6.37)$$

are the component of the shear γ .

→ The amplification is

$$A = J^{-1} = (\det \mathcal{T})^{-1} = \frac{1}{(\kappa - 1)^2 - \gamma^2}, \quad (6.38)$$

where $\gamma = \|\gamma\| = \sqrt{\gamma_1^2 + \gamma_2^2}$.

→ The convergence κ changes the size of the image, but not the shape. The shear γ is responsible for the distortion of the image.

→ A sufficient condition to produce multiple images is that at some point in the lens plane $\Sigma(\boldsymbol{\theta}) > \Sigma_{cr}$ (i.e. $\kappa(\boldsymbol{\theta}) > 1$)

→ Caustics are positions in the source plane in which $A \rightarrow \infty$. Critical lines are the corresponding positions in the lens plane in which $A \rightarrow \infty$.

6.5 Gravitational lensing by galaxy clusters

[MR:4.4, 4.5]

→ Lens: galaxy cluster. Sources: background galaxies.

6.5.1 Strong lensing

- Strong gravitational lensing: multiple images, strong distortion \implies image shape cannot be accounted for without lensing (\implies arcs).
- Arcs detected in $\sim 1/3$ of X-ray selected clusters.
- Arcs are formed in correspondence of critical lines: $A \rightarrow \infty$, i.e. $\kappa \sim 1 - \gamma$ ($\implies \Sigma \lesssim \Sigma_{cr}$ in the presence of shear).
- Spherical lens \implies arcs form on the Einstein ring at an angular distance from the centre $\theta_E \implies$ total projected mass within Einstein angle

$$M(\theta_E) = \pi(D_{OL}\theta_E)^2\Sigma_{cr} = \pi r_E^2\Sigma_{cr} = \frac{c^2}{4G} \frac{r_E^2 D_{OS}}{D_{OL} D_{LS}}. \quad (6.39)$$

- From location of tangential arcs \implies integrated mass within Einstein radius r_E
- From radial structure (radial arcs or width of tangential arcs) \implies mass density profile.
- Strong lensing \implies central parts of clusters (high Σ)

6.5.2 Weak lensing

- Weak gravitational lensing: no multiple images, weak distortion, $\Sigma < \Sigma_{cr}$ ($\kappa < 1 - \gamma$).
- Amplification components:

$$A_1 = \frac{1}{1 - \kappa + \gamma}, \quad (6.40)$$

$$A_2 = \frac{1}{1 - \kappa - \gamma}. \quad (6.41)$$

\implies shear induces tangential deformations.

- Average ellipticity $\langle (a - b)/(a + b) \rangle = 0$ for non-lensed galaxies.
- A circular source is deformed in an ellipse of axis ratio $b/a = A_1/A_2$.
 \implies Average ellipticity $\langle (a - b)/(a + b) \rangle = \gamma/(1 - \kappa)$ for lensed galaxies. (also statistical lensing)
- When $\kappa \ll 1$ ellipticity map \implies shear map.
- Using inversion techniques it is possible to derive κ from shear maps of $\langle \epsilon \rangle$. From $\kappa \implies \Sigma = \kappa \Sigma_{cr} \implies$ surface mass density profile
- Mass-sheet degeneracy: from shear alone we cannot detect the presence of a uniform Σ
- To break mass-sheet degeneracy one can estimate κ using estimate of the amplification.
- Weak lensing \implies outer parts of clusters (low Σ)

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7. Formation and evolution of clusters

7.1 Formation of clusters in cosmological models

[R:563, V:III.B.1]

→ Cluster form from perturbations in the density distribution of dark matter (DM). Hierarchical process.

→ State of the art modeling of cluster formation: cosmological N-body simulations \implies (angle-averaged) density distribution of cluster dark matter:

$$\rho_{DM} \propto r^{-p}(r + r_s)^{p-q} \quad (7.2)$$

weakly dependent on cluster mass.

→ $p = 1, q = 3 \implies$ Navarro Frenk & White (NFW) profile:

$$\rho_{DM}(r) = \frac{M_\Delta}{4\pi f(C_\Delta)} \frac{1}{r(r + r_s)^2}, \quad (7.3)$$

where r_s is the scale radius, and the distribution is truncated at the virial radius r_Δ . The average dark-matter density within r_Δ equals Δ times the critical density of the Universe ρ_{crit} ; $C_\Delta \equiv r_\Delta/r_s$ is the concentration parameter,

$$f(C_\Delta) = \ln(1 + C_\Delta) - \frac{C_\Delta}{1 + C_\Delta}, \quad (7.4)$$

and M_Δ is the total dark-matter mass (verify). (DM dominates $\implies M_\Delta$ also considered \sim total cluster mass)

→ By definition of the virial radius,

$$\frac{3M_\Delta}{4\pi r_\Delta^3} = \Delta\rho_{\text{crit}}, \quad (7.5)$$

where $\rho_{\text{crit}} = 3H_0^2/(8\pi G)$ is the critical density of the Universe and H_0 is the Hubble constant. Usually $\Delta = 200$ or $\Delta = 500 \implies$ total mass of cluster usually indicated as M_{200} or M_{500} .

→ TOTAL (DM+stars+gas) mass density profile of observed clusters: not far from $\rho \propto r^{-2}$. DM profile consistent with NFW if the central regions are excluded. In the central regions: baryons are important.

→ Cluster mass function: number of clusters at with mass in the range M to $M + dM$ at redshift z : $n(M, z)dM$. Used to constrain cosmological models.

7.2 X-ray scaling relations of clusters

[R:544]

→ Expected relations for a virialized cluster formed through pure gravitational collapse \implies no characteristic scales (gravity + bremsstrahlung+scale-free power spectrum) \implies self similarity

→ Cluster mass $M = 4\pi r^3 \rho_{\text{crit}} \Delta/3$, where $\rho_{\text{crit}} = 3H(z)^2/8\pi G$. $F_z \propto H(z)^2 \implies M \propto r^3 F_z$ or $r \propto M^{1/3} F_z^{-1/3}$. For a SIS $M \propto \sigma^2 r \propto T r \implies (M - T \text{ relation})$

$$T \propto M^{2/3} F_z^{1/3} \quad (7.6)$$

bolometric X-ray luminosity ($L_X - T$ relation)

$$L_X \propto V \rho_{\text{gas}}^2 T^{1/2} \propto M^2 r^{-3} T^{1/2} \propto T^2 F_z^{1/2} \propto M^{4/3} F_z^{7/6}, \quad (7.7)$$

entropy ($K - T$ relation)

$$K \propto \frac{T}{\rho^{2/3}} \propto T F_z^{-2/3} \quad (7.8)$$

Y parameter ($Y - T$ relation)

$$Y \propto M_{gas}T \propto MT \propto TF_z^{-2/3} \quad (7.9)$$

- At $z = 0$ $F_z = 1$.
- Observed $M - T$ relation (at $z = 0$): $M \propto T^{1.5} \implies$ consistent with self-similar.
- Observed $L_X - T$ relation (at $z = 0$): $L_X \propto T^3$ (even steeper at lower T) \implies steeper than self-similar.
- Observed $K - T$ relation (at $z = 0$): $K \propto T^{0.6} \implies$ shallower than self-similar.
- Interpretation of the observed scaling relations: entropy floor. There is no low entropy gas expected in lower mass systems.
- Possible explanations: (non-gravitational) pre-heating, cooling, AGN feedback, SN feedback (open question).
- Evolution of scaling relations with redshift can be used to test cosmological models (dependence on z through F_z)

7.3 Dynamics and evolution of the ICM

[R:563, V:III.B.1]

7.3.1 Cooling

[S:5.3.1, R:545, V:IV.B.2]

- Radiative cooling: energy loss by emission of radiation (bremsstrahlung + line emission important at lower T)

→ Cooling rate $\epsilon \propto n_e n_i \Lambda(T)$

→ $\Lambda(T)$ cooling function. It depends on metallicity. $\Lambda(T) \propto T^{1/2}$ at $T > 3 \times 10^7$ K.

→ Energy equation in the presence of cooling

$$\frac{p}{\gamma - 1} \left[\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right] \ln(p\rho^{-\gamma}) = -n_e n_i \Lambda(T). \quad (7.10)$$

where n_e and n_i are the electron and ion number densities, $\rho = \mu m_p n$, $n = n_e + n_i$ and $\gamma = C_P/C_V = 5/3$ is the ratio of specific heats.

→ Cooling time $t_{\text{cool}} = |E/\dot{E}| = |T/\dot{T}|$, where $E = (3/2)k_B T$ is the internal energy. Isobaric cooling time, assuming $p = \text{const}$ and $\mathbf{v} = 0$ (derive):

$$t_{\text{cool}}(p = \text{const}) = \frac{\gamma}{\gamma - 1} \frac{nk_B T}{n_e n_i \Lambda(T)} = \frac{5}{2} \frac{nk_B T}{n_e n_i \Lambda(T)} \quad (7.11)$$

Isochoric cooling time, assuming $\rho = \text{const}$ and $\mathbf{v} = 0$ (derive):

$$t_{\text{cool}}(\rho = \text{const}) = \frac{1}{\gamma - 1} \frac{nk_B T}{n_e n_i \Lambda(T)} = \frac{3}{2} \frac{nk_B T}{n_e n_i \Lambda(T)} \quad (7.12)$$

→ For $T \gtrsim 3 \times 10^7$ K:

$$t_{\text{cool}} \simeq 100 \left(\frac{n_e}{10^{-3} \text{ cm}^{-3}} \right)^{-1} \left(\frac{T}{10^8 \text{ K}} \right)^{1/2} \text{ Gyr} \quad (7.13)$$

⇒ cooling unimportant in the outskirts of clusters. But important in the central regions (higher density)!

7.3.2 Cool core clusters and the cooling flow problem

[S:5.7.1, R:545, V:IV.B.2, V:IV.B.4, McNamara & Nulsen 2007 (ARA&A):2]

→ Central $t_{\text{cool}} < \text{age} \sim t_H \implies$ 'cool-core' cluster; Central $t_{\text{cool}} > \text{age} \sim t_H \implies$ 'non-cool-core' cluster.

- Cooling radius r_{cool} such that $t_{cool} < \text{age}$ if $r < r_{cool}$.
- Cool-core clusters: bright X-ray emission from cool, dense gas in the central regions; central spike in X-ray SB \implies central gas density peak.
- If cooling is not balanced by heating, gas energy radiated away in $t_{cool} \lesssim 10^9$ yr.
- gas radiates \implies lower entropy $K \propto T n_e^{-2/3}$, (lower T and higher n) \implies flows inward. $t_{cool} \propto n_e^{-1} \implies$ cooling catastrophe.
- Hydrodynamic equations in the presence of cooling

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (mass) \quad (7.14)$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{\nabla p}{\rho} - \nabla \Phi, \quad (momentum) \quad (7.15)$$

$$\frac{p}{\gamma - 1} \left[\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right] \ln(p \rho^{-\gamma}) = -n_e n_i \Lambda(T), \quad (energy) \quad (7.16)$$

- Cooling time longer than free-fall time \implies cooling gas is nearly hydrostatic \implies steady-state.
- Hydrodynamic equations for a steady-state spherical cooling flow with $\dot{\rho} = 0$:

$$\frac{1}{r^2} \frac{d}{dr} (r^2 \rho v) = 0, \quad (7.17)$$

$$v \frac{dv}{dr} = -\frac{1}{\rho} \frac{dp}{dr} - \frac{d\Phi}{dr}, \quad (7.18)$$

$$\frac{pv}{\gamma - 1} \frac{d}{dr} [\ln(p \rho^{-\gamma})] = -n_e n_i \Lambda(T). \quad (7.19)$$

,

→ The equations above can be rearranged as

$$\dot{M} \equiv 4\pi r^2 \rho v = \text{const}, \quad (7.20)$$

$$v \frac{dv}{dr} = -\frac{1}{\rho} \frac{dp}{dr} - \frac{d\Phi}{dr}, \quad (7.21)$$

$$\dot{M} \frac{1}{4\pi r^2} \frac{d}{dr} \left[\frac{v^2}{2} + \frac{5k_B T}{2\mu m_p} + \Phi \right] = n_e n_i \Lambda(T). \quad (7.22)$$

→ For subsonic $v^2 \ll c^2 \sim kT/\mu m_p$, nearly isobaric flow the energy equation reduces to

$$\dot{M} \frac{5k_B dT}{2\mu m_p} = 4\pi n_e n_i \Lambda(T) r^2 dr. \quad (7.23)$$

or

$$dL_X(T) \simeq \frac{5}{2} \frac{k_B dT}{\mu m_p} \dot{M}, \quad (7.24)$$

the luminosity emitted by gas cooling from T to $T - dT$. ($dL_X(T) \equiv 4\pi r^2 n_e n_i \Lambda(T) dr$)

→ Total X-ray luminosity expected from the cooling region $r < r_{cool}$:

$$L_X \simeq \frac{5}{2} \frac{k_B T}{\mu m_p} \dot{M} \simeq 1.3 \times 10^{44} \frac{k_B T}{5 \text{ keV}} \frac{\dot{M}}{100 M_\odot / \text{yr}} \text{ erg/s}, \quad (7.25)$$

where T is the temperature of the gas at r_{cool} .

→ This classical cooling flow model fails in many respects: it predicts strong central peak in X-ray emission, distributed star formation in the cooling region, accumulated cooling material, plus intense emission lines at $E < 1 \text{ keV}$ in the X-ray spectra. NOT CONSISTENT WITH OBSERVATIONS!

→ In fact gas in the centre of clusters neither cools nor flows inward at a significant rate! Heating mechanisms required!

→ Different heating mechanisms have been considered: thermal conduction, heating by dynamical friction of galaxies, heating by AGN feedback.

→ AGN radio feedback is the most promising mechanism: central radio source \implies radio jets \implies radio lobes \implies cavities in the ICM \implies heating of the ICM (see McNamara & Nulsen 2007).